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Coefficient Diagram Method for Control System Design

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Preface

This book is written with two objectives. The first objective is to introduce a simple but powerful control design technique called Coefficient Diagram Method (CDM), whereby ordinary engineers without strong control and mathematics background can design a good controller for their specific plants. Also, control experts can solve such complicated design problems, which defy their best knowledge, in a consistent manner. The second objective is to clarify the “meaning” of the control theories currently in use. CDM is so general that any controller designed by various control theories can be expressed by an equivalent CDM design. Such equivalent design helps to clarify the “meaning” of each control theory by the common terminology used in CDM. As the result, control experts can correctly evaluate the merits and shortcomings of such control theories and use them wisely in practical application.

Various control theories have been developed so far, and many books are already written. However, the author feels that they hardly come up to the expectation of control designers. Firstly, a strong mathematical background is necessary to understand control theories. The emphasis is placed on analysis, and the design/synthesis problem is not properly addressed. Too much emphasis is placed on Linear Time Invariant (LTI) system, and the extension to time variant and nonlinear systems is not properly considered. For these reasons, the controller designed by such theories is not necessarily a good controller in a practical sense. Secondly, control theories have developed as the answers to specific needs of the industry more or less in an ad hoc manner under the strong influence of the computational power available at that time. Thus, they are so much diversified and lack unity and consistency. The standard textbooks describe these theories as they are in an exhaustive manner. Thus, it is very difficult to understand the meanings of these theories and to grasp the total picture of control theories.

This book is motivated by the need to remedy these situations. Although CDM emerged as a new control design technology in 1991, the basic ideas have been proposed and tested in practical application since the 1950s. The CDM only gives convenient expression and a mathematical basis to such practical ideas. The Kessler standard form [1] has been widely used in the steel industry since 1960 as the model of a good characteristic polynomial. The CDM adopts the Kessler standard form after some modification. The sufficient condition for stability by Lipatov [2], the simplification of the Routh stability condition, is not well known in the control community. However, the theory by Lipatov is very simple but effective and gives the design standard of the characteristic polynomials. The theory constitutes the mathematical basis of CDM. The center of the CDM is the coefficient diagram. The coefficient diagram shows the coefficients of the characteristic polynomial in logarithmic scale (ordinate) and the order of each coefficient in linear scale (abscissa). Its curvature represents stability. Its inclination represents response speed. The deformation of the diagram corresponding to the variation of a specific parameter shows robustness. Thus, the three key elements of the control system, namely stability, response, and robustness, are shown in a single diagram. Because the characteristic polynomial is the sum of the denominator and numerator polynomials of the open-loop transfer function, such polynomials are also shown in the coefficient diagram. With the help of these polynomials, the frequency response and time response can be roughly estimated. Thus, the coefficient diagram contains rich information about the system in a single diagram with an intuitively understandable visual form. In a sense, the coefficient diagram plays the same role as the Bode diagram in a more effective way for design/synthesis problems. The CDM belongs to an algebraic approach, which is the third control design approach between classical control and modern control. More specifically, it is an algebraic design approach on polynomial ring, and not on rational functions. Because rational functions are carefully avoided, differential equations are used directly. It is not necessary to use the Laplace transform.

In Chap. 1, we introduce the basic concept of CDM through a simple controller design example. Especially, "Sect. 1.2 A Simple Design Problem" will give the reader an overview of CDM design clearly. It describes the system representation used in CDM, the outline of the design process, and the control structure. Chapters 2 and 3 explain the basic theory and design method of CDM. In Chap. 4, we present four selected examples applying to advanced practical control design problems: a simple controller for an inverted pendulum mounted on a toy car, vibration suppression control for two-inertia system combined with a spring, a solution to the ACC benchmark problem where a two-mass-spring system with parameter uncertainty as well as the design constraints is considered, and a longitudinal control of a modern aircraft as a robust MIMO control design problem. In this approach, we need to calculate algebraic equations associated with the Diophantine equation and draw polynomial coefficients curves on the coefficient diagram. A CDM Toolbox for use with MATLAB for this purpose was developed and is given in the Appendix.

