Applied and Numerical Harmonic Analysis

Matthew Hirn
Shidong Li
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Editors

$$\hat{f}(\gamma) = \int f(x)e^{-2\pi i x\gamma}\,dx$$

# Excursions in Harmonic Analysis, Volume 6

## In Honor of John Benedetto's 80th Birthday

Birkhäuser

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ISSN 2296-5009 ISSN 2296-5017 (electronic)
Applied and Numerical Harmonic Analysis
ISBN 978-3-030-69636-8 ISBN 978-3-030-69637-5 (eBook)
https://doi.org/10.1007/978-3-030-69637-5

Mathematics Subject Classification: 43-XX



This book is published under the imprint Birkhäuser, www.birkhauser-science.com by the registered company Springer Nature Switzerland AG.
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

*Dedicated to Cathy*

# ANHA Series Preface

The *Applied and Numerical Harmonic Analysis (ANHA)* book series aims to provide the engineering, mathematical, and scientific communities with significant developments in harmonic analysis, ranging from abstract harmonic analysis to basic applications. The title of the series reflects the importance of applications and numerical implementation, but richness and relevance of applications and implementation depend fundamentally on the structure and depth of theoretical underpinnings. Thus, from our point of view, the interleaving of theory and applications and their creative symbiotic evolution is axiomatic.

Harmonic analysis is a wellspring of ideas and applicability that has flourished, developed, and deepened over time within many disciplines and by means of creative cross-fertilization with diverse areas. The intricate and fundamental relationship between harmonic analysis and fields such as signal processing, partial differential equations (PDEs), and image processing is reflected in our state-of-the-art *ANHA* series.

Our vision of modern harmonic analysis includes mathematical areas such as wavelet theory, Banach algebras, classical Fourier analysis, time-frequency analysis, and fractal geometry, as well as the diverse topics that impinge on them.

For example, wavelet theory can be considered an appropriate tool to deal with some basic problems in digital signal processing, speech and image processing, geophysics, pattern recognition, biomedical engineering, and turbulence. These areas implement the latest technology from sampling methods on surfaces to fast algorithms and computer vision methods. The underlying mathematics of wavelet theory depends not only on classical Fourier analysis, but also on ideas from abstract harmonic analysis, including von Neumann algebras and the affine group. This leads to a study of the Heisenberg group and its relationship to Gabor systems, and of the metaplectic group for a meaningful interaction of signal decomposition methods. The unifying influence of wavelet theory in the aforementioned topics illustrates the justification for providing a means for centralizing and disseminating information from the broader, but still focused, area of harmonic analysis. This will be a key role of *ANHA*. We intend to publish with the scope and interaction that such a host of issues demands.

Along with our commitment to publish mathematically significant works at the frontiers of harmonic analysis, we have a comparably strong commitment to publish major advances in the following applicable topics in which harmonic analysis plays a substantial role:

| | |
|---|---|
| *Antenna theory* | *Prediction theory* |
| *Biomedical signal processing* | *Radar applications* |
| *Digital signal processing* | *Sampling theory* |
| *Fast algorithms* | *Spectral estimation* |
| *Gabor theory and applications* | *Speech processing* |
| *Image processing* | *Time-frequency and* |
| *Numerical partial differential equations* | *Time-scale analysis* |
| | *Wavelet theory* |

The above point of view for the *ANHA* book series is inspired by the history of Fourier analysis itself, whose tentacles reach into so many fields.

In the last two centuries Fourier analysis has had a major impact on the development of mathematics, on the understanding of many engineering and scientific phenomena, and on the solution of some of the most important problems in mathematics and the sciences. Historically, Fourier series were developed in the analysis of some of the classical PDEs of mathematical physics; these series were used to solve such equations. In order to understand Fourier series and the kinds of solutions they could represent, some of the most basic notions of analysis were defined, e.g., the concept of “function." Since the coefficients of Fourier series are integrals, it is no surprise that Riemann integrals were conceived to deal with uniqueness properties of trigonometric series. Cantor’s set theory was also developed because of such uniqueness questions.

A basic problem in Fourier analysis is to show how complicated phenomena, such as sound waves, can be described in terms of elementary harmonics. There are two aspects of this problem: first, to find, or even define properly, the harmonics or spectrum of a given phenomenon, e.g., the spectroscopy problem in optics; second, to determine which phenomena can be constructed from given classes of harmonics, as done, for example, by the mechanical synthesizers in tidal analysis.

