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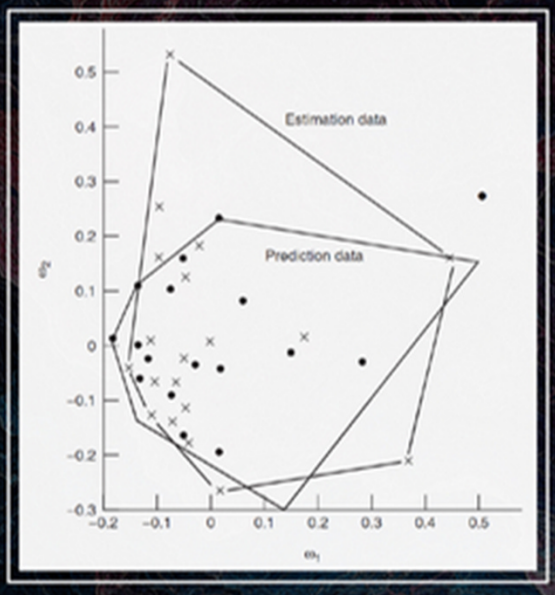
SIXTH EDITION

INTRODUCTION TO LINEAR REGRESSION ANALYSIS



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INTRODUCTION TO LINEAR REGRESSION ANALYSIS

WILEY SERIES IN PROBABILITY AND STATISTICS

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INTRODUCTION TO LINEAR REGRESSION ANALYSIS

Sixth Edition

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CONTENTS

PREFACE

Regression analysis is one of the most widely used techniques for analyzing multifactor data. Its broad appeal and usefulness result from the conceptually logical process of using an equation to express the relationship between a variable of interest (the response) and a set of related predictor variables. Regression analysis is also interesting theoretically because of elegant underlying mathematics and a well-developed statistical theory. Successful use of regression requires an appreciation of both the theory and the practical problems that typically arise when the technique is employed with real-world data.

This book is intended as a text for a basic course in regression analysis. It contains the standard topics for such courses and many of the newer ones as well. It blends both theory and application so that the reader will gain an understanding of the basic principles necessary to apply regression model-building techniques in a wide variety of application environments. The book began as an outgrowth of notes for a course in regression analysis taken by seniors and first-year graduate students in various fields of engineering, the chemical and physical sciences, statistics, mathematics, and management. We have also used the material in many seminars and industrial short courses for professional audiences. We assume that the reader has taken a first course in statistics and has familiarity with hypothesis tests and confidence intervals and the normal, t, χ^2, and F distributions. Some knowledge of matrix algebra is also necessary.

The computer plays a significant role in the modern application of regression. Today even spreadsheet software has the capability to fit regression equations by least squares. Consequently, we have integrated many aspects of computer usage into the text, including displays of both tabular and graphical output, and general discussions of capabilities of some software packages. We use Minitab®, JMP®, SAS®, and R for various problems and examples in the text. We selected these packages because they are widely used both in practice and in teaching regression and they have good regression. Many of the homework problems require software for their solution. All data sets in the book are available in electronic form from the

publisher. The ftp site ftp://ftp.wiley.com/public/sci_tech_med/introduction_linear_regression hosts the data, problem solutions, PowerPoint files, and other material related to the book.

CHANGES IN THE SIXTH EDITION

We have made a number of changes in this edition of the book. This includes the reorganization of text material, new examples, new exercises, and new material on a variety of topics. Our objective was to make the book more useful as both a text and a reference and to update our treatment of certain topics.

Chapter 1 is a general introduction to regression modeling and describes some typical applications of regression. Chapters 2 and 3 provide the standard results for least-squares model fitting in simple and multiple regression, along with basic inference procedures (tests of hypotheses, confidence and prediction intervals). Chapter 4 discusses some introductory aspects of model adequacy checking, including residual analysis and a strong emphasis on residual plots, detection and treatment of outliers, the PRESS statistic, and testing for lack of fit. Chapter 5 discusses how transformations and weighted least squares can be used to resolve problems of model inadequacy or to deal with violations of the basic regression assumptions. Both the Box–Cox and Box–Tidwell techniques for analytically specifying the form of a transformation are introduced. Influence diagnostics are presented in Chapter 6, along with an introductory discussion of how to deal with influential observations. Polynomial regression models and their variations are discussed in Chapter 7. Topics include the basic procedures for fitting and inference for polynomials and discussion of centering in polynomials, hierarchy, piecewise polynomials, models with both polynomial and trigonometric terms, orthogonal polynomials, an overview of response surfaces, and an introduction to nonparametric and smoothing regression techniques. Chapter 8 introduces indicator variables and also makes the connection between regression and analysis-of-variance models. Chapter 9 focuses on the multicollinearity problem. Included are discussions of the sources of multicollinearity, its harmful effects, diagnostics, and various remedial measures. We introduce biased estimation, including ridge regression and some of its variations and principal-component regression. Variable selection and model-building techniques are developed in Chapter 10, including stepwise procedures and all-possible-regressions. We also discuss and illustrate several criteria for the evaluation of subset regression models. Chapter 11 presents a collection of techniques useful for regression model validation.