This book is not intended to be a standard textbook for control education. Rather it is intended to supplement those textbooks at the design/synthesis stage. The author used the textbooks by Franklin [3] and Chen [4], and some of the examples are designed by CDM in comparison. To learn CDM, basic mathematics such as algebra is necessary. However, such mathematics as the Laplace transform or matrix algebra are carefully avoided, because such mathematics will give much burden to chemical or biological engineers who are interested in control.

S. Manabe wishes to express his sincere thanks to many people who supported his research on many occasions in the last fifty years. Professors Warren and Weimer at the Ohio State University introduced the author at an early age to "Automatic Control". Messrs. M. Yokosuka, H. Takeda, H. Morikawa, and N. Mitani helped the author to pursue the control research at Mitsubishi Electric. Professor A. G. J. MacFarlane of Cambridge University gave the author valuable suggestions about the meanings of the works of J. C. Maxwell and E. J. Routh. These works constitute the basis of CDM, and his suggestions helped the author to reorient the course of his research. Professor Y. Hori of Tokyo University and members of the Motion Control Committee (Institute of Electrical Engineers of Japan) were keenly interested in CDM and gave valuable advice on various occasions. Professors Y. Nozaka and M. Iida of Tokai University helped the author on many occasions. The students and colleagues of Tokai University helped in the development of CDM through lively discussion in the classroom and laboratory. Professor Young-Chol Kim of Chungbuk National University of Korea and his students had a keen interest in CDM. They made pioneering efforts to make CDM known in the international community. Without their help and efforts, CDM would not have come to this present stage. S. Manabe also wishes to express his sincere gratitude to his wife, Yasuko Manabe, for her spiritual and physical support especially at the advanced age.

Y. C. Kim would like to express his utmost respect and gratitude to Dr. Manabe. Since his first meeting at ASCC in 1997, Dr. Manabe has taught him CDM through numerous discussions and workshops and has also worked on this subject together. It is a great pleasure to express his gratitude to his gurus L. H Keel and S. P. Bhattacharyya for their friendships, encouragement, and valuable teachings. Dr. Kim wants to express his deep gratitude to his beloved wife Agnes Jaesook Kim for her love, patience, and support.

This book was first intended as a full text including more advanced contents. However, because there is a lot of interest in CDM recently, but there are health problems to the authors, they decided to publish the first part of the book. The final edition will be published in the later years. They hope this book will help make future textbooks in the field of control much easier to understand. This book inevitably has errors, and we welcome corrective feedback from the readers. We also apologize in advance for any omissions or inaccuracies in referencing and would want to compensate for them in the final edition.

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Contents

Chapter 1
Introduction

Abstract The overall picture of the Coefficient Diagram Method (CDM) is explained in this chapter. In **Basic Philosophy**, the basic philosophy of the author in writing this book is briefly introduced. In **Simple Design Problem**, the overall design procedure of CDM is shown by the design example of a simple position control system. In **System Representation**, the polynomial expression used in CDM is compared with the transfer function expression in classical control and the state-space expression in modern control. In **Outline of Design Process**, the design steps in CDM are explained in the order of design. In **Control Structure**, it is shown that various forms of controllers derived from various control theories can be represented by equivalent CDM controllers. In **Historical Background**, the history of CDM in the last 50 years is looked back. In **Summary**, important points are summarized and the six features of CDM are explained.

1.1 Basic Philosophy

In the beginning, the author wishes to express his basic philosophy through three topics; namely control, feedback control, and algebraic approach. "**Control is compromise**" is the basic philosophy of this book. The background of this philosophy is first explained. Then, the structure of control in a broad sense is presented, and it is shown that **Feedback control is only a small part of control**. Finally, **Algebraic Approach** is briefly explained and compared with classical control and modern control. It is shown that CDM, a specific type of algebraic approach, is developed as an answer to various design issues.

"**Control is compromise**." is the basic philosophy of this book. *Control* in Chinese character is composed of two characters; the first character means *trimming tree* and the second character means *riding horse*. You can trim branches of trees, but you cannot grow the branches on the tree. You have to wait until the branch grows again. In riding a horse, the horse cannot run as fast as 100 km/h or as slow as 1 km/h. He

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has a characteristic speed, which he likes best. The rider has to be satisfied with a compromise between his desire and the desire of the horse.

