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Wavelets in Neuroscience

Second Edition

 Springer

Springer Series in Synergetics

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ISSN 0172-7389

ISSN 2198-333X (electronic)

Springer Series in Synergetics

ISBN 978-3-030-75991-9

ISBN 978-3-030-75992-6 (eBook)

<https://doi.org/10.1007/978-3-030-75992-6>

1st edition: © Springer-Verlag Berlin Heidelberg 2015

2nd edition: © The Editor(s) (if applicable) and The Author(s), under exclusive license to Springer Nature Switzerland AG 2021

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Foreword

Modern technologies have provided unexpected new dimensions of data in size and variety with a strongly increasing tendency. Their processing and analysis is one of the basic challenges in our era of digitalization. However, most conventional data analysis techniques were designed for much smaller data sets. Wavelets are one of the most important exceptions; they have a very strong potential for applications to big data even when the data are non-stationary, noisy and high-dimensional. Wavelets have been very successfully applied to solve various scientific and engineering problems in which conventional methods, as correlation and spectral techniques, are ineffective or even fail. In recent decades, wavelet analysis has become one of the most successful and widespread tools for analyzing and synthesizing multivariate and spatio-temporal measurements, performing efficient image processing, compressing large amounts of data, or recognizing patterns, etc.

There is a huge and rapidly growing amount of publications devoted to wavelet techniques and their application to various fields. Therefore, it is hard for newcomers or interested users from other disciplines to understand important modern methodological directions and learn how to use wavelet tools appropriate for their specific problems. In this monography, a group of outstanding and very active Russian scientists presents the second edition on modern wavelet techniques and their application to urgent problems in neuroscience. Modern neuroscience is characterized by a rapidly increasing amount of measurements. Making this great potential accessible for the study of the brain and even for clinical practice is indeed a challenging problem of highest actuality. It is clearly demonstrated here that wavelets are very appropriate for this task.

This book gives first a clear and concise mathematical introduction to wavelet theory and secondly discusses how to use wavelets efficiently in neuroscience. In this second edition, the selected neuroscientific problems are well subdivided into exemplary and basic topics from single neurons to real-time classification of EEG patterns. Based on their manifold experience with such data, the authors provide an

excellent guidance how to treat such problems. This monography is a very useful introduction for starters in the field, but it provides also important information and suggestions for researchers applying advanced signal processing techniques in neuroscience as well as also for users from other fields.

Berlin/Potsdam, Germany

Jürgen Kurths

Preface

*If people do not believe that mathematics is simple,
it is only because they do not realize
how complicated life is*

John von Neumann

More than 30 years ago, Jean Morlet introduced for the first time the notion of a wavelet as a soliton-like function. At the beginning, he applied this function to the analysis of backscattered seismic signals, but soon he realized that wavelets have a significantly broader field of possible applications. In 1981, Alexander Grossmann interpreted wavelets as coherent states and gave an elegant proof of Morlet's reconstruction algorithm. Since then this technique has witnessed explosive growth and it now represents a universal mathematical tool with useful applications in many scientific and engineering studies.

Originally wavelets emerged as an alternative to the classical spectral analysis based on the Fourier transform, such as windowed Fourier analysis or the Gabor transform. In order to improve processing of transient components in complex signals, Morlet decided to replace Gabor functions, which have a fixed duration, by new building blocks or time—frequency atoms, which can have an arbitrarily small duration. Later this concept led to new insights and a mathematically rigorous foundation.

Nowadays, there is no doubt that the introduction of wavelets theory was one of the most important events in mathematics over the past few decades. This is probably the only concept that has been applied in practically all the fields of basic science. Moreover, wavelets are widely used for image recognition and compression, for analysis and synthesis of complex signals, in studies of turbulent flows and feature extraction from biological and medical data, etc.

This book is devoted to application of wavelet-based methods in neuroscience. We have attempted to illustrate how wavelets may provide new insight into the complex behavior of neural systems at different levels: from the microscopic dynamics of individual cells (e.g., analysis of intracellular recordings) to the macroscopic level of widespread neuronal networks (e.g., analysis of EEG and MEG recordings). Our main aim has been to show how and where wavelet-based tools can gain an advantage over classical approaches traditionally used in

neuroscience. We hope that the logical structure of the book as regards content (from micro to macro scale) represents a new approach to experiential data analysis and could be helpful in everyday use. The book describes several examples obtained by the authors in experimental neuroscience.

In the second edition of the monograph, we added new results of the development of wavelet-based methods for online processing of epileptic EEG for the creation system for prediction and prevention of epileptic events, which has been used for the closed-loop brain-computer interface to epilepsy control. We also considered, in the revised monograph, several new wavelet applications to analyze various neurophysiological processes associated with the processing of visual sensory information and the real and imaginary motor activity executions in humans. We mainly focus in the second edition on our results, which were obtained in the past few years on the study of multi-channel EEG. We consider the results obtained with the help of wavelets and leading to a deeper understanding of the human brain's processes during acts of sensorimotor integration. Moreover, we apply the discovered brain activity patterns to create brain-computer interfaces to monitor and improve a subjects' performance (brain-computer interface operators) under routine tasks (for example, prolonged classifying visual stimuli, increasing attention level, etc.)