The first 11 chapters are the nucleus of the book. Many of the concepts and examples flow across these chapters. The remaining four chapters cover a variety of topics that are important to the practitioner of regression, and they can be read independently. Chapter 12 in introduces nonlinear regression, and Chapter 13 is a basic treatment of generalized linear models. While these are perhaps not standard topics for a linear regression textbook, they are so important to students and professionals in engineering and the sciences that we would have been seriously remiss without giving an introduction to them. Chapter 14 covers regression models for time series data. Chapter 15 includes a survey of several important topics, including robust regression, the effect of measurement errors in the regressors, the inverse estimation or calibration problem, bootstrapping regression estimates, classification and regression trees, neural networks, and designed experiments for regression.

In addition to the text material, Appendix C contains brief presentations of some additional topics of a more technical or theoretical nature. Some of these topics will

be of interest to specialists in regression or to instructors teaching a more advanced course from the book. Computing plays an important role in many regression courses. Mintab, JMP, SAS, and R are widely used in regression courses. Outputs from all of these packages are provided in the text. Appendix D is an introduction to using SAS for regression problems. Appendix E is an introduction to R.

USING THE BOOK AS A TEXT

Because of the broad scope of topics, this book has great flexibility as a text. For a first course in regression, we would recommend covering Chapters 1 through 10 in detail and then selecting topics that are of specific interest to the audience. For example, one of the authors (D.C.M.) regularly teaches a course in regression to an engineering audience. Topics for that audience include nonlinear regression (because mechanistic models that are almost always nonlinear occur often in engineering), a discussion of neural networks, and regression model validation. Other topics that we would recommend for consideration are multicollinearity (because the problem occurs so often) and an introduction to generalized linear models focusing mostly on logistic regression. G.G.V. has taught a regression course for graduate students in statistics that makes extensive use of the Appendix C material.

We believe the computer should be directly integrated into the course. In recent years, we have taken a notebook computer and computer projector to most classes and illustrated the techniques as they are introduced in the lecture. We have found that this greatly facilitates student understanding and appreciation of the techniques. We also require that the students use regression software for solving the homework problems. In most cases, the problems use real data or are based on real-world settings that represent typical applications of regression.

There is an instructor's manual that contains solutions to all exercises, electronic versions of all data sets, and questions/problems that might be suitable for use on examinations.

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Douglas C. Montgomery
Elizabeth A. Peck
G. Geoffrey Vining

ABOUT THE COMPANION WEBSITE

This book is accompanied by an instructor companion website and a student companion website:

www.wiley.com/go/montgomery/introlinearregression6e

The instructor site includes PowerPoint slides to facilitate instructional use of the book.

The student site includes data sets.

CHAPTER 1

INTRODUCTION

1.1 REGRESSION AND MODEL BUILDING

Regression analysis is a **statistical technique** for investigating and **modeling the relationship between variables**. Applications of regression are numerous and occur in almost every field, including engineering, the physical and chemical sciences, economics, management, life and biological sciences, and the social sciences. Regression analysis is used extensively in data mining and is a basic tool of data science and analytics. Because of its wide applicability to a range of problems, regression analysis may be the most widely used statistical technique.

