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Kirsti Andersen

The Geometry of an Art

The History of the Mathematical Theory of Perspective from Alberti to Monge

Springer

Kirsti Andersen
Department of History of Science
The Steno Institute
University of Aarhus
Denmark

Sources and Series Editor:
Jesper Lützen
Institute for Mathematical Sciences
University of Copenhagen
DK-2100 Copenhagen
Denmark

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To Christian and Michael

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Introduction

Key Issues

Ever since the late 1970s when Pia Holdt, a student of mine at the time, and Jed Buchwald, a colleague normally working in another field, made me aware of how fascinating the history of perspective constructions is, I have wanted to know more. My studies have resulted in the present book, in which I am mainly concerned with describing how the understanding of the geometry behind perspective developed and how, and to what extent, new insights within the mathematical theory of perspective influenced the way the discipline was presented in textbooks. In order to throw light on these aspects of the history of perspective, I have chosen to focus upon a number of key questions that I have divided into two groups.

Questions Concerning the History of Geometrical Perspective

- How did geometrical constructions of perspective images emerge?
- How were they understood mathematically?
- How did the geometrical constructions give rise to a mathematical theory of perspective?
- How did this theory evolve?

In connection with the last question it is natural to take up the following themes.

- Was there any interplay between the developments of the mathematical theory of perspective and other branches of geometry?
- What was the status of the theory of perspective?

Questions Concerning Textbooks on Perspective

- What inspired the author of a particular work?
- How was the communication between the mathematicians and the practitioners of perspective?

In fact, I touch upon the latter issue so often that part of my book could be seen as a case study of the difficulties in bridging the gap between those who have a mathematical knowledge and the mathematically untrained practitioners who wish to use this knowledge.

In addition, for reasons I will come back to, I have found it important to ask:

- Were there regional differences in the treatment of geometrical perspective?

The Word 'Perspective'

The field now known as 'perspective' got its name from the optical sciences, *perspectiva*[1] being the Latin word Boethius chose as a translation of the Greek *optikē* (Carter[S] 1970, 840).[2] During the Middle Ages, perspective came to signify a science that has been characterized by A. Mark Smith as "in most respects the bastard offspring of the three basic Greek traditions in optics – the geometrical, the physical and the anatomical" (Witelo[S] *Pers*, 18). In the fifteenth century, *perspectiva* was associated with yet another discipline that was also called *scenographia* and deals with the art of representing spatial panoramas or objects graphically on two-dimensional surfaces.[3] For a time the expressions *perspectiva naturalis* (or *communis*) and *perspectiva artificialis* (or *pingendi*) were used to distinguish between optics and the geometrical discipline of representation.

From the Renaissance onwards, the branch of perspective dealing with representation on a two-dimensional surface was divided into various subdisciplines. The one dealing with the problem of depicting straight lines and lengths was called *linear perspective*. In general I use the word 'perspective' to mean linear perspective, or, more precisely, the art of using geometry for constructing images obtained by a central projection.

---

[1]'Perspective' comes from *perspicere*, which means to look through, into, or at, as well as to perceive clearly.

[2]To enable readers to survey my primary sources, I have placed them in a separate bibliography, which is included as the first. Other literature is listed in the second bibliography, and references to such works are indicated by an [S].

[3]The word *scenographia*, or scenography, was most likely taken from Vitruvius, who used it in his *De architectura* in a meaning that is not clear (Vitruvius[S] *Arch*/1955, book I, chapter 2, §2, and book VII, preface, §11; cf. appendix one in the present book, pages 728–729). Some authors used 'scenography' synonymously with perspective because Vitruvius was understood to have hinted at some kind of perspectival representation. A number of other writers, on the other hand, reserved the term scenography for the art of constructing theatre scenes in perspective.

