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Nardênio Almeida Martins
Douglas Wildgrube Bertol

Wheeled Mobile Robot Control

Theory, Simulation, and Experimentation

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Theory, Simulation, and Experimentation

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Theory without practice is useless, but practice without theory is naive.

When you are learning, the teacher acts just like a needle; the student is the thread. As your mentor, I can help you by pointing you in the right direction. But, like the needle in the thread, I must separate myself from you in the end, because the strength, the fiber, and the ability to bring all the parts together must be yours.

—Lance H. K. Secretan

To our families and all who believe, persist, and seek the fulfillment of their dreams.

To the treasures of my life, Maria Madalena, Natasha, Nícolas, and my second mother, Augusta de Oliveira Dias (⋆ 08/28/1928 – † 11/18/2020), a simple, wise, patient, tolerant and lovely person that I knew, lived and learned to admire, respect and love.

—Nardênio Almeida Martins

To Jaqueline, Regina, and Nubio.

—Douglas Wildgrube Bertol

Preface

In this book, differential-drive wheeled nonholonomic mobile robots (DWMRs) are considered to be the object of study because they are easy to implement physically (simple mechanical structure and low manufacturing cost), as well as serving several applications, as they adapt relatively well to operating conditions in indoor and outdoor paved environments, in which the soil irregularities are not very severe. Besides, DWMRs have awakened the scientific and technological interests of the international scientific community in the control area due to their simple kinematic model and, mainly, to their nonholonomic restrictions (bidirectional movement). Since DWMRs belong to a class of nonlinear, multivariable, underactuated, nonholonomic systems and, in practice, have uncertainties and/or disturbances, these constitute quite challenging control problems. Uncertainties and/or disturbances can be parametric or structured and nonparametric or unstructured. Among the control problems, the trajectory tracking control problem for DWMRs is treated as an object of study in this book, as it is particularly relevant in practical applications, considering that the control modules of the DWMR must, in general, follow a path previously planned and collision-free. However, as the control problems of the DWMRs are closely related to the kinematic and dynamic modeling and, consequently, with the difficulty in obtaining adequate models for the control purposes, it is considered necessary to elaborate, develop, and implement control projects based on Lyapunov's stability theory that are robust to compensate for the damaging effects of the incidence/influence of uncertainties and/or disturbances. Thus, the control system designs covered in this book use both the kinematic and dynamic models.

For the design of robust kinematic controllers, the first-order sliding control technique is used as a basis to deal with the problems of uncertainties and/or disturbances in the kinematic model in rectangular coordinates. The choice of this control technique is due to its fast response, its good performance in a transient regime, and its robustness concerning the uncertainties and/or disturbances that can be reached by the control inputs when in the sliding mode. The drawbacks of this control technique are the vibration phenomenon due to the discontinuous control portion and the need to know the limits of the disturbances to apply the adequate gain to compensate for them, thus saving the actuators from unnecessary efforts. To circumvent these drawbacks, continuous approximation methods are used, including soft computing techniques, specifically fuzzy logic and artificial neural networks, culminating in techniques of first-order quasi-sliding mode control, adaptive fuzzy sliding mode control, and adaptive neural sliding mode control. In addition to these drawbacks, sliding mode control projects have difficulty selecting sliding surfaces due to the lack of methodologies. In this book, an adequate choice of sliding surfaces is made based on the behavior of the reduced dynamics of the system using the equivalent control method to meet the desired performance and robustness requirements.

Most dynamic controllers of DWMRs generate torques as a control signal; however, commercial DWMRs usually receive velocity signals as control and not torques, as is the case of the following DWMRs: the PowerBot and Pioneer from Mobile Robots Inc., the Khepera from the company K-Team Corporation, the Merlin Miabot Pro Autonomous Mobile Robot from Merlin Systems Corporation, and the Soccer Robot from Microrobot. Therefore, in this book the dynamic model of the PowerBot DWMR is considered and the dynamic control is a PD control provided by the manufacturer, one of the reasons why this book deals with kinematic controller designs that are robust to the incidence of uncertainties and/or disturbances. Some of these robust kinematic controller designs are extended to introduce formation control of DWMRs using the leader–follower control strategy with the separation-bearing technique. Finally, to prove the effectiveness of the robust kinematic controllers contained in this book, simulations are carried out with MATLAB/Simulink software and/or with the MobileSim simulator, as well as experiments in the PowerBot DWMR to illustrate the successful practical application of the theory.

