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Jacopo Maria De Ponti

# Graded Elastic Metamaterials for Energy Harvesting



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Politecnico di Milano  
Milan, Italy

ISSN 2191-530X                      ISSN 2191-5318 (electronic)  
SpringerBriefs in Applied Sciences and Technology  
ISSN 2282-2577                      ISSN 2282-2585 (electronic)  
PoliMI SpringerBriefs  
ISBN 978-3-030-69059-5              ISBN 978-3-030-69060-1 (eBook)  
<https://doi.org/10.1007/978-3-030-69060-1>

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# Preface

Interconnection between machines, devices and people is one of the key aspects of the contemporary society and its working paradigms, which are driving the so called *Fourth industrial revolution* or *Industry 4.0* (Hannover Fair, 2011). In this perspective, *Machine to Machine communication* (M2M) and *Internet of Things* (IoT) are able to provide increased automation, improved communication and self-monitoring, in different environments and industrial processes. A driving central force of innovation is represented by smart sensors and devices, which generate the data and allow further functionality from self-monitoring and self-configuration to condition monitoring of complex processes. This new generation of sensors has to be small, economically feasible and autonomous. The reduced power requirements of recent small electronic components make on-chip energy harvesting solutions a promising alternative to batteries or complex wiring. Amongst others, vibration-based energy harvesting schemes are particularly attractive due to the numerous and continuous sources of vibration present in the environment. However, due to the low amount of energy involved in common ambient vibrations, it is interesting to focus, or trap, waves from a larger region outside the device into a confined region in the near vicinity of the sensor. This can be obtained by exploiting the unprecedented properties of metamaterials and structuring materials in wave manipulation; once the wave is localised, by using electromagnetic, electrostatic or piezoelectric effects, efficient conversion from elastic to electric energy can be achieved.

The aim of this book is to offer a conceptual roadmap to guide the reader in the field of metamaterials and wave manipulation devices for energy harvesting applications. Two separate bodies of literature investigate acoustic/elastic metamaterials and vibration energy harvesting technologies. This book has the ambition of amalgamating these two fields inside a common framework, guiding the reader through increasing complexity. In addition, for the first time, graded multiresonator designs are proposed for energy harvesting, quantifying their advantages with respect to conventional solutions. Since this book was written single-handedly, probably contains many mistakes, and misses certain developments and contributions. To all distinguished colleagues and collaborators, I wish to present my apologies for any omissions in my text. The aim of this book is to reach a broad audience, from graduates to researchers. For this reason, it cannot be considered as an exhaustive book on both wave manipulation

and energy harvesting. However, it can be useful for people approaching the world of metamaterials for the first time (first chapters), as well as researchers interested in graded metamaterial designs (last chapter).

As with every valuable research work, the results presented here would not have been possible without the continuous help and support of many people encountered in the last years. These people are dear friends, more than collaborators. A special thank goes to Dr. Gregory Chaplain, who has the merit of having formulated theoretical and analytical models for the reversed Umklapp conversion, rainbow trapping, and topological rainbow in SSH systems. He is for me a stimulating and interesting source of ideas, from whom I have much to learn. I thank Prof. Richard Craster: all this work would not have been possible without his contribution and continuous support. He is for me a great mentor, both on the scientific and human level, and this book wants to be a memory of the collaboration period spent together at Imperial College London. Last, but not least, I thank Dr. Andrea Colombi for the continuous suggestions and ideas, and Profs. Raffaele Ardito and Alberto Corigliano for their precious advice and trust my research work.

