Progress in Mathematical Physics

Volume 52

Editors-in-Chief
Anne Boutet de Monvel, *Université Paris VII Denis Diderot*
Gerald Kaiser, *The Virginia Center for Signals and Waves, Austin Texas, USA*

Editorial Board
Sir M. Berry, *University of Bristol, UK*
C. Berenstein, *University of Maryland, College Park, USA*
P. Blanchard, *University of Bielefeld, Germany*
A.S. Fokas, *University of Cambridge, UK*
D. Sternheimer, *Université de Bourgogne, Dijon, France*
C. Tracy, *University of California, Davis, USA*

Gravitation and Experiment

Poincaré Seminar 2006

Thibault Damour
Bertrand Duplantier
Vincent Rivasseau
Editors

Birkhäuser Verlag
Basel · Boston · Berlin

Author:

Thibault Damour
Institut des Hautes Études Scientifiques
Route des Chartres
91440 Bures-sur-Yvette
France
e-mail: damour@ihes.fr

Vincent Rivasseau
Laboratoire de Physique Théorique
Université Paris XI
91405 Orsay Cedex
France
e-mail: Vincent.Rivasseau@th.u-psud.fr

Bertrand Duplantier
Service de Physique Théorique
Orme des Merisiers
CEA - Saclay
91191 Gif-sur-Yvette Cedex
France
e-mail: bertrand.duplantier@cea.fr

2000 Mathematics Subject Classification: primary 83E30, 83D05, 83C35, 83C57
secondary 83Cxx, 83-03, 83-05, 85-99, 01A61

Library of Congress Control Number: 2007931898

Bibliographic information published by Die Deutsche Bibliothek
Die Deutsche Bibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data is available in the Internet at <http://dnb.ddb.de>.

ISBN 978-3-7643-8523-1 Birkhäuser Verlag, Basel – Boston – Berlin

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, re-use of illustrations, broadcasting, reproduction on microfilms or in other ways, and storage in data banks. For any kind of use whatsoever, permission from the copyright owner must be obtained.

© 2007 Birkhäuser Verlag, P.O. Box 133, CH-4010 Basel, Switzerland
Part of Springer Science+Business Media
Printed on acid-free paper produced of chlorine-free pulp. TCF ∞
Printed in Germany

ISBN 987-3-7643-8523-1
e-ISBN: 978-3-7643-8524-8

9 8 7 6 5 4 3 2 1
www.birkhauser.ch

Contents

Foreword

This book is the sixth in a series of lectures of the *Séminaire Poincaré*, which is directed towards a large audience of physicists and of mathematicians.

The goal of this seminar is to provide up-to-date information about general topics of great interest in physics. Both the theoretical and experimental aspects are covered, with some historical background. Inspired by the Bourbaki seminar in mathematics in its organization, hence nicknamed "Bourbaphi", the Poincaré Seminar is held twice a year at the Institut Henri Poincaré in Paris, with contributions prepared in advance. Particular care is devoted to the pedagogical nature of the presentations so as to fulfill the goal of being readable by a large audience of scientists.

This volume contains the ninth such Seminar, held in 2006. It is devoted to Relativity and Experiment.

This book starts with a detailed introduction to general relativity by T. Damour. It includes a review of what may lie beyond by string theorist I. Antoniadis, and collects up-to-date essays on the experimental tests of this theory. General relativity is now a theory well confirmed by detailed experiments, including the precise timing of the double pulsar J0737-3039 explained by M. Kramer, member of the team which discovered it in 2003, and satellite missions such as Gravity Probe B described by J. Mester. The search for detecting gravitational waves is also very much under way as reviewed by J.Y. Vinet.

We hope that the continued publication of this series will serve the community of physicists and mathematicians at professional or graduate student level.

We thank the Commissariat à l'Énergie Atomique (Division des Sciences de la Matière) and the Daniel Iagolnitzer Foundation for sponsoring the Seminar. Special thanks are due to Chantal Delongeas for the preparation of the manuscript.

