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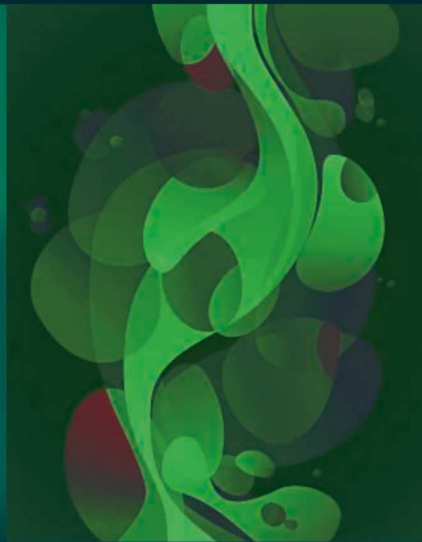

Len Pismen

ACTIVE MATTER WITHIN AND AROUND US

From Self-Propelled Particles to Flocks and Living Forms

Springer

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Active Matter Within and Around Us

From Self-Propelled Particles to Flocks and Living Forms

Springer

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To Yael, nothing without you

Preface

Active matter is within us, in our cells. It is around us, within living organisms and their gatherings, and even in sands and waters enlivened by winds, waves, and swimming plankton. The study of active matter is a new and rapidly expanding field on the frontier between physics and biology, also pertinent to fledgling biomorphic technologies. It is still young but already too extensive to be covered across-the-board.

The style of this book is less technical and less formal than in available reviews. I avoid mathematical equations (even though it is contrary to what I am used to in my own work) and include as many illustrations as possible, aspiring to reflect ideas with minimum technical details. Since the narrative is centered on modeling, I also avoid biochemical details, but always remember and at times remind the reader that molecular interactions, not yet understood in all relevant details, are the all-important players behind the scenes.

Studies of active matter, especially in their biological applications, are a part of mainstream twenty-first century science, which is an industry based on expensive precision experiments and computations, and dominated by large teams. Yet, the underlying ideas and methods of the theory of active matter are rooted in the nonlinear analysis that flourished late in the last century, and prominent figures in this field are schooled in statistical and nonlinear physics.

My own involvement in the field is marginal, but this makes it more interesting to write about it and leaves more freedom for unbiased judgement. I am awed by the inner workings of *life*, compared to which the field of active matter is a *game*. But it is a sophisticated and fascinating game, perhaps the most fascinating game played by the physicists of the early twenty-first century, between nine-eleven and the coronavirus pandemic. Its future stands open, as it penetrates ever deeper into the intricacies of *living* matter.

I appreciate the help of friends and colleagues for discussing details of their work and granting me permission to reproduce images in this book. My contacts with the group of Prof. Stanislav Shvartsman in New York and Princeton were particularly helpful in getting a better feel for the biological experiments.

Haifa, Israel, October 2021 Len Pismen

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Introduction

For the natural philosophers of the Enlightenment, the notion of *active matter* would be associated with the gnawing question of whether the newly discovered laws of mechanics could be extended to life itself. There were strong theological objections: the *élan vital* emanating from God had to be breathed into passive matter to bring it to life. Vitalism persisted throughout the rational 19th century; even Louis Pasteur supported his belief in vitalism by disproving the spontaneous generation of organisms from non-living matter. With time, matter became more and more familiar. The four classical elements, earth, water, air, and fire, could now be associated with the four states of matter, solid, liquid, gas, and plasma, governed on the macroscopic level by thermodynamic laws. Yet, it was not until the late 20th century that the wealth of behavior of far-from-equilibrium nonlinear systems was finally appreciated, and understood to be essential for life itself.

When does matter become active? The modern usage of this term is surprisingly novel: it first appeared in the paper by Ramaswamy and Simha (2006) as the appellation of the fledgling research field that has been rapidly expanding since then. It largely applies to interactions involving living organisms and their constituent parts, but also extends to biomorphic materials and designs. The studies of active matter could flourish only in this century: they are impossible without computer power and precision experiments, penetrating into the microscopic mechanisms of active motion.

Was there active matter before life? Perhaps, this is a matter of a definition. According to Chaté (2020), "*active matter physics is about systems in which energy is dissipated at some local level to produce work*". This definition, though part of a much more narrowly focused view, implies an affirmative answer, widening the notion of active matter to all non-equilibrium processes. On the other hand, Needleman and Dogic (2017) assert that "*active matter is different from traditionally studied non-equilibrium phenomena*", in which "*the entire system is driven away from equilibrium by energy provided through an external macroscopic boundary*". They cite the statement by Gottfried Wilhelm Leibniz in a letter to Damaris Masham: "*I define the Organism, or natural Machine, as a machine in which each part is a machine* [...], *whereas the parts of our artificial machines are not machines*".

Marchetti et al (2013), in an all-round review co-authored by seven physicists, define active matter as "*composed of self-driven units, active particles, each capable of converting stored or ambient free energy into systematic movement*", referring to Schweitzer (2003), who did not use this term, however.

