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(Editors)

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Russel Cooper, Gary Madden, Ashley Lloyd and Michael Schipp
December 2005

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Introduction

Russel Cooper, Gary Madden, Ashley Lloyd and Michael Schipp

This volume analyses the economics of online markets and ICT networks. The most recent version of the Web has become a platform, and building new business on this platform is no longer a multi-million dollar undertaking. Start-up companies are leveraging a decade of innovation of technology that is proven, or very nearly so. What is different is that the Web's most recent innovative wave is driven by entrepreneurs not financiers. Search has provided a fundamental business model for many Internet-enabled companies. Past lessons led to building a new service that provides a sustainable revenue base from which to refine service offerings and the definition of a viable business model. Within this context, the volume examines the micro-economics of platform structure and firm competition within and between online markets, modern theoretical treatments of regulatory intervention in online markets and the consideration of forward-looking experimental analysis of demand for yet to be provided services.

The volume is divided into three parts: innovation and competition in online markets; regulation, pricing and evaluation with real options; and empirical approaches to market analysis. The structure of the volume is guided by the basic themes considered at the International Telecommunications Society's Africa-Asia-Australasia Regional Conference 'ICT Networks—Building Blocks for Economic Development', which was held in Perth Western Australia on 28-30 August 2005. The volume contains a selection of parallel session papers presented at the conference as well as five invited papers commissioned to augment the volume. The invited papers are authored by Abraham Hollander and Thierno Diallo (Chapter 1), Ian Harper, Ric Simes and Craig Malam (Chapter 2), James Alleman and Paul Rappoport (Chapter 9), Nadine Bellamy and Jean-Michel Sahut (Chapter 11) and Aniruddah Banerjee and Harold Ware (Chapter 12).

The Conference was sponsored by: Arnold & Porter LLP, BAKOM, BT, CRA International, Curtin Business School, Commonwealth Department of Communications, Information Technology and the Arts, Edward Elgar Publishers, Elsevier Science Publishers, France Telecom, IDATE, InfoCom Research, Inc., KT, Optus, NERA Economic Consulting, NTT DoCoMo, Physica-Verlag, Telcordia Technologies, Telenor and Telus. The Conference also received endorsements from the National Office for the Information Economy and the Organisation for Economic Cooperation. The conference was attended by 200 researchers, practitioners and regulators involved in telecommunications market analysis.

Part I: Innovation and Competition in Online Markets

The volume begins with six chapters concerned with firm behavior, viz., product bundling and development, and competition within online markets. The analyses relate both to markets for information goods and the platforms that act as a conduit for service delivery and a means to conduct market transactions. In particular, in Chapter 1, ‘Pricing and Bundling of Shared Information Goods: The Case of Cable Channels’, Abraham Hollander and Thierno Diallo consider the pricing and bundling decisions of a firm that sells a product shared on a regular basis among household members, in particular the bundling of television channels. The analysis explores under what conditions a cable firm will let households choose among the channels they subscribe to, and under what conditions the cable firm will offer them an all-channels-or-nothing offer. This analysis postulates that a potential viewer derives the same utility from every channel. How pricing and bundling decisions are affected by the structure of the fee that cable distributors pay for content to cable networks and the heterogeneity of households are explored. Ian Harper, Ric Simes and Craig Malam (Chapter 2 ‘The Development of Electronic Payments Systems’) argue that electronic payments systems are transforming economic processes. However, e-payments system innovation must lead economy-wide diffusion, e.g., economy-wide diffusion requires established viable institutional structures. Namely, customers must be confident that payment instruments are secure and reliable. This chapter examines the forces driving the spread of e-payments instruments, and why the use of some instruments burgeoned while others struggle. Secondly, the analysis considers whether there are valid public policy reasons for authorities to intervene in these markets and, if so, how they should intervene. In Chapter 3 (‘Behavioral Frictions in Online Contracting: Evidence from Yankee Auctions’), Mira Slavova seeks to identify bidding behaviors in online Yankee auctions and clarify relationships between behavior and allocation and payment outcomes. Main study concerns are identification of persistent bidding behavior; monotonic correspondence between bidding behavior and auction outcome; and between payment outcomes and submitted bid numbers. The study establishes a structure of behavior for submitted bids and explores their economic rationalization. An observed monotonic correspondence between behavior and auction outcomes is consistent with mechanism design. Alternatively, evidence suggesting behaviors result from heuristic problem-solving or reduced payoffs for bidders submitting multiple bids favor the transaction costs framework. Sumi Cho and Sang-Ho Lee (Chapter 4 ‘Online Channel Competition in a Differentiated Goods Market’) introduce an online transaction channel into a Hotelling linear city model where online and offline firms coexist in equilibrium. To examine the competition effect of an online channel, a symmetric case of two offline firms is considered and welfare loss to online business measured. Compared to the pure online competition case, the introduction of an online channel by a hybrid firm may reduce welfare when consumer offline channel transport cost are ‘large’ relative to firms’ online channel delivery costs. The analysis is extended to the asymmetric case whereby two offline firms supply different quality goods via an online