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Contents

1	Mathematical Methods of Signal Processing in Neuroscience	1
1.1	General Remarks	1
1.2	Nonstationarity of Neurophysiological Data	2
1.3	Wavelets in Basic Sciences and Neuroscience	4
1.4	Automatic Processing of Experimental Data in Neuroscience	5
1.5	Brain-Computer Interfaces	6
1.6	Topics to Consider	7
	References	10
2	Brief Tour of Wavelet Theory	15
2.1	From Fourier Analysis to Wavelets	16
2.2	Continuous Wavelet Transform	26
2.2.1	Main Definitions. Properties of the Continuous Wavelet Transform	26
2.2.2	Mother Wavelets	31
2.2.3	Numerical Implementation of the Continuous Wavelet Transform	34
2.2.4	Visualisation of Wavelet Spectra. Wavelet Spectra of Model Signals	44
2.2.5	Phase of the Wavelet Transform	51
2.3	Discrete Wavelet Transform	61
2.3.1	Comparison of the Discrete and Continuous Wavelet Transforms	61
2.3.2	General Properties	64
	References	71
3	Analysis of Single Neuron Recordings	75
3.1	Introduction	75
3.2	Wavelet Analysis of Intracellular Dynamics	76

3.2.1	Interference Microscopy and Subcellular Dynamics	76
3.2.2	Modulation of High Frequency Oscillation by Low Frequency Processes	78
3.2.3	Double Wavelet Transform and Analysis of Modulation	79
3.2.4	Modulation of Spike Trains by Intrinsic Neuron Dynamics	81
3.3	Information Encoding by Individual Neurons	84
3.3.1	Vibrissae Somatosensory Pathway	84
3.3.2	Classification of Neurons by Firing Patterns	86
3.3.3	Drawbacks of the Traditional Approach to Information Processing	87
3.3.4	Wavelet Transform of Spike Trains	88
3.3.5	Dynamical Stability of the Neuronal Response	91
3.3.6	Stimulus Responses of Trigeminal Neurons	94
3.4	Wavelet Coherence for Spike Trains: A Way to Quantify Functional Connectivity	99
3.4.1	Wavelet Coherence of Two Point Processes	100
3.4.2	Measure of Functional Coupling Between Stimulus and Neuronal Response	101
3.4.3	Functional Connectivity of Gracilis Neurons to Tactile Stimulus	103
	References	115

4 Classification of Neuronal Spikes from Extracellular

	Recordings	119
4.1	Introduction	119
4.2	General Principles of Spike Sorting	120
4.3	Spike Detection Over a Broadband Frequency Activity	122
4.4	Naive Spike Sorting	125
4.5	Principal Component Analysis as Spike-Feature Extractor	128
4.5.1	How It Works	128
4.5.2	Possible Pitfalls	130
4.6	Wavelet Transform as Spike-Feature Extractor	132
4.6.1	Wavelet Spike Classifier	133
4.6.2	Potential Problems	133
4.7	Wavelet Shape-Accounting Classifier	135
4.8	Performance of PCA Versus WT for Feature Extraction	137
4.9	Sensitivity of Spike Sorting to Noise	140
4.9.1	Impact of High/Low Frequency Noise on PCA and WT	140
4.9.2	Proper Noise Filtering May Improve Spike Sorting	142

- 4.10 Optimal Sorting of Spikes with Wavelets and Adaptive Filtering 144
 - 4.10.1 Noise Statistics and Spike Sorting 145
 - 4.10.2 Parametric Wavelet Sorting with Advanced Filtering 146
- 4.11 Spike Sorting by Artificial Neural Networks 151
 - 4.11.1 General Approach 152
 - 4.11.2 Artificial Neural Networks 154
 - 4.11.3 Training the Artificial Neural Network 156
 - 4.11.4 Algorithm for Spike Sorting Using Neural Networks 157
- 4.12 Artificial Wavelet Neural Networks for Spike Sorting 160
 - 4.12.1 Structure of Wavelet Neural Networks 161
 - 4.12.2 Wavelet Neural Networks 161
- References 171
- 5 Analysis of Gamma-Waves in Multielectrode LFP Recordings 175**
 - 5.1 Introduction 175
 - 5.2 Disentanglement of Raw LFP Recordings into Pathway-Specific Generators 177
 - 5.2.1 LFP Recordings and Current-Source-Density Analysis 177
 - 5.2.2 Decomposition of LFPs into Pathway-Specific Generators 180
 - 5.3 Localization and Quantification of Gamma Waves in the Schaffer-Generator by Wavelet Analysis 184
 - 5.3.1 Method for Detecting Gamma Waves 184
 - 5.3.2 Elementary Micro-fEPSPs in Ongoing Schaffer Activity 188
 - 5.3.3 Detected Gamma Events Help to Establish Causal Relations Between CA3 and CA1 Pyramidal Cells 190
 - 5.4 Improved Identification of Micro-fEPSP Events 192
 - 5.4.1 Distortion of Micro-fEPSP Events by Wavelet Method 192
 - 5.4.2 Likelihood Enhanced Wavelet (LeW) Method 193
 - 5.5 Bilateral Integration of Gamma-Parsed Information 200
 - 5.5.1 Experimental Recordings and Retrieval of Bilateral Micro-fEPSP Events 202
 - 5.5.2 Analysis of Bilateral CA3-CA1 Pathways 203
 - 5.6 Conclusions 206
 - References 208

