YAN-GANG ZHAO | ZHAO-HUI LU

STRUCTURAL RELIABILITY

APPROACHES FROM PERSPECTIVES OF STATISTICAL MOMENTS

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Approaches from Perspectives of Statistical Moments

Yan-Gang Zhao and Zhao-Hui Lu

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Preface

This book provides a unified presentation of structural reliability approaches from the perspective of statistical moments. It outlines the unique framework of structural reliability theory centered around the methods of moment. The book is different from many of other books for structural reliability methods that generally more focus on the first-order reliability method (FORM).

The book is composed of the following 13 chapters.

In Chapter 1, the necessity and importance for dealing with the various uncertainties in structural design is emphasised, and different existing measures of safety are reviewed. After pointing out the limit of the traditional deterministic measure of structural safety, the importance of probabilistic measure of safety, i.e., the structural reliability theory is emphasised.

In Chapter 2, basic concepts of structural reliability such as performance function, failure probability, and reliability index are introduced through a fundamental case, which includes only two statistically independent random variables (i.e. a load effect variable S and a resistance variable R). Since an accurate integration for failure probability is almost impossible for many practical engineering problems, the Monte Carlo Simulation (MCS) is first introduced.

Chapter 3 presents the moment computation for performance functions. For some explicit functions, the first few moments can be easily computed from their definitions. For a good behaviour function with only one variable, point estimates with arbitrary numbers of points are developed. Using the point estimate method, the first few moments of a function of single variable can be quickly obtained with required accuracy. Two additional approximate methods are also introduced to obtain the first few moments of performance functions of multiple random variables.

In Chapter 4, direct methods of moment are presented for structural reliability, including the direct second-moment method (2M), the direct third-moment method (3M), and the direct fourth-moment method (4M). These methods are easy to implement for structural reliability analysis and have no issues associated with the design points that are necessary in FORM. No iteration or computation of derivatives is required. The applicable range is then determined for the 2M, 3M, and 4M reliability methods, and simple 3M and 4M reliability indices are suggested for structural reliability analysis in engineering.

Unlike the direct methods of moment in Chapter 4, where the first few moments of the original performance function are used, methods of moment based on the first-order and second-order approximation of the performance function are introduced in Chapter 5. Under the first order approximation of a performance function in standardised normal space, the first-order second-moment reliability index is equal to the shortest distance and therefore perpendicular to the hyper plane in the standardised normal space. While under the second-order approximation of performance function in standardised normal space, second-order second-moment method, second-order third-moment method, and the second-order fourth-moment method, are further developed.

In Chapter 6, reliability evaluation is discussed for the problems without using probability distributions of random variables. The information of the first few moments of the random variables will be used for structural reliability assessment instead of probability distributions.

Chapter 7 presents transformation of non-normal variables to independent normal variables, which are necessary tools in Chapters 3, 5, and 6. Normal tail transformation for a single random variable (or independent random variables) and Rosenblatt or Nataf transformation for correlated random variables are reviewed. The emphasis of this chapter is to introduce the so-called pseudo normal transformation methods for transformation of correlated non-normal random variables into independent standard normal random variables, which can be achieved even when the probability distributions of the basic random variables are unknown.

In Chapter 8, the concept of system reliability is introduced. In order to improve the narrow bounds of the failure probability of a series structural system, a point estimation method is introduced for calculating the joint probability of every pair of failure modes of the system. A moment-based method is presented for the system reliability assessment of series and non-series structures, with emphasis on series systems. The method directly calculates the reliability indices based on the first few moments of the system performance function. This method does not require the reliability analysis of individual failure modes, nor does it need the iterative computation of derivatives, any design points, or the mutual correlations among the failure modes.

Chapter 9 presents the application of the methods of moment for the determination of load and resistance factors. Derivative-based iteration, which is necessary in FORM, is not required in the method. For this reason, the methods of moments are easier for implementation. Although the obtained load and resistance factors are different from those by FORM, the target mean resistances are essentially the same for both methods.

In Chapter 10, the time-variant problems in structural reliability are discussed. Simulating stationary non-Gaussian vector process using the third-order polynomial normal transformation model is first described. The transformation is then applied to evaluate the first passage probability of stationary non-Gaussian structural responses in structural dynamical reliability. A moment-based approach is introduced for time-variant static reliability assessment, where both the resistance and load effects are time-dependent.

