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Mathematics in the Visual Arts

Edited by
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Introduction

The presence of mathematics in the arts has been plain since at least Pythagoras' time. This applies as much to music (rhythm, scales and chords) as to all the visual arts, which are addressed in this book. The visual arts are also related – more and more closely – to other sciences (material, life and cultural). However, in order to get to the very roots of the connections between art and science, we felt it appropriate to choose “the queen of sciences”.

Within the mathematical sciences themselves, geometry, born from the vision of space (geometry: “measuring the Earth”), is, in this respect, the first. In the words of Max Bill: “The primary element of any plastic work is geometry, in terms of relationships between positions in the plane or the space”[1]. Confronted with the forms they saw in nature, the early geometrists tried to understand them by drawing them in an idealized way, that is, by modeling them. In the artists' hands, these basic forms became the means of expression with universal scope.

Before characterizing this unquestionable presence of mathematics in the works of art in more detail, we should first note that mathematics, by its very nature, has a tendency towards plastic representations: *mathematical objects*, created for the purpose of translating scientific abstractions into visual terms.

Visual artists of modernity have often taken their inspiration from *mathematical models*, as if to delegate to them the task of speaking the unspeakable of art. Think of M.C. Escher, who exploited the riches of tiles from the hyperbolic plane, or Salvador Dali, who represented the crucified Jesus on a hypercube[2], or the constructivist sculptors Henry Moore, Naum Gabo and Barbara Hepworth.

A certain parallelism between mathematical and artistic approaches has often been argued – and equally often rebutted. Let us therefore say at the outset what, in our view, should be excluded, and that is the quest for beauty for its own sake. If mathematics happens to be “beautiful”, this is actually a consequence of its elegance, in other words, its simplicity. As for art, it renounced beauty as a determining criterion long ago.

What seems of greater interest in this respect is the search for truth. This is, without question, the ambition of mathematics, which is wholly intellectual in nature and based on axioms that are posited as true or on accepted assumptions. This ambition is more intuitive in art: in a picture, truth is not expressed in a “thinking way”; it can be simultaneously striking and inaccessible.

The best established point of convergence between the artistic and mathematical approaches (as with other sciences) is that they turn the subject, whatever it may be, into a heuristic form; that is, they make it thoughtprovoking. Moving away from a materialistic concept of painting, can mathematics help us to discover the “spiritual software” of a work of art?

Let us turn to the contemporary aspects of the “marriage” between mathematics and the visual arts.

Since the emergence of non-Euclidean geometries and new branches of physics (quantum and relativistic) that point to the importance of chance, or even uncertainty, in the material world, we have seen a gradual erasure of the boundaries between the logical understanding of phenomena and the intuitive approach. Max Bill’s

Mathematical Art represents a culmination of this convergence.

In the wake of conceptual art, digital art has driven the dematerialization of artwork still further. Now that a painting is nothing but a signal, devoid of any meaning of its own, the work's significance has shifted upstream, in other words, to its production processes, the algorithms or the thought processes that generated it.

But there is more: having gradually freed itself from the material (in favor of light, or other forms of energy or information and communication), the work of art tends nowadays to emancipate itself from its creator, with their assent, and win its autonomy. Randomness thus plays a role, not only in the decision-making processes of the artist, but within the work itself. Art appears to have accepted its own artificialization. It remains to be seen whether this will lead to its disappearance as a human construct, or to its reconfiguration as *total artificial art.*

Taking note of the fact that geometry, in all its forms, is the mathematical discipline that has contributed most to the visual arts, this book sets out to show the fruitfulness of their relationships throughout all eras.

Giuseppe Longo and Sara Longo, in their article "Infinity of God and Space of Men in Painting", evoke the contribution of the geometric perspective to Renaissance painting. From the mid-14th Century, armed with this new tool, artist-theologians were able to organize the space of men and symbolize their finitude, in the face of God's infinite act.