History shows that the word *Control* appeared first in *Records of Historian* written two thousand years ago by Szuma Chien in the Han Dynasty. In *The First Emperor of Chin*, the second emperor of Chin made the following remark as a reply to the advice of his wise ministers that the heavy tax should be reduced in order to keep peace in the country.

> But what is splendid about possessing an empire is being able to do as you please and satisfy your desires. By stressing and clarifying the laws, a ruler can stop his subjects from doing evil and so *control* the land within the seas.

The second emperor used *control* as controlling his subjects as he would. Such attitude is completely against the traditional wisdom for good government, and it is very natural that the dynasty ended very soon with the third emperor.

Good control is the result of a compromise between the desire of the controller and that of the object to be controlled. Or it is the compromise between *what should be done* and *what can be done*. The philosophies of present control design/synthesis theories are located somewhere in the spectrum of this compromise; classical control is more or less at *can be done* end, while modern control at *should be done* end.

In modern control, too much emphasis is placed on optimality. In a world where compromise is important, optimality is not a good philosophy. For one thing, in the process of seeking optimality, some kind of performance index is introduced. The effort to attain such optimality tends to give an answer, which is optimal only in that index and very poor in the other index. The common sense tells that if optimization is stopped at a certain point below the optimum, performance is fairly good even at the other index. It can be said that the second best is also the second best even at the other performance index. For another thing, the optimum is usually not a peak, but rather a range or plateau. Thus, the optimum cannot be obtained by mathematics and is chosen by the common sense of the designer. For these reasons, optimality is avoided as far as possible in this book, and compromise is more stressed.

Because compromise is very important in control system design, some mechanism to facilitate compromise has to be imbedded in the design process. Human being has a strong capacity of compromise when the problem is expressed in a graphical form. A graphical representation is a key to compromise, and serious attention has been paid to it. *Graph rather than mathematics* is also a feature of this book.

Feedback control is only a small part of control. This book is intended for the design of feedback control of dynamic systems. The main topic is on Linear Time Invariant (LTI) system. Although feedback control is very important, it is only a small part of total control. The total control system is shown in Fig. 1.1. It consists of a plant, feedback controller, and intelligent controller.

Input to the plant is applied through actuators, and the output of the plant is measured by sensors. The plant is a dynamic system, which can be expressed by differential equations. The plant considered here is an energy device in the sense that its performance is limited by energy. The feedback controller is an information device in the sense that sensor outputs are connected to controller inputs as information and

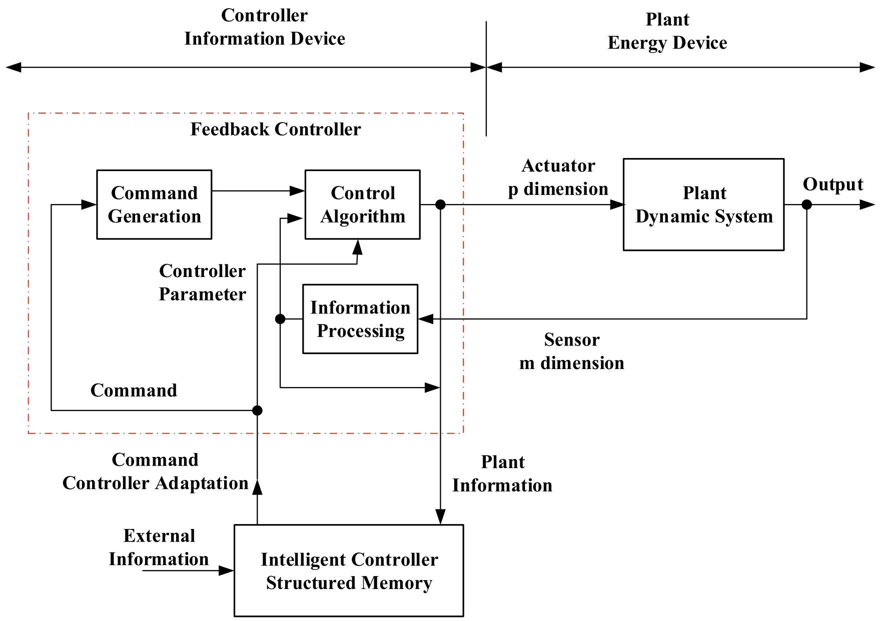

Fig. 1.1 Control system

controller outputs are fed to actuator inputs in the form of information. Information is usually in electronic form. The feedback controller has three functions; namely command generation, control algorithm, and sensor information processing.

