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Beyond Black Holes and Time-Travels

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To all who were not afraid to stay out
of the mainstream: Ludwig Boltzmann,
Cheikh Anta Diop, Antonio Gramsci,
Lev Davidovich Landau, Rosa Luxemburg,
Lewis F. Richardson, and the others.

Acknowledgments

The four authors of this book have worked together for decades, as part of a larger scientific enterprise of theoretical physicists—perhaps more a brotherhood than a research team—that bears the name of TNT group, based at Sapienza University in Rome. They owe a debt of gratitude to all friends, colleagues, and collaborators of TNT group for developing the vision which underlies the writing of this book. In particular, they wish to thank Fabio Cecconi and Massimo Falcioni, who participated in the early stage of this book and with whom they had long discussions on how to popularize scientific ideas which are often disregarded by the general public. The authors also acknowledge the invaluable help of friends such as Marco Baldovin, Francesco Borra, Claudio Castellano, Andrea Plati, and Marco Zannetti for reading parts of the book and suggesting improvements. They are also grateful to Gianni Battimelli, Luca Biferale, Roberto Natalini, Michele Buzzicotti, the Archivio Amaldi, and the Volterra Family Archive for providing us with some of the figures.

Contents

Chapter 1
A Sort of Introduction

> If we study the history of science we see produced two phenomena which are, so to speak, each the inverse of the other. Sometimes it is simplicity which is hidden under what is apparently complex; sometimes, on the contrary, it is simplicity which is apparent, and which conceals extremely complex realities. (Henri Poincaré)

> To develop the skill of correct thinking is in the first place to learn what you have to disregard. In order to go on, you have to know what to leave out: this is the essence of effective thinking. (Kurt Gödel)

The first duty of the authors of a book is to explain their reasons, which means answering the question *why this book?* In order to convince the reader that there are some good motivations, we start from an old story: Friedrich Wilhelm IV (king of Prussia in the period 1840–1861), owing to his enthusiasm about science, was often asking his royal astronomer *"Herr Argelander, anything new happening in the sky?"* Once the astronomer, having no new interesting result to offer to the king, bravely replied *"Does Your Majesty already know the old things?"* We have the feeling that after two centuries the situation did not change too much. Usually, people far from the academic world do not know (or, reading the scientific breaking news, only have a vague often misleading idea about) the latest scientific discoveries: well, in our opinion, this is not a serious problem. Unfortunately, they ignore also some basic results of older science as well as the ideas underlying applications they use in their everyday life. Moreover, often non-scientists have a foggy or stereotypical idea of what science and research are. A possible reason is the common belief of many authors of popular scientific books that all readers, as the King of Prussia, are avid only of the last discoveries, or of the most spectacular and exotic[1] ones. Therefore, as a net result, we have a plethora of popular books covering topics like black holes, time-travels, gravitational waves, big bang, Higgs boson, and multi-universes.

[1] Unfortunately, often the focus on exotic and far from everyday life science contributes to the common belief that most scientists conduct strange researches very far from practical (and economical) needs.