In short, this book focuses on the development and methodologies of trajectory control of DWMRs. The methodologies are based on kinematic models (posture and configuration) and dynamic models, both subject to uncertainties and/or disturbances. The control designs are developed in rectangular coordinates obtained from the first-order sliding mode control in combination with the use of soft computing techniques, such as fuzzy logic and artificial neural networks. Control laws, as well as online learning and adaptation laws, are obtained using the stability analysis for both the developed kinematic and dynamic controllers, based on Lyapunov's stability theory. An extension to the formation control with multiple DWMRs in trajectory tracking tasks is also provided. Results of simulations and experiments are presented to verify the effectiveness of the proposed control strategies for trajectory tracking situations, considering the parameters of an industrial and a research DWMR, the PowerBot.

The main objective of this book is the development of projects and the implementation of control systems for DWMRs in the performance of tasks of tracking trajectories with or without incidence of uncertainties and/or disturbances.

In addition to the main objective, other objectives are also established, such as:

- Address theoretical–scientific and practical aspects of DWMRs;
- Dealing with the mathematical modeling of the kinematics and dynamics of DWMRs;
- Present the design, implementation, and performance of kinematic and dynamic control systems and the stability analysis in trajectory tracking for engineering applications;

- Provide some methods of chattering phenomenon mitigation;
- Provide source codes of the control system designs that enable the developer to use creativity and imagination to modify, alter, or redo control system designs in both kinematic and dynamic scopes as well as proposing new control designs;
- Provide simulations in MATLAB/Simulink software, MobileSim simulator, and practical applications in DWMR PowerBot;
- Promote a tool for pedagogical applications to related areas.

This book is structured as follows. Chapter 1 contains an introduction to mobile robotics with an emphasis on the description and mathematical modeling of DWMRs, such as kinematic model, dynamic model, the inclusion of actuator dynamics, formulation of the dynamic state-space model, and controllability of the dynamic model. Chapter 2 provides the theoretical foundation on control problems and systems, introducing the trajectory tracking problem, PD dynamic control, posture error dynamics, robustness considerations, and generic model for nonlinear systems. Chapter 3 describes the simulation and experimentation environments, dealing with the implementation design considering data and information from the PowerBot DWMR, trajectory adopted, as well as the ideal, realistic, and experimental scenarios. It is emphasized that the contents of Chaps. 1–3 are deemed necessary for the development and understanding of the control projects covered in Chaps. 4–9. Chapter 4 deals with a control based on backstepping methodology very widespread in the technical–scientific literature that does not present aspects of robustness. Chapter 5 reports on the concept, characteristics, robustness, performance, and chattering phenomenon of the first-order sliding mode control and introduces four variants of this control technique. Chapter 6 considers four control variants by first-order quasi-sliding mode control, in which the chattering mitigation is treated using a continuous approximation method, i.e., fractional continuous approximation or proper continuous function, resulting in the loss of invariance property, but guaranteeing robustness. Chapter 7 discusses two variants of the adaptive fuzzy sliding mode control to mitigate the chattering phenomenon with the guarantee of robustness, whose difference between them is in the way in which the adaptation law of the consequences is elaborated by using a fuzzy system as a continuous approximation method. Chapter 8 presents an adaptive neural network sliding mode control, in which is used radial basis function neural network as continuous approximation method, thus mitigating the chattering phenomenon and ensuring robustness. Moreover, Chaps. 4–8 address the control design with the Lyapunov method for stability analysis, simulations using MATLAB/Simulink software and/or MobileSim simulator, experimental results using PowerBot DWMR, analysis and discussion of results, and general considerations. Chapter 9 extends one of the variants of the first-order sliding mode control, first-order quasi-sliding mode control, and adaptive fuzzy sliding mode control to the formation control of DWMRs by using the leader–follower strategy with the separation-bearing technique and considering the decentralized structure and homogeneous architecture. This chapter addresses the problem formulation, control design with Lyapunov method for stability analysis, simulations using MATLAB/Simulink software and/or MobileSim simulator, and general considerations.