Both points of view have their merit. Certainly, neither stars nor geologically active planets are driven externally. A star is also a structured "machine" driven by its own nuclear fusion energy, and governing its coterie of planets, some of which themselves are structured machines. Our Earth possesses, besides Sun's radiation, her own radioactive energy source that keeps her interior fluid, driving the magnetic dynamo and the continental plate tectonics. Her oceans are convective machines, with their network of mighty currents governing the climate, and occasional outbursts of hurricanes. And perhaps we should recall at this point Lovelock's (1979) *Gaia*, the living planet, of which we, as well as other living "machines", are constituent parts. Leibniz could not know anything like this, but if he were resurrected in our day, he couldn't fail to acquire a laptop and a smartphone and, with his natural curiosity, would agree that the parts of our artificial machines *are* machines. However, all this would bring us too far. As once remarked a fictitious Russian writer, one cannot embrace the unembraceable. The working definition of "active matter" may be confined to the problems investigated by scientists who are actually working in the field.

What does it include? First of all, indeed, self-driven "active" particles. What exactly constitutes a "particle" is interpreted very widely. It might just be a particle moving in a fluid under the action of a gradient of some field, e.g., the concentration of some species, or electric potential, or magnetic force. It might be a particle, not active in its own right, but entrained in a vibrating granular layer; part of a "dissipative structure" not unlike convective structures in the oceans, atmosphere, or laboratory studies. A granular layer was indeed the first medium in which a dissipative non-equilibrium structure was discovered, by Faraday (1831). It might be a microbe wandering in search of nutrients. It might be a bird or a fish, a part of a flock or a shoal. It might be a person in a crowd. Note that this list includes a rather loose notion of *self*-driving, but in any case, the "self" of the "particle" is always subjected to at least some degree to external forces and/or collective interactions.

There is a rather artificial distinction between "dry" and "wet" active matter, which originated more from the kind of modeling than from the nature of the "particles" themselves. Interactions in "wet" matter are mediated by the medium they are immersed in, while in the case of "dry" matter, the medium is ignored, sometimes rightly, when its influence is a minor factor, and sometimes just to make the problem tractable, e.g., ignoring fluid mechanics when modeling flocks.

The "particles" are often assigned certain intrinsic characteristics, most commonly, their *orientation*. The orientation may determine their preferred direction of motion, e.g., with respect to gradients of an external field, or the character of their interactions, e.g., the tendency to align with their neighbors. The basic types of orientation are *vector*, denoting the direction, or *nematic*, implying an alignment without a definite direction, as in a vector without an arrow. Both kinds of orientation may or may not be qualified by their strength.

But active matter is not necessarily dispersed in the form of particles. The notion of intrinsic orientation comes from the physics of condensed matter, which engaged with directed interactions in solids and fluids long before studies of active matter came on the scene. Moreover, most scientists active in these studies have been nurtured on problems in the physics of condensed matter. The orientation of magnetic or electric dipoles is a major factor determining the structure of crystalline solids – but solids, with their *solidity*, are not properly qualified to be active, unless dispersed. Liquids are more akin to active matter, and of all liquids, most kindred are partially ordered liquids, *liquid crystals*, combining various degrees of orientation with fluidity. This is the source of another research direction, active liquid crystals, which added the attribute of activity to the intrinsic properties of oriented liquids. The resulting models may involve assemblies of orientable particles but are largely "wet", and involve continuum rather than discrete description.

From here, the direct route for further expansion goes to *soft matter*, flexible solids and colloids endowed with some kind of activity. This area is most relevant for biology, as tissues, cells, and their components have a similar texture and consistency. Moreover, *activity* is their innate feature: they are machines consisting of machines, and themselves parts of greater machines, which are already beyond the realm of mere *matter*, even soft and active.

Living matter, driven by the complex interplay of genes and proteins, is far more complicated than models of active matter can afford. Specific molecular interactions are of most concern to biologists and biochemists, but it is gradually understood that factors of a general nature, common to active and living matter, play a substantial role in cellular and developmental processes. Finally, advanced biomorphic technology creates soft robots and artificial swimmers imitating, still on a basic level, soft natural machines. All of this will come out in our narrative.

Chapter 1
Polar Flocks

1.1 Birds of a Feather Fly Together

The most important feature of active "particles" (whatever their nature) is *collective motion*. It may arise surprisingly easily, without any leader, or external field, or geometrical constraints. The words "how birds fly together" appear in the title of a paper written by physicists (Toner and Tu, 1995), not by ornithologists. But what is a physicist's bird? It is a *vector* moving in the direction shown by its arrow. This is reasonable: animals move looking ahead, in the direction of their body axis. Animals are social, and capable of comprehending their surroundings and adjusting their behavior accordingly, thereby initiating collective motion.

The simplest example of the spontaneous emergence of directed motion based on these premises is the Vicsek model (Vicsek et al, 1995) sketched in Fig. 1.1. It lacks any specific physical mechanism, and just assumes that each particle (or "bird") adjusts its alignment to conform with the average alignment vector (shown by the arrows in the picture) of its neighbors within a certain range, and moves in the acquired direction. The adjustment is biased by random noise, and keeps repeating, as motion brings the particle to another neighborhood. The particles always move in the direction determined by their orientation with a constant speed; thus, polarity is