As an example of a problem in which regression analysis may be helpful, suppose that an industrial engineer employed by a soft drink beverage bottler is analyzing the product delivery and service operations for vending machines. He suspects that the time required by a route deliveryman to load and service a machine is related to the number of cases of product delivered. The engineer visits 25 randomly chosen retail outlets having vending machines, and the in-outlet delivery time (in minutes) and the volume of product delivered (in cases) are observed for each. The 25 observations are plotted in Figure 1.1*a*. This graph is called a **scatter diagram**. This display clearly suggests a relationship between delivery time and delivery volume; in fact, the impression is that the data points generally, but not exactly, fall along a straight line. Figure 1.1*b* illustrates this straight-line relationship.

If we let y represent delivery time and x represent delivery volume, then the equation of a straight line relating these two variables is

$$y = \beta_0 + \beta_1 x \tag{1.1}$$

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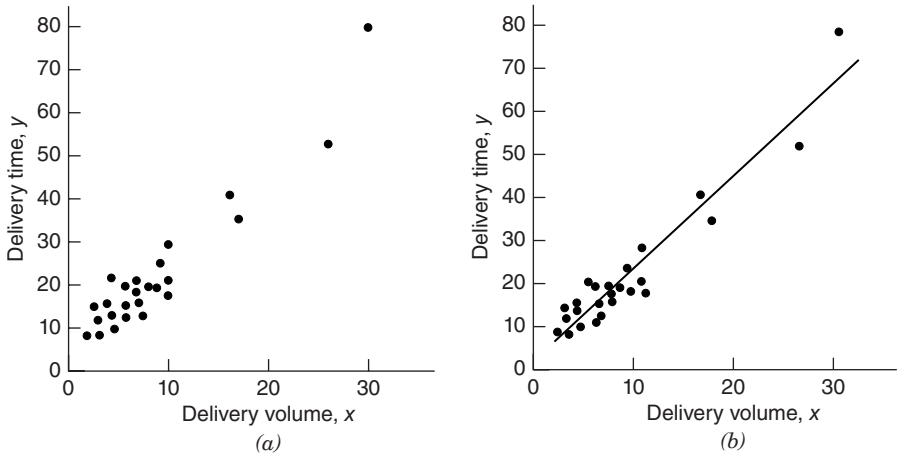

Figure 1.1 (*a*) Scatter diagram for delivery volume. (*b*) Straight-line relationship between delivery time and delivery volume.

where β_0 is the intercept and β_1 is the slope. Now the data points do not fall exactly on a straight line, so Eq. (1.1) should be modified to account for this. Let the difference between the observed value of y and the straight line $(\beta_0 + \beta_1 x)$ be an **error** ε. It is convenient to think of ε as a statistical error; that is, it is a random variable that accounts for the failure of the model to fit the data exactly. The error may be made up of the effects of other variables on delivery time, measurement errors, and so forth. Thus, a more plausible model for the delivery time data is

$$y = \beta_0 + \beta_1 x + \varepsilon \tag{1.2}$$

Equation (1.2) is called a **linear regression model**. Customarily x is called the independent variable and y is called the dependent variable. However, this often causes confusion with the concept of statistical independence, so we refer to x as the **predictor** or **regressor** variable and y as the **response** variable. Because Eq. (1.2) involves only one regressor variable, it is called a **simple linear regression model**.

To gain some additional insight into the linear regression model, suppose that we can fix the value of the regressor variable x and observe the corresponding value of the response y. Now if x is fixed, the random component ε on the right-hand side of Eq. (1.2) determines the properties of y. Suppose that the mean and variance of ε are 0 and σ^2, respectively. Then the mean response at any value of the regressor variable is

$$E(y \mid x) = \mu_{y|x} = E(\beta_0 + \beta_1 x + \varepsilon) = \beta_0 + \beta_1 x$$

Notice that this is the same relationship that we initially wrote down following inspection of the scatter diagram in Figure 1.1*a*. The variance of y given any value of x is

$$\mathrm{Var}(y \mid x) = \sigma^2_{y|x} = \mathrm{Var}(\beta_0 + \beta_1 x + \varepsilon) = \sigma^2$$

Thus, the true regression model $\mu_{y|x} = \beta_0 + \beta_1 x$ is a line of mean values, that is, the height of the regression line at any value of x is just the expected value of y for that

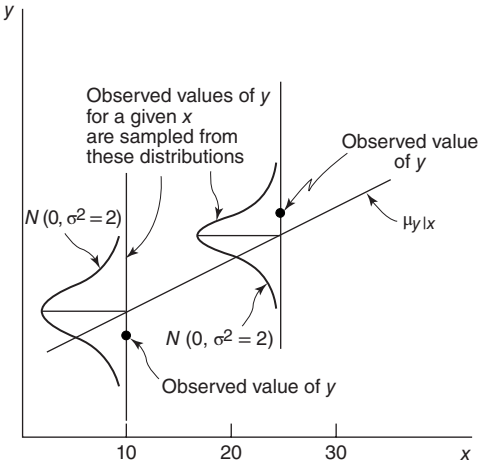

Figure 1.2 How observations are generated in linear regression.