channel. Finally, how the different quality goods supplied by hybrid firms impacts on the welfare of online markets is examined. Chapter 5 by Gary Madden, Truong Truong and Michael Schipp ('Competition and Growth in Virtual Markets') attempts to provide insights into the understanding of fundamental tradeoffs faced by B&M and Virtual firms competing in Virtual marketplaces for Type-2 customers. An innovation contained in the paper is the establishment of a link between firm relative locations within the Virtual market to investment and ultimately profit. This innovation enabled the derivation of some testable predictions. Some ways to move beyond current model assumptions could be the consideration of non-uniform distributions of Virtual consumers (in particular non-symmetric distributions due to the skewed age distribution of customers making purchases online), the introduction of market power and allowing both the B&M and Virtual markets to be made open to competition. Armando Calabrese, Massimo Gastaldi and Nathan Leviaidi Ghiron (Chapter 6, 'Mobile Network Prospects: A Multi-sided Platform Analysis of Competition') conclude Section I by analyzing telecommunication markets before and after the introduction of an OMTP standard. Operators supply the same phone as producers but sell the good combined with a SIM card. The analysis assumes that firms act as a cartel to maximize joint profit, and producer and operator coalitions are distinct only when their telephones have different technical standards. A sequential game analyses three scenarios. In Scenario 1 the market is characterized by separate coalitions—producers supply telephones only and operators sell their telephones with SIM cards. Scenario 2 considers a market whereby an operator coalition with an OMTP standard competes with producers that only sell standard telephones. In Scenario 3 producers adopt the OMTP standard and mobile phones are not differentiated. A goal of the analysis is to identify conditions for the transition from Scenario 2 to Scenario 3.

Part II: Regulation, Pricing and Evaluation by Real Options

The next five chapters analyze traditional areas of concern to firms and regulators in telecommunications markets, i.e., technology choice and project evaluation when future revenue streams are uncertain. Kris Funston (Chapter 7, 'Real Options and Telecommunications Regulation') argues that when real option theory is applied to telecommunications investment the impact on the regulated fair rate of return depends on the uncertainty of returns. When demand is reasonably certain there is less of a case to apply the real options approach. Also, when an asset is reversible an investor has a put option. The overall result is that the allowed NPV fair rate of return may exceed the true user cost. Additionally, the option to wait could be less important when making investments and can potentially be ignored. Further, it is questionable whether an access provider should be allowed to recover a call option when the option has zero value in a competitive framework. When new network investment is irreversible, there is substantial uncertainty associated with the technology and future demand for services then access regulation may lead to a truncation problem arising. Finally, when an investor is able to vertically

integrate downstream retail functions with network investment the problem associated with regulatory truncation may be less important. In Chapter 8, 'A Discrete Real Options Approach to Access Pricing', Guillermo Lozano and José María Rodríguez demonstrate that cost-based access pricing can be assimilated into traditional NPV calculations by using a lattice modeling approach to calculate access prices. The approach assumes a finite time horizon in discrete- rather than continuous-time. From a policy perspective, discrete-time models are easier to interpret and understand. Additionally finite-horizon models are closer to real world applications than perpetual models. Conversely, such models may omit important properties such as the accessibility of the valuation method, tractability of model values and flexibility to incorporate competitive interactions. This framework is convenient as delay options are readily built into the NPV framework without any additional assumption. The extended-NPV approach can be viewed as a correction to ensure that traditional NPV assumptions are consistently applied. Some numerical examples to illustrate and interpret the behavior of the model are provided. Chapter 9 by James Alleman and Paul Rappoport ('Optimal Pricing with Sunk Cost and Uncertainty') argues that in using static models of the firm regulators make errors in determining a proper wholesale price as the opportunity cost delay is neglected. For an incumbent, the option is exercised and represents an opportunity cost. For a potential entrant delay need not be exercised should the regulator allow the purchase of access at below economic cost. Thus service-based entry is excessive and facilities-based entry is suboptimal. When a regulated paradigm is dynamic the converse holds: optimal prices are higher than for static calculations; only efficient service entry occurs as prices are set at the correct economic marginal cost; facilities-based entrants receive correct price signals; social welfare is maximal; an incumbent's valuation by financial markets is higher; and the cost of capital is lower than for a regulated paradigm. In Chapter 10 ('Efficient Spectrum Policy Using Real Options and Game Theoretic Methods'), Tae-Ho Lyoo, Jongwook Jeong, Hyun-Jung Lee and Jeong-Dong Lee treat spectrum allocations as acquiring a right to provide service and propose a model to combine real options and game theory methods applied to the valuation of converging communications services including WCDMA, HSDPA and WiBro. Namely, a service provider with a spectrum allocation can decide when to begin operations. An option concerning the type of service provided over this spectrum is also available. The real options approach explicitly incorporates the value obtained from decision making flexibility. However, in modern competitive communications markets there are often several service providers that affect the revenue and cost streams of market participants. Clearly, interactions among market participants should be considered in any valuation process via game theory. Nadine Bellamy and Jean-Michel Sahut (Chapter 11, 'A Real Options Approach to Investment Evaluation with a Network Externality') conclude this section with development of an analytical framework to understand and quantify the valuation of an investment when network externalities are present. Modeling starts from a probabilistic demand modeling stance and the inclusion of an option to terminate the project. These elements allow the more accurate valuation of the investment. Expected free cash flow values are larger on average than values estimated via the

deterministic NPV procedures. However, probabilistic NPV estimates require the calculation of confidence intervals to evaluate project risk. Conversely, the integration of the project termination option (put option) increases the project's value that is linked to the termination probability and savings from the early abandonment of the project. Finally, while the proposed modeling approach appears more realistic it raises questions as to the best means to obtain values for the termination parameters for effective scenario implementation