Fourier analysis is also the natural setting for many other problems in engineering, mathematics, and the sciences. For example, Wiener’s Tauberian theorem in Fourier analysis not only characterizes the behavior of the prime numbers, but also provides the proper notion of spectrum for phenomena such as white light; this latter process leads to the Fourier analysis associated with correlation functions in filtering and prediction problems, and these problems, in turn, deal naturally with Hardy spaces in the theory of complex variables.

Nowadays, some of the theory of PDEs has given way to the study of Fourier integral operators. Problems in antenna theory are studied in terms of unimodular trigonometric polynomials. Applications of Fourier analysis abound in signal processing, whether with the fast Fourier transform (FFT), or filter design, or the adaptive modeling inherent in time-frequency-scale methods such as wavelet theory. The coherent states of mathematical physics are translated and modulated Fourier transforms, and these are used, in conjunction with the uncertainty principle, for dealing with signal reconstruction in communications theory. We are back to the raison d'être of the *ANHA* series!

University of Maryland
College Park

John J. Benedetto
Series Editor

# Foreword

John Benedetto was the first doctoral thesis student I supervised. It is sort of inadvisable to start out with a graduate student like that: it may saddle you, going forward, with unrealistically high expectations.

But, we were a good pair: John bright-eyed and brash in his twenties, just getting into the thesis student's role, and me bright-eyed and brash in my thirties, just getting into the thesis adviser's role. Actually, John was intent not only on writing a thesis on analysis and Banach spaces with me but also on learning all about Thomist philosophy from the eminent French philosopher Étienne Gilson on the other side of the Toronto campus. I did not know about this double ambition of his at the time—John reminisced about it to me only some years later, having found meanwhile that it was a bit much to juggle the two specialties at once. If I had known I was dealing with a mathematician cum medieval philosopher, actually, it would have made John seem even more akin, because I at the same stage of my education had set out not only to become a mathematician in the image of George Mackey and Garrett Birkhoff but at the same time to become a composer in the image of Irving Fine. And, I would not have been let down when John gave up his philosophical moonlighting, for I had had to give up my concentration on music in the same way.

So, John was declared Doctor of Philosophy and launched on his professional career with my blessing and that of the University of Toronto. I applauded his service in New York, Maryland, Pisa, and elsewhere. It may seem that our research emphases diverged a bit, but it does not feel to me that we got out of touch. In particular, we both welcomed the rise of wavelet theory with enthusiasm and without needing to consult each other. But, I remained mostly a spectator, while John threw himself into the amazing development of applied Fourier analysis. He became one of the leaders in forming the field and in making it known to a wider public, and he leads a large phalanx of creative Fourier analysts in the next generation. I have been duly appreciative of the achievements of the Norbert Wiener Center, though I have viewed them mostly from afar.

One has no right to take pride in the work of one's students and grandstudents, but I confess to feeling that pride in this case, however, unjustified. May they carry on whatever in my own life deserves to be carried on.

Toronto, ON, Canada  
May 2020

Chandler Davis

# Preface

“John J. Benedetto has had a profound influence not only on the direction of harmonic analysis and its applications, but also on the entire community of people involved in the field.” This statement can be found in the preface of the volume celebrating John’s 60th birthday and holds true even more so today. During the 20 years that follows, the world has witnessed that the breadth and depth of John’s influence continue to expand. Besides his enormously impactful scientific research contributions, John’s influence also lies in, for instance, advising 61 Ph.D. students (so far) and nurturing many other junior scholars; founding the Journal of Fourier Analysis and Applications (JFAA), and the book series of Applied and Numerical Harmonic Analysis (ANHA); establishing the renowned Norbert Winner Center, and fostering a wide range of highly relevant health and scientific research. All in all, John’s most profound influence lies in his building of a worldwide community of scholars in harmonic analysis and its applications. Advancing beautiful mathematical ideas and applications is an underlying theme of John’s illustrious career and is continuing in the latest forum of the annual February Fourier Talks (FFT). A full account of John’s influence on the field of harmonic analysis would require volumes.