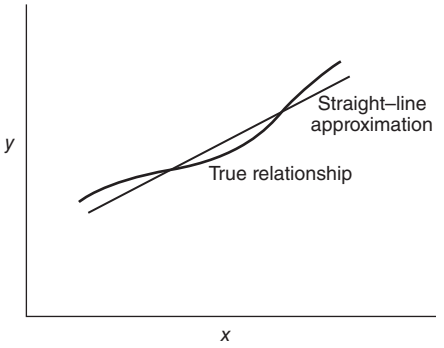

Figure 1.3 Linear regression approximation of a complex relationship.

x. The slope, β_1 can be interpreted as the change in the mean of y for a unit change in x. Furthermore, the variability of y at a particular value of x is determined by the variance of the error component of the model, σ^2. This implies that there is a distribution of y values at each x and that the variance of this distribution is the same at each x.

For example, suppose that the true regression model relating delivery time to delivery volume is $\mu_{y|x} = 3.5 + 2x$, and suppose that the variance is $\sigma^2 = 2$. Figure 1.2 illustrates this situation. Notice that we have used a normal distribution to describe the random variation in ε. Since y is the sum of a constant $\beta_0 + \beta_1 x$ (the mean) and a normally distributed random variable, y is a normally distributed random variable. For example, if $x = 10$ cases, then delivery time y has a normal distribution with mean $3.5 + 2(10) = 23.5$ minutes and variance 2. The variance σ^2 determines the amount of variability or noise in the observations y on delivery time. When σ^2 is small, the observed values of delivery time will fall close to the line, and when σ^2 is large, the observed values of delivery time may deviate considerably from the line.

In almost all applications of regression, the regression equation is only an approximation to the true functional relationship between the variables of interest. These functional relationships are often based on physical, chemical, or other engineering or scientific theory, that is, knowledge of the underlying mechanism. Consequently, these types of models are often called **mechanistic models**. For example, the familiar physics equation momentum = mass × velocity is a mechanistic model.

Regression models, on the other hand, are thought of as **empirical models**. Figure 1.3 illustrates a situation where the true relationship between y and x is relatively complex, yet it may be approximated quite well by a linear regression equation. Sometimes the underlying mechanism is more complex, resulting in the need for a more complex approximating function, as in Figure 1.4, where a "piecewise linear" regression function is used to approximate the true relationship between y and x.

Generally regression equations are valid only over the region of the regressor variables contained in the observed data. For example, consider Figure 1.5. Suppose that data on y and x were collected in the interval $x_1 \le x \le x_2$. Over this interval the

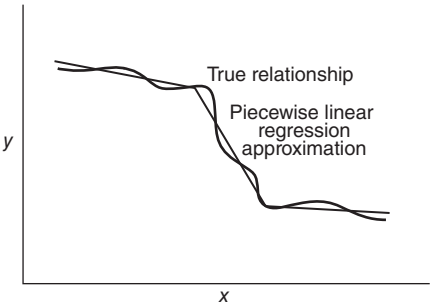

Figure 1.4 Piecewise linear approximation of a complex relationship.

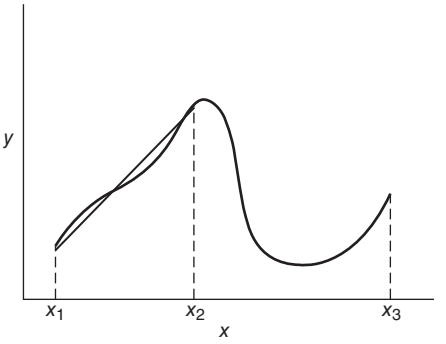

Figure 1.5 The danger of extrapolation in regression.

linear regression equation shown in Figure 1.5 is a good approximation of the true relationship. However, suppose this equation were used to predict values of y for values of the regressor variable in the region $x_2 \le x \le x_3$. Clearly the linear regression model is not going to perform well over this range of x because of model error or equation error.