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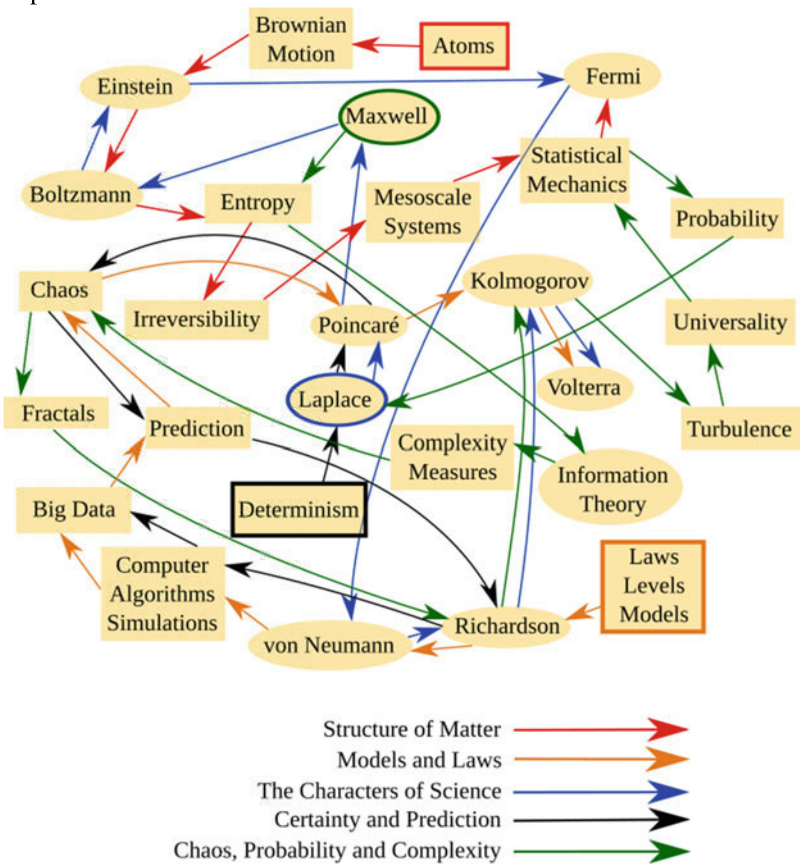
This book does not follow the mainstream: its purpose is to give an informal and easy account of some ideas of modern physics and mathematics that, in spite of their major conceptual and practical relevance, are not typically discussed in popular texts. Moreover, we present also a brief account of the life and personality of the scientists who contributed to the development of those ideas. The potential readers we had in mind are high school teachers, college and high school students, and—more generally—persons with just a basic knowledge of mathematics (high school), but with interest and curiosity in science. Our ambition is to provide the readers with some friendly tools to better understand the world around us as well as to get some history of the ideas that changed it. In our path we tried to avoid too technical details (only in a few cases we introduce some simple formulas or more technical footnotes), without the abdication to correctness of the conceptual aspects. We will mainly focus on a set of topics covering the following subjects (and their interplay): statistical mechanics, soft matter, probability, chaos, complexity, and the role of models and numerical simulations. The various topics have been selected for their conceptual and practical relevance and, of course, within our areas of interest and expertise.



The book consists of 28 entries, pictorially depicted in the above graph. Some of the entries are devoted to *specific arguments* (the rectangles), others to several *important scientists* (the ellipses), including those who, in spite of the importance of their contributions to science and the history of thought, are almost unknown to the general public. Unlike artists and writers, with very few exceptions (A. Einstein, S. Hawking and very few others), the life of scientists is not well known among the non-experts. We do not understand why: many scientists had an intense and interesting life, sometimes with vivid (occasionally tough) contrasts with their competitors. As a paradigmatic example we can mention Ludwig Boltzmann and his battle for the atomistic hypothesis against Ernst Mach and the energetic school. A short presentation of the main aspects of life of some important protagonists of science can also be useful to understand and appreciate their contributions and the way science proceeds in practice.

Deliberately the entries appear in alphabetic order. With this choice there is no attempt to do something similar to an encyclopedia. On the contrary, our intention is to avoid an exceedingly systematic and potentially boring approach: we invite the reader to perform a *random walk* among the entries, jumping from one entry to another at will. Even if it is not required to follow a specific order in the reading of the book parts, in the figure above we suggest some possible reading paths, linking a few entries according to a common theme or *fil rouge*. As clear from the graph, the itineraries are not disconnected and some entries are common in different paths, in addition their precise order in the sequences is not particularly relevant. We invite the readers to create their own road map of the book. Below we offer a short guide to the above-suggested routes.

Structure of Matter

Atoms → Brownian Motion → Einstein → Boltzmann → Entropy →
Irreversibility → Mesoscale Systems → Statistical Mechanics → Fermi.