Thibault Damour
Bertrand Duplantier
Vincent Rivasseau

Gravitation and Experiment, 1–49
© 2007 Birkhäuser Verlag Basel
Poincaré Seminar 2006

General Relativity Today*

Thibault Damour

Abstract. After recalling the conceptual foundations and the basic structure of general relativity, we review some of its main modern developments (apart from cosmology): (i) the post-Newtonian limit and weak-field tests in the solar system, (ii) strong gravitational fields and black holes, (iii) strong-field and radiative tests in binary pulsar observations, (iv) gravitational waves, (v) general relativity and quantum theory.

1. Introduction

The *theory of general relativity* was developed by Einstein in work that extended from 1907 to 1915. The starting point for Einstein's thinking was the composition of a review article in 1907 on what we today call the *theory of special relativity.* Recall that the latter theory sprang from a new kinematics governing length and time measurements that was proposed by Einstein in June of 1905 [1], [2], following important pioneering work by Lorentz and Poincaré. The theory of special relativity essentially poses a new fundamental framework (in place of the one posed by Galileo, Descartes, and Newton) for the formulation of physical laws: this framework being the chrono-geometric space-time structure of Poincaré and Minkowski. After 1905, it therefore seemed a natural task to formulate, reformulate, or modify the then known physical laws so that they fit within the framework of special relativity. For Newton's law of gravitation, this task was begun (before Einstein had even supplied his conceptual crystallization in 1905) by Lorentz (1900) and Poincaré (1905), and was pursued in the period from 1910 to 1915 by Max Abraham, Gunnar Nordström and Gustav Mie (with these latter researchers developing *scalar* relativistic theories of gravitation).

Meanwhile, in 1907, Einstein became aware that gravitational interactions possessed particular characteristics that suggested the necessity of *generalizing* the framework and structure of the 1905 theory of relativity. After many years of intense intellectual effort, Einstein succeeded in constructing a *generalized theory*

*Translated from the French by Eric Novak.

of relativity (or *general relativity*) that proposed a profound modification of the chrono-geometric structure of the space-time of special relativity. In 1915, in place of a simple, neutral arena, given a priori, independently of all material content, space-time became a physical "field" (identified with the gravitational field). In other words, it was now a dynamical entity, both influencing and influenced by the distribution of mass-energy that it contains.

This radically new conception of the structure of space-time remained for a long while on the margins of the development of physics. Twentieth century physics discovered a great number of new physical laws and phenomena while working with the space-time of special relativity as its fundamental framework, as well as imposing the respect of its symmetries (namely the Lorentz-Poincaré group). On the other hand, the theory of general relativity seemed for a long time to be a theory that was both poorly confirmed by experiment and without connection to the extraordinary progress springing from application of quantum theory (along with special relativity) to high-energy physics. This marginalization of general relativity no longer obtains. Today, general relativity has become one of the essential players in cutting-edge science. Numerous high-precision experimental tests have confirmed, in detail, the pertinence of this theory. General relativity has become the favored tool for the description of the macroscopic universe, covering everything from the big bang to black holes, including the solar system, neutron stars, pulsars, and gravitational waves. Moreover, the search for a consistent description of fundamental physics in its entirety has led to the exploration of theories that unify, within a general quantum framework, the description of matter and all its interactions (including gravity). These theories, which are still under construction and are provisionally known as string theories, contain general relativity in a central way but suggest that the fundamental structure of space-time-matter is even richer than is suggested separately by quantum theory and general relativity.

2. Special Relativity

We begin our exposition of the theory of general relativity by recalling the chrono-geometric structure of space-time in the theory of *special* relativity. The structure of Poincaré-Minkowski space-time is given by a generalization of the Euclidean geometric structure of ordinary space. The latter structure is summarized by the formula $L^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$ (a consequence of the Pythagorean theorem), expressing the square of the distance L between two points in space as a sum of the squares of the differences of the (orthonormal) coordinates x, y, z that label the points. The symmetry group of Euclidean geometry is the group of coordinate transformations $(x, y, z) \to (x', y', z')$ that leave the quadratic form $L^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$ invariant. (This group is generated by translations, rotations, and "reversals" such as the transformation given by reflection in a mirror, for example: $x' = -x$, $y' = y$, $z' = z$.)