This feedback controller is connected to the intelligent controller, which is also an information device and consists of large structured memory, which stores many procedures and large past experiences. The main purpose of the intelligent controller is to give proper instruction to the feedback controller as to what kind of control is to be performed at the specific moment. Such instruction is usually composed of one for command generation and the other for modification of control algorithm. The input to the intelligent controller is the plant information obtained from the actuator and sensor information. Some exterior information is fed to the intelligent controller as the basic command of overall control. Usually, many feedback controllers are under the control of an intelligent controller. The intelligent controller is placed at a higher layer than feedback controllers in the hierarchy.

The feedback controller and the intelligent controller constitute the controller in a broad sense. Usually, they are in a control computer. In the olden age before the control computer was introduced, the feedback controller was an analog type, and the relay sequence circuit performs the function of the intelligent controller. It is interesting to note that, in the actual system, the majority of the control function is devoted to the intelligent control and only a small portion, 5 to 10%, is used for feedback control. This means that feedback control is only a small part of control in a broad sense. For this reason, too much sophistication of feedback control is not justified.

Because feedback control is in the hierarchy under the intelligent control, the controller can be adapted to plant condition, and it can be provided with some measure to cope with the nonlinear nature of the plant. Thus, the feedback controller in the LTI environment must be as simple as possible. Otherwise, the addition of adaptive and nonlinear function by intelligent control will become difficult.

In olden times, controller and plant are both energy devices as seen in Watt's fly-ball governor and the steam engine controlled by the governor. In recent years, the plant may be an information system. Then controller and plant are both information devices. The energy limitation problem will not become evident. This book is concerned with a specific system where the controller is an information device and the plant is an energy device. Under this circumstance, the actuator usually has the severest power and response limitation. These limitations have the greatest influence on the performance of the closed-loop system. Because such actuators are expensive, the number of actuators, expressed as dimension p, is usually much smaller than the number of sensors, expressed as dimension m. This specific circumstance tends to make the system in the form of a loop instead of a mesh, where the actuator plays the role of a common node for various loops.

This total control system may be compared with control of human body motion. The human body is the plant. The actuators are feet and hands. The sensors are eyes, ears, skin, sense of muscle force, sense of balance, etc. The feedback controller may be the cerebellum and brain stem, and the intelligent controller may be the cerebrum. The human body is an energy device, and the performance is limited by the energy limitation. The number of actuators is much smaller than that of sensors. The vast memory accumulated in the cerebrum through the past education and experience works as the intelligent controller with much flexibility and complexity.

Algebraic Approach lies between classical control and modern control as shown in Fig. 1.2. It is the third control theory. It is often called the polynomial approach because polynomials are used in system representation. In classical control, frequency response design and root locus design are currently used. Both methods use transfer function in the system representation. The relation of controller parameters and closed-loop characteristics is graphical; the Bode/Nyquist diagram for frequency response design, and the root locus diagram for root locus design. In the design process, the basic controller structure with undetermined parameters is first assumed, and such parameters are adjusted so that the closed-loop system meets design specifications. Such design process is called as *Outward Approach* [1].

In modern control, pole assignment, optimal control (LQR, LQG), and H-infinity design are currently used. They all use state space in system representation. The relation of controller parameters and closed-loop characteristics is expressed in equations. In pole assignment, pole location representing closed-loop characteristics is related to controller parameters in algebraic matrix relation such as Ackermann's formula [2]. In optimal control and H-infinity, closed-loop characteristics are related to controller parameters through the Riccati equation. In the design process, an overall closed-loop system, to meet design specifications, is first chosen, and the controller