Within our daily life, we experience matter has a continuum: apparently it seems that we can divide an iron bar of 200 g, in two bars of 100 g, and then in four bars of 50 g and so on. Today we all know that such an experience is illusory, as matter has a granular, discrete structure because of the existence of atoms. The atomic hypothesis started with a philosophical intuition of Leucippus and Democritus (fifth century BC). However, we have been able to play with atoms and sub-atomic particles only since the first decades of twentieth century AD. The path from Leucippus and Democritus to modern particle accelerators has been tortuous and not straightforward both for some general philosophic (as well as religious) reasons and for the difficulties inherent to the visualization of atoms. At the end of the nineteenth century even eminent scientists, e.g., Planck and Mach, did not believe in the physical existence of atoms. Only after the works of Einstein and Perrin on the Brownian Motion in the first decades of 1900—explaining the erratic behavior of colloidal particles, whose dimension are small (compared with macroscopic objects) but much larger than the molecular sizes—the physical existence of atoms had been accepted by the whole scientific community. The Brownian Motion had an important role not only in the history of the modern physics, as it constituted the starting point of an interesting chapter of probability theory (stochastic processes) with applications in a wide range of fields from biology to finance. In addition, colloidal particles had been the prototype of the so-called mesoscopic systems.

Models and Laws
Laws, Levels, and Models → Richardson → von Neumann → Computer, Algorithms, Simulations → Big Data → Prediction → Chaos → Poincaré→ Komogorov → Volterra.

In science one can find terms as laws, theories and models; for instance the law of gravitation, quantum theory, and Lotka-Volterra model. Someone could think that models are less noble or less important than theories, but they are actually unavoidable in scientific practice. We can say that even wonderful theories as Newton's mechanics, Maxwell's electromagnetism, and Quantum Mechanics are actually very accurate and general models. They are able to describe reality only at a certain scale and resolution. Many interesting systems have a multiscale structure, involving very different characteristic temporal and spatial scales, important examples are climate dynamics and proteins. Due to the difficulties in dealing with the variables involved in multiscale systems, the unique way to handle such phenomena is to find a description at a certain level of resolution. On the other hand, it is usually rather difficult to find a methodology to build effective equations and there are no systematic (general purpose) recipes to achieve such a goal. Therefore, it is always necessary to use the previous understanding of the considered system and, often, analogy and intuition. A perfect example of this procedure is given by the history of the weather forecasting, starting from the first empirical approaches, based on weather maps, then to the ingenious ideas of Lewis Fry Richardson to finish with the Meteorological Project at the Institute for Advanced Study (Princeton) led by John von Neumann, Charney, and colleagues in the 1940s–1950s, which gave birth to the numerical weather forecasting today in use.

The characters of Science
Laplace → Poincaré → Maxwell → Boltzmann → Einstein → Fermi → von Neumann → Richardson → Kolmogorov → Volterra.

Science has many faces and the researches on the specific topics can have different origins, from pure curiosity to understand very general aspects of the Nature, to very practical applications. In a similar way, the scientists can have rather different interests, attitudes, and approaches. Remarkably, some topics originally motivated by very abstract problems, after some decades (or even centuries) had been relevant even for very practical applications. Here a short list. Boltzmann's equation, originally introduced to understand thermodynamic irreversibility, is now used for projecting electronic devices and space capsules. The methods introduced by Poincaré for the study of chaotic systems, today are used for the planning of missions in space. Quantum mechanics, which was born to explain black body radiation and certain behaviors of matter at atomic scale, today has a major role in many aspects of our life. Fractals had been introduced for rather sophisticated mathematical problems, now are used even by the movie industry. Looking at the history of science it is not difficult to find several eminent scientists who had an important role even in the political life of their time. Everybody knows the story (legend?) of Archimedes in the war between Syracuse and the Roman army. In more recent times several scientists had been deeply involved in military projects, as John von Neumann, while others, as Vito Volterra and Lewis Fry Richardson, used their prestige and intelligence to contrast the power and the politicians of their times. Pierre-Simon Laplace, for a short period was minister under Napoleon, and Henri Poincaré was involved in the famous Dreyfus affaire. John von Neumann had an important role in the Manhattan project for the first nuclear bomb as well as in other important military activities, while Richardson was a coherent pacifist.