In general, the response variable y may be related to k regressors, $x_1, x_2, \ldots, x_k$, so that

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k + \varepsilon \tag{1.3}$$

This is called a **multiple linear regression model** because more than one regressor is involved. The adjective linear is employed to indicate that the model is linear in the parameters $\beta_0, \beta_1, \ldots, \beta_k$, not because y is a linear function of the x's. We shall see subsequently that many models in which y is related to the x's in a nonlinear fashion can still be treated as linear regression models as long as the equation is linear in the β's.

An important objective of regression analysis is to **estimate the unknown parameters** in the regression model. This process is also called fitting the model to the data. We study several parameter estimation techniques in this book. One of these techmques is the method of least squares (introduced in Chapter 2). For example, the least-squares fit to the delivery time data is

$$\hat{y} = 3.321 + 2.1762x$$

where $\hat{y}$ is the fitted or estimated value of delivery time corresponding to a delivery volume of x cases. This fitted equation is plotted in Figure 1.1*b*.

The next phase of a regression analysis is called **model adequacy checking**, in which the appropriateness of the model is studied and the quality of the fit ascertained. Through such analyses the usefulness of the regression model may be determined. The outcome of adequacy checking may indicate either that the model is reasonable or that the original fit must be modified. Thus, regression analysis is an **iterative** procedure, in which data lead to a model and a fit of the model to the data is produced. The quality of the fit is then investigated, leading either to modification

of the model or the fit or to adoption of the model. This process is illustrated several times in subsequent chapters.

A regression model does not imply a cause-and-effect relationship between the variables. Even though a strong empirical relationship may exist between two or more variables, this cannot be considered evidence that the regressor variables and the response are related in a cause-and-effect manner. To establish causality, the relationship between the regressors and the response must have a basis outside the sample data—for example, the relationship may be suggested by theoretical considerations. Regression analysis can aid in confirming a cause-and-effect relationship, but it cannot be the sole basis of such a claim.

Finally it is important to remember that regression analysis is part of a broader data-analytic approach to problem solving. That is, the regression equation itself may not be the primary objective of the study. It is usually more important to gain insight and understanding concerning the system generating the data.

1.2 DATA COLLECTION

An essential aspect of regression analysis is data collection. Any regression analysis is only as good as the data on which it is based. Three basic methods for collecting data are as follows:

- A retrospective study based on historical data
- An observational study
- A designed experiment

A good data collection scheme can ensure a simplified and a generally more applicable model. A poor data collection scheme can result in serious problems for the analysis and its interpretation. The following example illustrates these three methods.

Example 1.1

Consider the acetone–butyl alcohol distillation column shown in Figure 1.6. The operating personnel are interested in the concentration of acetone in the distillate (product) stream. Factors that may influence this are the reboil temperature, the condensate temperature, and the reflux rate. For this column, operating personnel maintain and archive the following records:

- The concentration of acetone in a test sample taken every hour from the product stream
- The reboil temperature controller log, which is a plot of the reboil temperature
- The condenser temperature controller log
- The nominal reflux rate each hour

The nominal reflux rate is supposed to be constant for this process. Only infrequently does production change this rate. We now discuss how the three different data collection strategies listed above could be applied to this process. ■