In honor of John’s 80th birthday, this book is another assemblage of community’s appreciation to John’s deep impact on the field of harmonic analysis and applications and to the scientific community. Needless to say, the original articles collected in this volume are all highly relevant and written by prominent, well-respected scholars in the field. This volume covers an invited chapter and the following five parts:

1. John Benedetto’s Mathematical Work,
2. Harmonic Analysis,
3. Wavelets and Frames,
4. Sampling and Signal Processing,
5. Compressed Sensing and Optimizations.

As such, this book shall be once again an excellent reference and resource for graduate students and professionals in the field. Contributors of the volume include A. Abtahi, A. Aldroubi, C. Cabrelli, P.G. Casazza, J. Cahill, D.-C. Chang,

E. Cordero, W. Czaja, S.B. Damelin, S. Data, M. Dorfler, N. Dyn, M. de Gosson, Y. Han, C. Hegde, C. Heil, J.A. Hogan, Y. Hu, R. Johnson, D. Joyner, F. Keinert, J.D. Lakey, C. Leonard, W. Li, Y. Li, R.D. Martin, F. Marvasti, I. Medri, K.D. Merrill, D.G. Mixon, U. Molter, F. Nicola, A. Olevskii, M. Pekala, I.Z. Pesenson, A. Petrosyan, D.L. Ragozin, T. Strohmer, J. Stueck, T.T. Tran, A. Ulanovskii, E.S. Weber, M. Werman, T. Wertz, X. Wu, S. Zheng, and X. Zhuang.

To close, we would like to thank Radu V. Balan, Wojciech Czaja, Luke Evans, Alfredo Nava-Tudela, and Kasso Okoudjou, for organizing the conference celebrating John's birthday, and Jean-Pierre Gabardo, Christopher Heil, Emily King, Götz Pfander, and David Walnut for putting together an outstanding scientific program. We also acknowledge the financial support of the Institute for Mathematics and its Applications and the Department of Mathematics at the University of Maryland.

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November 2020

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# Acronyms

| | |
|---|---|
| AC | Absolutely continuous |
| ACO | Approximately controllable |
| AOB | Approximately observable |
| ART | Algebraic Reconstruction Technique: a method of image reconstruction in computerized tomography |
| BEC | Bose–Einstein condensation |
| BIBD | Balanced incomplete block design |
| BJ | Born–Jordan distribution |
| BJDn | Born–Jordan distribution of order $n$ |
| BV | Bounded variation |
| CNN | Convolutional neural network |
| CPU | Central processing unit |
| CS | Compressive sensing |
| CSC | Convolutional sparse coding |
| DCP | Deep coding problem |
| DCPP | Deep coding problem with pooling |
| DOA/DOD | Directions of arrival/departure |
| DS | Dynamical sampling |
| ECO | Exactly controllable |
| EF | Equiangular frame |
| ENR | Ratio of total transmitted energy to the noise energy |
| EOB | Exactly observable |
| ETF | Equiangular tight frame |
| ETFF | Equiangular tight fusion frame |
| FFT | February Fourier Talks |
| FTC | Fundamental Theorem of Calculus |
| GFT | Graph Fourier transform |
| IBM | International Business Machines |
| JB | John Benedetto |
| Lip | Lipschitz continuous |
| LMS | License in Mediaeval Studies |

| | |
|---|---|
| MIMO Radar | Multiple-input multiple-output radar |
| MIT | Massachusetts Institute of Technology |
| ML | Maximum likelihood |
| MRA | Multiresolution analysis |
| MTER | Multiscale transform based on an even-reversible subdivision |
| MUBs | Mutually unbiased bases |
| NLS | Nonlinear Schrödinger equation |
| NP-hard | Nondeterministic polynomial-time hard |
| NWC | Norbert Wiener Center for Harmonic Analysis and Applications |
| NYU | New York University |
| PDE | Partial differential equation |
| RCA | Radio Corporation of America |
| RGS | Reynolds Gauss–Seidel method |
| ROC | Receiver operating characteristic |
| RNLS | Rotational nonlinear Schrödinger equation |
| RS | Random sampling |
| SNR | Signal-to-noise ratio |
| SOR | Successive over-relaxation method |
| SPIE | Society of Photographic Instrumentation Engineers |
| SSL | Spatio-spectral limiting |
| SSN | Spectral neutral neighbor |
| SVM | Support vector machine |
| TA | Teaching assistant |
| UMCP | University of Maryland at College Park |

# Part I
# Introduction

The first part of this volume serves as its introduction and contains a single chapter in which D. Joyner summarizes the mathematical work of John, including an exhaustive list of his students and publications.