Part III: Empirical Approaches to Market Analysis

The final five complementary chapters concern innovative empirical approaches to market analysis. Andiruddah Banerjee and Harold Ware (Chapter 12, 'Mixed Logit Analysis of Carrier Market Share with Stated-preference Data') recognize that as the dramatic transformation of the telecommunications industry continues business planning and product management must be continually informed of new service demand. Stated-preference survey techniques are a means to elicit consumer preferences in telecommunications markets. The study illustrates the use of ranked stated-preference data to obtain market share and choice elasticity estimates for local telecommunications services purchased by business customers. A key study finding is that, in the newly competitive business telecommunications markets, incumbent service providers face more elastic demand responses than previously believed. Yong Yeop Sohn and Hun-Wha Yang (Chapter 13, 'Information Technology, Corporate Performance and Firm Size') develop several models to show that firm performance improves with IT adoption and use. Data on firms are obtained from a survey of Korean manufacturing establishments. Study findings indicate that adoption and use of IT is positively related to profitability and sales. Also, large-size firms typically use IT systems relatively more, and so the impact of IT use on performance is greater. However, no such difference between small and medium size firms is apparent. Actually, there is a divide in adoption of IT by firm size. Further, Korean firms use IT mainly for reducing costs and not marketing and customer management improvements. Finally, Korean informatization is significantly affecting firm performance when pursued to increase productivity and firm value. In Chapter 14 ('Contingent Valuation of Terrestrial DMB Services') Sangkyu Byun, Hongkyun Bae and Hanjoo Kim assess the potential economic value of emerging T-DMB services. Due an absence of market data, experimental data are obtained from a survey of potential users. A quantitative CVM analysis estimates the subscription fee users are willing to pay conditional on their demographic characteristics, experience with telecommunications services and attitude toward T-DMB. The estimated virtual market for T-DMB services is based on a monthly subscription fee provided to respondents in a DBDC questionnaire. The analysis concludes that DMB has a promising business model based on a relative high (compared to terrestrial TV) subscription fee. Revenue per frequency for T-DMB is higher compared to that for terrestrial TV, but is lower relative to mobile telephony service. Consequently, T-DMB has the potential to provide a new

catalyst to growth for Korean IT industry. Jae-Hyeon Ahn, Sang-Pil Han, Kyoung-Yong Jee and Moon-Koo Kim (Chapter 15, 'Consumer Preference for New Wireless Data Services') develop a hierarchical decision structure of consumer choice for emerging mobile services by breaking down the choice problem into a hierarchical decision structure for interrelated service attributes. The analytic hierarchy process allows analysis of consumer preferences for service attributes to determine the relative attractiveness of alternative new mobile data services. Results indicate that the economic costs are perceived most important. Also, the monthly charge is most important among costs, while transmission speed and service coverage are more the important among benefits. To successfully introduce PIS services transmission speed must be improved, especially considering that WLAN has an enhanced transmission speed of up to 54Mbps. Finally, as T-DMB service is the preferred mobile service there is merit in PIS providers offering bundled services, with DMB focusing on the broadcasting and PIS on mobile Internet service. The final chapter in this volume (Chapter 16, 'An International SME E-marketplace Networking Model') by Jaechon Park and Jemin Yang examines the *APEC Global B2B Interoperability Project* where a repository system using ebXML and Web Service is constructed to resolve technical interoperability issues. While the Project demonstrated the potential of e-marketplace networking, an M2M business model, in terms of technology and business is feasible although critical mass of buyers, sellers and products is not achieved. Clearly, post-incubation e-marketplaces must gain the trust of participants to attract more SMEs. Several benefits should be realized from the e-marketplace networking of agents including the lowering of SME international market entry barriers, reduction in the stagnation of information flow from excessive competition and improved competitiveness, and international trade facilitation.