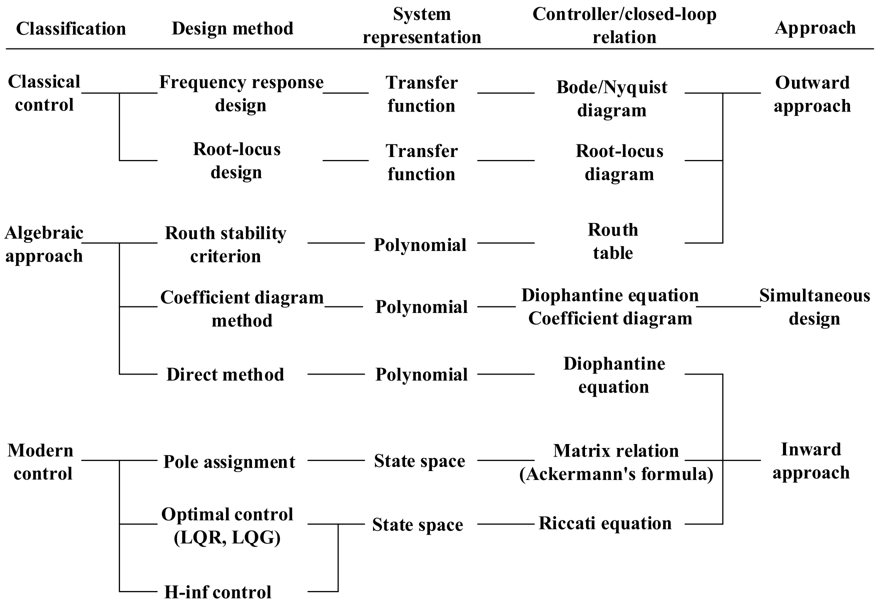

Fig. 1.2 Comparison of control theories

is obtained as the solution of equations. Such design process is called as *Inward Approach* [1].

Contrary to classical and modern control, the algebraic or polynomial approach is not clearly defined, because it is at the developing stage. In a broad sense, it includes the design approach by Routh stability criterion. It uses polynomial as system representation, and controller parameters are related to closed-loop stability through the Routh table. The design process is definitely an outward approach. In a narrow sense, the algebraic design is characterized by the direct design method [2]. The system representation is by the polynomial. The closed-loop characteristics are represented by the characteristic polynomial. It is related to the controller parameters through the Diophantine equation. In the usual design process, the characteristic polynomial is chosen first, and the controller is obtained as the solution of the Diophantine equation. Thus, it is definitely an inward approach. The problem is extended to the MIMO case by extension of the polynomial to the polynomial matrix. Various researchers have made intensive study already [1–4], but no serious study has been made for the selection of the characteristic polynomial. The characteristic polynomial is usually specified by pole assignment.

The transfer function expression is easy to understand but is inaccurate at pole-zero cancellation, while the state-space expression is accurate but difficult to understand. The polynomial expression is easy to understand like the transfer function, while it is accurate as the state-space expression. This is an advantage of the algebraic approach over classical control and modern control. The other advantage is that the Diophantine equation is linear and easy to solve, while the Riccati equation is quadratic.

The coefficient diagram method differs from the direct design method in that controller parameters and characteristic polynomial are related through the Diophantine equation and *Coefficient Diagram*. The relation is expressed mathematically and graphically in such an effective manner that *Simultaneous Design* is possible, whereby controller and characteristic polynomial are simultaneously designed. In the outward approach, the problem is that the obtainable closed-loop characteristics are limited by the assumption of controller structure at outset. Thus, when the assumption is not appropriate, the desired closed-loop characteristics may not be obtained. Even if the design is completed, there always remains the doubt that the controller might not be optimum. In the inward approach, although the closed-loop characteristics are guaranteed, there is no guarantee that the designed controller retains commonly acceptable features such as simplicity and robustness. In the outward approach, such features are usually designed into the controller assumption, and controllers designed by classical control are usually robust. The robustness issue only came up to the surface with the introduction of modern control and inward approach.

By simultaneous design of CDM, simple and robust controllers corresponding to the specified closed-loop characteristics can be easily designed. The graphical expression of the coefficient diagram makes intuitive design possible as in classical control with the Bode/Nyquist diagram. In control design, the simplicity of the controller and closed-loop performance are trade-off issues. The controller represents *what can be done*, while the closed-loop performance represents *what should be done*. To find a good compromise between the two is the key to good control. Compromise is the most important in control design, and CDM gives some answers to it.