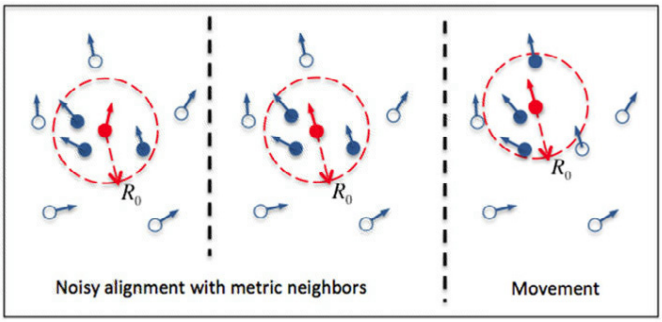

Fig. 1.1 Scheme of the Vicsek model. A test particle (*red*) aligns imperfectly with neighbors (*dark blue*) within a range shown by the *dashed circle*, and then moves in this direction (Ginelli, 2016)

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Fig. 1.2 (**a**) The V-formation of Canada geese (Spedding, 2011). (**b**) A vortex trail shed by a fish (Weihs, 1973). (**c**) A flock of pigeons (Spedding, 2011)

not distinguished in this model from velocity. Vicsek's flock is a basic representative of what is called "dry" active matter abstracted from a surrounding medium.

The model seems to be oversimplified, but this was the key to its success (over 3600 citations at the time of writing). An advantage (or, depending on your point of view, a disadvantage) of models of this kind is that they may produce pictures bearing a superficial resemblance with observations even when they do not reflect the actual way the system in question operates. Alignment of migrating birds or fish is often motivated by hydrodynamics: in this way, they save propulsion effort. Canada geese migrate in a characteristic V-formation (Fig. 1.2a), just as plane squadrons fly, since such an orderly arrangement reduces drag. Fish also save energy when swimming in a shoal. A vortex trail shed by a fish or a bird induces immediately behind it a stream opposite to the swimming or flying direction (Fig. 1.2b), which would require the immediate follower to exert extra energy. However, if the follower's position is shifted laterally, it comes into the zone where the induced velocity is directed favorably. Weihs (1973) calculated that the best position is midway between two fish of the preceding row, obtaining a difference in relative speed of up to 30% between the best and worst lateral positions.

On the other hand, Usherwood et al (2011) have found that, for pigeons (Fig. 1.2c), flocking is energetically costly, so social factors apparently overrule hydrodynamics in this case. A simple universal model is the most reasonable choice when the alignment is of a social origin, arising from sensory inputs and information exchange, essential, e.g., when cohesion of the group is a necessary means of defense against attack by a predator.

1.2 Phenomenology of Vicsek's model

The Vicsek model is amenable to agent-based numerics. Depending on the average density and the level of noise, the particles may be perfectly aligned, or disordered, or ordered locally but disordered elsewhere (Fig. 1.3); the ordering can also be intermittent when particles align for a while and then succumb to noise. Even when the ordered state persists, the interaction network is permanently rearranged due to the active motion of individuals, continuously changing their neighbors.

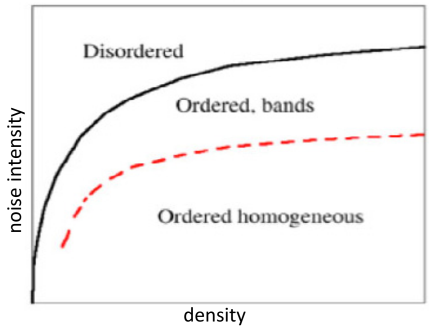

Fig. 1.3 Qualitative phase diagram showing the dependence of ordering on the flock density and the strength of noise (Ginelli, 2016)

Aligned particles tend to gather into dense self-propelling blobs, while particles in dilute surroundings move at random. Dense blobs, in their turn, tend to gather into ordered bands transverse to the alignment and, hence, propagation direction, whereupon they move more or less uniformly, but may dissipate if the local coherence is lost. Such a global ordering precedes slower velocity alignment. In the snapshot of the simulation of the Vicsek model (Katyal et al, 2020) shown in Fig. 1.4a, separation of dense and dilute domains is still very far from forming ordered bands. Nevertheless, the propagation directions are already well aligned in the two clusters, joined only by a narrow track of particles and far from forming dense ordered bands. After long transients, bands such as those shown in Fig. 1.4b periodically arrange in space, similar to the smectic phase in liquid crystals on a

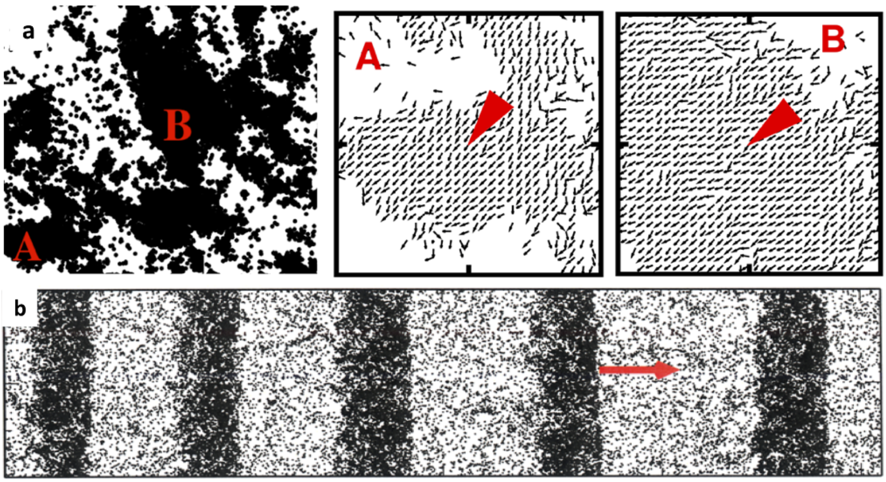

Fig. 1.4 (**a**) Snapshot of a part of the Vicsek flock, showing zooms of the density clusters A, B in the two panels on the right. The *red arrows* show the propagation direction (Katyal et al, 2020). (**b**) Propagating bands (Chaté et al, 2008)

macroscopic scale. The number of bands increases with increasing density at constant noise (Solon, Chaté, and Tailleur, 2015).