- 6 Wavelet Approach to the Study of Rhythmic Neuronal Activity . . . 211**
 - 6.1 Introduction 211
 - 6.2 Basic Principles of Electroencephalography 212
 - 6.2.1 Electrical Biopotential: From Neuron to Brain 213
 - 6.2.2 Application of EEG in Epilepsy Research 214
 - 6.3 General Principles of Time–Frequency Analysis of EEG 216
 - 6.3.1 The Need for Mathematical Analysis of EEG 216
 - 6.3.2 Time–Frequency Analysis of EEG: From Fourier Transform to Wavelets 217
 - 6.3.3 Time–Frequency Analysis of Spike-Wave Discharges by Means of Different Mother Wavelets 221
 - 6.4 Applications of Wavelets in Electroencephalography 229
 - 6.4.1 Time–Frequency Analysis of EEG Structure 230
 - 6.4.2 Automatic Detection of Oscillatory Patterns and Different Rhythms in Pre-recorded EEG 230
 - 6.4.3 Classification of Oscillatory Patterns 231
 - 6.4.4 Real-Time Detection of Oscillatory Patterns in EEG 231
 - 6.4.5 Multichannel EEG Analysis of Synchronization of Brain Activity 232
 - 6.4.6 Artifact Suppression in Multichannel EEG Using Wavelets and Independent Component Analysis 232
 - 6.4.7 Study of Cognitive Processes 233
 - References 235

- 7 Wavelet-Based Diagnostics of Paroxysmal Activity in EEG and Brain-Computer Interfaces for Epilepsy Control 243**
 - 7.1 Introduction 243
 - 7.2 Mother Wavelet Function in the Continuous Wavelet Transform 245
 - 7.3 Detection of Spike-Wave Discharges (Absence Epilepsy) in WAG/Rij Rats 247
 - 7.4 Spindle-Like Oscillations and Spike-Wave Epilepsy 252
 - 7.4.1 Time–Frequency Analysis of Spindle-Like Oscillatory Patterns 255
 - 7.4.2 Wavelet-Based Approach for Detecting Sleep Spindles and 5–9 Hz Oscillations in EEG 262
 - 7.4.3 Classification of Normal and Abnormal Spindle Oscillations by Means of Adaptive Wavelet Analysis 265
 - 7.5 Pro-epileptic Activity and Undeveloped Spike-Wave Seizures in Genetically Prone Subjects 275

7.5.1	Time–Frequency Characteristics of Pro-epileptic Patterns in EEG in WAG/Rij Rats	275
7.5.2	Algorithm for the Automatic Detection of Pro-epileptic Patterns in EEG	276
7.6	Brain–Computer Interface for On-Line Diagnostics of Epileptic Seizures	280
7.6.1	On-Line SWD Detection Algorithm	281
7.6.2	Experimental Verification of the Algorithm and On-Line SWD Diagnostics	284
7.7	Brain Stimulation Brain–Computer Interface for Prediction and Prevention of Epileptic Seizures	287
7.7.1	Precursor Wavelet-Based On-Line Detection	287
7.7.2	Absence Seizure Control by a Brain Computer Interface	291
	References	294
8	Analysis of Visual Sensory Processing in the Brain and Brain–Computer Interfaces for Human Attention Control	303
8.1	Introduction	303
8.2	Ambiguous Stimuli as a Tool to Study Visual Perception	306
8.3	Local and Integrative Neural Activity During Visual Sensory Processing	308
8.3.1	Local Activity	309
8.3.2	Functional Connectivity	315
8.4	Visual Sensory Processing and the Human Factors	324
8.4.1	Different Scenarios of Visual Perception	325
8.4.2	Spectral Properties of the Different Scenarios	327
8.4.3	Single-Trial Analysis	329
8.5	BCIs for the Control of Human Condition During Sensory Processing Tasks	331
8.5.1	Wavelet-Based Approach to Estimate Attention in BCI	332
8.5.2	Testing the Feedback Effect	334
8.5.3	Cognitive Load Distribution via BCI	337
	References	344
9	Analysis and Real-Time Classification of Motor-Related EEG and MEG Patterns	351
9.1	Real and Imagery Movements	351
9.1.1	Wavelet-Transform Modulus Maxima (WTMM)	353
9.1.2	Time–Frequency Analysis	357