1.2 A Simple Design Problem

In order to show the general picture of the Coeffcient Diagram Method, a simple design example of a position control system is presented in this section. Five topics, namely **Problem Statement**, **Definition of Stability Index and Equivalent Time Constant**, **Design Process**, **Characteristics of Control System**, and **Coefficient Shaping** will be represented in order.

Problem Statement will be first made. The system considered is a generic position control system shown in Fig. 1.3. The plant consists of a power amplifier and a motor. It can be represented in the differential equation form as

$$(0.25s + 1)(s + 1)sy = u, \tag{1.1}$$

$$v = sy, \tag{1.2}$$

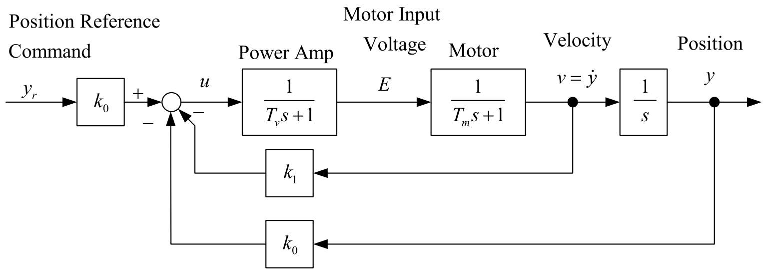

Fig. 1.3 Position control system

where u is the input to the power amplifier; v the velocity; y the position. The s stands for differential operator d/dt. The controller is a PD controller, where the velocity and position sensors obtain the necessary feedback signals. It can be represented by the differential equation form as

$$u = k_0(y_r - y) - k_1 v, \tag{1.3}$$

where y_r is the position reference command; k_0 position gain; k_1 velocity gain. By eliminating u from these equations, the equation which relates y to y_r is obtained as

$$(0.25s^3 + 1.25s^2 + s + k_1 s + k_0)y = k_0 y_r. \tag{1.4}$$

The term preceding y of the left side is the characteristic polynomial $P(s)$.

$$P(s) = 0.25s^3 + 1.25s^2 + (1 + k_1)s + k_0. \tag{1.5}$$

The problem is to find the controller gains k_1 and k_0 such that the system has good characteristics in terms of stability, response, and robustness

Definition of Stability Index and Equivalent Time Constant is next made as preparation for design. The characteristic polynomial is generally expressed in the following form.

$$P(s) = a_n s + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0 = \sum_{i=0}^{n} a_i s^i. \tag{1.6}$$

The stability index γ_i and the equivalent time constant τ are defined as follows:

$$\gamma_i = a_i^2/(a_{i+1}a_{i-1}), \quad i = 1, \cdots, n-1. \tag{1.7}$$

$$\tau = a_1/a_0. \tag{1.8}$$

The stability index is an important measure to indicate the stability of the system. Its importance has been known for many years, and different names were given to this term by many authors. Kessler [5] called it *damping factor*; Naslin [6] *characteristic ratio*; Brandenburg [7] *double ratio*. The damping factor represents the exact nature of the term, but may be misleading, because it is currently used in a different meaning. The characteristic ratio may not represent the nature of the term properly. The double ratio represents how it is made of $(a_i/a_{i-1})/(a_{i+1}/a_i)$, but does not represent what it is. For these reasons, a new name *stability index* is given here. The standard values recommended in CDM are as follows:

$$\gamma_{n-1} = \cdots = \gamma_3 = \gamma_2 = 2, \qquad \gamma_1 = 2.5. \tag{1.9}$$

The reason why these values are recommended is the key to CDM and will be made clear in later chapters. The *equivalent time constant* is a measure of the response speed. When γ_i takes the standard values, all transient response settles within 2.5–3 times of τ. CDM design is based on stability index and equivalent time constant.

Design Process will be shown next. Because the problem is simple, the standard values of γ_i ($\gamma_2 = 2$, $\gamma_1 = 2.5$) can be chosen. By the definition of stability index, the following relations are derived.

$$a_1 = 1 + k_1 = a_2^2/(a_3\gamma_2) = 1.25^2/(0.25 \times 2) = 3.125, \tag{1.10}$$

$$a_0 = k_0 = a_1^2/(a_2\gamma_1) = 3.125^2/(1.25 \times 2.5) = 3.125. \tag{1.11}$$

Then, the design results are as follows:

$$k_1 = a_1 - 1 = 2.125, \qquad k_0 = 3.125, \qquad \tau = 1. \tag{1.12}$$

The resulting controller shows good characteristics, namely no-overshoot, fast response, and good robustness. The settling time is about 2.5–3 times of τ. It should be noted that simple arithmetic suffices for the design with no need for higher mathematics.