Another contribution of geometry to painting is highlighted by Jean-Pierre Crettez in his two articles on "internal geometry" – a concept created by the author to show how classical painters such as Leonardo da Vinci and Georges de La Tour used geometry (invisible but revealed through its structural mesh) to ensure the coherence and harmony of their pictorial space.

Since the early 20th Century, geometry has had a plurality of forms: non-Euclidean geometry, catastrophe theory, algorithmic geometry, fractal theory, etc. In her article "Geometry and the Life of Forms", Ruth Scheps explains how these various geometric currents have inspired geometric abstract artists – from suprematism to digital art, via optical art, kinetic art, conceptual art and minimalism.

A special case of artistic inspiration, derived from geometric and natural forms, is provided by Giuseppe Longo and Sara Longo in their article "Among the Trees: Iterating Geneses of Forms, in Art and Nature", which presents the fractal geometric structure as a source of inspiration for the "Komorebi" project (an untranslatable Japanese word that refers to the effect of sunlight through the foliage).

At the interface of art and science, or belonging to both, scientific drawings and photography have also given rise to artistic works, as illustrated by the article by Bruno Chanetz, "The Passion of Flight: from Leonardo da Vinci to Jean Letourneur", and by Jean Letourneur himself, "Sculptor of Fluid Movement", whose drawings and sculptures draw their inspiration from visualizations created by an engineer at ONERA.

Finally, the most contemporary tools for experimental visual art are, no doubt, provided by advances in computer modeling and simulation. Sophie Lavaud, in her article "*Emergilience*, an Art Research Project", explains her project: exploring the conditions necessary for the emergence, through self-organization, of "infinite dynamic picture-systems" composed of collective and global shapes and phenomena.

Introduction written by Ruth SCHEPS and Marie-Christine MAUREL.

1 Bill, M. (1949). The mathematical way of thinking in the visual art of our time. *Werk*, 3.

2 Dali, S., *Crucifixion (Corpus Hypercubus)*, oil on canvas, 194.3 x 123.8 cm, 1954. The Metropolitan Museum of Art, New York.

1
Infinity of God and Space of Men in Painting, Conditions of Possibility for the Scientific Revolution[1]

1.1. A brief introduction to infinity

There is no space in Greek geometry. By drawing lines, using a ruler and a compass as we would say today, measurements are made and *figures* are constructed, with no mathematical "infinite container" – a plane or a space – "behind" them. Symmetries – rotations and translations – provide proof in the finite. And potential infinity (*apeiron*, without limit, without bounds) is constructed by using extensions and iterations: a segment can be extended *with no finite limit* in a straight line (the second axiom), *eis apeiron.* If we take a set of prime numbers, we can construct a new prime which is greater than each of the elements in that set (Euclid's theorem on the infinitude of primes). An extension and an endless iteration of the finite, from the act of drawing a line to the construction of integers. Time is infinite in this sense, never present in its entirety in our minds. Infinity is not beyond that in which there is nothing, Aristotle tells us in his *Physics*, that in the beyond there is always something. It is a becoming, a potentiality.

Paolo Zellini[2] explains that the Aristotelian distinction between this mathematical infinity, which must be constructed step by step, potential, and the infinity which is "already" there, in actuality, and is all-encompassing, was to resurface in medieval metaphysical debate. God is an allenveloping, all-inclusive infinity, beyond which nothing is a given. However, this concept of actual infinity is not an easy matter. For Aristotelians, it was embodied in negation, as in Aristotle, and God cannot have a negative attribute. However, St. Thomas convinced people by excluding the existence of this kind of infinity in actuality, *except* as an attribute of God and God alone. And this concept of actual infinity was to grow in strength and acquire a *positive* identity in people’s minds. This reached the point where, in 1277, the Bishop of Paris, Etienne Templier, decreed that actual infinity was a positive attribute of God and His Creation. God, when He so wishes, introduces actual infinity into the world; for example, by bestowing Full and Infinite Grace upon a finite being, a woman, Mary – and for those who disagreed, burning at stake awaited. There is no doubt that this uncompromising “axiomatic posture” helped stabilize the concept of actual infinity.