- 9.2 Visual and Kinesthetic Motor Imagery 360
 - 9.2.1 Wavelet Analysis 362
 - 9.2.2 Cluster Analysis 362
 - 9.2.3 Neurophysiological Aspects of Motor Imagery 365
- 9.3 Age-Related Distinctions in EEG Signals During Execution of Motor Tasks Characterized in Terms of Wavelet Spectra 366
 - 9.3.1 Experimental Study and Motor Brain Response Time Analysis 367
 - 9.3.2 Time–Frequency Analysis of Brain Response on Motor Activity 369
 - 9.3.3 Classification of Wavelet Spectra by Machine Learning Techniques 374
- References 377
- 10 Conclusion 383**
 - Reference 384

Acronyms

ANN	Artificial neural networks
BCI	Brain-computer interface
CNS	Central nervous system
CSD	Current source density
CWT	Continuous wavelet transform
DWT	Discrete wavelet transform
EA	Elderly adult
ECoG	Electrocorticogram
EEG	Electroencephalogram
ERD	Event-related desynchronization
ERWP	Event-related wavelet power
FFT	Fast Fourier transform
GAERS	Genetic rats with absence epilepsy
HF	High frequency
HPF	High-pass filter
ICA	Independent component analysis
icEEG	Intracranial EEG
ISI	Interspike interval
LF	Low frequency
LFP	Local field potential
LH	Left hand
MBRT	Motor brain response time
MEG	Magnetoencephalogram
NN	Neuronal network
OSDS	On-line SWD detection system
PCA	Principal component analysis
PSTH	Peristimulus time histogram
PWAF	Parametric wavelet sorting with advanced filtering
RH	Right hand
RTN	Reticular thalamic nucleus

rWF	Representative waveform
SD	Standard deviation
sdEEG	Subdural EEG
SI	Somatosensory
SWD	Spike-wave discharge
TC	Thalamocortical
TSS	Time scale synchronization
VPM	Ventroposteromedial
WAG/Rij	Wistar Albino Glaxo/Rijswijk
WF	Waveform
wICA	Wavelet independent component analysis
WMSPC	Wavelet method with superparamagnetic clustering
WNN	Wavelet neuronal network
WPOD	Wavelet power over domain
WSAC	Wavelet shape-accounting classifier
WSC	Wavelet-based spike classifier
WT	Wavelet transform
YA	Young adult

Chapter 1

Mathematical Methods of Signal Processing in Neuroscience



Abstract This chapter offers a brief introduction to the novel advanced mathematical methods of analysis and processing of neurophysiological data. First, we give the rationale for the development of specific mathematical approaches for decoding information from non-stationary neurophysiological processes with time-varying features. Second, we focus on the development of mathematical methods for automatic processing and analysis of neurophysiological signals, more specifically, in the development of brain-computer interfaces (BCIs). Finally, we give an overview of the main applications of wavelet analysis in neuroscience, from the *microlevel* (the dynamics of individual cells or intracellular processes) to the *macrolevel* (dynamics of large-scale neuronal networks in the brain as a whole, ascertained by analyzing electro- and magnetoencephalograms).

1.1 General Remarks

Neurodynamics is a contemporary branch of interdisciplinary neuroscience that examines mechanisms of the central nervous system based on the mutual experience of chemists, biologists, physicists, mathematicians, and specialists in the nonlinear theory of oscillations, waves, and dynamical chaos [1–6]. Practical applications of modern methods in neuroscience facilitate an interdisciplinary approach to brain functions and attract experts in experimental and theoretical neurobiology, psychophysiology, cognitive neuroscience, biophysics, physics, nonlinear dynamics, etc. This interdisciplinary collaboration provides unique methods for analyzing the functional activity of the central nervous system (CNS) that focus on the basic principles of the neuronal dynamics of individual cells and neural networks.

Recent progress in understanding molecular and ionic mechanisms of neuronal activity [7] encourages further investigation of certain key problems in modern physics, such as exploration of the functional properties and principles of infor-

mation coding, as well as its representation and the processing of sensory data in the central nervous system. Perception and information processing are important functions of the CNS. Visual, acoustic, tactile, and gustatory stimuli are transformed by the sensory receptors of the first order neurons into a sequence of electrical pulses. These first-order sensory neurons are therefore involved in primary processing of sensory information [8–12]. Sensory information is then passed through relay stations (brain stem and thalamic nuclei) that transform and convolve the information code, until finally it reaches the cerebral cortex which shapes the “fingerprint” of the external world [13–15]. At each subsequent stage, the processes of information transfer become increasingly difficult to study. The question of how the totality of nervous impulses (action potentials or spikes) generated by single neurons can reflect the full complexity and diversity of the external world remains one of the biggest challenges in fundamental science [13, 16–18].