Under some circumstances, slower responses are permissible. Then design proceeds for $\gamma_2 > 2$ and $\gamma_1 = 2.5$ with $k_1 = 2.125 \sim 0$ as a free parameter.

$$k_0 = a_0 = a_1^2/(a_2\gamma_1) = (1 + k_1)^2/(1.25 \times 2.5) = 0.32(1 + k_1)^2, \tag{1.13}$$

$$\gamma_2 = a_2^2/(a_3a_1) = 1.25^2/[0.25(1 + k_1)] = 6.25/(1 + k_1) > 2, \tag{1.14}$$

$$\tau = a_1/a_0 = 3.125/(1 + k_1). \tag{1.15}$$

When k_1 becomes 0, the response is the slowest. Then $k_0 = 0.32$, $\gamma_2 = 6.25$, and $\tau = 3.125$. If further slow response is allowed, design proceeds for $k_1 = 0$, $\gamma_2 = 6.25$, and $\gamma_1 > 2.5$ with $k_0 = 0.32 \sim 0$ as a free parameter.

$$\gamma_1 = a_1^2/(a_2 a_0) = 1/(1.25k_0) > 2.5, \tag{1.16}$$

$$\tau = a_1/a_0 = 1/k_0 > 3.125. \tag{1.17}$$

The above design procedure shows that CDM design is simple and flexible in meeting the specific need of design. The effectiveness of CDM comes from the standard values of the stability indices into which experiences of past design efforts are crystallized.

Characteristics of Control System will be discussed hereafter. Table 1.1 shows the case when τ is increased from the designed nominal value 1 to 2, 3.125, and 5. These are the cases where slower responses are permissible. In order to retain favorable responses, the stability indices are increased in a systematic manner. The first step is to increase γ_2 with fixed $\gamma_1 = 2.5$ until k_1 becomes 0. The second step is to increase γ_1 with fixed $\gamma_2 = 6.25$ and $k_1 = 0$ by decrease of k_0. The coefficient diagram is shown in Fig. 1.4. In coefficient diagram, the coefficients a_i of the characteristic polynomial is shown in the ordinate in logarithmic scale, while order i is shown in abscissa in decreasing order. In the nominal case #1,

$$a_i = [a_3 \;\; a_2 \;\; a_1 \;\; a_0] = [0.25 \;\; 1.25 \;\; 3.125 \;\; 3.125]. \tag{1.18}$$

The step responses with different τs are shown in Fig. 1.5. All responses are smooth and show no-overshoot.

Table 1.2 shows the case when γ_1 is made one half and two times with k_0 made twice and one half. The coefficient diagram is shown in Fig. 1.6. The step responses are shown in Fig. 1.7. When γ_1 is small, the stability deteriorates and conspicuous overshoot is observed.

From these observations, we can see that the response speed is mainly affected by τ, and the waveform of the step response is largely influenced by γ_1. These two parameters seem to characterize the step response. The stability index γ_i is represented by the curvature of the coefficient diagram at the specific order i, as is clear from the definition of the stability index. The equivalent time constant is represented by the inclination of the diagram at the rightmost ends. The coefficient diagram and the step response are closely related through stability indices and equivalent time constant. If the coefficient diagram is given, the approximate step response can be visualized and vis-á-vis. This problem will be discussed in later chapters.