Zellini quite rightly stressed the significance of this debate for the birth of a cosmology of infinity, that was to find fulfillment, first mystical and then scientific, in the infinite Universe and “*gli infiniti mondi*” of Nicolas of Cusa (1401–1464) and Giordano Bruno (1548–1600).

1.2. Infinity in painting and the invention of mathematical space

The concept of actual infinity was clarified in a metaphysical debate, circumscribing infinity as a single “entity” and forcing the mind to envisage it in its totality. How would the “entity” pass into mathematics, where it will be turned into a specific object of discourse, and indeed an element of proof?

The transition came about through the invention of perspective (*prospettiva*) in Italian Renaissance painting[3].

The problem of depicting the scenes where narrative figures were to be placed became a central issue for painters from the late 13th Century onwards. Giottesque "boxes" (dolls' houses with one wall missing, exposed to the viewer) are scenes whose purpose is to contain the *historia* and to render its theological teachings intelligible. In a contiguous arrangement, the spatial scenes (boxes, landscape, hills) punctuate the narrative – we will come back to this later.

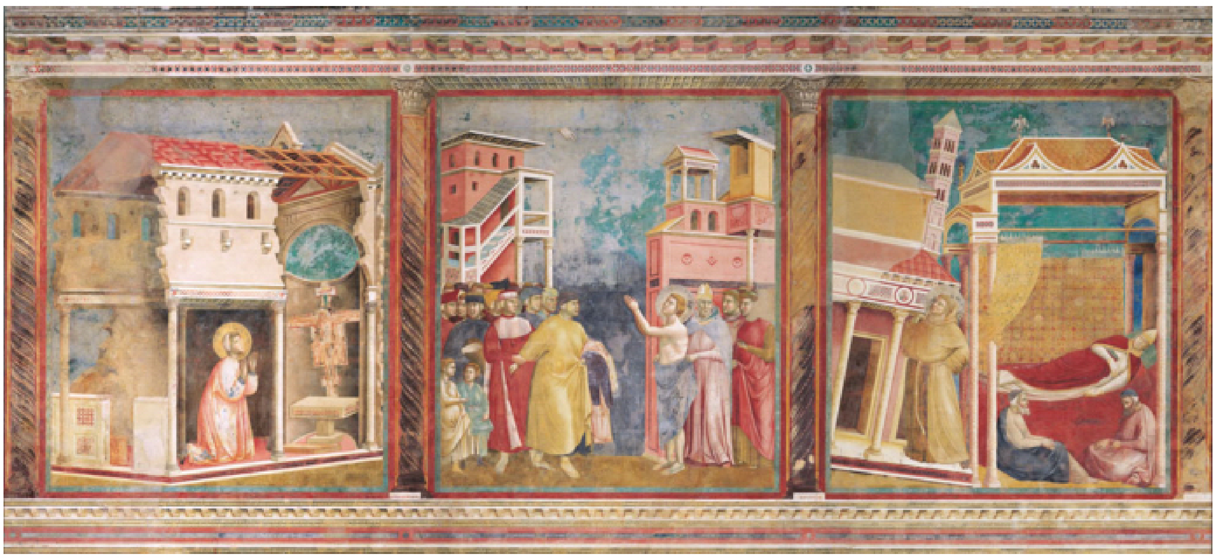

Figure 1.1. *Giotto di Bondone,* Life of St. Francis*, fresco, around 1290. Assisi, Basilica of St. Francis*

The geometrical perspective which Filippo Brunelleschi experimented with in 1417, and which was defined in 1435 by Leon Battista Alberti, is a revolution: not only does it construct a single compositional space (and thus, with a few rare exceptions, a unified narrative) but, above all, it is the result of a construction where man is the source of every measurement (see Alberti, *De Pictura*, I, 19) and where actual infinity, the point of convergence of the orthogonal lines at the bottom of the painting, is contained, enclosed within the representational framework. Since the second half of the last century, in response to Erwin

Panofsky’s inaugural article (*Perspective as Symbolic Form*, 1925), art historians including Pierre Francastel, Hubert Damisch and Louis Marin from the École des Hautes Etudes en Sciences Sociales, in Paris, have highlighted the importance of this pictorial revolution.