Experimental methods have recently been developed for registering the neuronal activity underlying processes of information encoding-decoding at different levels of the nervous system—from molecular changes in membrane properties of receptor cells to changes in the local (electrical) field potentials in the cerebral cortex. Traditional and noninvasive methods for registering electrical brain activity, such as electroencephalography (EEG) with electrodes arranged on the skin of the head, offer several advantages, and this method is still commonly used in neurophysiology and medicine. EEG is often used in various studies of brain functions in humans and animals [19, 20]. There are also invasive methods using implanted electrodes which provide better spatial resolution, and these are advantageous when examining neuronal activity in small groups of neurons in superficial (cortex) and deep (subcortical) structures. Another advantage of invasive recording techniques is that implanted electrodes can also be used for electrical stimulation with different research purposes, e.g., suppression of epileptic discharges [21–23]. The relatively new noninvasive recording technique known as magnetic encephalography (MEG) has become more popular over the last few years, because it provides better spatial resolution than EEG and better quality of signals reflecting brain activity [24–26].

1.2 Nonstationarity of Neurophysiological Data

Despite technical progress in developing new methods of data acquisition in experimental neurophysiology, mathematical methods of experimental data analysis could not be readily applied, and this may impede further progress. In the vast majority of experimental studies in neuroscience, only a few statistical methods of data analysis are used, e.g., calculation of the mean spike frequency, construction of various correlation characteristics and distribution functions, etc. Traditional methods of statistical analysis are undoubtedly useful, but most of them unable to evaluate the relevant information regarding complex processes in the CNS. In order to illustrate this fact, we give an example that demonstrates the response of a sensory neuron to periodic stimulation. From a mechanical point of view, the response of the neuron

to a sequence of equal external stimuli could be identical, so periodic stimulation of a neuron with a series of impulses could elicit a periodic sequence of spikes (action potentials, for example, 2 or 3 spikes per stimulus). However, in the experimental situation, we often obtain time- and activity-dependent variations in the neuron's response (the neuron does not demonstrate an equal response to repeated identical stimuli) which reflect neuronal plasticity. The phenomenon of synaptic neuronal plasticity (the basic mechanism underlying memory and learning) reflects adaptation to external afferent activity modified by the internal characteristics of individual cells and the global dynamics of the wider neuronal network interactions [27, 28]. It is known that a neuron can even stop responding to the next stimulus from a certain moment.

Figure 1.1 illustrates the adaptive response of a neuron of the trigeminal complex to periodic stimulation. Maximum neuron activity (27 spikes/s) is observed at the onset of stimulation; it falls to an average of 10 spikes/s within a few seconds and varies thereafter, exhibiting a slow negative drift. On the one hand, such behavior of a living cell makes it extremely difficult to define characteristic forms/patterns of neural activity associated with the peculiar properties of a given stimulus. On the other hand, such complexity in neuronal activity encourages the development of more relevant (complex) methods of data analysis, in addition to the simple description of statistical characteristics of neuronal responses that is one of the tasks of neurodynamics. We conclude that more specific mathematical methods must be applied, such as wavelets

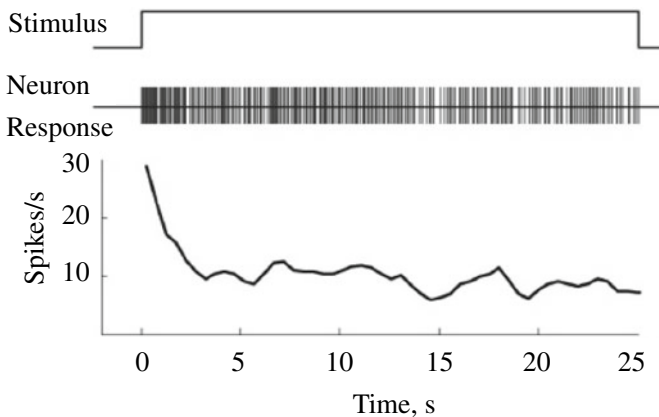


Fig. 1.1 Illustration of adaptation reaction of neuronal firing activity to a repeated stimulation. This neuron was recorded in a rat in the trigeminal sensory nuclear complex which receives tactile information from vibrissae. Stimulation was performed by periodic mechanical deflection of one whisker by a series of short directed air puffs (duration of each air pulse 5 ms). *From top to bottom:* start and end of stimulation by the sequence of periodic impulses, firing activity of a single neuron (train of spikes), and dynamics of the mean spike frequency (averaging over a sliding time window of 500 ms duration)

[29–31], the Hilbert–Huang transform [32–34], and the Wigner–Ville transform [35–37], which are more suitable for decoding information about non-stationary processes with time-varying features.

1.3 Wavelets in Basic Sciences and Neuroscience

Wavelet analysis [29, 38–41] is unique in the sense that even the first practical application to neurophysiological data analysis produced prominent results [30, 42–46]. For this reason, it is considered a very powerful analytical tool for studying the dynamics of neural systems.