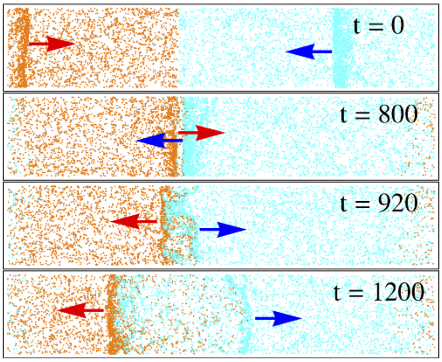

Fig. 1.5 Two counter-propagating bands passing each other in a soliton-like fashion (Ihle and Chou, 2014)

A rather unrealistic feature, arising in simulations by Ihle and Chou (2014) and demonstrated in Fig. 1.5, is a soliton-like behavior of counter-propagating bands, which pass each other with a minimal distortion following a collision. When the front of brown particles hits the front of turquoise particles, small groups of highly aligned brown particles tunnel through that front and continue going in the same direction. Once behind the turquoise front, they re-orient the oncoming turquoise particles, forming a new dense band. At the same time, brown particles that are left further behind their front, and hence less ordered and less dense, are also forced to return by groups of aligned turquoise particles. While returning, the freshly reoriented brown particles form a new dense front going in the opposite direction[1]. Real birds would hardly change their plans in such a way; anyway, two flocks, making use of the third dimension, would avoid each other.

Inhomogeneities (though not in the form of ordered bands) are prominent in actual bird flocks studied in the field. Field studies challenge the way the Vicsek model quantifies the interactions in animal aggregations. As a flock rearranges, sometimes even temporarily splitting, its density and structure are continuously changing but its coherence is never lost, as it would be in the Vicsek model when mutual distances exceed the interaction range. Ballerini et al (2008) confirmed, by quantifying their observations of large starling flocks, that interactions are actually based on *topological* rather than metric distance: each individual interacts with a fixed number of neighbors, commonly six to seven, irrespective of their spacing. This interaction mechanism allows the flock to maintain cohesion against strong perturbations. Of course, metric interactions should be relevant for inanimate active particles tied by physical forces, especially in "wet" active matter where interactions are carried by a surrounding medium.

Simulations of the "topological" version of the Vicsek model with metric-free interactions (Ginelli and Chaté, 2010) support the fact that they have a cohesive tendency. Unlike the metric model, the phase transition to collective motion with reduced noise is almost abrupt. There is no segregation into an ordered "liquid" and a disordered "gas" phase, because neighbors in dilute regions are never disconnected, and therefore low density does not necessarily induce disorder. The simulation with

[1] Thomas Ihle, private communication

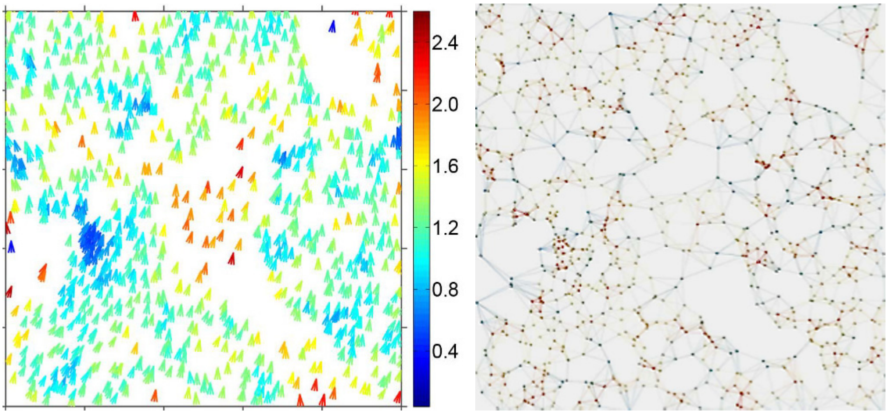

Fig. 1.6 Non-metric interactions with seven nearest neighbors. *Left*: A snapshot of particle positions; the colors show the interaction range. *Right*: An instantaneous interaction network (Komareji and Bouffanais, 2013)

non-metric interactions among seven nearest neighbors illustrated by Fig. 1.6 reveals many different metric interaction ranges correlated with density fluctuations.

Of course, reality is more complicated than anything that a basic model can predict. Observations showed waves of "information transfer" within a group (Attanasi et al, 2014), influenced by the rank of individual birds (Nagy et al, 2010). Leadership was also shown to be important in cooperative transport of large cargo by ants that necessarily requires alignment of their pulling force (Feinerman et al, 2018). We may recall the tendency of humans to obey their leaders. Experiments with groups of volunteers (Dyer et al, 2009) showed that a small informed minority (5%) could effectively guide a large uninformed group to a target by making decisions by consensus. We know from political experience that humans often follow even incompetent leaders, with or without coercion. Animal herds and flocks manage to sustain group integrity during long migrations with no verbal communication and no enforcement.