Two typical problems of hierarchical models encountered in structural reliability are discussed in Chapter 11, one is structural reliability analysis considering the uncertainties in distribution parameters, and another is the dynamic reliability considering uncertainties contained in input parameters. The application of FORM, methods of moment, and the point-estimate method to evaluate the overall probability of failure with consideration of both the two levels of uncertainties is introduced. The point-estimate method for evaluating the quantile of the conditional failure probability is also introduced.

In Chapter 12, the reliability evaluation using the first few linear moments (L-moments) of random variables is discussed. The chapter introduces the definition of the first four L-moments and the computation of the first four L-moments from the probability distributions and statistical data of random variables. The second- and third-order polynomial normal transformation techniques are then investigated using the first three and four L-moments, respectively, and FORM based on the transformation techniques using L-moments is demonstrated to be sufficiently accurate in structural reliability assessment.

Chapter13 presents the methods of moment combined with Box-Cox transformation, which may significantly abate the non-normality of the performance functions, for structural reliability analysis. A criterion for determining the Box-Cox transformation parameter is introduced, and the procedure of the methods with Box-Cox transformation for structural reliability is presented.

A large number of examples are provided with step-by-step procedures to better illustrate the concepts and methodology. Numerical simulations with more significant digits are also provided to track and calibrate these procedures. This book can be used as textbook for undergraduate and graduate student as well as reference for readers in the areas of structural reliability, reliability engineering and risk management.

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1

Measures of Structural Safety

1.1 Introduction

Engineering structures are generally built in a severe environment. During the service life of some structures, there may be extreme loads caused by earthquakes, wind, snow, fire, etc. To ensure the safety of such structures, engineers do their best to predict the extreme loads and design the structures accordingly. Despite this, some structures may still not be able to withstand disasters because of factors such as overloading, poor quality of structural materials, poor workmanship, or human error. Nevertheless, the reason for structural failure is that there is a gap between our predictions of structural performance and actions and the reality; and this gap generally presents as uncertainties.

Along the development of engineering structural designs, decisions are often required to be made under the conditions of such uncertainties, in the sense that the consequence of a given decision cannot be determined with complete confidence. Therefore, decision making under conditions of uncertainties involves risks. How to quantitatively evaluate the risk and provide a measure of safety is an important problem in engineering.

1.2 Uncertainties in Structural Design

1.2.1 Uncertainties in the Properties of Structures and Their Environment

First, consider a simple beam with a rectangular section of $b \times h$ and a span of l. The mechanical model is illustrated in Figure 1.1a, with its moment diagram shown in Figure 1.1b.

The maximum moment caused by a load P is given as $Pl/4$, and the plastic moment is calculated as $\sigma_y bh^2/4$, where σ_y is the yield stress of the material. Assume that the beam is made of an ideal elastic-plastic material. When the load P becomes sufficiently large to make the maximum moment $Pl/4$ equal to the plastic moment $\sigma_y bh^2/4$, the beam will form a mechanism. The safety of the beam can be checked if the following equation is satisfied during the design of the beam:

$$\frac{Pl}{4} \leq \frac{bh^2}{4}\sigma_y \tag{1.1}$$

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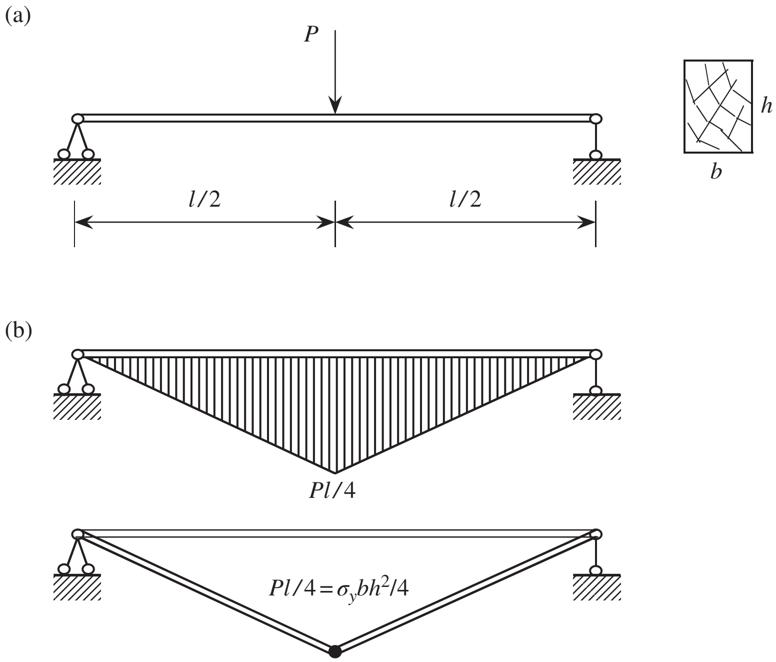

Figure 1.1 A simple beam. (a) Load bearing condition and cross section of the beam. (b) Bending moment diagram and the plastic moment.