Part I: Innovation and Competition in Online Markets

1 Pricing and Bundling of Shared Information Goods: The Case of Cable Channels

Abraham Hollander and Thierno Diallo

Introduction

This chapter looks at the pricing and bundling decisions of a firm that sells a product shared on a regular basis among household members. Examples of such products are computer software, telephone service and cable television subscription. The pricing of shared goods has attracted recent attention in the case of academic journals. An academic journal is a product sold to an individual subscriber who does not share it and to libraries that make the journal available for consultation to readers. Ordover and Willig (1978) have characterized the welfare maximizing combination of personal and institutional subscription prices. Liebowitz (1985) has shown that journal publishers rely increasingly on discriminatory pricing to capture revenues from library visitors whose benefit from sharing is enhanced by improved access to photocopying. Besen and Kirby (1989) found that a seller's profit increases as the result of sharing when it is less costly for consumers to distribute a work via sharing than for producers to do so by making copies. Varian (2000) has found a condition under which readers and publishers are better off when a portion of books in circulation is made available for sharing in libraries. Further, Bakos et al. (1999) showed that small scale sharing influences profits by affecting the disparity of buyers' reservation prices. They also establish that when the disparity of reservation prices among members of a group that share is larger than the disparity of the willingness to pay across groups, a seller can set prices that leave less surplus to consumers than in the absence of sharing. In this regard, sharing within groups achieves a result that is akin to bundling.¹

This chapter examines the bundling of television channels. The analysis explores when a cable firm will let households choose among the channels on offer, and when it will make an all-channels-or-nothing proposition. Some issues related to the bundling of channels by cable providers have been considered by Chae (1992) who examined the interplay between the costs of production of content and distribution per subscriber. Crampes and Hollander (2005) investigated the composition of channel bundles when subscribers differ in regard to their preferred content mix. Crawford (2004) showed that by bundling each of the top-15 cable networks in the US, cable distributors could increase profits by 4%.

This chapter focuses on a different set of determinants of bundling. The analysis assumes away differentiation among channels by postulating that a potential viewer derives the same utility from every channel. It examines how the pricing

¹ See, e.g., Adams and Yellen (1976) and Salinger (1995).

and bundling decisions are affected by: (a) the structure of the fee that cable distributors pay for content to cable networks; and (b) the heterogeneity of households. A priori, the relevant household characteristics include: the probability that individual household members will like particular programs; the utility that individual household members derive from watching certain programs; households' income; the number of individuals per household; and the number of television sets per household member. The latter is important because a household's willingness to pay depends on the probability of congestion. Congestion can occur when there are fewer television sets in a household than potential viewers. In such cases there is a positive probability that at least one household member is unable to watch a favourite program shown on a channel to which the household is subscribed. This means that the household's willingness to pay for cable, which aggregates the reservation prices of individual members, depends on the number of television sets in the household. With respect to the issue of congestion, the chapter is somewhat related to the literature on clubs.² In clubs, however, congestion arises when membership increases. In the case of cable television congestion correlates positively not with the number of subscriptions but with the number of household members per television set.

Next the chapter introduces notation, states basic assumptions and specifies the household's utility. A discussion of possible equilibria for the case where all potential viewers are identical follows. It serves to develop some intuition before examining the more general case where households are heterogeneous. The case where all viewers have the same probability of obtaining positive utility from programming but differ in the amount of utility they derive from it is then examined. That section shows that when payments to cable networks do not depend on the number of subscribers, distributor profits are highest when all channels are offered as a bundle. That is, the cost of content only determines the number of channels that the firm offers. When payment for content depends on the number of subscribers, the distributor may set prices that ensure that some households subscribe to fewer channels than the maximum number available. While such an outcome is possible, it emerges only for a small set of parameter values. The section also explores how the probability of this outcome depends on the probability distribution of the number of television sets per household, and on the probability that household members will obtain positive utility from watching television. A final section summarizes the results and discusses the relevance of this exploratory chapter to markets other than cable television.

Notation and Assumptions

A cable firm sells television channel subscription, and each channel shows a variety of programs. The firm may decide whether to supply enough programming for a single channel or for two channels. Use the index $i \in \{1, 2\}$ to identify the chan-

² See, e.g., Buchanan (1965) and Boadway (1980).

nels. Households that subscribe to cable television have either one or two members, indexed $j = \{a, b\}$. Household member j derives utility θ_j per unit of time from watching a program he or she likes, and a utility of zero from other programs. Let z_j^i denote the probability that household member j likes the programming shown on channel i .³ The probability that a household member likes a program is independent of the probability that another household member likes the same program. Member j who watches a liked program for a portion z of the time on channel i receives gross utility $\theta_j z_j^i$. Equivalently, $\theta_j z_j^i$ denotes the expected utility per unit of time that member j obtains from viewing channel i . Some households have one television set while other households have two television sets. When members of a single-set household differ with respect to the program they would like to watch at a particular time, they choose the program by tossing a coin. In a two-set household, each person watches the program on a different television set. The number of sets in the household is denoted $k = \{1, 2\}$.

The amount a household is willing to pay to gain access to a single channel or two channels equals the sum of expected utility that household members derive from access to the subscribed channels. Denote by p_1 and p_2 the prices at which the firm sells subscriptions for a single channel and for two channels when it adopts a policy whereby households can choose the number of channels they wish to subscribe.⁴ The price under a bundling policy is p_B . The cable firm pays the cable network a fee $T_i = m_i q_i + c_i$ for the content shown on channel i , where q_i is the number of channel subscribers and m_i is the per subscriber charge. This fee does not depend on size of the household or number of television sets in the household. The proportion of subscribers with two television sets is denoted λ .