1.3 The Flock as a Continuum

Models defining motion and interaction rules are very handy for carrying out computations; cheap computer power has even encouraged the study of classical kinetic and hydrodynamic problems by racing millions of simulated molecules in the bowels of a computer (even though any number even the most powerful computer can handle is diminutive compared with the actual number of molecules in a water droplet we can see with the naked eye, and molecular interaction potentials are far more sophisticated than those commonly used in computer models). Nevertheless, there is something that cannot be obtained in this way: the analytical insight gained by the great physicists of the past, which necessitates a continuum description. Toner and

Tu (1995) undertook to translate the Vicsek model (still fresh from the press at the time) into the language of the continuum.

It is straightforward to write down hydrodynamic equations of motion with anisotropic viscosity and added noise in such a way as to define the local velocity and density of a set of particles. The task is simplified by a special feature of the Vicsek model: it identifies orientation with velocity. This eliminates a difficult task of combining hydrodynamic equations with “elastic" equations that take care of keeping the alignment intact (more on this in Sect. 2.1). The continuum model immediately suggests a heuristic argument for the stabilizing effect of motion. If the equations are rewritten in the coordinate frame moving with the average velocity of the “flock” (still unknown), it becomes clear that neighbors of a particular “bird” will be different at different moments of time, depending on inhomogeneities in the velocity field. Therefore originally distant “birds” may interact at a later time, which effectively extends the interaction range and stabilizes the ordered phase.

Yet, there are inconvenient facts, which make the entire undertaking questionable. Active fluids have no equation of state, and *pressure* cannot be defined in a constructive way, except within a narrow class of models (Solon, Fily, et al, 2015). One can compress an active fluid, increasing its average density. In a common fluid, the required work is unequivocally determined by pressure, but in an active medium it depends on the way particles interact with the confining walls. Different forces and hence different amounts of work are needed to reach the same final density when compressing with a hard wall or with a soft enclosure, into which particles bump gently. This can be demonstrated quantitatively by separating two parts of a container by a mobile wall with asymmetric interaction potentials on its two sides. The partition moves to equalize the two wall-dependent pressures, resulting in a steady state with unequal densities in the two chambers (Fig. 1.7). In equilibrium fluids, even oriented ones like liquid crystals, the normal force per unit area on any part of the boundary is independent of its orientation. This is not so in active media, as long as the propulsion speed is anisotropic, even if the particles are oriented isotropically.

Moreover, realignment of Vicsek’s particles *does not conserve momentum*, while the hydrodynamic equations of Toner and Tu, like those of standard hydrodynamics, are based on momentum conservation, which is presumed to be valid in some average sense, and include the gradient of this ill-defined variable, pressure. The Vicsek model does not even possess the Galilean invariance inherent in classical

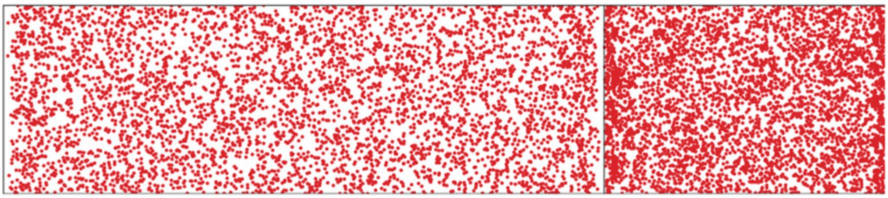

Fig. 1.7 Simulated spontaneous compression/expansion of an active fluid due to an anisotropic wall potential (Solon, Fily, et al, 2015).

mechanics, since the particles move with a constant velocity defined in a specific ("laboratory") frame of reference. An easy way to forget these objections is to tell oneself that the Vicsek model is after all a model rather than a law of nature, and the continuous equations of Toner and Tu can be viewed as just another model, related in some respects but different in others.

Once a system of differential equations, justified or not, is in place, it can be studied in a standard way. One can test the linear stability of its "trivial" homogeneous solutions propagating with a certain speed, and find their symmetry-breaking bifurcations leading to a family of propagating bands. Solon, Caussin, et al (2015) worked in this way with a modified system, simplified in some respects compared with that of Toner and Tu but including a nonlinear term that tends to keeps the absolute value of the velocity constant, as assumed in the Vicsek model. They obtained what Chaté (2020) calls an *embarrassingly* large family of linearly stable solutions: periodic patterns, solitary bands, phase-separated domains. All this variety disappears when a noise term, missing in the simplified system, is reintroduced, causing a unique solution to be selected. This is consistent with the role of noise in equilibrium systems where noise facilitates transition from metastable states to a state with the minimal energy, but energy is neither well defined nor conserved in active matter. Chaté (2020) lists this among current riddles, asking: "How do we understand the selection of a unique solution observed at microscopic and fluctuating hydrodynamic levels but which is not present at the deterministic hydrodynamic level?"

Clearly, noise is an essential component, and formulating phenomenological hydrodynamic equations (even if fully justified) is only the beginning of the road. Theories of phase transitions (Landau et al, 1980) have to account not just for fluctuations but for their *correlations* in time and space. This cannot be done in a straightforward way, since pair correlation functions depend on triple correlation functions, and so on, necessitating a cut-off under some assumptions. A powerful method in the theory of critical phenomena is the renormalization group, based on invariance to scale transformations (Goldenfeld, 1992). The theory is precise in 4D, and applications to physical dimensions are commonly based on the $4-\epsilon$ expansion, where ϵ is assumed to be small, even though $4-3=1$ is not what would normally be treated as a small parameter. Toner and Tu (1998) boldly applied the renormalization group further down, in 2D, even though admitting that some parameters of their theory are not scale-invariant, and in spite of all above mentioned shortcomings, claimed that their predictions (or retrodictions?) are in agreement with the computations by Vicsek et al (1995).