If the quantities of P, l, b, h, and σ_y in Eq. (1.1) are known exactly, from the comparison of the two sides in Eq. (1.1), we can deterministically understand whether the beam is absolutely safe or not. However, the problem arises when we cannot know these quantities exactly. For example, if the beam is used as a bridge, people who step on the bridge may be a child with a weight of 20 kg, an adult with a weight of 70 kg, or in an extremely rare case, a Japanese sumo wrestler with a weight of 200 kg. Therefore, the load P is clearly an uncertain variable. This raises a similar question about the yield stress σ_y. For a specific material, one can obtain the yield stress from tests. However, different values of yield stress will be obtained from different specimens, and it is difficult, if not impossible to determine the yield exact stress of the material used for the beam. As for the parameters l, b, and h, they are generally considered to be known. However, in reality, they cannot be known exactly because of inevitable errors in the process of construction.

If we define $\sigma_y bh^2/4$ as resistance R and $Pl/4$ as load effect S, then Eq. (1.1) becomes

$$S \leq R \tag{1.2}$$

When we check the safety of the beam using Eq. (1.2) under deterministic conditions, if S is less than R, the beam will be absolutely safe. Since the quantities of P, l, b, h, and σ_y in Eq. (1.1) are uncertain, R and S are also uncertain. This uncertainty may lead to a possible reversal of the balance between R and S, and ultimately, the beam may collapse. For example, when $S = 100$ and $R = 150$, the bridge will be considered to be absolutely safe under deterministic conditions. However, since the values of R and S are uncertain, there is a possibility of $S = 140$ and $R = 130$; in this case, the beam may fail.

The example above indicates that not all the parameters in the design process can be exactly known and they generally contain uncertainties. Uncertainty refers to imprecise and incomplete information about the phenomenon being investigated. For example, if it is known that a structure will be subjected to an earthquake load, the exact magnitude of the earthquake is unknown. At least at the present stage, we cannot expect to know exactly when, where, and through what process an earthquake will occur. Furthermore, the duration, the peak ground acceleration, and the frequency of the earthquake waves are also uncertain. Figure 1.2 shows the time histories of ground motion records during the Chuetsu earthquake 2004, Japan (Li and Zhao 2005), and it has been observed that no earthquake histories are the same. Generally, uncertainty is significant in the load and environment. Uncertainties exist in almost all the aspects of a load, e.g. the live load varies every day and the wind load changes all the time.