# John Benedetto's Mathematical Work

**David Joyner**

**Abstract** John Joseph Benedetto (JB) has been at the University of Maryland, College Park, since 1965. In this chapter, I will submit data that attests to JB's (a) large number of PhD students, (b) large number of papers (As a linear regression computation shows, the number of PhD students (per year) he advises and the number of papers (published per year) are both increasing, on average. See below.), and (c) remarkable outreach into the business sector, inviting cooperation between industry and his group of UMCP mathematicians that became the Norbert Wiener Center.

## 1 Brief Biography

On June 17, 1933, Vienna DiTonno married John ("Zip") Benedetto in Wakefield, Mass., the working class town just north of Boston where they were born and raised. Zip and Vienna were children of the depression and never got past 8th grade in school. Their only child, JB was born there six years later, on July 16, 1939.

Zip ran a pool hall in downtown Wakefield. While JB was an excellent student, in high school he got no further than trigonometry and solid geometry, as they did not teach calculus at the time. After school, to his mom's dismay, JB would visit the pool hall almost daily to help his dad run his business (and to play a little pool!). Another person who frequented Zip's pool hall was Robert McCloskey, a Harvard professor[1] and a collegiate billiards champion as an undergraduate. Seeing JB's academic talent, McCloskey told Zip[2] to encourage JB to apply to Harvard. However, after JB graduated from Malden Catholic High School in 1956, he applied

[1]According to archives of "The Crimson," McCloskey was appointed Chair of the Government Department at Harvard in 1958.

[2]Sadly, Zip passed away at age 44 in May of 1956, when JB was 16.

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M. Hirn et al. (eds.), *Excursions in Harmonic Analysis, Volume 6*, Applied and Numerical Harmonic Analysis, https://doi.org/10.1007/978-3-030-69637-5_1

(and was accepted) to Boston College instead of Harvard. At Boston College, he had inspirational teachers for his first- and second-year mathematics courses, convincing JB to major in mathematics. As a nod perhaps to McCloskey, as a senior, JB applied to Harvard for graduate school, and nowhere else. Fortunately for mathematics, he was accepted and, after graduating from Boston College in 1960, began to take courses from Gleason (real analysis), Widder (Laplace transforms), Mackey, and Walsh (of Walsh functions fame), among others. His master's degree was awarded by Harvard in 1962.

In the fall of 1962, JB left Harvard for the University of Toronto, where he studied with Chandler Davis,[3] who JB did not know of at Harvard. The reason for this move to Canada is not as simple as it sounds. It has really nothing to do with the fact that both Walsh and Davis had advisors in the Birkhoff family (and both on the Harvard faculty). At Boston College and Harvard, JB was very interested in Thomistic philosophy (the philosophy of Thomas Aquinas), and he knew the Pontifical Institute of Medieval Studies at St. Michael's College was a subset of the University of Toronto. His plan was to get a PhD in mathematics in 1964 and an LMS from the Pontifical Institute[4] along the way. JB even knew what he wanted to work on for his PhD: the Laplace transform of distributions and topological vector spaces.[5] So, in the summer of 1962, JB is a man who knows what he wants. However, once JB arrived in Toronto that fall, life had other plans. First, JB was assigned as a TA to Chandler Davis. That is how they met and started working together. Second, he started taking classes at the Pontifical Institute, but after the first philosophy course dropped his plan to get an LMS. In fact, JB was Chandler Davis' first PhD student, and they got along very well.[6] JB's PhD degree was awarded by the University of Toronto a few years later in 1964, and a revised version of his thesis was published in [1966b] (Fig. 1).

In 1964, after graduating, JB took a tenure-track job at New York University.[7] While a graduate student, during the summers JB worked at RCA in Burlington, MA. However, starting the summer of 1964 and part time during the academic year, JB worked at IBM Cambridge, instead of RCA. On a whim, JB left NYU for a tenure-track position at UMCP the following year. Except for visiting positions at MIT, the Mittag-Leffler Institute, and Scuola Normale Superiore, JB has been at UMCP since 1965. Once at UMCP, JB continued to consult for industry but, of course, eventually this work came under the umbrella of the Norbert Wiener Center (more on that below).

[3]From the it's-a-small-world department, Chandler Davis' PhD advisor was Garrett Birkhoff, son of George David Birkhoff, who was Joseph Walsh's PhD advisor.