1.4 Towards Statistical Description

The central theoretical question is understanding both similarities to and distinctions from the behavior of passive matter obeying thermodynamic laws. The phenomenology of the Viscek model, as reflected in Fig. 1.3, looks at first sight to be not so very different from that of ordinary fluids: the liquid and gas phase differ by density, and the phase transition happens with a changing temperature that determines the level of random noise. Liquid and gas can coexist, as dense (ordered) and dilute (disordered) domains in Fig. 1.4. Even giant intermittent fluctuations are possible in common fluids near a critical point.

Another passive system similar in some respects to the Vicsek model is the magnetic XY model describing interactions of spins on a lattice (Kosterlitz and Thouless, 1973). Its microscopic "particles" are vectors of a fixed length, like Vicsek's "birds", and likewise the orientation of these vectors tends to adjust to their immediate environment, but there are no density inhomogeneities and, of course, no active motion. It is known that fluctuations at *any* non-zero temperature prevent formation of a phase with the long-range order in this system. A typical simulation of the XY model, like the one in Fig. 1.8, shows a disordered state with a number of topological defects – vortices with orientation rotating by 2π around their cores. Although activity often brings about disorder, it turns out in this case that it is *motion* which is responsible for the long-range order in the Vicsek model, and topological defects do not appear in simulated flocks.

Contrary to all distinctions between active and equilibrium systems, the notion of *entropy* appears to play a special role in collective motion. Bialek et al (2012) posed the question of how to derive the overall distribution of velocities, given the matrix of correlations between velocities of individuals measured in an actual flock. Infinitely many distributions are consistent with the measured correlations, but the successful choice was the one describing a system that is as random as it can be, i.e., having the maximal entropy, while still matching the data. The maximum entropy model correctly predicted, with no free parameters, the propagation of order throughout a flock of starlings based on pairwise interactions between birds. It also confirmed the conclusion by Ballerini et al (2008) that interactions are ruled by topological rather than metric distance.

Yet, the absence of meaningful definitions of such basic thermodynamic variables as energy and pressure is a clear sign that thermodynamics is actually a misnomer when it comes to active matter. Thermodynamics of equilibrium processes is based on statistical mechanics, but activity violates basic principles like equipartition of energy among various degrees of freedom and detailed balance, and noise in assemblies of

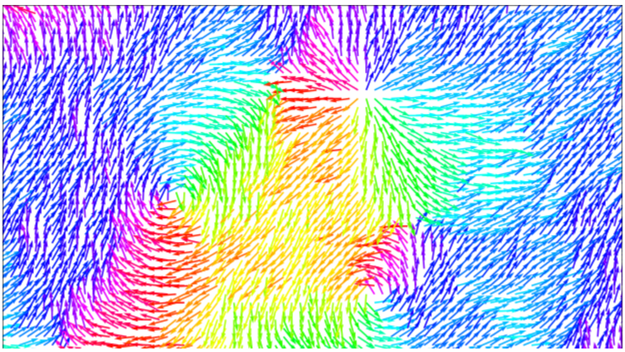

Fig. 1.8 Monte Carlo simulation of the XY model, showing a disordered state with a number of topological defects (vortices). The coloring shows the direction of vectors (by ChrisJLygouras - Own work, CC)

active particles is likely to be more structured than thermal noise. It makes it even more challenging to derive macroscopic dynamics from underlying microscopic interaction rules. This has prompted sophisticated statistical theories aspiring to explain on a deeper level the dynamics of the utterly simple and not too realistic model central to this chapter. The motivation was that the results might be applicable to a wider class of phenomena in "dry" active matter.

Bertin et al (2006) set the aim of building a statistical description of a modified version of the Vicsek model, wherein particles may change their propagation direction (but not the magnitude of their velocity) either by a random "kick" or as a result of a binary collision that aligns the velocities of the two particles to their average direction shifted by random noise. Recall that the original model allows interactions between many particles within a set distance, but is less realistic in another respect, as each particle adjusts its direction independently, while in the model by Bertin et al, binary collisions conserve the momentum before being biased by thermal noise. Restricting to binary interactions is common in molecular dynamics, and momentum conservation, with the random factors accounted for by effective viscosity, leads to hydrodynamic equations far better justified than those of Toner and Tu. All coefficients, including inertial effects, linear and nonlinear viscosities, and even a cubic term setting the magnitude of the velocity, are computed from the microscopic parameters of the model.

Yet, the theory is problematic in one important aspect: it is based on Boltzmann's hypothesis of "molecular chaos", which assumes that colliding particles are *uncorrelated*. This is justified when the mean free path is large compared to the radius of interactions, something which may be true in the "gas" phase but is not true at realistic densities when order is established. Indeed, Boltzmann's hypothesis fails even in equilibrium liquids, and the theory of the liquid state has to involve sophisticated approximations accounting for molecular correlations. Ihle (2011) pointed out this drawback and put forward an alternative theory based on the weak gradient expansion, mirroring the theory of weakly inhomogeneous media (Chapman and Cowling, 1970). His derivations led to more complicated nonlinear hydrodynamic equations than those of Bertin et al, but with coefficients also expressed through the microscopic parameters of the model.