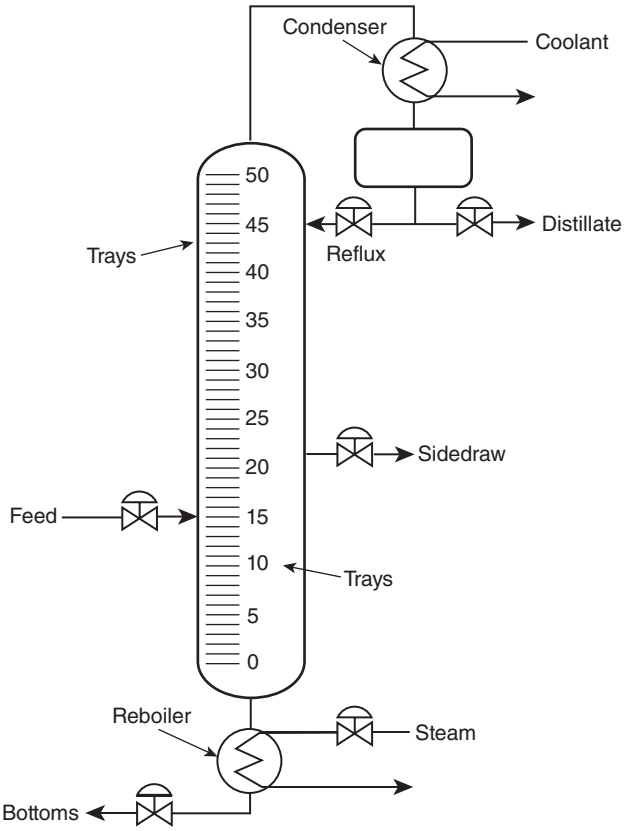

Figure 1.6 Acetone–butyl alcohol distillation column.

Retrospective Study We could pursue a retrospective study that would use either all or a sample of the historical process data over some period of time to determine the relationships among the two temperatures and the reflux rate on the acetone concentration in the product stream. In so doing, we take advantage of previously collected data and minimize the cost of the study. However, there are several problems:

1. We really cannot see the effect of reflux on the concentration since we must assume that it did not vary much over the historical period.
2. The data relating the two temperatures to the acetone concentration do not correspond directly. Constructing an approximate correspondence usually requires a great deal of effort.
3. Production controls temperatures as tightly as possible to specific target values through the use of automatic controllers. Since the two temperatures vary so little over time, we will have a great deal of difficulty seeing their real impact on the concentration.
4. Within the narrow ranges that they do vary, the condensate temperature tends to increase with the reboil temperature. As a result, we will have a great deal

of difficulty separating out the individual effects of the two temperatures. This leads to the problem of **collinearity** or **multicollinearity**, which we discuss in Chapter 9.

Retrospective studies often offer limited amounts of useful information. In general, their primary disadvantages are as follows:

- Some of the relevant data often are missing.
- The reliability and quality of the data are often highly questionable.
- The nature of the data often may not allow us to address the problem at hand.
- The analyst often tries to use the data in ways they were never intended to be used.
- Logs, notebooks, and memories may not explain interesting phenomena identified by the data analysis.

Using historical data always involves the risk that, for whatever reason, some of the data were not recorded or were lost. Typically, historical data consist of information considered critical and of information that is convenient to collect. The convenient information is often collected with great care and accuracy. The essential information often is not. Consequently, historical data often suffer from transcription errors and other problems with data quality. These errors make historical data prone to **outliers**, or observations that are very different from the bulk of the data. A regression analysis is only as reliable as the data on which it is based.

Just because data are convenient to collect does not mean that these data are particularly useful. Often, data not considered essential for routine process monitoring and not convenient to collect do have a significant impact on the process. Historical data cannot provide this information since they were never collected. For example, the ambient temperature may impact the heat losses from our distillation column. On cold days, the column loses more heat to the environment than during very warm days. The production logs for this acetone–butyl alcohol column do not record the ambient temperature. As a result, historical data do not allow the analyst to include this factor in the analysis even though it may have some importance.

In some cases, we try to use data that were collected as surrogates for what we really needed to collect. The resulting analysis is informative only to the extent that these surrogates really reflect what they represent. For example, the nature of the inlet mixture of acetone and butyl alcohol can significantly affect the column's performance. The column was designed for the feed to be a saturated liquid (at the mixture's boiling point). The production logs record the feed temperature but do not record the specific concentrations of acetone and butyl alcohol in the feed stream. Those concentrations are too hard to obtain on a regular basis. In this case, inlet temperature is a surrogate for the nature of the inlet mixture. It is perfectly possible for the feed to be at the correct specific temperature and the inlet feed to be either a subcooled liquid or a mixture of liquid and vapor.

In some cases, the data collected most casually, and thus with the lowest quality, the least accuracy, and the least reliability, turn out to be very influential for explaining our response. This influence may be real, or it may be an artifact related to the inaccuracies in the data. Too many analyses reach invalid conclusions because they

lend too much credence to data that were never meant to be used for the strict purposes of analysis.

Finally, the primary purpose of many analyses is to isolate the root causes underlying interesting phenomena. With historical data, these interesting phenomena may have occurred months or years before. Logs and notebooks often provide no significant insights into these root causes, and memories clearly begin to fade over time. Too often, analyses based on historical data identify interesting phenomena that go unexplained.