[4]The LMS, a License in Mediaeval Studies or "Licentiate," is a kind of post-graduate degree awarded by The Pontifical Institute of Mediaeval Studies. There is no analogous degree offered in the United States.

[5]Inspired by Widder's course on the Laplace transform and the reading course with Mackey on distributions and topological vector space that he took at Harvard.

[6]JB has written about his connection with Chandler Davis in [2014a].

[7]That year, the Courant Institute moved to its current location, in Weaver Hall.


**Fig. 1** JB getting his PhD, with mom Vienna and grandpa Mr. DiTonno in 1964

As far as the arc of his career is concerned, JB's main mathematical inspirations are:

- Chandler Davis (PhD advisor) (Fig. 2),
- N. Wiener (whom JB never met),
- A. Beurling (whom JB never met),
- A. Gleason, one of his teachers at Harvard.

While JB has told me that many of the ideas he gets for papers are from thinking about mathematics while on a walk or traveling, I know there is another source: lots of hand computations. To illustrate this, I will tell a story connected with my PhD thesis (in 1983, shortly after the publication of my favorite paper of his, [1980a]). As a graduate student, he assigned me a problem connected with his Mathematische Annalen 1980 paper. I do not remember the problem, but I remember that after I solved it, I did not want to take credit for it if he already solved it but just did not, for whatever reason, add it to his paper. So one day, we had a meeting in his office about this psychological problem I was having. He said to resolve the matter, I could read the notes he made while writing the paper. Apparently, for each paper JB writes, he keeps his notes (or at least, did at the time) in a notebook. So, JB pulls out this massive notebook (the kind with the extra large rings) full of hand-written computations. That JB keep such a massive set of detailed notes for each paper was amazing to me at the time, and still is.

In his career, JB has been a Senior Fulbright-Hays Scholar, a SPIE Wavelet Pioneer, a Fellow of the American Mathematical Society, and a SIAM Fellow. His

**Fig. 2** Chandler Davis and JB in 1964

paper [1989b] won MITRE's Best Paper Award, and he was named Distinguished Scholar-Teacher by the University of Maryland in 1999.

Currently, JB is the Director of the Norbert Wiener Center for Harmonic Analysis and Applications (NWC), which he founded in 2004. It serves as an interface between funding agencies and industry with problems that can be solved using harmonic analysis by mathematicians at the NWC.[8] In its 15 years of existence, the mathematicians at the NWC have brought in over 7 million dollars in grants and have worked with over 15 industrial partners.[9] Besides dollar grants, many of these industrial partners have also supported numerous student internships. Hundreds have spoken at or attended the annual NWC conference, the February Fourier Talks, or FFT. The NWC is also connected with the *Journal of Fourier Analysis and its Applications*[10] and the *Applied and Numerical Harmonic Analysis* book series.[11]

As of summer 2019, JB has directed 58 PhD students (with several more in the pipeline). As of this writing, JB is in the top 100 of all PhD advisors worldwide.[12] JB does not co-author published PhD theses of his PhD students. Nonetheless, he has over 200 publications, of this writing, and over 80 co-authors (many of which are his former PhD students, if they do research with JB going beyond their thesis). What is even more impressive is that none of JB's academic publications were co-authored until 1983. At the time of this writing, JB's most frequent co-author (by far) is his UMCP colleague, Wojciech Czaja.

[8]Currently, JB, Radu Balan, Wojciech Czaja, and Kasso Okoudjou.

[9]For example, NIH, AFOSR, Siemens, MITRE, DARPA, ONR, NSF, and many more.

[10]For which JB is the Founding Editor-in-Chief.

[11]For which JB is the Series Editor.

[12]According to the database "Mathematics Genealogy Project."

## 2 Coda

In summary, JB's piercing intellectual curiosity has led to over 200 hundred refereed publications and about 60 PhD students, so far. Which reminds me of the old joke, "Great mathematicians never die, they just tend to infinity."

## 3 PhD Theses

Here is a list of the 61 (and counting) PhD students that JB has advised.