Certainty and Prediction

Determinism → Laplace → Poincaré → Chaos → Prediction → Richardson → Computer, Algorithms and Simulations → Big Data.

Usually science is considered the spring of certainty. The prototypical example begin astronomy, in fact we use to say "astronomical precision" as a synonym of certainty. In the twentieth century the discovery of the planet Neptune, using the laws of motion and gravitation theory, had been seen as the triumph of the ideas of Laplace on determinism and the power of Newton's mechanics. After the discovery of chaos by Poincaré (ironically occurred during the investigation of an astronomical problem), we know that also "astronomical precision" cannot be considered as exact. Even in a deterministic system, the evolution can be chaotic, meaning that infinitesimally small perturbations in the initial state, for instance, a slight change in one body's initial position might lead to dramatic differences in the later states of the system. Such a result at a first glance can sound rather negative, somehow a limit of science, on the other hand, the understanding of the specific aspects of a given chaotic system allows us to focus only on the problems which can have a serious answer. We can say that the real power of science is in understanding the limits of the theories and models, as explained very clearly by Confucius: *"To know that you know when you do know, and know that you do not know when you do not know: that is knowledge"*.

Chaos, Probability and Complexity

Maxwell → Entropy → Information Theory → Complexity measures → Chaos → Fractals → Richardson → Kolmogorov → Turbulence → Universality → Statistical Mechanics → Probability → Laplace

Terms as certainty and probability seem to belong to different realms; in a similar way determinism and complexity appear to be in opposite positions. Surely in all the situations where a large number of causes are involved (e.g., in a gas) we expect a certain degrees of "complexity", and the use of probabilities appears natural. On the other hand, with the discovery of chaos, we understood that even in deterministic chaotic systems, with only a few number of variables, one can have high levels of complexity in which statistical approaches are mandatory. On the other hand, even in the realm of probabilities, one can find something rather similar to deterministic behaviors: for instance, the true essence of the limit theorems (such as the law of the large numbers) is the fact that in presence of many independent variables (causes) one can have a result which is practically sure—in the appropriate limits with probability arbitrarily close to one, in the jargon of probabilists. Such ideas date back to Jakob Bernoulli, at the end of the eighteenth century, with his motto *“Something is morally certain if its probability is so close to certainty that shortfall is imperceptible”*. This result gives a sensible connection between probability and empirical world, stressing the non-abstract character of probability.

Besides the 28 entries in alphabetic order, the reader can find three extra entries. At the beginning a presentation of the Random Walk, a topic which has a relevant role in many fields of modern science and appears in the title of the book. At the end a *divertissement* where a few popular fiction works (books and movies) are put in contact with the 28 entries, suggesting that many concepts and scientists discussed here—even if not frequently present in popular science books—have emerged from the ivory towers of science and already reached a wider audience. Finally, at the very end, we provide the readers with some further readings (technical and non-technical articles and books) organized along the suggested paths, which can be used to deepen some topics of interest.

Chapter 2
Random Walk

In this book we have collected a group of scientists and a set of scientific topics most of which are outside the mainstream of popular science. We have chosen those arguments since they are fundamental elements of knowledge for a large part of modern science even though they are not very much popularized. In this context, the random walk (RW), that gives the title to the book, is a classical example of a scientific topic with very important applications in many different fields from biology to chemistry, from economics to sociology, and we deem it appropriate to start this journey briefly presenting this subject.

The RW, sometimes called drunkard's walk, is a simple random process in which the subject of the action, the walker or the drunk, at each time interval chooses a direction at random, for example, by flipping a coin, and take a step in that direction. Step by step the walker generates a random path. The simplest instance of RW considers a walker that can only move on a line by making steps of fixed amplitude randomly choosing to go to the right or to the left. In this case, assuming that the walker starts from an initial position, by using elementary combinatorics (essentially the binomial coefficients) it is possible to calculate the probability of finding the walker at a certain time in a certain position relative to where it started (Fig. 2.1).