The electronic supplementary material contains the applications made in each chapter of the book, such as:

- Simulation codes implemented in MATLAB/Simulink software;
- Scripts for generating graphics and visualizing the results;
- Source codes of the implemented control techniques.

These extra materials of the book can be found at http://extras.springer.com/, which are available to authorized users. The extra material is only distributed to Springer customers. We do not have the mentioned github repo.

Thus, this book is mainly intended for researchers, and undergraduate and graduate students in the areas of robotics with control applications, but it is also recommended for researchers, teachers, students, and professionals in the areas of computer science, informatics, mathematics, physics, electrical engineering, electronic engineering, mechanical engineering, computer engineering, control and automation engineering, mechatronics engineering, who are interested in the knowledge of the state-of-the-art theory and practice, in the control field applied in robotics, specifically the sliding mode control technique.

Maringá, Brazil Nardênio Almeida Martins
April 2021 Douglas Wildgrube Bertol

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We always thank God, especially for the health given to us. Our sincere wishes of gratitude to all those people who somehow contributed directly or indirectly to the development and conclusion of this book.

We, the authors, are not geniuses of humanity, but we are grateful for the phrases below from some of them, in this case, those are from Albert Einstein, which makes us have more motivation, dedication, discipline, perseverance, persistence, resilience, pride, and love in the exercise of our profession as a teacher to be even better people, as well as educating better citizens for humanity.

"Certainly my career was not determined by my own will, but by countless factors over which I have no control."

"When we accept our limits, we can go beyond them."

"We were unable to solve a problem based on the same reasoning used to create it."

"A person who has never made a mistake has never experienced anything new."

"Imagination is more important than knowledge. Knowledge is limited. Imagination surrounds the world."

"I have no special talent. I'm just passionately curious."

"Don't try to be a successful person. Try to be a person of value."

Maringá, Brazil
April 2021

Nardênio Almeida Martins
Douglas Wildgrube Bertol

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Chapter 1
Background on DWMR System Modeling

1.1 Introduction

In mobile robot applications, three mechanisms of locomotion are well widespread: wheels, track plates, and legs. In this book, only the wheel locomotion mechanism will be considered, since it is the most widely used, easy to implement physically, and relatively well adaptive to the operating conditions in paved internal and external environments where ground irregularities are not very severe. For environments with marked irregularities and tasks that need obstacles transposition, the last two mechanisms of locomotion are more appropriate.

Mobile robots equipped with wheels that theoretically do not suffer deformations constitute a class of mechanical systems that are characterized by nonintegrable kinematic constraints (nonholonomic constraints) and therefore can not be eliminated from the equations of the model.

The derivation of the kinematic and/or dynamic models for differential wheeled mobile robots (DWMRs) are available in the literature [8, 14, 17] of mobile robots and for generic mobile robots equipped with wheels of various types. A systematic procedure for model derivation can be found in [1, 16]. Generally, the dynamic model of a DWMR is performed using the Newton-Euler method or the Lagrangian approach [9]. However, in [19, 20] the modeling of a DWMR is performed through the Kane approach, which emphasizes the advantages of this method for nonholonomic systems and presents the resulting dynamic model.

In [3, 4, 10], the DWMRs can be divided into four different models, namely: posture kinematic model, configuration kinematic model, configuration dynamic model, and the posture dynamic model. The kinematic models describe the DWMR by a function of the velocity and the orientation of the wheels, while the dynamic models describe the DWMR by a function of generalized forces applied by the actuators. The posture models consider as state variables only the position and orientation of the robot, unlike the configuration models which consider, besides the posture variables, other internal variables, such as angular displacement of the wheels. Moreover, an analysis of the structure of the kinematic and dynamic models of DWMRs are

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performed, which are divided into five classes, characterized by the general structure of the equations of the models. Thus, for each class, structural properties of the kinematic and dynamic models are derived, considering mobility constraints.