Wavelet terminology was introduced in the 1980s [38, 47, 48]. This mathematical approach was initially proposed as an alternative to classical spectral analysis based on the Fourier transform. Wavelet theory is considered to be one of the most important events in mathematics of the past decades. Indeed, it appears to be the sole new mathematical concept that was immediately recognized as a tool in practically all branches of basic science (first and foremost, in physics and related disciplines) and many technical fields [31, 49–56]. In fact, introduction of the wavelet theory itself was not entirely unexpected. It was developed to meet the very real needs of experimental investigations, particularly in geophysics and seismology. Contemporary wavelet analysis combines various pre-existing ideas and methods. For example, fast wavelet transform algorithms are based on the subband coding ideology known from radio and electric engineering [57]. Some ideas were borrowed from physics (coherent states [58], etc.) and mathematics (studies on Caldéron–Zygmund integral operators [59]). Wavelet analysis is logically related to the theory of diffusion differential equations [60].

Today, wavelets are widely used for the analysis and synthesis of various signals, image processing and recognition, compression of large volumes of information, digital filtration, the study of fully developed turbulence, and the solution of certain differential equations. This list can certainly be extended [55, 60–68]. The new theory aroused great interest from the very beginning. According to well-known estimates [49], since the 1990s, the number of publications using wavelets in physics has been growing continuously. The number of references to Internet sources containing the term “wavelet” has reached several million. In fundamental science, this mathematical approach is mostly applied to study complex temporally non-stationary or spatially nonhomogeneous nonlinear processes. Wavelet analysis is well adapted for studying the complex structure of signals from living systems, since other traditional computation techniques can be applied only to processes with time (or space)-constant parameters (i.e., stationary in time or spatially homogeneous). Despite the fact that wavelet analysis has long been regarded as a standard tool for studying complex processes and practical application of this method in neuroscience and medicine is just beginning, prognoses for its successful application are rather optimistic. In this monograph we highlight recent advances made by practical application of wavelet in neurodynamics and neurophysiology.

1.4 Automatic Processing of Experimental Data in Neuroscience

An important field of wavelet applications in neurophysiology and neuroscience is the development of methods for automatic processing and analysis of brain signals. Electrical signals that can be recorded from the brain (EEG) represent a linear mixture of coexisting oscillatory components, i.e., nonlinear effects do not complicate the process of recognition. The development of expert systems for automatic EEG analysis is of particular interest for both fundamental neuroscience and clinical practice due to a wide spectrum of possible applications (classified in Fig. 1.2). One must distinguish between on-line and off-line analysis. Automatic (i.e., without the attention and control of an operator) analysis of pre-recorded EEG signals (*off-line* diagnostics) aims to reduce routine work, for example, to suppress artifacts in the recorded EEG. EEG analysis in real time (*on-line*) aims at fast detection of certain EEG events and the organization of closed-loop control systems. Clinically-oriented applications are the most effective field of on-line analysis of neurophysiological signals, including EEG monitoring with predictive diagnostic purposes, e.g., for the suppression of epileptic activity, the so-called spike-wave discharges [21].

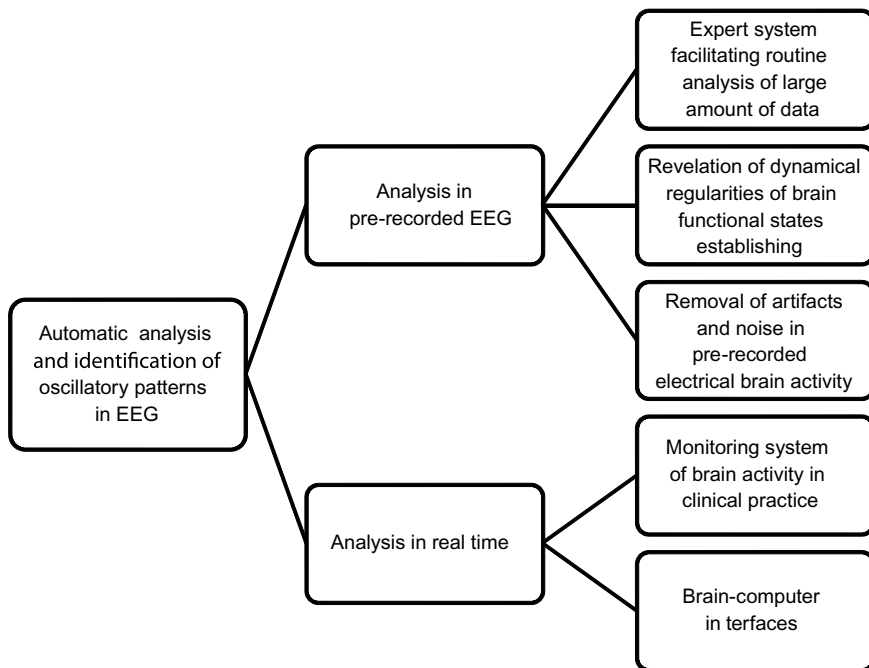


Fig. 1.2 Wavelet-based methods of automatic EEG diagnostics, processing, and analysis

1.5 Brain-Computer Interfaces

One of the most exciting applications of wavelets is to use it for *mental control* of brain functions, which, as a matter of fact, is a new form of human-computer interaction [69, 70]. The specific dynamics of electrical brain activity characterizes mental activity that includes compilation of imaginary commands (“mental action”). This “mental action” is associated with specific changes in the time–frequency characteristics and spatial structure of EEG [71–74]. In the brain-computer interface, mental control systems must perform the following steps (see Fig. 1.3):

- Recognize and select characteristic changes in the EEG (event-related *oscillatory patterns*).
- Decrypt their meaning (associated with a specific operation).
- Convert this meaning into commands for hardware control.