Unlike Bertin et al, Ihle took as the basis of his theory the original Vicsek model evolving, as in standard agent-based computations, by discrete steps – indeed, by large steps, as he relied on the assumption that the free path between collisions was much larger than the interaction range. Such a highly discrete character of the motion is a questionable point in Ihle's theory, as time steps of agent-based computations are relatively small and, of course, real flocks interact continuously. The continuum statistical theory of flocks (or, more generally, of "dry" active matter) remains unsettled, as reflected, in particular, by the two contending papers in a discussion issue on active matter (Peshkov et al, 2014; Ihle, 2014), which contain both justification of these efforts and convincing mutual criticisms of the rival theories.

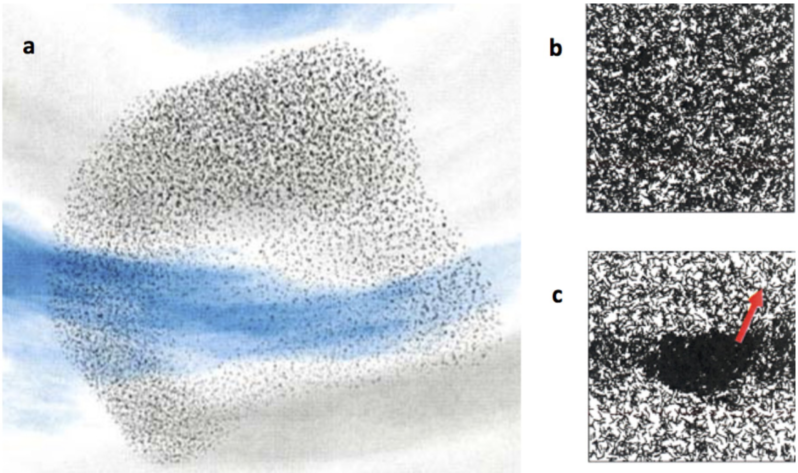

Fig. 1.9 (**a**) Snapshot of a 3D flock, with short-range attractive interactions; sky and cloud colors are added to make it look like a real bird flock. (**b**) A homogeneous 2D flock with medium strength long-range cohesive interactions. (**c**) Consolidated flock with strong cohesion. The *red arrow* indicates the mean direction of motion (Chaté et al, 2008)

1.5 Variations on Vicsek's model

There have been a number of attempts to make the basic Vicsek model model more realistic and versatile. Collective motion is possible only at finite density: if the Vicsek model evolved in an infinite domain, the "birds" would eventually fly apart. Grégoire et al (2003) added pairwise short-range attractive interactions to the Vicsek model. Simulations of this model produced compact flocks superficially similar to a dense flock of birds, especially when images of clouds and blue sky are added, as in Fig. 1.9a. An alternative attempt to consolidate a flock was to add long-range cohesive interactions of hydrodynamic origin, introduced in a rather vague way through advection by a fluid flow generated by the particles themselves and applicable to bacteria rather than to birds (Chaté et al, 2008). In the simulations of

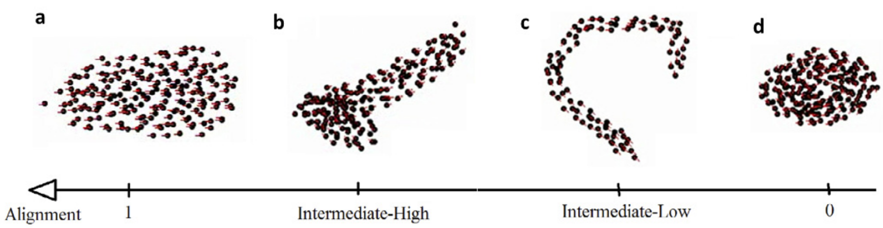

Fig. 1.10 Changes in the shape of a cohesive flock with changing alignment strength (Strömbom et al, 2015)