In this chapter, the kinematic and dynamic DWMR models used in the course of the book are presented and for ease of understanding see [3, 4, 10], which describes a systematic procedure for derivation of kinematic and dynamic models (using the Lagrangian approach) for DWMRs, taking into account mobility constraints and only conventional fixed wheels. Still, it is important to emphasize that the posture models of the DWMR are used as the basis for the synthesis of the controllers only, while configuration models of the DWMR are considered in the simulations realized in Matlab/Simulink software only.

1.2 DWMR Modeling and Description

The DWMR, shown in Fig. 1.1, is a typical example of a nonholonomic mechanical system. This system consists of a rigid body (base) having two conventional fixed wheels driven by independent actuators (for example, direct current motors or DC motors) to perform the movement and orientation, and a third wheel that rotates freely (passive wheel) whose function is only to support the DWMR, and their effects are negligible in the DWMR dynamics.

The posture vector is characterized by the triple $\boldsymbol{\xi} = [x\ y\ \theta]^{\mathrm{T}}$ where x and y are the coordinates of the point C (which is also the mass center or guidance point) in the inertial coordinate system $\mathrm{OX_O Y_O}$ and θ is the orientation angle of the mass center coordinate system of the DWMR $\mathrm{CX_C Y_C}$ concerning to the inertial coordinate system $\mathrm{OX_O Y_O}$.

The following parameter notation is used in the DWMR study presented in Fig. 1.1 and listed in Table 1.1.

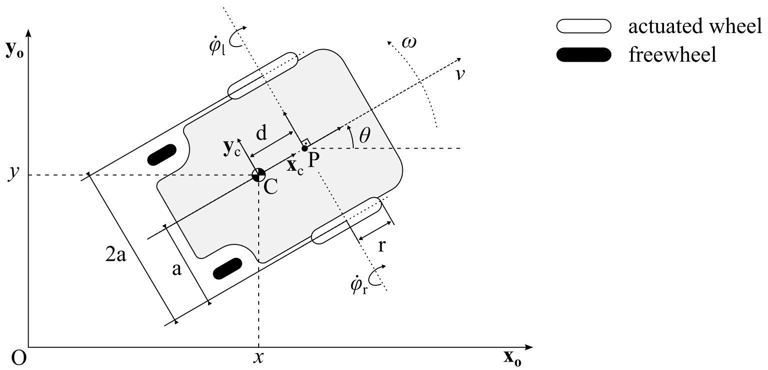

Fig. 1.1 DWMR and coordinate systems

Table 1.1 DWMR parameters

Parameter	Description
P	Intersection of the symmetry axis with the axis of the wheels
C	Mass center or guidance point
d	Distance between C and P
r	Right and left wheel radius
2a	Distance between the actuated wheels and the symmetry axis
m_c	Mass of the DWMR without wheels and motors
m_w	Mass of each wheel and motor assembly
m_t	Total mass of the DWMR
I_c	Moment of inertia of the DWMR without wheels and motors about the vertical axis through P
I_w	Moment of inertia of each wheel and motor about the wheel axis
I_m	Moment of inertia of each wheel and motor about the vertical axis parallel the wheel plane
I	Total inertia moment of the DWMR
$\dot{\varphi}_r, \dot{\varphi}_l$	Angular velocity of the right and left wheels
υ, ω	Linear and angular velocities of DWMR
$\boldsymbol{q}$	Generalized coordinate vector

1.2.1 Kinematic Model

The local coordinates of mechanical systems can be described in terms of the generalized coordinate vector $\boldsymbol{q}$, being $\boldsymbol{q} = [q_1 \ q_2 \ \ldots \ q_n]^T \in \Re^n$. In many situations, the movement of mechanical systems is subject to various constraints that are permanently satisfied during movement and that take the form of algebraic relations between positions and velocities of points of the system [5].