The expected utility of a household that subscribes to a single channel i is:⁵

$$(\theta_a + \theta_b)z_a^i z_b^i + \theta_a z_a^i (1 - z_b^i) + \theta_b z_b^i (1 - z_a^i) = \theta_a z_a^i + \theta_b z_b^i, \tag{1.1}$$

whether it has one or two television sets. The expected utility of a household that subscribes to both channels depends on the number of television sets owned. This is a result of the existence of a positive probability that one household member will like a program on one channel while another person likes a program simultaneously shown on the other channel. In the Appendix it is shown that on defining

³ Alternatively z_j^i can be thought of as the portion of the time that channel i provides programming that member j likes.
⁴ Because of the symmetry assumptions it is not sensible to make a distinction between a channel that is part of a basic service and another that is optional.
⁵ The first term in Eq. 1.1 represents the contribution to gross utility from programs liked by household members a and b . The remaining terms are the contribution to utility of programs enjoyed by either a or b , but not both.

$\Theta \equiv \theta_a + \theta_b$ the expected gross utility of a two-set household from both channels is:

$$U_2^2(\Theta, z_a, z_b) = \Theta z_a z_b (4 - 2z_a - 2z_b + z_a z_b) + \theta_a z_a (1 - z_b)^2 (2 - z_a) + \theta_b z_b (1 - z_a)^2 (2 - z_b). \quad (1.2)$$

The expected utility of a one-set household from both channels is:

$$U_2^1(\Theta, z_a, z_b) = \Theta z_a z_b (3 - z_a - z_b) + \theta_a z_a (1 - z_b)^2 (2 - z_a) + \theta_b z_b (1 - z_a)^2 (2 - z_b). \quad (1.3)$$

Assume henceforth that the probability that a household member likes a program is the same for both channels, i.e., $z_a = z_b$. Also assume that $m_i = m$ and $c_i = c$, for $i = \{a, b\}$. To gain some intuition into the pricing and bundling decision, the special case where all consumers are the same is examined next.

Pricing and Bundling with Identical Consumers

From Eq. 1.1 through Eq. 1.3, Table 1.1 is constructed and displays household gross utility as a function of the number of channels subscribed and number of television sets. Table 1.1 shows that a household which has a single television set gains the amount $\Theta z(1-z)[1-z(1-z)]$ in gross utility when it subscribes to a second channel. A household that has two television sets adds $\Theta z(1-z)$ to utility by subscribing to a second channel. The contribution is larger for a two television set household because there is a positive probability that one household member watches a program on the first set while another watches on the second television set.⁶

Table 1.1. Gross Household Utility and Number of Channels and Television Sets

	One television	Two televisions
One channel	$U_1^1(\Theta, z) = \Theta z$	$U_1^2(\Theta, z) = \Theta z$
Two channels	$U_2^1(\Theta, z) = \Theta z[(2-z) - z(1-z)^2]$ $= \Theta z\{1 + (1-z)[1-z(1-z)]\}$	$U_2^2(\Theta, z) = \Theta z(2-z)$

⁶ Note that for a household with single-channel subscription, a second television set does not contribute to utility. However, an extra set contributes $\Theta z^2(1-z)^2$ to a household with two-channel subscription, viz., $\Theta z(1-z) - \Theta z(1-z)[1-z(1-z)] = \Theta z^2(1-z)^2 > 0$.

It is apparent from Table 1.1 that the contribution to utility from the second channel is a non-monotonic function of z . When z is very low, the second channel cannot substantially add to the probability that either one or two household members find a program they enjoy. When z is sufficiently high, subscribing to a single channel ensures a high probability that household members will be able to find a program they like. Therefore, the contribution to utility from the second channel is highest for intermediate values of z . When the firm sells a single channel, it is priced at the household's reservation price. When the firm offers two channels a la carte, and prices them uniformly, each channel is sold for a rate equal to the household's gain in gross utility from the second channel. The latter is $\Theta z(1-z)[1-z(1-z)]$ when $k=1$. This means that when the household has a single set, revenue from the sale of two channels is larger than revenue from the sale of a single channel if and only if $2(1-z)[1-z(1-z)] > 1$, or $z < 0.35$. For $k=2$, revenue from the sale of two channels is $2\Theta z(1-z)$. This revenue is larger than the revenue from the sale of a single channel when $z < 0.5$.⁷ When the firm sells both channels as a bundle, the price is set equal to the household's gross utility from two channels. Revenue is clearly higher than for the a la carte sale for all $z > 0$.