1. 1971, George Benke, *Sidon sets and the growth of $L^p$ norms*
2. 1977a, Wan-Chen Hsieh, *Topologies for spectral synthesis of the space of bounded functions*
3. 1977b, Fulvio Ricci, *Support preserving multiplication of pseudo-measures*
4. 1980, Ward Evans,[13] *Beurling's spectral analysis and continuous pseudo-measures*
5. 1983, W. David Joyner, *The harmonic analysis of Dirichlet series and the Riemann zeta function*
   NSF Post-Doc IAS 1984.
6. 1987, Jean-Pierre Gabardo, *Spectral gaps and uniqueness problems in Fourier analysis*
   Sloan Dissertation Fellowship 1986.
7. 1989, David Walnut, *Weyl–Heisenberg wavelet expansions: existence and stability in weighted spaces*
   Sloan Dissertation Fellowship 1988
8. 1990a, Christopher Heil, *Wiener amalgam spaces in generalized harmonic analysis and wavelet theory*
   NSF Post-Doc MIT 1990
9. 1990b, Rodney Kerby, *Correlation function and the Wiener–Wintner theorem in higher dimensions*
10. 1990c, George Yang, *Applications of Wiener–Tauberian theorem to a filtering problem and convolution equations*
11. 1991a, William Heller, *Frames of exponentials and applications*
12. 1991b, Joseph Lakey, *Weighted norm inequalities for the Fourier transform*
13. 1992, Erica Bernstein, *Generalized Riesz products and pyramidal schemes*
14. 1993a, Shidong Li, *The theory of frame multiresolution analysis and filter design*
15. 1993b, Sandra Saliani, *Nonlinear wavelet packets*
16. 1993c, Anthony Teolis, *Discrete signal representation*
17. 1994, Georg Zimmermann, *Projective multiresolution analysis and generalized sampling*

[13] Now named Celia Evans.

18. 1998a, Melissa Harrison, *Frames and irregular sampling from a computational perspective*
19. 1998b, Hui-Chuan Wu, *Multidimensional irregular sampling in terms of frames*
20. 1999a, Manuel Leon, *Minimally supported frequency wavelets*
21. 1999b, Götz Pfander, *Periodic wavelet transforms and periodicity detection*
22. 1999c, Oliver Treiber, *Affine data representations and filter banks*
23. 2000, Sherry Scott, *Spectral analysis of fractal noise in terms of Wiener's generalized harmonic analysis and wavelet theory*
24. 2001a, Matthew Fickus, *Finite normalized tight frames and spherical equidistribution*
25. 2001b, Ioannis Konstantinidis, *The characterization of multiscale generalized Riesz product measures*
26. 2002a, Anwar A. Saleh, *A finite dimensional model for the inverse frame operator*
27. 2002b, Jeffrey Sieracki, *Greedy adaptive discrimination: Signal component analysis by simultaneous matching pursuit with application to ECoG signature detection*
28. 2002c, Songkiat Sumetkijakan, *A fractal set constructed from a class of wavelet sets*
29. 2003a, Alexander M. Powell, *The uncertainty principle in harmonic analysis and Bourgain's theorem*
    Dissertation Fellowship 2000
30. 2003b, Shijun Zheng, *Besov spaces for the Schrödinger operator with barrier potential*
    Dissertation Fellowship 2000
31. 2004, Joseph Kolesar, $\Sigma - \Delta$ *modulation and correlation criteria for the construction of finite frames arising in communication theory*
32. 2005a, Andrew Kebo, *Quantum detection and finite frames*
33. 2005b, Juan Romero, *Generalized multiresolution analysis: construction and measure-theoretic characterization*
34. 2006a, Abdelkrim Bourouihiya, *Beurling weighted spaces, product-convolution operators, and the tensor product of frames*
35. 2006b, Aram Tangboondouangjit, *Sigma-Delta quantization: number-theoretic aspects of refining error estimates*
36. 2007a, Somantika Datta, *Wiener's generalized harmonic analysis and waveform design*
37. 2007b, Onur Oktay, *Frame quantization theory and equiangular tight frames*
38. 2008, David Widemann, *Dimensionality reduction for hyperspectral data* (Co-adviser, W. Czaja)
39. 2009a, Matthew Hirn, *Enumeration of harmonic frames and frame based dimension reduction* (Co-adviser, K. Okoudjou)
    Wylie Dissertation Fellowship 2009
40. 2009b, Emily King, *Wavelet and frame theory: frame bound gaps, generalized shearlets, Grassmannian fusion frames, and p-adic wavelets* (Co-adviser, W. Czaja)
    Wylie Dissertation Fellowship 2008