Observational Study We could use an observational study to collect data for this problem. As the name implies, an observational study simply observes the process or population. We interact or disturb the process only as much as is required to obtain relevant data. With proper planning, these studies can ensure accurate, complete, and reliable data. On the other hand, these studies often provide very limited information about specific relationships among the data.

In this example, we would set up a data collection form that would allow the production personnel to record the two temperatures and the actual reflux rate at specified times corresponding to the observed concentration of acetone in the product stream. The data collection form should provide the ability to add comments in order to record any interesting phenomena that may occur. Such a procedure would ensure accurate and reliable data collection and would take care of problems 1 and 2 above. This approach also minimizes the chances of observing an outlier related to some error in the data. Unfortunately, an observational study cannot address problems 3 and 4. As a result, observational studies can lend themselves to problems with collinearity.

Designed Experiment The best data collection strategy for this problem uses a designed experiment where we would manipulate the two temperatures and the reflux ratio, which we would call the factors, according to a well-defined strategy, called the experimental design. This strategy must ensure that we can separate out the effects on the acetone concentration related to each factor. In the process, we eliminate any collinearity problems. The specified values of the factors used in the experiment are called the levels. Typically, we use a small number of levels for each factor, such as two or three. For the distillation column example, suppose we use a "high" or +1 and a "low" or −1 level for each of the factors. We thus would use two levels for each of the three factors. A treatment combination is a specific combination of the levels of each factor. Each time we carry out a treatment combination is an experimental run or setting. The experimental design or plan consists of a series of runs.

For the distillation example, a very reasonable experimental strategy uses every possible treatment combination to form a basic experiment with eight different settings for the process. Table 1.1 presents these combinations of high and low levels. This experimental arrangement is called a factorial design.

Figure 1.7 illustrates that this factorial design forms a cube in terms of these high and low levels. With each setting of the process conditions, we allow the column to reach equilibrium, take a sample of the product stream, and determine the acetone concentration. We then can draw specific inferences about the effect of these factors. Such an approach allows us to proactively study a population or process.

TABLE 1.1 Designed Experiment for the Distillation Column

Reboil Temperature	Condensate Temperature	Reflux Rate
−1	−1	−1
+1	−1	−1
−1	+1	−1
+1	+1	−1
−1	−1	+1
+1	−1	+1
−1	+1	+1
+1	+1	+1

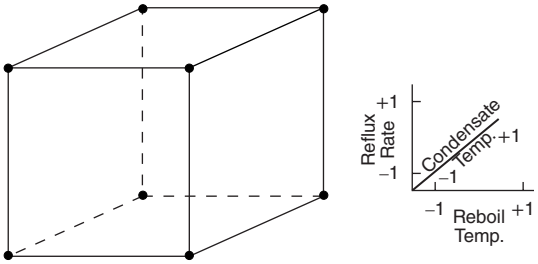

Figure 1.7 The designed experiment for the distillation column.

1.3 USES OF REGRESSION

Regression models are used for several purposes, including the following:

1. Data description
2. Parameter estimation
3. Prediction and estimation
4. Control

Engineers and scientists frequently use equations to summarize or describe a set of data. Regression analysis is helpful in developing such equations. For example, we may collect a considerable amount of delivery time and delivery volume data, and a regression model would probably be a much more convenient and useful summary of those data than a table or even a graph.

Sometimes parameter estimation problems can be solved by regression methods. For example, chemical engineers use the Michaelis–Menten equation $y = \beta_1 x/(x + \beta_2) + \varepsilon$ to describe the relationship between the velocity of reaction y and concentration x. Now in this model, β_1 is the asymptotic velocity of the reaction, that is, the maximum velocity as the concentration gets large. If a sample of observed values of velocity at different concentrations is available, then the engineer can use regression analysis to fit this model to the data, producing an estimate of the maximum velocity. We show how to fit regression models of this type in Chapter 12.

Many applications of regression involve prediction of the response variable. For example, we may wish to predict delivery time for a specified number of cases of soft drinks to be delivered. These predictions may be helpful in planning delivery activities such as routing and scheduling or in evaluating the productivity of delivery operations. The dangers of extrapolation when using a regression model for prediction because of model or equation error have been discussed previously (see Figure 1.5). However, even when the model form is correct, poor estimates of the model parameters may still cause poor prediction performance.