Other Publications

The first comprehensive survey of the history of geometrical perspective is found in Noël Germinal Poudra's *Histoire de la perspective ancienne et moderne* (History of ancient and modern perspective, Poudra[S] 1864). In this work, written almost one and a half centuries ago, Poudra outlined the mathematical contents of some of the literature on the optical theory of appearance, perspectival representations, and descriptive geometry from antiquity to the publication in 1849 of his own book on descriptive geometry (*ibid.*, 576). Presumably for linguistic reason, Poudra did not pay sufficient attention to literature written in German, but otherwise he provided a helpful survey. Poudra's methodology is, however, outdated.

Many excellent books and papers on the history of perspective have appeared since Poudra's publication, not least over the last five decades, but no one after Poudra has composed a comprehensive work focussing on the entire development of the mathematical theory of perspective. With the present book I attempt to fill this gap for the period up to 1800.

The Period and Regions Examined

To make my project feasible, I stop my account of the development of the mathematical theory of perspective around 1800. There is no reason to choose 1800 precisely, but terminating somewhere around that year is fairly natural, for three reasons: By 1759 the mathematical theory of perspective had been fully worked out, following Johann Heinrich Lambert's creation of perspective geometry. In order to describe the impact of this event I find it relevant to include literature from a few decades after 1759. In addition, the development in Britain took an interesting turn in the second half of the eighteenth century. Finally, choosing a year around 1800 allows me to report on how Gaspard Monge incorporated perspective into his descriptive geometry.

Continuing into the nineteenth century would – among other things – involve a study of how the mathematical theory of perspective was absorbed by projective geometry. This is a very interesting topic, but essentially different from the one I have chosen to focus upon, namely the development of the mathematical theory of perspective as an independent discipline.

To be able to characterize the development in a certain region, I have also introduced geographical limitations and only examine areas in which a considerable number of publications on perspective appeared. These were the Italian, German, French, Dutch, and English-speaking parts of Europe.

The Sources and How They Are Used

My account is based on more than two hundred books, booklets, and pamphlets on perspective written, and in most cases also published, before 1800 (and listed in my first bibliography). I am sure I have not found all titles that might be relevant[4] – nor have I aspired to. Still, I am confident that my material is so comprehensive that adding further publications would not change my conclusions in any significant way.

My intention has by no means been to produce an annotated bibliography. Nevertheless, I have written about each of the primary sources I have seen.[5] Some of the small comments on rather insignificant publications may seem distracting in relation to the overall objective, yet I have included them because I find them helpful in providing a general picture of how perspective was transmitted in the literature.

It goes without saying that I have only read the most important sources in the total material from cover to cover. In studying the other publications I have concentrated on a few select subjects relevant to the cardinal issues: How does the work of the author relate to other literature on perspective? Which perspective constructions did he chose? How did he describe them and, in particular, to what extent was his presentation based on a geometrical background?

In addition, I have a few pet topics, the most conspicuous being the question of how to represent a row of columns in a picture, which I call the *column problem* (and describe in section II.15). This theme was actually much debated in the considered period, and I find it highly relevant, for it provides an example of some artists revolting against the solution given by the mathematicians, while other artists defended the mathematical approach.

Contexts and Restrictions

The history of perspective can be seen in many different contexts. Covering such a long period and such a vast area I cannot, in any realistic way, include social aspects such as patronage, education, vogues in art, or interests in and capacity to buy art. Instead I have mainly chosen to address aspects of the mathematical theory of perspective that are documented by the pre-1800 writings on perspective, and in particular to investigate how the protagonists gained a geometrical understanding of perspective. In addition, I have found it important to look at each text on perspective within the framework of its possible sources of inspiration. As these were, in most cases, local, I have chosen not to organize my book chronologically, but by geographical regions – incor-

[4]In searching for literature I have been greatly helped by Jones[S] 1947, Schüling[S] 1973, and Vagnetti[S] 1979.

[5]While undertaking my final revision, I became aware of a number of rare books that had not previously been included in my bibliography. At this point I lacked both the time and the funding to travel to the libraries where these volumes might have been accessed.

porating a split around 1600 because of a small revolution in the history of the mathematical theory of perspective that took place that year.