Milan, Italy  
January 2021

Jacopo Maria De Ponti

# Contents

<b>1</b>	<b>Introduction</b>	1
1.1	Preliminary Comments and Outlines	1
1.2	An Introduction to Inhomogeneous Media and Metamaterials	3
	References	6
<b>2</b>	<b>State-of-the-Art of Engineered Materials for Energy Harvesting</b>	7
2.1	Parabolic Acoustic Mirror (PAM)	7
2.2	Resonant Defect Inside a Phononic Crystal (PnC)	9
2.3	Acoustic Funnel	10
2.4	Multifunctional Device Based on Local Resonance	11
2.5	Lensing	15
2.6	Acoustic Black Hole (ABH)	17
2.7	Graded Designs: The Rainbow Effect	19
	References	22
<b>3</b>	<b>One-Dimensional Inhomogeneous Media</b>	27
3.1	The Propagation of Mechanical Waves	27
3.2	One-Dimensional Periodic Structures	29
3.3	One-Dimensional Aperiodic Structures	39
3.4	Physical Interpretation of Local Resonance	51
	References	59
<b>4</b>	<b>Graded Elastic Metamaterials</b>	61
4.1	Problem Statement in Dynamic Elasticity	61
4.2	Wave Propagation in Thin Elastic Plates	65
4.3	Wave Propagation in Thin Elastic Plates with Resonators	69
4.4	Wave Propagation in Unbounded Media and Half-Spaces	71
4.5	Wave Propagation in Half-Spaces with Resonators	75
4.6	Reversed Mode Conversion Using Umklapp	80
	References	87



- 5 Advanced Multiresonator Designs for Energy Harvesting** ..... 91
  - 5.1 An Introduction to Piezoelectricity ..... 91
  - 5.2 Graded Metasurface for Energy Harvesting ..... 93
  - 5.3 From Rainbow Reflection to Rainbow Trapping ..... 104
  - 5.4 Topological Rainbow Trapping in Graded SSH Systems ..... 110
  - References ..... 118
  
- Conclusions** ..... 121

# Chapter 1

## Introduction



**Abstract** In recent years the world is facing an extraordinary diffusion of the Internet of Things (IOT) concept which is the idea of building smart and autonomous sensors networks which can help us in sensing, understanding and controlling our environment. For this idea to be effective, new sensors should be small, barely costless and autonomous. Recent advances in low-power consumption circuitry have enabled ultrasmall power integrated circuits which can run with extremely low amount of power. For these reasons, the area of energy harvesting has captivated both academics and industrialists, to self power, or at least compensate, the power consumption of small electronic devices, using ambient waste energy. The integration of such systems with recent metamaterial technologies allows to dramatically increase the energy available for harvesting, and the operational bandwidth.

### 1.1 Preliminary Comments and Outlines

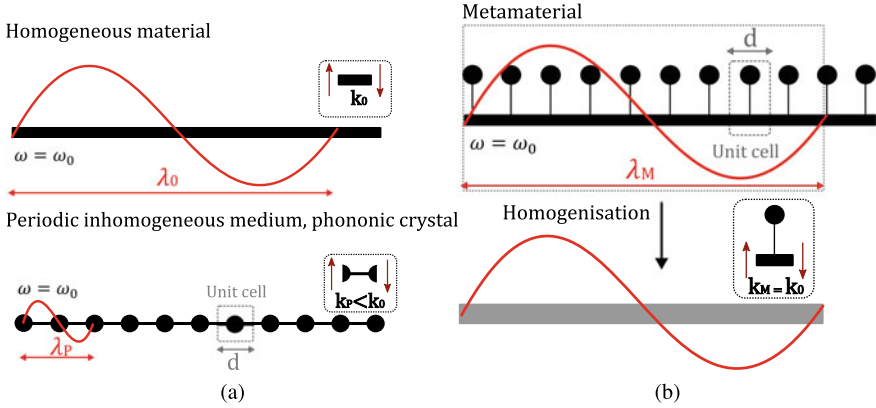
A very widespread form of energy is represented by mechanical vibrations, which exist with variable intensity in almost every environment. However, one of the main issues is that the energy involved is usually very low, and spread over a broadband low frequency spectrum. For these reasons, in order to fully take advantage of this form of energy, it is required a device that: (i) **focus or localise waves**: it is possible to increase the absorbed energy since it comes from a larger spatial region, or due to confinement in specific positions; (ii) **work in a broadband regime**: the energy in common ambient spectra can be completely used, and the performance is less affected by input changing; (iii) **can be easily manufactured**: mass scale production is possible with affordable costs.