41. 2010, Christopher Flake, *The multiplicative Zak transform, dimension reduction, and wavelet analysis of LIDAR data* (Co-adviser, W. Czaja)
42. 2011a, Enrico Au-Yeung, *Balayage of Fourier transforms and the theory of frames*
43. 2011b, Avner Halevy, *Extensions of Laplacian eigenmaps for manifold learning* (Co-adviser W. Czaja)
44. 2011c, Nathaniel Strawn, *Geometric structures and optimization on finite frames* (Co-adviser, R. Balan)
45. 2012a, Kevin Duke, *A study of the relationship between spectrum and geometry through Fourier frames and Laplacian eigenmaps*
46. 2012b, Alfredo Nava-Tudela, *Image representation and compression via sparse solutions of systems of linear equations*
47. 2013, Rongrong Wang, *Global geometric conditions on dictionaries for the convergence of $\ell^1$ minimization problems* (Co-adviser W. Czaja)
48. 2014a, Travis Andrews, *Frame multiplication theory for vector-valued harmonic analysis*
49. 2014b, Alex Cloninger, *Exploiting data-dependent structure for improving sensor acquisition and integration* (Co-adviser W. Czaja)
    Wylie Dissertation Fellowship 2013
    NSF Postdoctoral Fellowship to Yale
50. 2014c, Tim Doster, *Harmonic analysis inspired data fusion with applications in remote sensing* (Co-adviser W. Czaja)
51. 2014d, Wei-Hsuan Yu, *Spherical two-distance sets and related topics in harmonic analysis* (Co-adviser A. Barg)
52. 2015a, Gokhan Civan, *Identification of operators on elementary locally compact abelian groups*
53. 2015b, Paul Koprowski, *Graph theoretic uncertainty principles*
54. 2015c, James Murphy, *Anisotropic harmonic analysis and integration of remotely sensed data* (Co-advisor W. Czaja)
55. 2016, Matthew Begué, *Expedition in data and harmonic analysis on graphs* (Co-advisor K. Okoudjou)
56. 2018a, Weilin Li, *Topics on harmonic analysis, sparse representations, and data analysis* (Co-advisor W. Czaja)
    Wylie Dissertation Fellowship 2017
57. 2018b, Mark Magsino, *Constant amplitude zero-autocorrelation sequences and single pixel camera imaging*
58. 2018c, Franck Njeunje, *Computational methods in machine learning: transport model, Haar wavelet, DNA classification, and MRI* (Co-advisor W. Czaja)
59. 2020a, Shujie Kang, *Generalized frame potential and problems related to SIC-POVMs* (Co-advisor K. Okoudjou)
60. 2020b, Chenzhi Zhao, *Non-harmonic Fourier analysis and applications*
61. 2020c, Kung-Ching Lin, *Nonlinear sampling theory and efficient signal recovery*