Among the important questions I have had to leave unexplored is what role the academies of art played in promoting perspective. A few of the books I present were written in connection with academy courses, and they give the impression that perspective was considered important for the artists, but was taught at a rather elementary level. There is, however, much more work to be done in this field (Siebel[S] 1999 offers an interesting case study for the German situation around 1800).

Similarly, my primary sources do not give an adequate background for discussing thoroughly the highly pertinent question of the actual use of perspective in paintings, architectural illustrations, and other drawings. I do touch upon the subject a few times, but for the most part I refer to the literature on art history in which the topic has been eminently discussed – and to such a degree, in fact, that I refrain from a general listing of the literature dealing with the question of the application of perspective.

Conclusions

Although I am not writing a suspense novel, I do not wish to reveal any of my conclusions at this premature stage. My answers to the key issues I have addressed will turn up in various chapters, and in the final chapter, XIV, the reader will find some overall conclusions. To those who would like to follow the main line of mathematical development, I can disclose that for this purpose the most relevant parts of the book are those concerning Italy up to and including 1600, France in the seventeenth century and the Netherlands in the seventeenth and early eighteenth century, Britain in the eighteenth century, and the contributions made by Lambert. Finally, let me divulge that in addition to Lambert, the main protagonists in this story are Guidobaldo del Monte, Simon Stevin, Willem 'sGravesande, and Brook Taylor.

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One of the things that kept up the momentum in my work was receiving invitations to talk about perspective on numerous occasions, for which I am very thankful. There is no point in listing all the meetings here, but I would like to draw attention to a series of interesting and animated meetings arranged by a French group of ardent historians of perspective, including *Didier Bessot*, *Rudolf Bkouche*, *Christian Guipaud*, *Roger Laurent*, and *Jean Pierre Le Goff*, and additionally counting *Rocco Sinisgalli* from Italy. One member of the group, *Roger Laurent*, generously provided me with copies of texts that was almost impossible to access otherwise.

A number of colleagues have helped me with specific jobs or drawn my attention to particular publications. These kind people I have thanked in footnotes at the specific places where I have benefited from their services. My students have also contributed to my work in various ways. First, as I already noted in the introduction, *Pia Holdt* opened my eyes to the history of perspective. Later, during courses I taught on the history of the mathematical theory of perspective, many students have posed interesting questions and pointed to fascinating problems. In this connection I would particularly like to mention my thought-provoking talks with *Lise Husted Kjelstrøm* and *Rikke Schmidt Kjærgaard*. Additionally, I have learned a good deal from students who contacted me to discuss their theses. In particular I thank *Paola Marchi* for inspiring dialogues, and *Sabine Siebel* for an interesting correspondence. They both produced recommendable theses (Marchi[S] 1998; Siebel[S] 1999) – quite independently of me.

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I am deeply thankful to the institutions that have housed and helped me while I have been carrying out my research, first of all my home institute, the *History of Science Department*, *the Steno Institute, University of Aarhus*, and additionally the following: *Accademia di Danimarca*, Rome; *The Dibner Institute for the History of Science and Technology*, Cambridge, Massachusetts; *Fondation Danoise*, Paris; *Herzog August Bibliothek Wolfenbüttel*; *Institut für Geschichte der Naturwissenschaften*, *Universität München*; *Department of Mathematics*, *Utrecht University*.

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Notes to the Reader

In this section I have collected helpful remarks on the various practical solutions I have chosen for presenting diagrams, concepts, mathematical arguments, references, and protagonists.

Drawings and Notation

To facilitate the reading of the geometrical figures illustrating perspective projections, I have introduced a practice that applies to all the diagrams I have created and to most of my adaptations of diagrams drawn by the perspectivists.