The Poincaré-Minkowski space-time is defined as the ensemble of *events* (idealizations of what happens at a particular point in space, at a particular moment in time), together with the notion of a *(squared) interval* S^2 defined between any two events. An event is fixed by four coordinates, x, y, z, and t, where (x, y, z) are the spatial coordinates of the point in space where the event in question "occurs," and where t fixes the instant when this event "occurs." Another event will be described (within the same reference frame) by four different coordinates, let us say $x + \Delta x$, $y + \Delta y$, $z + \Delta z$, and $t + \Delta t$. The points in space where these two events occur are separated by a distance L given by the formula above, $L^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$. The moments in time when these two events occur are separated by a time interval T given by $T = \Delta t$. The squared interval S^2 between these two events is given as a function of these quantities, by definition, through the following generalization of the Pythagorean theorem:

$$S^2 = L^2 - c^2\,T^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - c^2(\Delta t)^2\,, \tag{1}$$

where c denotes the speed of light (or, more precisely, the maximum speed of signal propagation).

Equation (1) defines the *chrono-geometry* of Poincaré-Minkowski space-time. The symmetry group of this chrono-geometry is the group of coordinate transformations $(x, y, z, t) \to (x', y', z', t')$ that leave the quadratic form (1) of the interval S invariant. We will show that this group is made up of linear transformations and that it is generated by translations in space and time, spatial rotations, "boosts" (meaning special Lorentz transformations), and reversals of space and time.

It is useful to replace the time coordinate t by the "light-time" $x^0 \equiv ct$, and to collectively denote the coordinates as $x^\mu \equiv (x^0, x^i)$ where the Greek indices $\mu, \nu, \ldots = 0, 1, 2, 3$, and the Roman indices $i, j, \ldots = 1, 2, 3$ (with $x^1 = x$, $x^2 = y$, and $x^3 = z$). Equation (1) is then written

$$S^2 = \eta_{\mu\nu}\,\Delta x^\mu\,\Delta x^\nu\,, \tag{2}$$

where we have used the Einstein summation convention[1] and where $\eta_{\mu\nu}$ is a diagonal matrix whose only non-zero elements are $\eta_{00} = -1$ and $\eta_{11} = \eta_{22} = \eta_{33} = +1$. The symmetry group of Poincaré-Minkowski space-time is therefore the ensemble of Lorentz-Poincaré transformations,

$$x'^\mu = \Lambda^\mu_\nu\,x^\nu + a^\mu\,, \tag{3}$$

where $\eta_{\alpha\beta}\,\Lambda^\alpha_\mu\,\Lambda^\beta_\nu = \eta_{\mu\nu}$.

The chrono-geometry of Poincaré-Minkowski space-time can be visualized by representing, around each point x in space-time, the locus of points that are separated from the point x by a unit (squared) interval, in other words the ensemble of points x' such that $S^2_{xx'} = \eta_{\mu\nu}(x'^\mu - x^\mu)(x'^\nu - x^\nu) = +1$. This locus is a one-sheeted (unit) hyperboloid.

If we were within an ordinary Euclidean space, the ensemble of points x' would trace out a (unit) sphere centered on x, and the "field" of these spheres

[1] Every repeated index is supposed to be summed over all of its possible values.

centered on each point x would allow one to completely characterize the Euclidean geometry of the space. Similarly, in the case of Poincaré-Minkowski space-time, the "field" of unit hyperboloids centered on each point x is a visual characterization of the geometry of this space-time. See Figure 1. This figure gives an idea of the symmetry group of Poincaré-Minkowski space-time, and renders the rigid and homogeneous nature of its geometry particularly clear.