Heterogeneous Subscriber Preferences

Now assume that Θ has a distribution $F(\cdot)$ with corresponding density $f(\cdot)$ on the interval $[0, \Theta^{\max}]$. Also assume that z is the same for all viewers. Some households may subscribe to one channel, others to two channels. The choice of a household depends on the number of television sets it has, and on prices. When the firm sets prices p_1 and p_2 , households that subscribe to two channels have the preference parameters $\{\Theta, z, z\}$ that satisfy the conditions given by Eq. 1.4:

$$U_2^k(\Theta, z, z) - p_2 \geq 0 \quad \text{and} \quad U_2^k(\Theta, z, z) - p_2 \geq U_1^k(\Theta, z, z) - p_1, \quad (1.4)$$

for $k = \{1, 2\}$. The parameters of households that subscribe to a single channel satisfy Eq. 1.5 conditions:

$$U_1^k(\Theta, z, z) - p_1 \geq 0 \quad \text{and} \quad U_2^k(\Theta, z, z) - p_2 \leq U_1^k(\Theta, z, z) - p_1, \quad (1.5)$$

⁷ It is easily shown that when $k=1$ selling a second channel makes the largest contribution to profits when for $z=0.5$. When $k=2$ the contribution to profits of selling the second channel is largest when $z=0.25$. Subsequent sections assume non-linear rather than uniform pricing.

for $k = \{1, 2\}$. When the firm bundles, subscribers' parameters satisfy Eq. 1.6:

$$U_2^k(\Theta, z, z) - p_B \geq 0. \quad (1.6)$$

The preference index that leaves a household indifferent between subscribing to a single channel and not subscribing is Θ . Further, the preference indexes of households indifferent between subscribing to two channels and not subscribing are denoted Θ' and Θ'' , respectively for households that have one television set and households that have two sets. Finally, the preference indexes of households indifferent between subscribing to a single channel and subscribing to two channels are $\bar{\Theta}$ and $\tilde{\Theta}$, respectively, for households that have one set and households that have two sets. These preferences indexes depend on prices as follows:

$$\Theta \equiv \frac{p_1}{z},$$

$$\tilde{\Theta} \equiv \frac{p_2 - p_1}{z(1-z)},$$

$$\bar{\Theta} \equiv \frac{p_2 - p_1}{z(1-z)[1-z(1-z)]},$$

$$\Theta'' \equiv \frac{p_2}{z(2-z)}$$

and

$$\Theta' \equiv \frac{p_2}{z[(2-z) - z(1-z)^2]}.$$

When the distributor offers two channels, some households may subscribe to one channel, others to both channels. Three configurations are possible a priori. One configuration is where both the households that have two television sets and the households that have one set are split into a segment that subscribes to one channel, and a segment that subscribes to both channels (Case 1). Another configuration has all households with two television sets subscribing to both channels, and households with one television set divided into a segment that subscribes to one channel and a segment that subscribes to both channels (Case 2). A final configuration has all households with two television sets subscribing to both channels, and all households with one television set subscribing to a single channel (Case 3). Clearly, an equilibrium where households that own two sets subscribe to a single channel or are segmented into subscribers to both channels and subscriber to a single channel, while all households with one set subscribe to both channels, cannot arise.

Case 1: One-set and Two-set Households Are Segmented

This type of segmentation requires that $\underline{\Theta} < \Theta'' < \Theta' < \bar{\Theta} < \bar{\bar{\Theta}} < \Theta^{\max}$. Among two-set households, those that subscribe to both channels have $\Theta \in [\bar{\bar{\Theta}}, \Theta^{\max}]$, while those that subscribe to a single channel have $\Theta \in [\underline{\Theta}, \bar{\bar{\Theta}}]$.⁸ Among households with a single television set, those that subscribe to both channels have $\Theta \in [\bar{\Theta}, \Theta^{\max}]$, whereas those that subscribe to a single channel have $\Theta \in [\underline{\Theta}, \bar{\Theta}]$.⁹ The corresponding profit is:

$$\begin{aligned} \Pi = & (p_1 - m) \{ [F(\bar{\bar{\Theta}}) - F(\underline{\Theta})] + (1 - \lambda) [F(\bar{\Theta}) - F(\bar{\bar{\Theta}})] \} \\ & + (p_2 - 2m) \{ \lambda [F(\bar{\Theta}) - F(\bar{\bar{\Theta}})] + [1 - F(\bar{\bar{\Theta}})] \} - 2c. \end{aligned}$$

First-order necessary conditions are:

$$1 - F(\underline{\Theta}) - p_1 f(\underline{\Theta}) \frac{\partial \underline{\Theta}}{\partial p_1} + m f(\underline{\Theta}) \frac{\partial \underline{\Theta}}{\partial p_1} = 0 \quad (1.7)$$

and

$$\begin{aligned} & \lambda [1 - F(\bar{\bar{\Theta}}) - \bar{\bar{\Theta}} f(\bar{\bar{\Theta}}) + m f(\bar{\bar{\Theta}}) \frac{\partial \bar{\bar{\Theta}}}{\partial p_2}] \\ & + (1 - \lambda) [1 - F(\bar{\Theta}) - \bar{\Theta} f(\bar{\Theta}) + m f(\bar{\Theta}) \frac{\partial \bar{\Theta}}{\partial p_2}] = 0. \end{aligned} \quad (1.8)$$