Concepts Related to the Eye Point and the Picture Plane

The reader will often meet the following situation (figure 1). Given are an *eye point* O (from *oculus*), a horizontal plane of reference γ – called the *ground plane* (in former times the *geometrical plane*) – and a picture plane π. Usually the latter is tacitly assumed to be vertical, but we will also meet situations in which π forms an oblique angle with γ. The line of intersection of π and γ is called the *ground line* and denoted GR. The orthogonal projection of O upon γ is called its *foot*, and the letter F is used to denote this point. Moreover, the orthogonal projection of O upon π is called the *principal vanishing point* and denoted P, and finally the orthogonal projection of P upon GR, or equivalently the orthogonal projection of F upon GR, is denoted by the letter Q. There is no common name for this point, but I call it the *ground point*.

The line through P parallel to the ground line is called the *horizon,* and denoted by HZ. On this line, two points D are marked. These are the so-called *distance points*, defined by $PD = OP$. They have been given this name because the distance OP between the eye point O and the picture plane π is frequently called, simply, the *distance* – which I write in italics throughout the book. Sometimes it is handy to be able to distinguish between the two distance points, and hence I have introduced the terms right and left distance point, the latter of which, when seen from the eye point, is the point lying to the left of P.

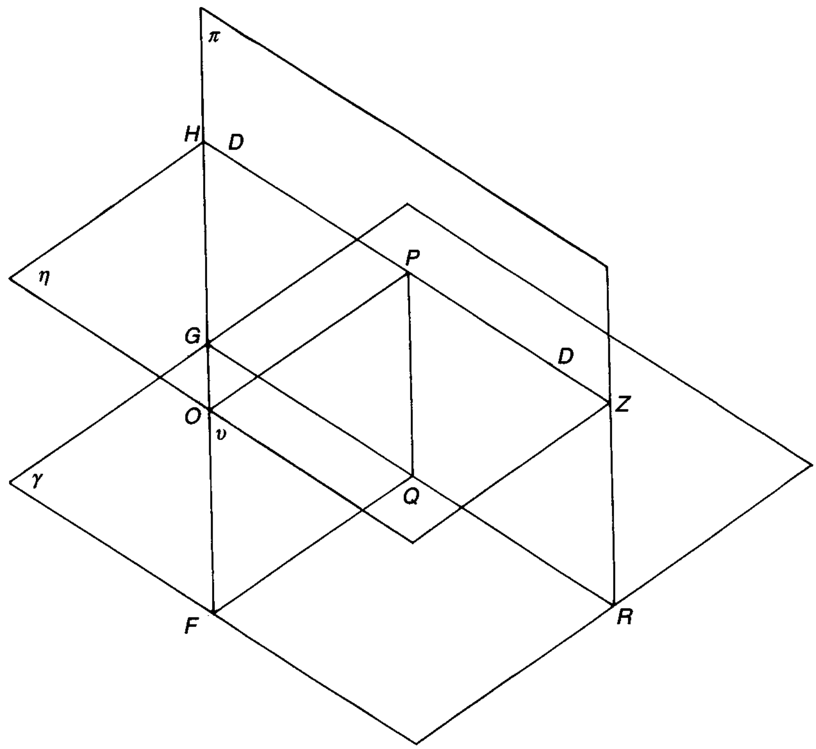

FIGURE 1. Concepts related to the eye point and the picture plane.

Following Nicolas Louis de Lacaille, I call the horizontal plane η which passes through the eye point O, the *horizontal plane*, and name the plane υ defined by the points O, P, and Q the *vertical plane*.

Concepts Related to Images of Points, Lines, and Planes

It is practical to have a term for points and lines (figure 2) situated behind the picture plane π, such as A and l, whose perspective images are constructed. Inspired by Brook Taylor, I introduce the name *original* objects for such elements – occasionally stressing the fact that object is original by adding a subscript o, as in A_o.