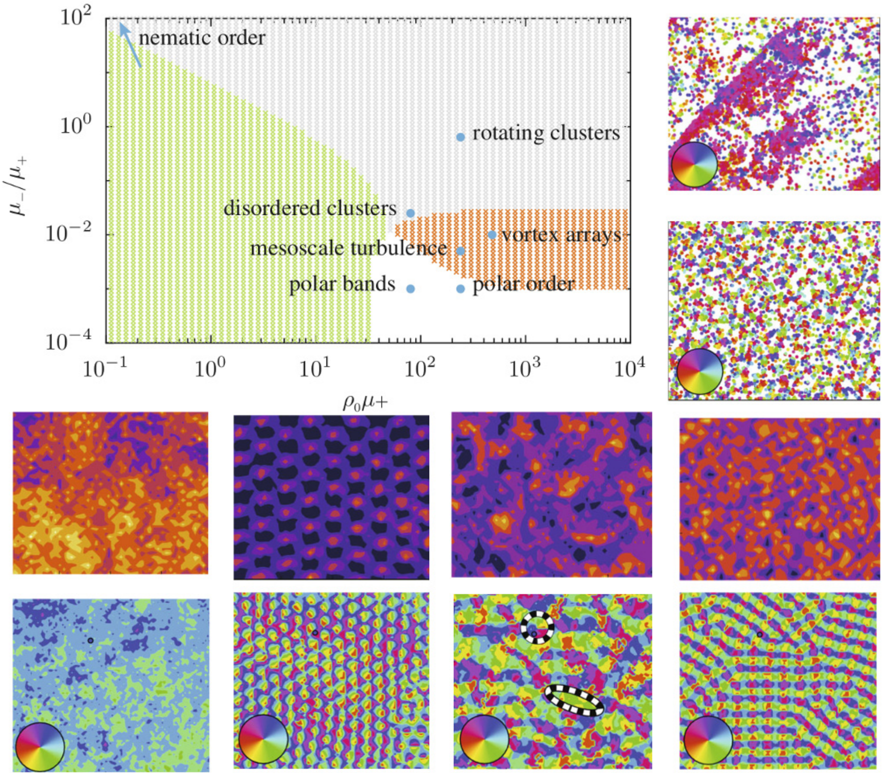

Fig. 1.11 Patterns obtained in the model with competing alignment and anti-alignment interactions. *Upper left*: Estimated domains of different patterns in the parametric plane spanned by the average density ρ_0 multiplied by the strength of the aligning interaction μ_+ and the ratio μ_-/μ_+ of anti-aligning to aligning interactions. *Blue points* indicate the parameter values for which simulation results are shown. *Upper right*: Alignment fields in the regimes of polar bands (*top*) and disordered clusters (*bottom*). Density (*middle row*) and alignment (*lower row*) maps in other regimes, from *left to right*: large-scale polar order; vortex lattice; mesoscale turbulence, with the black-white ellipse and ring indicating a jet and a transient vortex, respectively; and dense rotating clusters. In the density color maps, density grows from dark to light shading; the orientation field is color-coded as shown in the insets (Großmann et al, 2015)

this model, both the polar order and the band structure were destroyed with growing interaction strength (Fig. 1.9b), but for still greater cohesion, order was restored and the flock consolidated into a compact group (Fig. 1.9c).

Combining attraction to the center of a local group, defining the alignment with local repulsive interactions, and grading the alignment strength produces different shapes of simulated flocks (Strömbom et al, 2015). In the snapshots shown in Fig. 1.10, the shape changes from a compact rigid propagating flock at high alignment (a) to flocks of a more irregular shape with lively internal dynamics (b and c) as the strength of alignment decreases, and to a stationary but fluctuating group (d) when the alignment is switched off.

Großmann et al (2015) modified the Vicsek model still more radically, combining velocity alignment with nearest neighbors and *anti-alignment* with particles which are further away, such as might be caused by some unspecified complex hydrodynamic interactions. The inspiration for this model was the symmetry-breaking in reaction–diffusion systems due to competing short-range activation and long-range inhibition (Turing, 1952). Replacing here activation for alignment and inhibition for anti-alignment gives a ready recipe for complex patterning, and this is indeed what this model provides. A variety of patterns coming up in simulations of this model are shown in Fig. 1.11.

1.6 Crowding Impedes Motion

Physicists are not particularly interested in animal social relations, even when they are trying to amend the basic model to fit field observations – but human crowds behave in a special way, being driven by psychology rather than by mechanical forces. Though vectorial in their orientation and motion, freely moving humans are largely motivated by avoidance rather than alignment. Helbing and Molnár (1995) based their model of pedestrian motion on the *social force*, including three principal components: acceleration to the desired velocity of motion, avoiding other pedestrians and borders, and attraction toward the target. The paper gained over 2000 citations, and the model has been tested in both normal and emergency situations.

When moving in a constrained space like a corridor, people self-organize in lanes with a uniform walking direction (Fig. 1.12a). However, the lanes are not necessarily well ordered, and as shown in Fig. 1.12b, dynamically disordered multiple lanes, marked by green crosses, prevail at higher densities. Over some critical density,

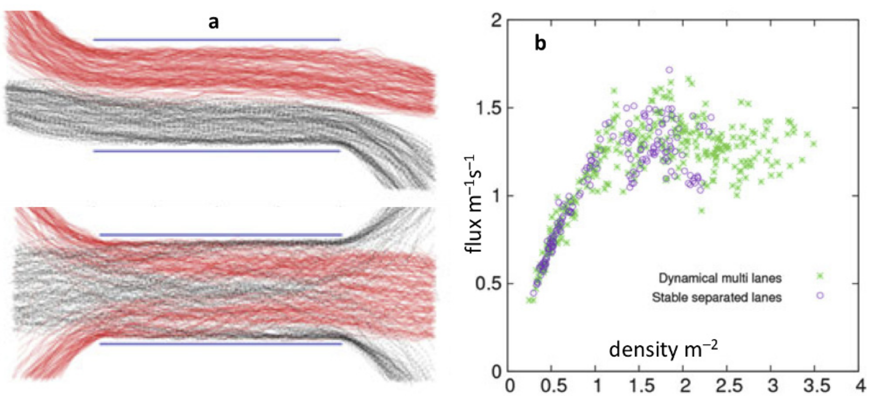

Fig. 1.12 (**a**) Recorded trajectories of pedestrians in a corridor, color-coded according to their direction, with stable separated lanes (*top*) and dynamical multiple lanes (*bottom*). (**b**) Density–flux diagrams for the two types of bidirectional flow (Zhang et al, 2014)