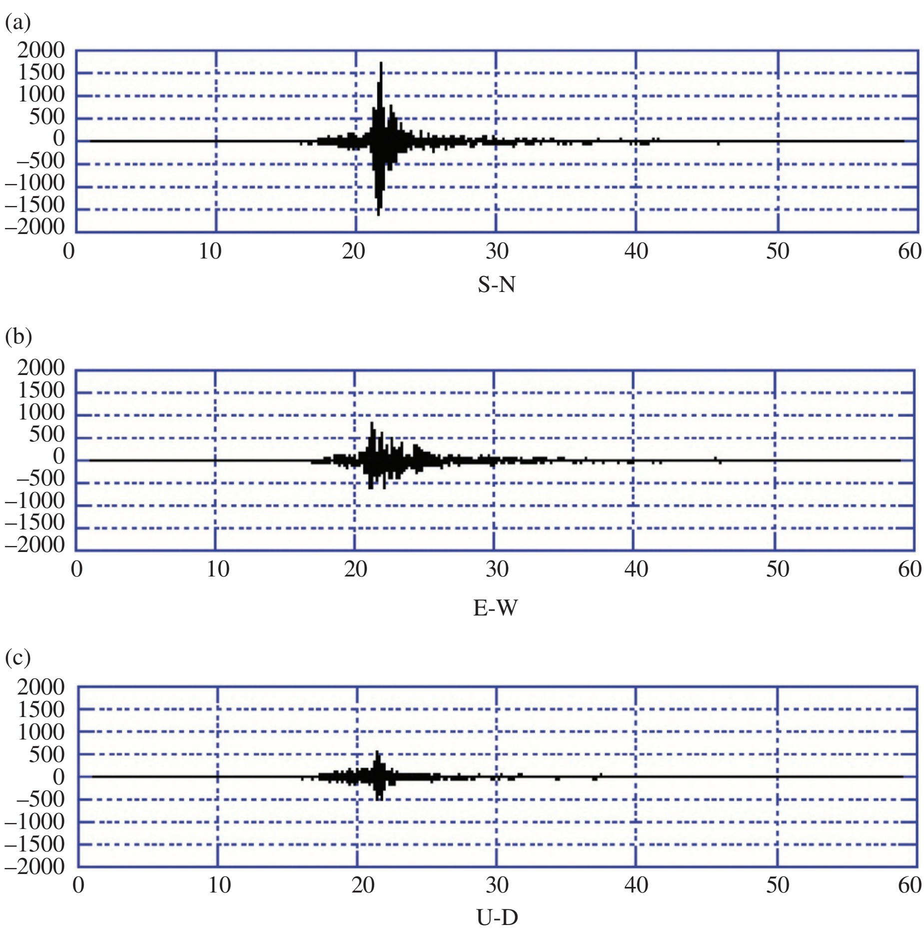

Figure 1.2 Time histories of earthquake records. (a) S–N. (b) E–W. (c) U–D.

Other than the load and environmental effects, there are also many uncertainties associated with structures themselves. It can be expected that the grade of concrete will be, for example, C30, but it is not known what the values of the compressive strength in a specific cross-section will be, unless a test is carried out for that section. Assume that 15 concrete cylinders are sampled from the concrete mixes used in construction and subsequently tested in compression with the following results (in MPa): 30.5, 29.8, 31.2, 30.8, 28.9, 31.6, 30.7, 30.9, 30.4, 30.5, 30.8, 30.3, 30.5, 31.1, and 30.9.

Based on these observations, the sample mean (average) is obtained as

$$\overline{f_c} = \frac{1}{15}\sum_{i=1}^{15} f_{ci} = 30.6 \tag{1.3}$$

where f_{ci} is the concrete compressive strength of the ith samples; and the $\overline{f_c}$ is the average of the concrete compressive strength.

The sample mean is, of course, an estimate of the true concrete strength (which remains unknown). The fact that the observed strength of the 15 different cylinders is significantly scattered gives rise to an uncertainty in the actual strength of any given section of any part of the structure. Furthermore, the strength of structural members may change with time, as most of them degrade in the corrosive environment.

Another type of uncertainty is human error. Human error is an inappropriate or undesirable human decision or behaviour that reduces or has the potential of reducing the effectiveness or safety of a system. Human error includes badly designed or faulty equipment, poor management practices, inaccurate or incomplete procedures, and inadequate or inappropriate training.

1.2.2 Sources and Types of Uncertainties

Uncertainties in engineering may be associated with physical phenomena that are inherently random, or with predictions and estimations of reality performed with incomplete or inadequate information. From this standpoint, uncertainties may be associated with the inherent variability of a physical process or with imperfections in the modelling of a physical process. That is, randomness in a physical process contributes to uncertainty because inherent errors of an imperfect prediction model cannot be entirely corrected deterministically. Furthermore, prediction or modelling errors may contain two components; a systematic component and a random component. In measurement theory, these are known as the systematic error and random error, respectively. From a practical standpoint, inherent variability is essentially a state of nature, and the resulting uncertainty may not be controlled or reduced, i.e. the uncertainty associated with inherent variability is something that we have to live with. The uncertainty associated with prediction or modelling errors may be reduced using more accurate models or the acquisition of additional data. In some literatures, the uncertainties above are also classified as aleatory and epistemic uncertainties (ISO 2394 2015; Ang and Tang 2006).

1.2.3 Treatment of Uncertainties

Since we cannot exactly predict the outcomes of uncertain phenomena, such uncertain phenomena are characterised by experimental observations that are invariably different from one to another (even if performed under apparently identical conditions). In other words, there is usually a range of measured or observed values. Furthermore, within this range, certain values may occur more frequently than others. The characteristics of such experimental data can be portrayed graphically in the form of a *histogram* or *frequency diagram*. Such examples are shown in Figure 1.3 for the residual stress of H-shaped steel (JIS-G-3192 1977) and the Young's modulus of SS400 steel (JIS-G-3101 1976; Ono et al. 1986). Therefore, the histogram or frequency diagram is a graphical description of the variability of experimental information.

For a specific set of experimental data, the corresponding histogram may be constructed as follows.