My notation for the perspective image of a point A is A_i, and for the perspective image of a line l I use the symbol l_i. When l cuts π, the point of intersection is called its *intersection point* and is denoted I_l. To l is also assigned a *vanishing point*, V_l, defined as the point of intersection of π and the line through O parallel to the line l. As we shall see later, it is a fundamental result in the theory of perspective that the image l_i passes through V_l. For this

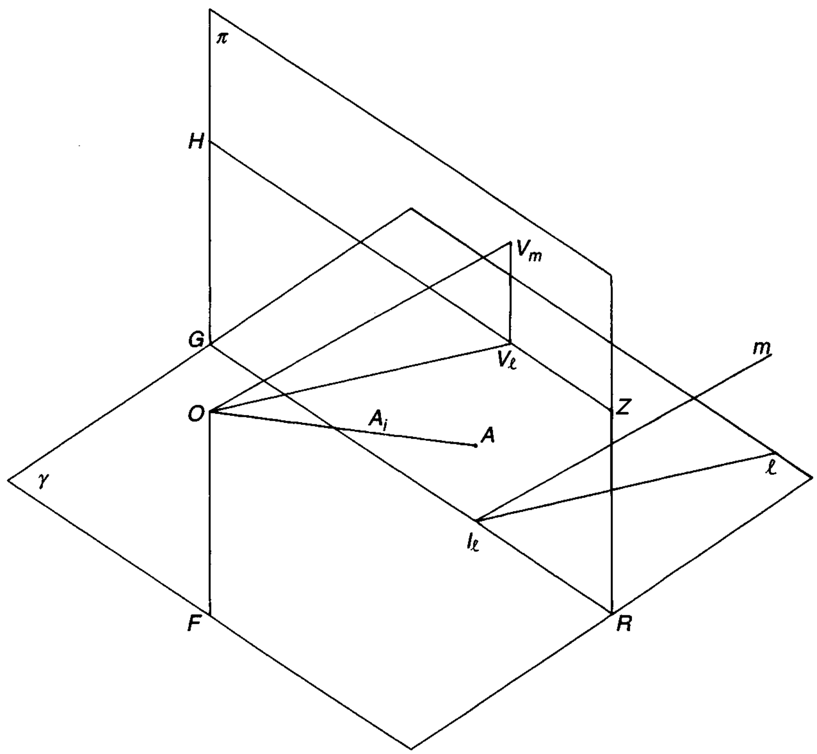

FIGURE 2. Concepts related to the images of a point and a line.

reason many perspectivists, and I with them, also call the point V_l the vanishing point of l_i.

Let α be a plane that cuts π (figure 3). Just as two points were assigned to an original line, two lines are associated to α, its *intersection* i_α, and its *vanishing line* v_α. The first is the intersection of π and α, and the second is the intersection of π and the plane through O parallel to α.

Orthogonals, Transversals, and Verticals

In the interest of brevity, I use the word *orthogonals* to refer to lines orthogonal to the picture plane π, *transversals* for lines parallel to the ground line, and *verticals* for lines perpendicular to the ground plane γ. Many readers will presumably keep the book, and hence the figures, in a horizontal position and therefore not always experience the vertical lines as vertical. However, the term 'vertical' is such a handy concept for describing directions in the three-dimensional space that I use it quite often, asking the readers to rotate the drawings either in their minds or in reality.