This simple exercise originates a number of interesting consequences. For instance, at large values of times, starting with many walkers in the same initial position, one could wonder how many walkers have reached a given distance from the initial position. Intuitively, one can expect that many walkers remain near the initial position, but the exact answer to this problem is given by the Gaussian probability distribution that is able to tell in which positions walkers are most likely to be found (see the entry Probability). Figure 2.1 illustrates these features. Moreover, considering very small time and jump intervals (in a given and fixed proportion) one obtains the diffusion equation, namely, one has that the average square distance covered by the walker is proportional to the time lapsed (see the entry Brownian Motion). Application of RW can be found in many other fields: in the percentage increases of stock prices, in

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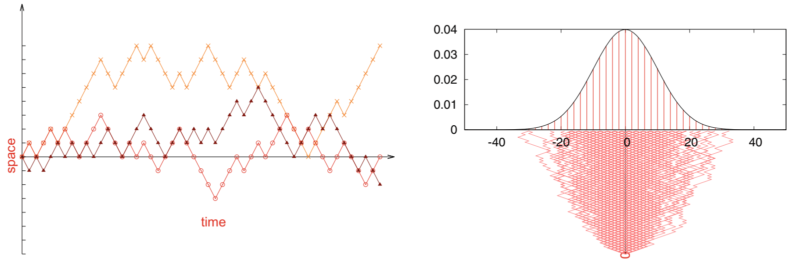

Fig. 2.1 On the left three examples of random path, on the right the limiting Gaussian probability density describing a large number of walker

the exploratory movements of animals in population dynamics, or also in the genetic drift of a population.

In simple cases the walker is limited in his movement in a one-dimensional line or in a two- or three-dimensional space, but it is possible to generalize the path of a walker among different sites (or states). For instance, one can consider a walker performing a journey among N different sites $1, 2, \ldots, N$, which are connected with the following rule: if the walker is in the site i it goes into the site j with a given probability $P_{i\to j}$. If $P_{i\to j} = 0$ there is no direct link between site i and site j. A possible real-world application of such a model describes an aimless journey among villages in a mountainous regions: a non-zero $P_{i\to j}$ means that there exists a road between the village labeled with i and that one labeled with j, and the value of probability $P_{i\to j}$ indicates how dangerous is the road between the two villages, i.e., the closer to zero the probability value the more dangerous the road between the two villages and the walker would not want to face it. Further generalization of RW, in which the walker jumps between connected nodes, can be useful in various and different contexts, from the epidemic spreading to the internet monitoring, not to mention that the idea of RW is at the heart of Google search engine, at least in its first implementation. Moreover, random walk processes are exploited in artificial intelligence algorithms, in which the criteria for decisions are based on testing at random different procedures, and also in the social media context, in which advertisements and suggestions mainly follow the tracked characteristics of the user, but sometimes they need RW to vary suggestions that reach users. Therefore, the random walk, seemingly a mathematical game, is rather a functional instrument in many different theoretical and applied fields.

Finally, discussing the title of the book, we thought the reader can perform a random walk between the various authors and the various aspects of modern physics presented in the book. Clearly some entries are more linked than others, e.g., a connection between Determinism and Laplace sounds more reasonable than Determinism and Volterra. Therefore we suggested few possible paths in the Introduction that can guide the reader to follow and deepen some topic that might be of interest to him. Anyway, we invite the willing reader to look for her/his personal walks.

Chapter 3
Atoms

Surely nowadays nobody doubts about the existence of atoms and that they are the basic building blocks of ordinary matter, this is now a part of the basic scientific knowledge. The enormous relevance of atoms in science has been vividly expressed by R. Feynman in his famous quote: *"If, in some cataclysm, all of scientific knowledge were to be destroyed, and only one sentence passed on to the next generations of creatures, what statement would contain the most information in the fewest words? I believe it is the atomic hypothesis (or the atomic fact, or whatever you wish to call it) that all things are made of atoms-little particles that move around in perpetual motion, attracting each other when they are a little distance apart, but repelling upon being squeezed into one another. In that one sentence, you will see, there is an enormous amount of information about the world, if just a little imagination and thinking are applied"*.