When $m = 0$ these conditions are satisfied for $\bar{\bar{\Theta}} = \bar{\Theta} = \underline{\Theta}$. This means that the firm sets prices in a way that insures that all households subscribe to both channels. Specifically, it means that the profit maximizing p_2 equals the profit maximizing p_B , viz., profit maximization requires bundling. Under a uniform distribution of Θ , bundling yields a profit of $(z(2-z)/4)\Theta^{\max} - 2c$. By contrast, the profit from the sale of a single channel is $(z/4)\Theta^{\max} - c$. That is, when the fee paid by the cable distributor for content does not depend on the number of sub-

⁸ The condition $z > 2 - (p_2/p_1)$ implies $p_2/z < p_2/z(2-z) < (p_2 - p_1)/z(1-z)$. Also note that the condition $(p_2 - p_1)/z(1-z) < \Theta^{\max}$ cannot be met for $z \rightarrow 1$ or $z \rightarrow 0$, because the left-hand term converges to ∞ . That is, subscribing to a second channel only pays off for intermediate values of z .

⁹ It is straightforward to show that the condition $(1-z)[1-z(1-z)] < (p_2 - p_1)/p_1$ insures that $p_1/z < p_2/z[2-z-z(1-z)^2] < (p_2 - p_1)/z(1-z)[1-z(1-z)]$.

scribers, offering two channels generates higher profits than offering a single channel when $(z(1-z)/4)\Theta^{\max} > c$.

Now consider the case where $m > 0$. Note first that $\bar{\Theta} = \bar{\Theta} = \underline{\Theta}$ cannot be a solution to Eq. 1.7 and Eq. 1.8. Indeed, if this was a solution then:

$$[1 - F(\bar{\Theta}) - \bar{\Theta} f(\bar{\Theta}) + m f(\bar{\Theta}) \frac{\partial \bar{\Theta}}{\partial p_1}] = 0 < [1 - F(\bar{\Theta}) - \bar{\Theta} f(\bar{\Theta}) + m f(\bar{\Theta}) \frac{\partial \bar{\Theta}}{\partial p_2}],$$

as $\frac{\partial \bar{\Theta}}{\partial p_2} > \frac{\partial \bar{\Theta}}{\partial p_1}$. This would also imply $[1 - F(\bar{\Theta}) - \bar{\Theta} f(\bar{\Theta}) + m f(\bar{\Theta}) \frac{\partial \bar{\Theta}}{\partial p_2}] > 0$, be-

cause $\frac{\partial \bar{\Theta}}{\partial p_2} > \frac{\partial \bar{\Theta}}{\partial p_1}$. But then, the condition in Eq. 1.8 could not be true. For the special case where F is uniform on $[0, \Theta^{\max}]$ conditions in Eq. 1.7 and Eq. 1.8 become, respectively:

$$\frac{1}{\Theta^{\max}} [\Theta^{\max} - 2\underline{\Theta} + \frac{m}{z}] = 0 \quad (1.9)$$

and

$$\begin{aligned} & \frac{1}{\Theta^{\max}} \left\{ \lambda \left[\Theta^{\max} - 2\bar{\Theta} + \frac{m}{z(1-z)} \right] \right. \\ & \left. + (1-\lambda) \left[\Theta^{\max} - 2\bar{\Theta} + \frac{m}{z(1-z)[1-z(1-z)]} \right] \right\} = 0. \end{aligned} \quad (1.10)$$

Jointly Eq. 1.9 and Eq. 1.10 yields:

$$p_1 = \frac{1}{2} (z \Theta^{\max} + m) \quad (1.11)$$

and

$$\begin{aligned} p_2 &= \frac{z \Theta^{\max}}{2} \left[1 + \frac{(1-z)[1-z(1-z)]}{1-\lambda z(1-z)} \right] + m \\ &= p_1 + \frac{1}{2} \left[\frac{z(1-z)[1-z(1-z)]}{1-\lambda z(1-z)} \Theta^{\max} + m \right]. \end{aligned} \quad (1.12)$$

The prices given by Eq. 1.11 and Eq. 1.12 only constitute an equilibrium pair if: (a) the margins on the sale of the first and second channel are positive; and (b) the

prices generate the ranking $\underline{\Theta} < \Theta'' < \Theta' < \bar{\Theta} < \bar{\Theta} < \Theta^{\max}$. The existence of a positive margin on the sale of the first and second channel requires $m < \min[p_1, p_2 - p_1] = p_2 - p_1 = \frac{1}{2} \left[\frac{z(1-z)[1-z(1-z)]}{1-\lambda z(1-z)} \Theta^{\max} + m \right]$. This condition is met by virtue of Eq. 1.11 and Eq. 1.12.¹⁰ In regard to the ranking of the Θ s, $\bar{\Theta} < \Theta^{\max}$ requires $m < z(1-z)[1-z(1-z)] \frac{1-2\lambda z(1-z)}{1-\lambda z(1-z)} \Theta^{\max} \equiv \alpha_{\max}$. Also, $\Theta'' > \underline{\Theta}$ requires $p_2 > (2-z)p_1$ or $m > \frac{(1-\lambda)z(1-z)^2}{1-\lambda z(1-z)} \Theta^{\max} \equiv \alpha_{\min}$. The remaining conditions on the ranking of the Θ s are met by Eq. 1.11 and Eq. 1.12. The existence of a solution requires that $0 < \alpha_{\min} < m < \alpha_{\max}$. Note first that α_{\min} and α_{\max} are positive because $\lambda \in (0,1)$ and $z(1-z) < 1/4$. It is straightforward to show that $\text{sign}[\alpha_{\max} - \alpha_{\min}] = \text{sign}[z^2 - \lambda(1-z)(2z^2 - 1)]$. The latter entails that the prices given by Eq. 1.11 and Eq. 1.12 may constitute an equilibrium for some m and $\lambda \in (0,1)$ when $z < \sqrt{1/2}$, and that they cannot be in equilibrium when $z \geq \sqrt{1/2}$.