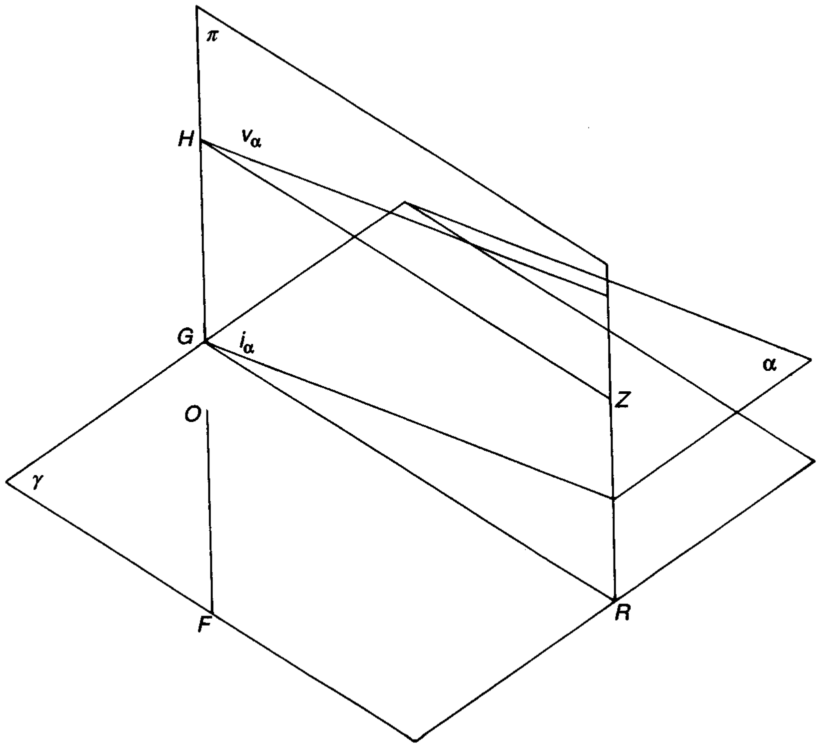

FIGURE 3. Concepts related to the image of a plane.

Rabatment

Since drawing paper is only two-dimensional and a large part of the problems in perspective deal with three-dimensional configurations, different authors have used a variety of procedures for depicting two or more planes on the plane of the paper. For such procedures I use the French term *rabatment*. I myself generally choose parallel projections to illustrate three-dimensional situations.

Mathematical Terminology, Results, and Techniques

Lines and Line Segments

The mathematical perspectivists I am dealing with kept to a tradition dating back to Euclid: they seldom distinguished between a line and a line segment. When they wrote the line *AB*, for instance, they most often meant the line segment *AB* – defined by the endpoints *A* and *B*. When I am sure of what they had in mind, I write either 'line segment' or 'line', sometimes using the symbol l for the latter. It often happens that my authors have called a line AB and then only later defined the precise position of the point A or B on the line; in a few cases, I do the same.

As to ratios, I denote the ratio between two line segments AB and CD as $AB : CD$. Modern readers may expect a special sign, as in $|AB|:|CD|$, that shows that the comparison concerns the lengths of the line segments. However, the protagonists followed the classical Greek custom of making 'geometrical calculations' in which they did not assign a number to a line segment, but considered it as a magnitude that could form a ratio with another one-dimensional geometrical magnitude.

Results from the Theory of Proportion

Students trained in classical mathematics learned how to manipulate with ratios and proportions. As this is no longer common knowledge, I have listed below the results referred to in this book. They occur either explicitly as propositions in book five of Euclid's *Elements* or can be obtained by combining Euclid's theorems.

When $r : s = t : u$, then

$$(r+s) : s = (t+u) : u \tag{1}$$

$$r : (r+s) = t : (t+u) \tag{2}$$

$$(r+s) : r = (t+u) : t, \tag{3}$$

and for $r > s$

$$(r-s) : s = (t-u) : u \tag{4}$$

$$(r-s) : r = (t-u) : t, \tag{5}$$

whereas for $s > r$

$$(s-r) : s = (u-t) : u. \tag{6}$$

Again, for $r > s$,

$$s : (r-s) = m : n \text{ implies that } (r+s) : (r-s) = (2m+n) : n. \tag{7}$$

Two proportions can be 'multiplied' as follows:

$$r : s = t : u \text{ and } s : a = u : b \text{ imply that } r : a = t : b. \tag{8}$$

Finally, when r, s, t, and u are of the same kind in the sense that they can all form ratios with each other, then

$$r : s = t : u \text{ implies that } r : t = s : u. \tag{9}$$

Mathematical Techniques

Many of the perspectivists presented in this book complained about how their predecessors treated mathematical techniques. A number of them found that it was presented much too abstractly, and others felt that it was not presented concisely enough. In fact, no solution can satisfy everyone.