Mental control systems should be able to solve two main problems. First, the technical problem of precise recognition of an EEG pattern, subsequent formulation of a “command”, and transmission to control. Second, cognitive and psychological tasks in which the operator (a person) should learn to keep specific mental states that can be recognized from analysis of the spatial-temporal structure of his/her EEG. An additional problem is that the system should work in real time. Earlier control systems were suggested to use information about complex physical activity expressed as

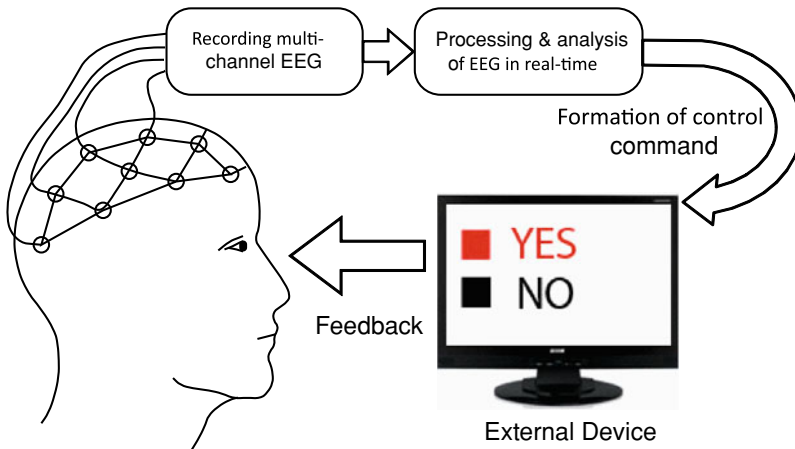


Fig. 1.3 General scheme of a simple brain-computer interface. Modern IBC is a system that registers and analyzes signals of electrical brain activity (usually EEG) from the user and “converts” them into a “machine” command for external device control. The central point of such a system is the development of algorithms for real-time recognition of EEG patterns corresponding to certain cognitive operations. Note the importance of the feedback loop in the BCI. This is necessary to adapt the aforementioned algorithms to recognize the specific patterns of electrical brain activity based on EEG features. Also the operator (user) must learn to evoke and control the relevant mental state, which is impossible without the use of feedback

body movements of the operator, e.g., the trajectory when moving a hand in the process of equipment handling. These interfaces encountered many problems, including registration of complex information, isolation of relevant information from the general data stream, and correct interpretation. Besides that, such interfaces require a system of sensors for registration of motor activity and a wireless device for data transmission from operator to computer. Therefore, simple brain-computer interfaces (BCI) are of particular interest, such as interfaces that are able to monitor electrical brain activity and detect the mental intentions of the operator. For example, simple stimulus-symbol interfaces conceived by the operator [75, 76] open up new prospects for resolving the problem of mental control.

Thus, algorithms of automatic EEG pattern recognition associated with specific cogitative operations in real time help to effectively perform the first step (pattern recognition) in brain-computer interfaces. Wavelet-based methods are perfectly suited to pattern recognition tasks [77–80].

Note that brain-computer interfaces have already been used as an alternative to traditional devices for inputting information into the computer. So for certain categories of users, for example, people with motor function disabilities, this way of interacting with the computer can improve their quality of life, at least partly, opening the way to a full-fledged life in society [81–84]. One of the first successfully worked BCIs was developed at Emory University by Roy Bakay and Phillip Kennedy, who used implanted depth electrodes in the brain motor center of a paralyzed 53-year-old patient, who was able to move the cursor on a computer screen, and thus communicate with doctors (writing several simple sentences) [85]. Rapid progress in neuroscience and technology suggests that brain-computer interfaces could be widely used for control of artificial limbs, manipulators, and robot technical devices (for example, wheelchairs), and also in the gaming industry [86–89].

1.6 Topics to Consider

A mathematically rigorous description of wavelet analysis can be found in numerous textbooks and monographs (see, for example, [29, 54, 56, 61, 90–94]) as well as in reviews in scientific journals [17, 52, 53, 95]. This book focuses on the new possibilities provided by the wavelet approach for decoding information from signals recorded on the level of individual neurons and groups of neurons, as well as neural network activity. A large number of the aforementioned scientific publications aimed to identify the most important problems in the field of wavelet applications to neurodynamics and neurophysiology. On this topic, we distinguish the following three areas of wavelet applications in neuroscience:

- **Microlevel** (cellular/intracellular)—wavelet analysis of the dynamics of individual cells or intracellular processes.
- **Mesolevel** (groups of cells)—analysis of information processes in small neuronal ensembles.

- **Macrolevel** (brain activity)—analysis of macrodynamics in widespread neural networks (EEG/MEG, neuroimaging data).