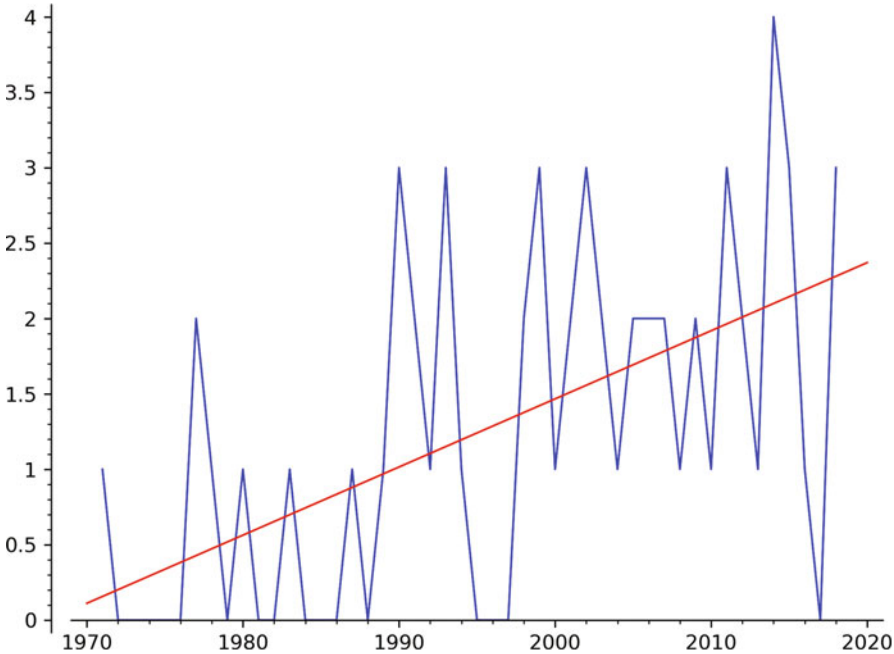


**Fig. 3** Linear regression on the number of JB's PhD students graduating per year

This is an average of about 1.2 PhD students per year. The list of pairs (year, number of JB's PhD students graduating that year) between 1971 and 2017 has best linear fit[14] $y = ax + b$, where $a = 0.0451\ldots$, $b = -88.8937\ldots$. In rough terms, the number of PhD students JB graduates per year increases by about 4.5% per year. The graph is in Fig. 3.

## 4 Papers

The majority of mathematical papers by JB deal with the representation of an "arbitrary" function[15] (typically on $\mathbb{R}$ or $\mathbb{R}^n$ and subject to some conditions), in one way or another (by a Fourier series, wavelet expansion, integral transform, and so on). The functions JB considers can be pretty general, but the point is that he represents them for us in a nice way and then uses such a representation to derive

[14] Again, thanks to SageMath.

[15] Of course, the question "what's a function?" immediately arises. Here, we include both "generalized functions" (e.g., a distribution in the sense of Schwartz) and Radon measures as functions.

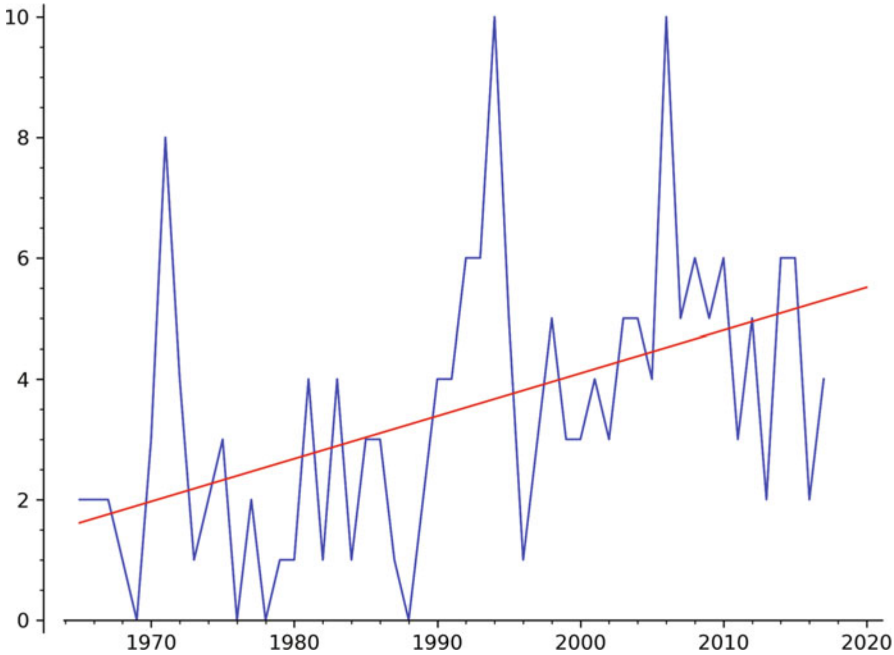


**Fig. 4** Linear regression on the number of JB's papers published per year

something useful. In many of his papers, JB takes such a representation and either (a) analyzes it to obtain estimates of a related quantity, or (b) applies it to an engineering problem, or (c) uses it to investigate a question in another field such as graph theory or analytic number theory.

Firstly, the list below includes some repetition (which I have tried to indicate). For example, some "technical reports" were revised and then submitted to a journal for publication. Secondly, some technical reports were not even submitted (e.g., they might have a more expository flavor). Finally, we note that some papers have very similar, or even identical, titles but are essentially unrelated (unless indicated).

Numerically, there is an average of about 3.48 papers per year. The list of pairs (year, number of papers published that year) between 1965 and 2017 has best linear fit[16] $y = ax + b$, where $a = 0.0708\ldots$, $b = -137.6162\ldots$. In rough terms, the number of papers JB publishes per year increases by about 7% per year. The graph is in Fig. 4.

[16]Thanks to the SageMath command `find_fit`.