Regression models may be used for control purposes. For example, a chemical engineer could use regression analysis to develop a model relating the tensile strength of paper to the hardwood concentration in the pulp. This equation could

then be used to control the strength to suitable values by varying the level of hardwood concentration. When a regression equation is used for control purposes, it is important that the variables be related in a causal manner. Note that a cause-and-effect relationship may not be necessary if the equation is to be used only for prediction. In this case it is only necessary that the relationships that existed in the original data used to build the regression equation are still valid. For example, the daily electricity consumption during August in Atlanta, Georgia, may be a good predictor for the maximum daily temperature in August. However, any attempt to reduce the maximum temperature by curtailing electricity consumption is clearly doomed to failure.

1.4 ROLE OF THE COMPUTER

Building a regression model is an iterative process. The model-building process is illustrated in Figure 1.8. It begins by using any theoretical knowledge of the process that is being studied and available data to specify an initial regression model. Graphical data displays are often very useful in specifying the initial model. Then the parameters of the model are estimated, typically by either least squares or maximum likelihood. These procedures are discussed extensively in the text. Then model adequacy must be evaluated. This consists of looking for potential misspecification of the model form, failure to include important variables, including unnecessary variables, or unusual/inappropriate data. If the model is inadequate, then adjustments must be made and the parameters estimated again. This process may be repeated several times until an adequate model is obtained. Finally, model validation should be carried out to ensure that the model will produce results that are acceptable in the final application.

A good regression computer program is a necessary tool in the model-building process. However, the routine application of standard regression computer programs often does not lead to successful results. The computer is **not** a substitute for creative thinking about the problem. Regression analysis requires the **intelligent** and **artful** use of the computer. We must learn how to interpret what the computer is telling us and how to incorporate that information in subsequent models. Generally, regression computer programs are part of more general statistics software packages, such as Minitab, SAS, JMP, and R. We discuss and illustrate the use of

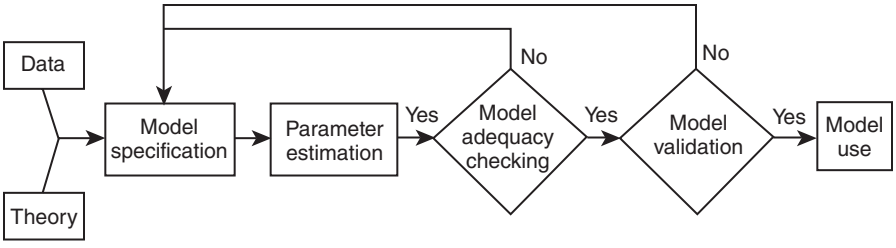

Figure 1.8 Regression model-building process.

these packages throughout the book. Appendix D contains details of the SAS procedures typically used in regression modeling along with basic instructions for their use. Appendix E provides a brief introduction to the R statistical software package. We present R code for doing analyses throughout the text. Without these skills, it is virtually impossible to successfully build a regression model.

CHAPTER 2

SIMPLE LINEAR REGRESSION

2.1 SIMPLE LINEAR REGRESSION MODEL

This chapter considers the **simple linear regression model**, that is, a model with a single regressor x that has a relationship with a response y that is a straight line. This simple linear regression model is

$$y = \beta_0 + \beta_1 x + \varepsilon \tag{2.1}$$

where the intercept β_0 and the slope β_1 are unknown constants and ε is a random error component. The errors are assumed to have mean zero and unknown variance σ^2. Additionally we usually assume that the errors are uncorrelated. This means that the value of one error does not depend on the value of any other error.

It is convenient to view the regressor x as controlled by the data analyst and measured with negligible error, while the response y is a random variable. That is, there is a probability distribution for y at each possible value for x. The mean of this distribution is

$$E\left(y|x\right) = \beta_0 + \beta_1 x \tag{2.2a}$$

and the variance is

$$\text{Var}\left(y|x\right) = \text{Var}\left(\beta_0 + \beta_1 x + \varepsilon\right) = \sigma^2 \tag{2.2b}$$

Introduction to Linear Regression Analysis, Sixth Edition. Douglas C. Montgomery, Elizabeth A. Peck, and G. Geoffrey Vining.

Companion website: www.wiley.com/go/montgomery/introlinearregression6e