The above results are shown on the parameter space by k_0 and k_1 as in Fig. 1.8. From the Routh stability condition, $a_0 > 0, a_2 a_1 > a_3 a_0$, the stable region is given as follows:

$$0 < k_0 < 5(1 + k_1). \tag{1.19}$$

Because such a stable region is too broad, proper parameter selection cannot be done. When $\gamma_1 = a_1^2/(a_2 a_0)$ is specified as 2.5, and γ_2 and τ are allowed to change, the parameter satisfy the following equation.

$$k_0 = 0.32(1 + k_1)^2. \tag{1.20}$$

Table 1.1 τ variation

Case	k_1	k_0	τ	γ_2	γ_1
#1	2.125	3.125	1	2	2.5
#2	0.5625	0.78125	2	4	2.5
#3	0	0.32	3.125	6.25	2.5
#4	0	0.2	5	6.25	4

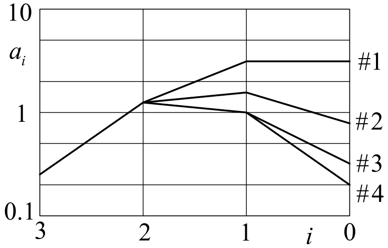

Fig. 1.4 Coefficient diagram for τ variation

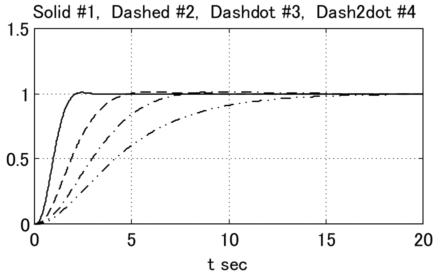

Fig. 1.5 Step responses to τ variation

Table 1.2 γ_1 variation

Case	k_1	k_0	τ	γ_2	γ_1
#1	2.125	3.125	1	2	2.5
#5	2.125	6.25	0.5	2	1.25
#6	2.125	1.5625	2	2	5

The responses are smooth on every point on the curve, oscillatory on the right side, corresponding to small γ_1, and sluggish on the left side. Although the parameters can be chosen at any point on the curve, the best choice is at point #1, where τ is smallest and the response is fastest.

Coefficient Shaping is a graphical design approach where the close relation between the coefficient diagram and the step response is fully utilized. By this approach, the characteristic polynomial and the controller can be simultaneously

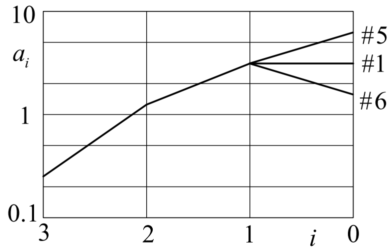

Fig. 1.6 Coefficient diagram for γ_1 variation

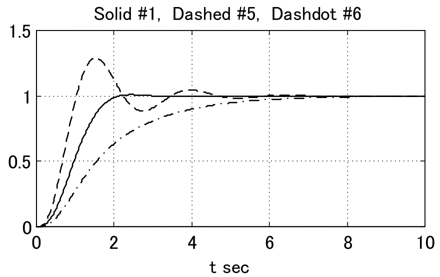

Fig. 1.7 Step responses to γ_1 variation

designed. This is the most conspicuous feature of CDM, and its implication will be discussed later. A brief description of this approach will be now shown. In this example, the characteristic polynomial is composed of two component polynomials.

$$\begin{aligned} P(s) &= P_0(s) + P_k(s), \\ P_0(s) &= 0.25s^3 + 1.25s^2 + s, \\ P_k(s) &= k_1 s + k_0, \end{aligned} \tag{1.21}$$

where $P_0(s)$ is the characteristic polynomial without controller, and $P_k(s)$ is the contribution by the controller. The coefficient diagram is shown in Fig. 1.9. The $P_0(s)$ is shown by small circles and dash-dot line. The $P_k(s)$ is shown by small square and dotted line. With this decomposed coefficient diagram, the variation of $P(s)$ due to parameter variation can be easily visualized. Because the general shape of $P(s)$ in the coefficient diagram is closely related to the step response, the variation of the step response can be easily estimated.

The first step in the design is to draw $P_0(s)$ in the coefficient diagram. We draw a rough sketch of $P(s)$ based on the required step response. The $P_k(s)$ is designed to fill the gap between $P(s)$ and $P_0(s)$. The final $P(s)$ is obtained as the sum of $P_0(s)$ and $P_k(s)$. Thus $P(s)$ and $P_k(s)$ are simultaneously designed. This approach is very effective in defining the basic control structure. The stability indices and equivalent time constant best fitted to the purpose can be easily estimated. Because this is a graphical approach, the intuition of the designer is fully utilized and design is more efficient.