As shown in Chap. 2, several works have been reported in literature to partially or totally address the aforementioned key requirements. Most of them rely on the design of structuring materials and metamaterials, i.e. engineered systems able to show efficient wave manipulation properties. These peculiar properties usually come from the concept of *band gap*, i.e. the existence of frequency ranges from which the propagation of waves is not allowed. For this reason, Chaps. 3 and 4 thoroughly analyse this concept in lumped systems and continuous media respectively. In Chap. 3, basic concepts on wave propagation in homogeneous and inhomogeneous media are introduced for both periodic and aperiodic structures, providing an interpretation of band gap through energy considerations and phase diagrams. Comparable attenuation capabilities are demonstrated for periodic and aperiodic structures through analytical lumped models, numerical and experimental results. Finally, a physical interpretation of the phenomenon of local resonance is provided by energy and phase considerations. This theory is generalised in Chap. 4 to elastic continua, with specific reference to plates and half-spaces. Wave propagation in thin elastic plates is firstly considered using the *Kirchhoff–Love* theory. This is followed by a numerical (using the Finite Element Method, FEM) study of wave propagation in thin elastic plates with resonators, with specific attention on the effect of the plate thickness and rod height on the band gap performance. The concept of grading is then introduced as a way to obtain broadband band gaps and the *rainbow effect*, i.e. the spatial signal separation depending on frequency. Similarly, the problem of wave propagation in elastic half-spaces is considered, going from the classical theories of Rayleigh, Shear (S) and Pressure (P) wave propagation, to the problem of the so called *metawedge*, i.e. an array of resonators able to provide rainbow effect or mode conversion depending on the direction of the incident Rayleigh wave with respect to the array grading. Finally, reversed mode conversion from surface Rayleigh to S and P bulk waves is demonstrated leveraging on the Umklapp phenomenon. This mechanism allows to manipulate surface waves, focusing the elastic energy in specific regions of space for a broadband input frequency spectrum. In this part, analytical, numerical (FEM) and experimental results for a device at the ultrasonic frequencies are reported and compared. All these concepts converge together with piezoelectric materials, to study and design piezo-augmented arrays of resonators. The choice of using as working paradigm graded arrays of resonators is motivated by their superior characteristics and versatility of use for wave manipulation, as widely shown in the recent scientific literature of the field. In addition, because such systems already contain a collection of resonators, the inclusion of vibrational energy harvesters is straightforward, leading to truly multifunctional metastructures combining vibration insulation with harvesting. First, a *rainbow reflection* device made of an array of rods with increasing height on an elastic beam is considered. By introducing a harvester inside this system, it is possible to harvest more energy with respect to the case of a single harvester to the same structure and at the same location. In addition, the harvesting bandwidth can be enlarged by introducing other harvesters in different positions. The same mechanism is also found when placing the same array of rods on an elastic half-space. In this part, analytical, numerical and experimental results are reported and compared. A second design shows that an increase of the total transduced energy along time can be

obtained using *rainbow trapping* instead of *rainbow reflection*. This is demonstrated designing a graded symmetry broken array of resonators on an elastic beam, and numerically modeling (FEM) the trapping properties for energy harvesting. Finally, a third design shows that it is possible to locally increase the transduced energy by exploiting *topological edge modes* in a graded Su–Schrieffer–Heeger system. Broadband energy harvesting performances and strong robustness to impurity defects are demonstrated through numerical models (FEM) on an attractively compact device.