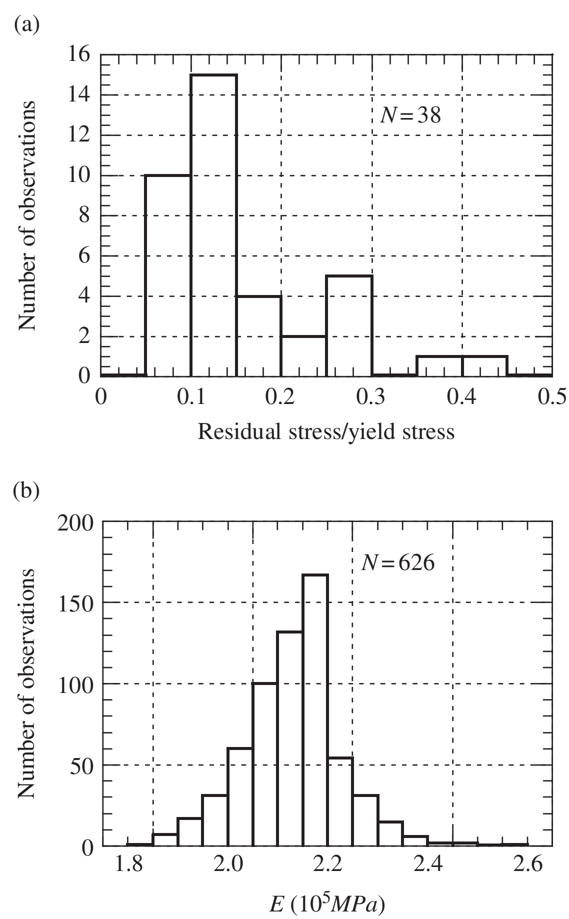

Figure 1.3 Examples of histograms. (a) Histogram of residual stress of H-shaped steel (JIS) (over a number of observations). (b) Histogram of the Young's modulus of SS400 steel (JIS) (over a number of observations).

Table 1.1 Data of residual stress in H-shaped steel (residual stress/nominal yield stress).

Sample No.	Sample values	Sample No.	Sample values	Sample No.	Sample values	Sample No.	Sample values
1	0.109	11	0.102	21	0.102	31	0.15
2	0.143	12	0.101	22	0.131	32	0.403
3	0.143	13	0.127	23	0.086	33	0.277
4	0.1	14	0.091	24	0.106	34	0.266
5	0.139	15	0.106	25	0.09	35	0.277
6	0.076	16	0.098	26	0.082	36	0.266
7	0.082	17	0.184	27	0.117	37	0.202
8	0.143	18	0.184	28	0.194	38	0.394
9	0.096	19	0.072	29	0.275		
10	0.086	20	0.123	30	0.20		

From the observed range of experimental measurements, choose a range on the abscissa sufficient to include the largest and smallest observed values, and divide this range into convenient intervals. Then, count the number of observations within each interval, and draw vertical bars with heights representing the number of observations in the respective intervals. Alternatively, the heights of the bars may be expressed in terms of the fractions of the total number of observations in each interval. For example, consider the data of residual stress in H-shaped steel in Table 1.1. It can be seen that the data ranges from 0.072 to 0.403. Choosing a uniform interval of 0.05, between 0.05 and 0.45, the number of observations and the corresponding fraction of total observations within each interval are shown in Table 1.2.

Table 1.2 Data for histogram and a frequency diagram.

Interval	Number of observations	Fraction of total observations
0.05–0.1	10	0.263158
0.1–0.15	15	0.394737
0.15–0.2	4	0.105263
0.2–0.25	2	0.052632
0.25–0.3	5	0.131579
0.3–0.35	0	0
0.35–0.4	1	0.026316
0.4–0.45	1	0.026316
Total	38	1

Plotting the number of observations in the given interval, we can obtain a histogram of the residual stress as shown in Figure 1.3a, whereas, in terms of the fraction of the total observations, the histogram would be as shown in Figure 1.4a.

For comparing an empirical frequency distribution with a theoretical probability density function, the corresponding frequency diagram is required. This may be obtained from the histogram by simply dividing the ordinates of the histogram by its total area. In the case of the histogram in Figure 1.4, we can obtain the corresponding frequency diagram by dividing the ordinates in Figure 1.3a by $38 \times 0.05 = 1.9$, or alternatively, by dividing the ordinates in Figure 1.4a by $1 \times 0.05 = 0.05$. The result would be as shown in Figure 1.4b, which is the frequency diagram for the residual stress.