In spite of the great relevance of atoms, until one century ago their real physical existence was rather controversial and even eminent scientists, such as E. Mach, P. Duhem, and W. Ostwald, did not believe that atoms were the basic constituents of the world. Surprisingly even Planck, one of the fathers of modern physics, changed his mind about the physical existence of atoms only at the end of the nineteenth century.

There is no risk to overestimate the relevance of atoms in science. In the following we summarize the development of atomism from its origin up to the modern time. The basic idea of atoms is rather old, as far as we can know it dates back to Leucippus and Democritus (fifth century BC). The Greek adjective "atomos" means "uncuttable"; now we know that atoms have an internal structure (electrons, protons, and neutrons, and, at even finer scales, quarks, strings, etc.). However, this is a marginal aspect that does not alter the relevance of the visionary intuition of the two Greek philosophers about the fact that the world of macroscopic objects is based on the motion of very small entities (atoms) and that the very nature of matter is discrete.

Of course, the atomism in the original formulation of ancient Greeks is rather far from a modern scientific theory, it was mainly a metaphysical thesis whose aim was to establish the ultimate nature of material reality by philosophical arguments.

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We cannot resist to cite the famous fragment by Democritus about the distinction between perceived properties like tastes and colors, whose existence is only "by convention", in contrast to the "true" reality, which is atoms and void: *"Sweet exists by convention, bitter by convention, heat by convention, cold by convention, color by convention; but atoms and the void exist in truth"*.

In the old times (say before Galileo and Newton) atomism was not particularly popular, mainly due to the negative opinion about atomism of the two main philosophical schools (of Plato and Aristotle). Moreover atomism, considered a dangerous belief from both Catholic and Protestant Churches, was subject to censorship for a long time. In the fifteenth century a copy of the poem *De Rerum Natura* of the Roman poet Lucretius was discovered by the Italian Poggio Bracciolini in a German monastery. This wonderful poem, which explains the natural world and our place in it in the framework of the atomism of Epicurus (who basically followed Leucippus and Democritus), was considered a dangerous example of atheism and, for several centuries, its reading was forbidden in many universities.

We can say that both Galileo and Newton shared the main ideas emerged in ancient Greece that atoms constitute the ultimate nature of material reality, even if Galileo had not a precise theory of them while Newton thought about atoms in a much more modern perspective.[1] However, the real impact of atomism for science started only in the eighteenth century, and its relevance increased a lot in the second half of the nineteenth century. Roughly speaking, before modern times, the word "atom" has been used with two meanings: what chemists called "chemical atom", that in modern terms corresponds to chemical elements, and what physicists (and more general natural philosophers) indicated as "physical atom", which now corresponds to indivisible particles.

In chemistry, a key step was due to J. Dalton, in the second half of the eighteenth century, with his formulation of chemical atomism: the basic assumption was that chemical elements are composed of "ultimate objects", i.e., atoms; his theory had relevant implications for the way chemicals combine by weight. We can summarize the chemical atomism in the following way: for each element there is a unique indivisible unit which enters into combination with similar units of other elements in small integer multiples. Remarkably, chemical atoms provide a very solid basis to explain the empirical data of stoichiometry, namely, the fact that quantities of reactants and products form a ratio of positive integers, e.g., two moles of nitrogen and one of oxygen (H_2O) are needed to form water.

In physics, the theory of atomism had an empirical (and theoretical) support owing to the kinetic theory of gases which successfully predicted experimental laws, for instance, the celebrated result due to Maxwell that the viscosity of gases is independent of the density. Kinetic theory, which aims to deduce the properties of gases in terms of the collisions between the particles (atoms or molecules) composing

[1] *"Have not the small Particles of Bodies certain Powers, Virtues or Forces, by which they act at a distance, not only upon the Rays of Light for reflecting, refracting, and inflecting them, but also upon one another for producing a great part of the Phenomena of Nature?"*, Newton, [Opticks, Book 3, Part 1].