According to Erwin Panofsky’s foundational essay, published in Germany in 1927 but translated into English in 1991, the representation of a space by the geometry of orthogonal lines has led to the development of “the concept of an infinity, an infinity not only prefigured in God, but indeed actually embodied in empirical reality”[4]. Erwin Panofsky noted that Ambrogio Lorenzetti’s *Annunciation* (below), painted a century before Alberti formulated his theory, is the first geometrical construction where the receding lines converge not towards a single point but towards a single vertical axis (in the picture plane, the column separating Gabriel from Mary). Daniel Arasse went further, extending this insight to the quite remarkable upsurge in complex geometric constructions, in scenes of the Annunciation to Mary.

His argument is very relevant to our discussion topic: the special affinity that existed in the 15th Century between Annunciation and perspective is due to the fact that in Christian history, the moment at which the infinite enters into the finite is the moment when the son of God miraculously appears in human flesh, through the meeting of God and the Madonna, full of Grace. Daniel Arasse discusses this idea by highlighting what he calls a “theological-pictorial” problem, which toys with the effects and the effectiveness of images: with a back and forth between depth and surface, the paradoxes internal to the spatial structures of certain *Annunciations* demonstrate the impossibility of depicting God within the space of human geometry. This research in painting could be closely linked to a conception of the divine that is not excluded, as Panofsky said, but present in the picture.

To support his argument, Arasse makes particular reference to a sermon delivered by St. Bernardine of Siena in 1427: the Annunciation is the moment when "immensity comes into measure [...], the unfigurable into figure, the uncircumscribable into place, the invisible into vision [...], length into brevity, width into narrowness, height into lowness"[5] ... all these conceptual paradoxes have given rise to spatial paradoxes from painters. Daniel Arasse also highlights how the most ingenious perspectivists enjoy toying with the rules of geometric perspective in order to show the paradox of the infinite entering into the finite.

Figure 1.2. *Ambrogio Lorenzetti,* Annunciation*, tempera on wood, Siena, Pinacoteca Nazionale, 1344*

In this Annunciation, there is a column, often a symbol of Christ, very substantial at floor level and becoming fainter towards the top where it overlays and obscures the receding *axis*, which we could say, at infinity, is an explicit reference to God. Here, in 1344, we have an extraordinary innovation: a rigorously drawn projective space. And then, through the effect of the geometry of the floor that goes from (wo)man to God, a new scene unfolds: God has His place here, hidden, far away at infinity, but present in the story that is being told. The Madonna, too, has a new human depth: her solid, three-dimensional body ushers in the expression of an emerging humanism. Perspective introduces God as the actual limit, at infinity, thus as the limit of a space that everything encompasses, including human spaces that replenish themselves. The very first pictures painted in *prospettiva* were annunciations, unique scenes where infinite meets finite[6]. Then, with Piero della Francesca, this metaphysical dissertation in paint went on to also become a technique, without necessarily losing its religious essence. Piero's book *De Prospectiva Pingendi* (1475) is actually a treatise on "practical" projective geometry, and was the most significant mathematical text of his time, as Vasari wrote.

Hence, *prospettiva* allows the painter to arrange the space of men and objects and to choose a viewpoint. The choice of where to place a vanishing point determines the spectator's *viewpoint*; it proposes/imposes a line of sight – for instance, viewing, humbly from below, Antonello da Messina's Saint Sebastian the Martyr (1476).

Figure 1.3. *Antonello da Messina,* St. Sebastian*, tempera on wood transposed onto canvas, Dresden, Gemäldegalerie, 1476*

And now this metaphysical and religious cosmology became a geometry of space: God, the stars and men found a new place in it, arranged in a unifying and *changeable* viewpoint. We have come a long way from the absolutes outside of space and the world of Byzantine mosaics, if we