The histogram, or frequency diagram, provides a graphical picture of the relative frequency of various observations or measurements. For most engineering purposes, certain

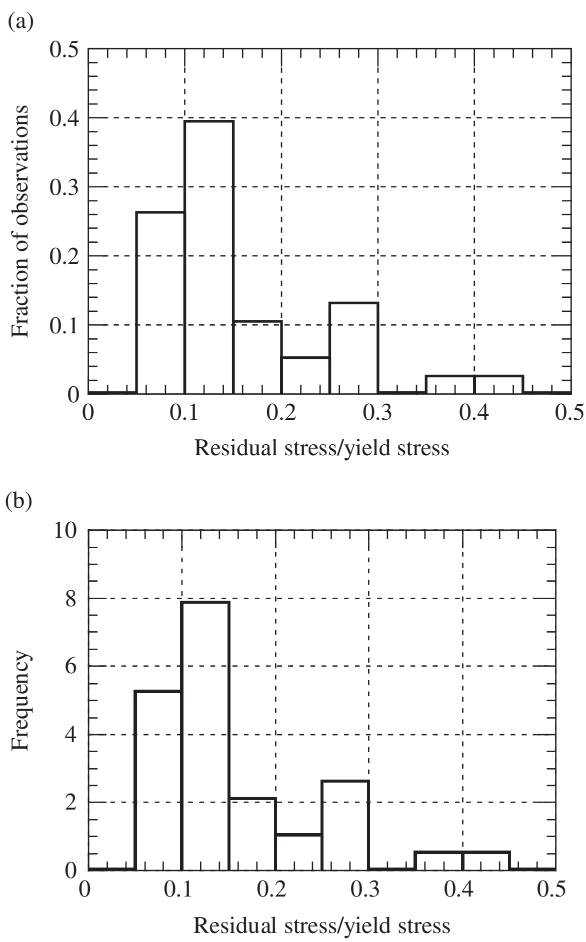

Figure 1.4 Histogram and frequency diagram of residual stress in H-shape steel. (a) Histogram for fraction of observations. (b) Frequency diagram.

aggregate quantities from the set of observations are more useful than the complete histogram. These include, in particular, the mean value (or average) and the measure of dispersion. Such quantities may be evaluated from a given histogram; statistically however, these are usually obtained in terms of the sample mean and the sample standard deviation, which are described in detail in Appendix A.3.

Clearly, if the recorded data of a variable exhibits scatter or dispersion, such as those illustrated in Figures 1.3 and 1.4, the value of the variable cannot be predicted with certainty. Such a variable is known as a random variable, and its value, or range of values, can be predicted only with an associated probability.

When two or more random variables are involved, the characteristics of one variable may depend on the value of the other variable. Pairs of observed data for the two variables, when plotted on a two-dimensional space, as shown in Figure 1.5, are characterised by scatter or dispersion in the data points, and these are called scattergrams. In view of such scatter, the value of one variable, given that of the other, cannot be predicted with certainty. The degree of predictability will depend on the degree of mutual dependency or the correlation between the variables.

When we deal with information such as that illustrated in Figures 1.3 through 1.5, which requires a probabilistic description, proper utilisation of this information will necessarily require concepts and methods of probability for engineering decision making. For example, if a design equation involves random variables, such as those described in Figures 1.3 through 1.5, quantitative analysis of the effects on the design and the formulation of the design will necessarily involve probabilistic concepts. In this book, basic theories of statistics and probability are included in the Appendix A. Alternatively, one may also refer to other textbooks.

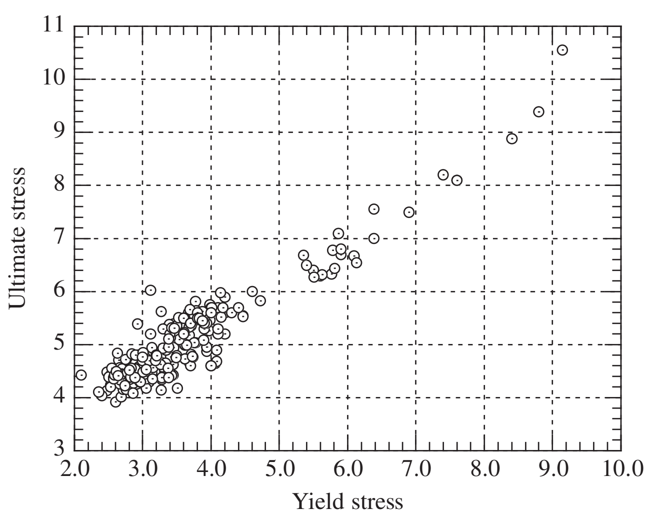

Figure 1.5 Scattergram of yield stress and ultimate stress for steel bars.