the gas, had its origin with the seminal work of Daniel Bernoulli (first half of the eighteenth century) who was able to explain pressure in terms of the many collisions of atoms with the wall of a vessel. Then the theory had been developed, mainly by Maxwell and Boltzmann (see the related entries), in the second half of the nineteenth century. Kinetic theory was an alternative to the imponderable material called caloric, used until the beginning of the nineteenth century to explain heat. After chemistry and kinetic theory offered explanations of a wide class of phenomena in the framework of atomism, the basic idea was widely accepted and the atomic-molecular hypothesis gained in plausibility for the evidence that gas laws also apply to solutions, which are homogeneous mixtures containing two or more chemical substances, the solute and the solvent.

In spite of these important results, in the second half of the nineteenth century, the success of phenomenological thermodynamics was at the origin of a school (known under the name of "energetics") that rejected atomism. Such an approach, which now may seem surprising, at that time was supported by important scientists and philosophers, e.g., P. Duhem, E. Mach, and W. Ostwald. In their positivistic point of view, the champions of the energetics believed that macroscopic phenomena (such as chemical reactions) should be treated solely in terms of a phenomenological approach based on the conservation of energy and the spontaneous increasing of entropy, and considered atoms just as a useful mathematical tool, with no real ground. In their opinion, since atoms and molecules are invisible, a decisive evidence of the atomic structure was impossible, therefore, the atomic theory must be considered just a physically unverifiable hypothesis and atoms as a mere notion of practical convenience but of no physical reality. The followers of energetics considered phenomenological thermodynamics superior to atomism in the explanation of the second law of thermodynamics. One of the reasons for such an opinion is the fact that Newtonian mechanics, owing to its time reversal property, cannot be exploited to distinguish between past and future, see the entries Boltzmann and Irreversibility.

The controversy about atomism was still open at the end of the nineteenth century: the majority of the British physicists were followers of the atomism, on the contrary energetics was particularly popular in the German scientific community. The main enemy of the energetics was Boltzmann, his confrontations with the opponents of the atomic theory became almost legendary. A rather famous fight took place in September 1895 in Lübeck, at the Congress of German Scientists. Years after, the famous German physicist Sommerfeld described what happened as follows: *"Helm was the champion of energetics; then came Ostwald and, afterward, the philosophical theories of Mach (who was not present at the event). In the opposite corner was Boltzmann, supported by Felix Klein. The skirmish between Boltzmann and Ostwald looked pretty much like a duel between a hefty bull and a trembling bullfighter".* A remark by A. Einstein on this debate is particularly illuminating: *"the prejudices of these scientists against atomic theory can be undoubtedly attributed to their positivistic philosophical views. This is an interesting example of how philosophical prejudices hinder a correct interpretation of facts even by scientists with bold thinking and subtle intuition".*

In the early twentieth century, after Einstein and Smoluchovsky theory of Brownian Motion (see related entry) and the experiments of Perrin which validated the theory, even the last exponents of energetics capitulated to the increasingly compelling evidence on the existence of atoms. In 1909, Ostwald did acknowledge that he had been wrong and Arrhenius, another opponent of the atomism, summarizing Einstein's and Perrin's work on Brownian Motion, during a 1911 congress in Paris declared that *"after this we can no longer question the essentiality of the existence of atoms"*. The last one who remained to oppose atomism was Mach, who espoused the lost cause of energetics till the end. Atoms really exist even if we cannot see them (as Mach liked to say), and Perrin had in fact counted them using Einstein's formula on Brownian Motion. As discussed in the entry Brownian motion, the diffusion coefficient, which is accessible experimentally, can be expressed in terms of several known (or measurable) macroscopic quantities (such as the gas constant, the viscosity, the temperature, the radius of the colloidal particle) and, more importantly for atomism, the Avogadro number N_A.[2] The validity of the relation between the diffusion coefficient and macroscopic quantities allowed for a decisive conclusion about the existence of atoms; in the words of Einstein *"if the prediction of this motion were to be proven wrong, this fact would provide a weighty argument against the molecular-kinetic conception of heat"*.