The DWMR shown in Fig. 1.1 presents three kinematic constraints [3, 4, 7, 10, 21]. The first constraint is that the DWMR can not slide sideways (non-slipping constraint), i.e., only move in the normal direction to the symmetry axis of the actuated wheels. This constraint can be written as:

$$\dot{y}\cos(\theta) - \dot{x}\sin(\theta) = 0, \qquad \text{for C} = \text{P}, \tag{1.1}$$

$$\dot{y}\cos(\theta) - \dot{x}\sin(\theta) - \text{d}\dot{\theta} = 0, \qquad \text{for C} \neq \text{P}. \tag{1.2}$$

The other two constraints are related to the rotation of the wheels (pure rolling constraints), i.e., the actuated wheels can not rotate in false, and are given by:

$$\dot{x}\cos(\theta) + \dot{y}\sin(\theta) + \text{a}\dot{\theta} - \text{r}\dot{\varphi}_r = 0, \qquad \text{for C} = \text{P and C} \neq \text{P}, \tag{1.3}$$

$$\dot{x}\cos(\theta) + \dot{y}\sin(\theta) - \mathrm{a}\dot{\theta} - \mathrm{r}\dot{\varphi}_{\mathrm{l}} = 0, \quad \text{for C} = \text{P and C} \neq \text{P}, \tag{1.4}$$

where φ_{r} and φ_{l} are angular displacements of the right and left wheels, respectively.

Equations (1.3) and (1.4) can be rewritten as follows:

$$\upsilon + \mathrm{a}\omega = \mathrm{r}\dot{\varphi}_{\mathrm{r}}, \quad \text{for C} = \text{P and C} \neq \text{P}, \tag{1.5}$$

$$\upsilon - \mathrm{a}\omega = \mathrm{r}\dot{\varphi}_{\mathrm{l}}, \quad \text{for C} = \text{P and C} \neq \text{P}, \tag{1.6}$$

since

$$\upsilon = \dot{x}\cos(\theta) + \dot{y}\sin(\theta), \tag{1.7}$$

$$\omega = \dot{\theta}. \tag{1.8}$$

Using Eqs. (1.5) and (1.6), one can then correlate the right and left angular velocities of the DWMR ($\dot{\varphi}_{\mathrm{r}}$ and $\dot{\varphi}_{\mathrm{l}}$) with the linear and angular velocities of the mass center of the DWMR (υ and ω), resulting in:

$$\begin{bmatrix} \dot{\varphi}_{\mathrm{r}} \\ \dot{\varphi}_{\mathrm{l}} \end{bmatrix} = \boldsymbol{\Omega} \begin{bmatrix} \upsilon \\ \omega \end{bmatrix} = \begin{bmatrix} \frac{1}{\mathrm{r}} & \frac{\mathrm{a}}{\mathrm{r}} \\ \frac{1}{\mathrm{r}} & -\frac{\mathrm{a}}{\mathrm{r}} \end{bmatrix} \begin{bmatrix} \upsilon \\ \omega \end{bmatrix}, \tag{1.9}$$

and vice versa:

$$\begin{bmatrix} \upsilon \\ \omega \end{bmatrix} = \boldsymbol{\Omega}^{-1} \begin{bmatrix} \dot{\varphi}_{\mathrm{r}} \\ \dot{\varphi}_{\mathrm{l}} \end{bmatrix} = \begin{bmatrix} \frac{\mathrm{r}}{2} & \frac{\mathrm{r}}{2} \\ \mathrm{b} & -\mathrm{b} \end{bmatrix} \begin{bmatrix} \dot{\varphi}_{\mathrm{r}} \\ \dot{\varphi}_{\mathrm{l}} \end{bmatrix}, \tag{1.10}$$

where $\mathrm{b} = \frac{\mathrm{r}}{2\mathrm{a}}$.

In most applications, kinematic constraints Eqs. (1.1)–(1.4) are linear relations with the generalized coordinate vector. Such relations can be described in matrix form as:

$$\boldsymbol{A}(\boldsymbol{q})\dot{\boldsymbol{q}} = 0. \tag{1.11}$$

Since the state vector is represented by five generalized coordinates,

$$\boldsymbol{q} = \left[\boldsymbol{\xi}^{\mathrm{T}} \; \boldsymbol{\varphi}^{\mathrm{T}}\right]^{\mathrm{T}} = \left[x \; y \; \theta \; \varphi_{\mathrm{r}} \; \varphi_{\mathrm{l}}\right]^{\mathrm{T}}, \tag{1.12}$$