1.2.4 Design and Decision Making with Uncertainties

If engineering decisions must be made under conditions of uncertainties, how should designs be formulated, or how can decisions affecting a design be resolved? Presumably, one can consistently assume the worst scenarios and develop conservative designs. From the standpoint of system performance and safety, this approach may be suitable. However, the resulting design may be too costly as a consequence of compounded conservatism. On the other hand, a conservative design may not ensure the desired level of performance or safety. Therefore, decisions should be made as a trade-off between costs and benefits. The most desirable solution would be optimal in the sense of the minimum cost and/or maximum benefits. If the available information and/or the evaluation models contain uncertainties, the required trade-off analysis should include the effects of such uncertainties for any decision making.

1.3 Deterministic Measures of Safety

The traditional definition of safety is through a *factor of safety*, usually associated with the following allowable stress design formula:

$$\sigma \leq [\sigma] \tag{1.4}$$

where σ is the applied stress; and $[\sigma]$ is the allowable stress.

The allowable stress is usually defined in structural design codes. It is derived from the material strength expressed in ultimate stress σ_u (or yield stress σ_y) but is lowered through a multiplier F:

$$[\sigma] = \frac{\sigma_u}{F} \tag{1.5}$$

where F is the factor of safety or coefficient of safety. The greater the uncertainty about the strength of materials, the higher the coefficient of safety. One may also call it a factor of prudence or if one likes, of ignorance. The factor F is usually selected based on experimental observations, previous practical experience, economic, and perhaps political considerations. The selection of F is one of the responsibilities of code committees, and F generally varies with different load cases for different types of structures.

In the allowable stress design formula described above, the safety factor F is introduced to deal with the uncertainties contained in the applied stress and the ultimate strength. However, as a deterministic measure of safety, the factor F may not provide a quantitative measure of safety. If different members of the structure use different safety factors, then the safety of the entire structure may be difficult to evaluate. In contrast, a structure designed using a larger factor of safety F may not mean that it is safer than those designed with a smaller factor F. For example, the safety factor for steel structures is generally smaller than that for reinforced concrete (RC) structures. However, this does not mean that RC structures are safer than steel structures. Similarly, the safety factor under an earthquake load is larger than that under a wind load. It does not mean that the structure is safer under an earthquake load.

There are also some other disadvantages for the allowable stress design formula, e.g. since the factor F is usually selected on the basis of previous experience, it may be unsuitable for checking the safety of structures under loads that are difficult to predict. Furthermore, the allowable stress design code lacks formula invariance. An investigation has been performed in detail by Melchers (1987). Since it is not a quantitative measure of safety, it is obviously difficult to determine a trade-off between safety and economy.

1.4 Probabilistic Measure of Safety

In the probabilistic measure of safety, the quantification of uncertainty, and the evaluation of its effects on the performance and design of an engineering structure are conducted using concepts and methods of probability. This measure is also referred to as structural reliability theory.

When an engineering structure is loaded in some way, its response would depend on the type and magnitude of the load, and the strength and stiffness of the structure. Whether the response is considered satisfactory depends on the requirements that must be satisfied. Such requirements may include safety of the structure against collapse, damage limitations, deflections limitations, or any of a range of other criteria. Each of these requirements may be termed as a limit state. The violation of a limit state can then be defined as the attainment of an undesirable condition for the structure.

The study of structural reliability is concerned with the calculation and prediction of the probability of limit state violation for engineering structures at any stage during their service life. In particular, structural safety is concerned with the ultimate or safety limit states of a structure.

The probability of the occurrence of an event such as limit state violation is a numerical measure of the chance of its occurring. This measure may either be obtained from measurements of the long-term frequency of the occurrence of the event for generally similar structures, or may simply be a subjective estimation of the numerical value. In practice, it is usually not possible to observe other similar structures for a sufficiently long period of time, and a combination of subjective estimation and frequency observation for structural components and properties is usually used to predict the probability of limit state violation for the structure as a whole.

In probabilistic assessments, any uncertainty about a variable is explicitly taken into account. This is not the case in traditional methods to measure safety, such as the factor of safety. These are deterministic measures in that the variables describing the structure and its strength are assumed to take on known values for which no uncertainty is assumed.

1.5 Summary

In this chapter, the necessity of dealing with various uncertainties in engineering decision making is emphasised and different measures of safety are reviewed. Because the traditional deterministic measure cannot provide a quantitative basis for safety, a probabilistic measure of safety, i.e. the structural reliability theory, needs to be used.