Case 2: One-set Households Are Segmented and Two-set Households Subscribe to Both Channels

Such an equilibrium requires that $\bar{\Theta} < \Theta'' < \underline{\Theta} < \Theta' < \bar{\Theta} < \Theta^{\max}$. Among households with a single television set, those that subscribe to a single channel have $\Theta \in [\underline{\Theta}, \bar{\Theta}]$ and those that subscribe to both channels have $\Theta \in [\bar{\Theta}, \Theta^{\max}]$. Among households with two sets those with a preference parameter $\Theta \in [\Theta'', \Theta^{\max}]$ subscribe to both channels, those with a lower value of Θ do not subscribe. Therefore, a firm that offers both channels has profit:

$$\Pi = (p_1 - m)(1 - \lambda)[F(\bar{\Theta}) - F(\underline{\Theta})] + (p_2 - 2m) \left\{ \lambda [F(\bar{\Theta}) - F(\Theta'')] + [1 - F(\bar{\Theta})] \right\} - 2c.$$

First-order necessary conditions are:

$$1 - F(\bar{\Theta}) - \bar{\Theta} f(\bar{\Theta}) + m f(\bar{\Theta}) \frac{\partial \bar{\Theta}}{\partial p_1} = 1 - F(\underline{\Theta}) - \underline{\Theta} f(\underline{\Theta}) + m f(\underline{\Theta}) \frac{\partial \underline{\Theta}}{\partial p_1} \quad (1.15)$$

and

¹⁰ When the condition is not met it is unprofitable to seek subscribers for a second channel.

$$\begin{aligned} & \lambda[1 - F(\Theta'') - \Theta'' f(\Theta'') + 2m f(\Theta'') \frac{\partial \Theta''}{\partial p_2}] \\ & + (1 - \lambda)[1 - F(\bar{\Theta}) - \bar{\Theta} f(\bar{\Theta}) + m f(\bar{\Theta}) \frac{\partial \bar{\Theta}}{\partial p_2}] = 0. \end{aligned} \quad (1.16)$$

Observe that when $m = 0$, Eq. 1.15 and Eq. 1.16 are satisfied only when $\bar{\Theta} = \underline{\Theta} = \bar{\Theta}$. Note also that the latter cannot be a solution for $m > 0$. For the special case where F is uniform on $[0, \Theta^{\max}]$ Eq. 1.15 and Eq. 1.16 yields:

$$\frac{1}{\Theta^{\max}} [\Theta^{\max} - 2\bar{\Theta} + \frac{m}{z(1-z)[1-z(1-z)]}] = \frac{1}{\Theta^{\max}} [\Theta^{\max} - 2\underline{\Theta} + \frac{m}{z}] = 0 \quad (1.17)$$

and

$$\begin{aligned} & \lambda \frac{1}{\Theta^{\max}} [\Theta^{\max} - 2\Theta'' + \frac{2m}{z(2-z)}] \\ & + (1 - \lambda) \frac{1}{\Theta^{\max}} [\Theta^{\max} - 2\bar{\Theta} + \frac{m}{z(1-z)[1-z(1-z)]}] = 0. \end{aligned} \quad (1.18)$$

Jointly Eq. 1.17 and Eq. 1.18 yields:

$$p_1 = \frac{1}{2} \left[\frac{z\lambda(2-z)}{2-z-\lambda z(1-z)^2} \Theta^{\max} + m \right] \quad (1.19)$$

and

$$\begin{aligned} p_2 &= \frac{z(2-z)}{2\lambda} \left[\frac{(2-z)[1-z(1-\lambda)] - z\lambda(1-z)^2}{2-z-z\lambda(1-z)^2} \right] \Theta^{\max} + m \\ &= \frac{1}{\lambda} p_1 + \frac{z(2-z)}{2\lambda} \left[\frac{(1-z)(1-z+\lambda z^2) - z(1-\lambda)}{2-z-z\lambda(1-z)^2} \right] \Theta^{\max} + \frac{2\lambda-1}{2\lambda} m. \end{aligned} \quad (1.20)$$

From Eq. 1.19 and Eq. 1.20 $\bar{\Theta} < \Theta''$ requires:

$$m < \frac{2-z}{\lambda[2-z-\lambda z(1-z)^2]} \Theta^{\max} A,$$

where