This monograph discusses the progress made on each of these levels in a consistent manner. The book contains eight chapters:

- Chapter 2 provides a mathematical introduction to wavelet analysis, including the basic concepts and definitions of wavelet theory, and considers practically significant questions related to the effective numerical implementation of the wavelet transform (both discrete and continuous). Special attention is paid to the importance of the relationship between wavelet and Fourier analysis. This chapter explicitly addresses those readers who are not familiar with the mathematical concepts of complex signal processing.

The next two chapters describe methods for wavelet investigation of neurophysiological systems.

- Chapter 3 discusses the application of wavelets to analyze cellular dynamics at the microscopic level (individual cells or intracellular processes). This chapter also presents the principles for analyzing the information from a single cell using electrical signals of individual neurons.
- Chapter 4 describes the main aspects of the wavelet analysis of the impulse shapes of individual neurons (action potentials) obtained by extracellular recordings of single-unit activity. We consider different approaches to classifying neuronal spikes, some based solely on wavelets and others involving combined methods, such as wavelet neural networks.
- Chapter 5 provides an introduction to the analysis of local field potentials (LFPs). LFPs are generated by synaptic currents excited by presynaptic neuronal assemblies in target cells. Thus, their study can shed light on the information processing on the circuit level. However, the multi-source nature of LFPs significantly complicates the analysis and requires advanced mathematical methods. We discuss how ongoing LFP activity can be disentangled into pathway-specific contributions. Then, we present wavelet-based methods that enable the identification and quantification of gamma waves (rhythmic patterns) generated by the CA3–CA1 pathways in the hippocampus while the compound LFPs are irregular.

The last four chapters of the book consider the macrodynamics of neuronal networks using wavelet analysis of electroencephalograms (EEGs).

- Chapter 6 considers the main definitions and principles of electroencephalography that are required for a better understanding of Chaps. 7, 8 and 9. We describe general physical and mathematical approaches to time–frequency analysis of rhythmic EEG activity using continuous wavelet transforms. We also review some recent achievements of wavelet-based studies of electrical brain activity, including (i) time–frequency analysis of EEG structure, (ii) automatic detection of oscillatory patterns in pre-recorded EEG, (iii) classification of oscillatory patterns, (iv) real-time detection of oscillatory patterns in EEG, (v) detection of synchronous states

of electrical brain activity, (vi) artifact suppression/rejection in multichannel EEG, and (vii) the study of cognitive processes.

- Chapter 7 considers basic problems of automatic diagnostics and processing of epileptic EEG. We discuss the wavelet-based techniques in order to fully automatize routine operations, such as visual inspection of EEG. In addition to that, we exemplify some practical applications of wavelet methods for automatic analysis of pre-recorded signals of neuronal activity (off-line diagnostics), and also some examples of wavelet-based EEG analysis in real-time (on-line). We also discuss principles of fast and precise detection of transient events in EEG and organization of close-loop control systems that can be used in brain-computer interface (BCI).
- Chapter 8 considers using wavelet analysis to study mechanisms of visual perception. First, we introduce an ambiguous visual stimulus, the Necker cube, a useful visual perception analysis tool. Second, we demonstrate how the wavelet-based methods reveal the local and network properties of the percept-related brain activity. Then, we considered the effect of the human condition (motivation and alertness) on the perceptive process. Finally, we review the basic principles of the BCIs that use the wavelet-based algorithm to evaluate the human state in visual perception tasks.
- Chapter 9 describes wavelet analysis of the motor-related cortical activity. First, this chapter introduced real and mental motor activity in the young and the middle-aged healthy subjects. The real motor acts, or motor execution (ME), enables interaction with the environment and induces the motor-related changes in 8–12 Hz and 15–30 Hz wavelet power in the motor cortex. The mental motor acts, or motor imagery (MI), did not include muscle control but may have a motor-planning stage, similar to ME. Detecting the ME and MI brain states underlies the BCI for motor control. Second, we described two types of motor imagery: kinesthetic and visual. Visual imagery corresponds to the self-visualization of the subject moving a limb that does not require special training. Kinesthetic imagery is the feeling of muscle movement that can only be realized by athletes or specially trained persons. Finally, we considered how the ME brain states change with age representing criteria for an objective assessment of the motor abilities in elderly adults.

This book is based primarily on the fundamental results in neurodynamics obtained recently by the authors—physicists, mathematicians, and biologists in close collaboration with specialists in experimental neurophysiology. At the same time, the book contains a relatively complete bibliography (over 400 sources) characterizing the application of wavelets in neurophysiological research. In general, this book overviews theoretical and practical knowledge and, in our opinion, demonstrates the advantages of powerful analytical tools and novel mathematical methods of signal processing and nonlinear dynamics in order to address neurophysiological problems. Moreover, wavelet analysis helps to reveal important information and facilitates a deeper understanding of the investigated phenomena. More intensive studies in

this area can contribute to interdisciplinary interactions between physics, nonlinear dynamics, applied mathematics, and neurophysiology and promote further mutual research in these areas.

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