The value of the Avogadro number N_A measured by Perrin was later confirmed by different measurements not directly tied to kinetic theory. For instance, Rayleigh speculated that the color blue of the sky was due to the scattering of sunlight with gas molecules present in the atmosphere, rather than suspended particles (such as water drops).[3] In a way rather similar to the case of the Brownian Motion, Rayleigh scattering theory allows to establish a relation between the Avogadro number and quantities which can be measured in optical experiments. Remarkably the value of the Avogadro number obtained with such an approach is in good agreement with the one found in Perrin experiments on Brownian particle diffusion.

In the twentieth century, with the progress of quantum physics, nuclear physics, modern chemistry, and so on, we had a complete triumph of the atomism. Beyond the results on physical reality of atoms from Brownian Motion and light scattering, we can mention many other clear evidences in the modern physics, e.g., phenomena as ionization, cathode rays, and radioactive decay. In addition, very important from a theoretical point of view, the periodicity of the relations between the properties of the elements and their atomic weight indicates in a clear way that the atoms are composed of smaller particles (electrons and nuclei). The accurate description of the behavior of elements at varying their atomic weight is surely one of the great success

[2]The Avogadro number is the number of elementary entities (atoms or molecules) comprising one mole of a given substance, for instance, the number of atoms present in 12 g of isotopically pure carbon-12, its numerical value is 6.0221415×10^{23}.

[3]The mechanism proposed by Rayleigh, now called Rayleigh scattering, which is responsible for the blue of the sky, involves the scattering of light with particles smaller than the wavelength of the light, while scattering with particles larger than the wavelength of light (called Mie scattering) is responsible, e.g., of the red color of the sunset, which can be very intense over the sea where a lot of water vapor droplets and small aerosol particles are typically present.

of quantum mechanics as well of atomism. A discussion of some of these issues and, most importantly, of the many discoveries occurred when studying the internal structure of atoms, can be found in the entry Fermi.

We conclude by mentioning that, nowadays, we can actually “see” the atoms and the ultimate discrete structure of the matter, for instance, with the help of scanning tunneling microscopy we can observe real-space images of surfaces at atomic-scale resolution.

Chapter 4
Big Data

Usually it is assumed that the research activity of the hard sciences (such as physics and chemistry) is based on three pillars, namely, theory and experiments (which are the traditional ones, say since Galilei) to which during the twentieth century it has been added the use of numerical computations (see entry Computer, algorithms and simulations), which nowadays is recognized as the third pillar. In the last decades, at least according to some authors, we had the rise of a fourth paradigm which is data mining, i.e., the exploration of a large amount of data through powerful tools of analysis.

As a precursor of such a trend we can mention that in the 1980s some researchers in the field of artificial intelligence (AI) devised BACON (after the British philosopher Francis Bacon), a computer program "able" to automate scientific discoveries. Apparently the program "discovered" some physical laws, including Kepler's third law. It is interesting to look at the details of the procedure used by BACON. The program received as input the numerical values of the distances from the Sun, D, and the revolution periods, P, of planets. BACON, then, discovered that D^3 is proportional to P^2. While this is surely interesting, we tend to think that it is difficult to claim that this represents a direct inductive approach only from data. Actually for Kepler the raw observables were not D and P, but a huge list of planetary positions seen from the Earth at different times, i.e., something much more difficult to interpret than the (clean) data given to BACON. Remarkably in his discovery, Kepler, who was guided by strong beliefs in mathematical harmonies as well as the controversial (at that time) heliocentric theory of Copernicus, was able to guess the "right" variables D and P.

More recently, some scientists trained an algorithm that, using a learning strategy just based on the knowledge of the past evolution of the system, succeeded in discovering the laws of motion of a chaotic double pendulum, which is a rather complex system. Therefore, despite being given no scientific knowledge by the researchers, the machine had, apparently, deduced certain not trivial laws of physics. Actually examples of this kind are nowadays more and more reported in the scientific literature,

M. Cencini et al., *A Random Walk in Physics*,
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