2

Fundamentals of Structural Reliability Theory

2.1 The Fundamental Case

One of the principle objectives of structural design is the assurance of structural performance, including safety, within the economical constraint. The reliability of a structure can be defined as its capability to fulfil its design purpose for a specified reference period. The problems of structural reliability may essentially be cast as a problem of supply versus demand. In other words, problems of structural reliability may be formulated as the determination of the capacity of a structural system (supply) to meet certain requirements (demand). In consideration of the safety of a steel bar as shown in Figure 2.1, we are concerned with ensuring that the resistance of the bar (supply) is sufficient to withstand the maximum tensional load (demand).

Traditionally, the structural reliability is achieved through using factors or margins of safety and adopting conservative assumptions in the process of design. That is to ascertain that a 'worst' or minimum supply condition will remain adequate (by some margin) under a 'worst' or maximum demand requirement. What constitutes minimum supply and maximum demand conditions, however, is often based on subjective judgments. Moreover, the adequacy or inadequacy of the applied margins may also be evaluated or calibrated only in terms of the past experiences with similar systems. As described in the chapter entitled Measures of Structural Safety, the traditional approach is difficult to quantify and lacks the logical basis to address the effects of uncertainties. Consequently, the level of safety or reliability cannot be assessed quantitatively. Moreover, the problem of assuring performance would obviously be difficult for new systems when there is no prior basis for calibration.

In fact, it is of course not a simple problem to determine the available supply as well as maximum demand. Estimation and prediction are invariably necessary and uncertainties are unavoidable for the simple reason that engineering information is invariably incomplete. Under the uncertainties, the available supply and actual demand cannot be determined precisely. Instead, the available supply and required demand may be modelled as random variables. In other words, structural reliability may be more realistically measured in terms of probability.

Consider a simple case of the steel bar above, where the reliability of a structure or a structural element is determined by only two statistically independent random variables, i.e. a load effect variable S and a resistance variable R as shown in Figure 2.1. In this case, the

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Yan-Gang Zhao and Zhao-Hui Lu.

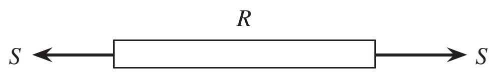

Figure 2.1 A steel bar with tensional load.

objective of reliability analysis is to ensure the event ($R > S$) throughout the service life, or some specified life of the engineering system. The assurance is expressed in terms of probability $P(R > S)$. For discrete random variables R and S, the probability of failure P_F may be formulated as follows:

$$P_F = P(R \leq S) = \sum_{\text{all } s} P(R \leq S | S = s) P(S = s) = \sum_{\text{all } s} P(R \leq s) P(S = s), \tag{2.1}$$

and for continuous R and S, if R and S are statistically independent, then the joint probability density function (PDF) of R and S can be expressed as $f_{R,\ S}(r, s) = f_R(r) f_S(s)$, then the probability of failure is

$$P_F = P(R \leq S) = \int_{r \leq s} f_R(r) f_S(s) dr ds = \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{s} f_R(r) dr \right] f_S(s) ds,$$

i.e.

$$P_F = \int_{-\infty}^{+\infty} F_R(s) f_S(s) ds \tag{2.2a}$$

Equation (2.2a) is the convolution with respect to s and may be explained with reference to Figure 2.2 as follows: for $S = s$, the conditional probability of failure would be $F_R(s)$, but since $S = s$ is associated with probability $f_S(s)ds$, the integration over all values of S yields Eq. (2.2a). Alternatively, the probability of failure may also be formulated by convolution with respect to r, yielding

$$P_F = \int_{-\infty}^{+\infty} [1 - F_S(r)] f_R(r) dr \tag{2.2b}$$

As portrayed graphically in Figure 2.2, the overlapping of the curves $f_R(r)$ and $f_S(s)$ represents a qualitative measure of the failure probability P_F, where we can observe that the following (Ang and Tang 1984); the measure of safety or reliability should be a function of the

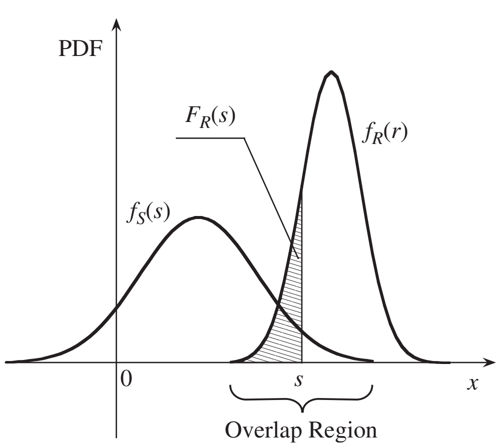

Figure 2.2 PDFs of R and S.