## 1.2 An Introduction to Inhomogeneous Media and Metamaterials

We introduce here the concept of *metamaterial*, as intended in this work in the setting of wave propagation phenomena. The word *metamaterial* etimologically means, from the greek prefix  $\mu\epsilon\tau\alpha$ , a material with properties beyond what we expect to find in naturally occurring, or conventional materials. Unfortunately, the general nature of this definition could lead to an improper use, arriving paradoxically to say that every material, from a certain perspective, is a metamaterial. Firstly, it is important to notice that there is a strong dependance on the level at which the phenomenon is observed. In wave propagation phenomena (the ones considered here), it is reasonable to take as a reference scale of observation the *wavelength*  $\lambda$ , i.e. the wave spatial period. In other terms, we can adopt a material Representative Elementary Volume (REV) of the size of  $\lambda$  for the *homogenised* material, i.e. an homogeneous material with equivalent global properties. If  $\lambda$  is much larger than the smallest constitutive material element (called *unit cell*, in analogy with atoms at smaller scale) we can consider the REV as homogeneous, as typically done in continuum mechanics. If this REV shows unusual physical properties (with respect to conventional materials), we denote it as a metamaterial. Using this convention, a material is considered as a metamaterial if shows unusual properties at strong *subwavelength* scale. On the contrary, it is simply an inhomogeneous medium. In the setting of elasticity, inhomogeneous media with a periodic structure are usually called *Phononic Crystals* (PnC), coming from the term *phonon*, i.e. the quantum vibration, and *crystal* which suggest the idea of something regularly repeated in space. Specifically, the term phononic is used to say that the phenomenon involves phonons, i.e. vibrations, and that it occurs at the wavelength scale between the unit cells. The simplest example of a phononic crystal is a spring mass chain [1]. However, due to generality, the term inhomogeneous media will be adopted here instead of phononic crystal, adding the specific term of *periodic* or *aperiodic* to define the spatial arrangement of the cells. If we define the medium as inhomogeneous, we implicitly assert that inhomogeneity is a constitutive property of the material and then the wave is interacting with each unit cell, i.e. its wavelength is comparable to the unit cell size. For this reason, we adopt the term medium instead of material, since the term material belongs to something on which *average macroscopic properties* can be defined, without looking at the specific



**Fig. 1.1** **a** Homogeneous material and inhomogeneous periodic medium, i.e. phononic crystal where the wavelength (from the homogenised system) is of the order of the lattice size  $d$ . **b** Metamaterial concept: due to local resonance, an interaction with large wavelengths is possible, thus defining an equivalent homogeneous material with the same effective properties. While the stiffness of the PnC is lower with respect to the homogeneous material ( $k_P < k_0 \rightarrow \lambda_P < \lambda_0$ ), this is not true for the metamaterial ( $k_M = k_0 \rightarrow \lambda_M \approx \lambda_0$ )

microstructure. Looking this in a more engineering rather than physical perspective, the term *structure* can be adopted, meaning an assembly of subelements, represented by the unit cells. Figure 1.1a compares a homogeneous material and inhomogeneous periodic medium (PnC). Even if they are made of the same material, the wavelength associated to the same frequency is remarkably different, mainly due to the different global stiffness. On the other hand, Fig. 1.1b shows the concept of metamaterial. The addition of resonators on the homogeneous material maintains the same global stiffness, thus slightly changing the wavelength. In this way, the system can be regarded as nearly homogeneous. Since the mechanical concept of continuum is meaningful if the behaviour is strongly subwavelength (Fig. 1.1b), metamaterials can be based in essence only on resonance effects, in accord with [2] (this mechanism will be explained in detail in this chapter).

This is coherent with the seminal work of the group of Ping Sheng at HKUST [3], that provided the first numerical and experimental evidence of a localised resonant structure for elastic waves propagating in three-dimensional arrays of thin coated spheres. Adopting this interpretation, metamaterials, contrary to phononic crystals, can be even aperiodic. However, they are usually periodically defined, due to the peculiar properties given by periodicity, as well as the reduced computational complexity and the existence of analytical closed form solutions.

The work of Liu in acoustics opened the door to the design of elastic metamaterials, but this concept was preceded by important discoveries in electromagnetism and optics. In 1967, the Russian physicist Victor Veselago published a visionary paper [4] in which electromagnetic media with simultaneously negative permittivity  $\epsilon$  and magnetic permeability  $\mu$  were shown to be characterized by a negative refractive