the three constraints can be rewritten in the form of Eq. (1.11), i.e.,

$$\boldsymbol{A}(\boldsymbol{q})\dot{\boldsymbol{q}} = \begin{bmatrix} -\sin(\theta) & \cos(\theta) & 0 & 0 & 0 \\ -\cos(\theta) & -\sin(\theta) & -\mathrm{a} & \mathrm{r} & 0 \\ -\cos(\theta) & -\sin(\theta) & \mathrm{a} & 0 & \mathrm{r} \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\varphi}_{\mathrm{r}} \\ \dot{\varphi}_{\mathrm{l}} \end{bmatrix}, \quad \text{for C} = \text{P}, \tag{1.13}$$

$$\boldsymbol{A}(\boldsymbol{q})\dot{\boldsymbol{q}} = \begin{bmatrix} -\sin(\theta) & \cos(\theta) & -\mathrm{d} & 0 & 0 \\ -\cos(\theta) & -\sin(\theta) & -\mathrm{a} & \mathrm{r} & 0 \\ -\cos(\theta) & -\sin(\theta) & \mathrm{a} & 0 & \mathrm{r} \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\varphi}_{\mathrm{r}} \\ \dot{\varphi}_{\mathrm{l}} \end{bmatrix}, \quad \text{for C} \neq \text{P}. \tag{1.14}$$

It is emphasized that the referential of DWMR velocity is given by the angular velocity of the right and left wheels ($\dot{\varphi}_{\mathrm{r}}$ and $\dot{\varphi}_{\mathrm{l}}$) respectively, i.e.,

$$\boldsymbol{v} = \begin{bmatrix} \dot{\varphi}_{\mathrm{r}} \\ \dot{\varphi}_{\mathrm{l}} \end{bmatrix}. \tag{1.15}$$

If the mass and inertia of the wheels and motors are disregarded, it is assumed that the DWMR satisfies the pure rolling and non-slipping conditions [11]. Thus, the matrix $\boldsymbol{A}(\boldsymbol{q})$ containing the nonholonomic constraints reduces to:

$$\boldsymbol{A}(\boldsymbol{q}) = \begin{bmatrix} -\sin(\theta) & \cos(\theta) & 0 \end{bmatrix}, \quad \text{for C} = \text{P}, \tag{1.16}$$

$$\boldsymbol{A}(\boldsymbol{q}) = \begin{bmatrix} -\sin(\theta) & \cos(\theta) & -\mathrm{d} \end{bmatrix}, \quad \text{for C} \neq \text{P}, \tag{1.17}$$

so that the displacements occur only in the direction of the symmetry axis of the actuated wheels and

$$\boldsymbol{q} = \boldsymbol{\xi} = \begin{bmatrix} x & y & \theta \end{bmatrix}^{\mathrm{T}}. \tag{1.18}$$

Also, it is pointed out that the referential velocity is supplied by the linear and angular velocities of the DWMR (υ and ω), i.e.,

$$\boldsymbol{v} = \begin{bmatrix} \upsilon \\ \omega \end{bmatrix}. \tag{1.19}$$

It is important to emphasize that the system has, as a configuration space n, the generalized coordinate vector $\boldsymbol{q}$ and the number of constraints given by p, so that the velocity vector is of dimension $\mathrm{m} = \mathrm{n} - \mathrm{p}$, in this case, $\mathrm{m} = 2$ (corresponds to the degrees of freedom of the system).

Be the annihilator [1] of these constraints [5] the Jacobian matrix $\boldsymbol{S}(\boldsymbol{q})$ of full rank $(\mathrm{n} - \mathrm{p})$ formed by a set of linearly independent and smooth vector fields distributed in the null space of $\boldsymbol{A}(\boldsymbol{q})$, i.e.,

$$\boldsymbol{A}(\boldsymbol{q})\boldsymbol{S}(\boldsymbol{q}) = 0. \tag{1.20}$$

According to Eqs. (1.11) and (1.20), it is possible to find an auxiliary velocity vector as a function of time $\boldsymbol{v} \in \Re^{\mathrm{p} \times 1}$ such that for all t:

---

[1] Note that $\boldsymbol{q}$ is in the null space of $\boldsymbol{A}(\boldsymbol{q})$. In the same way, all columns of the matrix $\boldsymbol{S}(\boldsymbol{q})$ of Eq. (1.20) are also in the null space of $\boldsymbol{A}(\boldsymbol{q})$, which justifies the term *annihilator*.