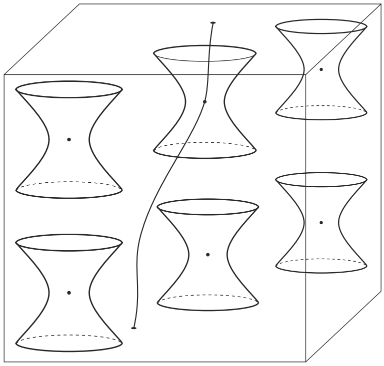

FIGURE 1. Geometry of the "rigid" space-time of the theory of special relativity. This geometry is visualized by representing, around each point x in space-time, the locus of points separated from the point x by a unit (squared) interval. The space-time shown here has only three dimensions: one time dimension (represented vertically), $x^0 = ct$, and two spatial dimensions (represented horizontally), x, y. We have also shown the 'space-time line', or 'world-line', (moving from the bottom to the top of the "space-time block," or from the past towards the future) representing the history of a particle's motion.

The essential idea in Einstein's article of June 1905 was to impose the group of transformations (3) as a symmetry group of the fundamental laws of physics ("the principle of relativity"). This point of view proved to be extraordinarily fruitful, since it led to the discovery of new laws and the prediction of new phenomena. Let us mention some of these for the record: the relativistic dynamics of classical particles, the dilation of lifetimes for relativistic particles, the relation $E = mc^2$ between energy and inertial mass, Dirac's relativistic theory of quantum spin $\frac{1}{2}$ particles, the prediction of antimatter, the classification of particles by rest mass and spin, the relation between spin and statistics, and the CPT theorem.

After these recollections on special relativity, let us discuss the special feature of gravity which, in 1907, suggested to Einstein the need for a profound generalization of the chrono-geometric structure of space-time.

3. The Principle of Equivalence

Einstein's point of departure was a striking experimental fact: all bodies in an external gravitational field fall with the same acceleration. This fact was pointed out by Galileo in 1638. Through a remarkable combination of logical reasoning, thought experiments, and real experiments performed on inclined planes,[2] Galileo was in fact the first to conceive of what we today call the "universality of free-fall" or the "weak principle of equivalence." Let us cite the conclusion that Galileo drew from a hypothetical argument where he varied the ratio between the densities of the freely falling bodies under consideration and the resistance of the medium through which they fall: "Having observed this I came to the conclusion that in a medium totally devoid of resistance all bodies would fall with the same speed" [3]. This universality of free-fall was verified with more precision by Newton's experiments with pendulums, and was incorporated by him into his theory of gravitation (1687) in the form of the identification of the inertial mass m_i (appearing in the fundamental law of dynamics $\boldsymbol{F} = m_i\, \boldsymbol{a}$) with the gravitational mass m_g (appearing in the gravitational force, $F_g = G\, m_g\, m'_g / r^2$):

$$m_i = m_g \,. \tag{4}$$

At the end of the nineteenth century, Baron Roland von Eötvös verified the equivalence (4) between m_i and m_g with a precision on the order of 10^{-9}, and Einstein was aware of this high-precision verification. (At present, the equivalence between m_i and m_g has been verified at the level of 10^{-12} [4].) The point that struck Einstein was that, given the precision with which $m_i = m_g$ was verified, and given the equivalence between inertial mass and energy discovered by Einstein in September of 1905 [2] ($E = m_i\, c^2$), one must conclude that all of the various forms of energy that contribute to the inertial mass of a body (rest mass of the elementary constituents, various binding energies, internal kinetic energy, etc.) do contribute in a strictly identical way to the gravitational mass of this body, meaning both to its capacity for reacting to an external gravitational field and to its capacity to create a gravitational field.

In 1907, Einstein realized that the equivalence between m_i and m_g implicitly contained a deeper equivalence between inertia and gravitation that had important consequences for the notion of an inertial reference frame (which was a fundamental concept in the theory of special relativity). In an ingenious thought experiment, Einstein imagined the behavior of rigid bodies and reference clocks within a freely falling elevator. Because of the universality of free-fall, all of the objects in such a "freely falling local reference frame" would appear not to be accelerating with respect to it. Thus, with respect to such a reference frame, the exterior gravitational field is "erased" (or "effaced"). Einstein therefore postulated what he called the "principle of equivalence" between gravitation and inertia. This principle has two

[2] The experiment with falling bodies said to be performed from atop the Leaning Tower of Pisa is a myth, although it aptly summarizes the essence of Galilean innovation.

parts, that Einstein used in turns. The first part says that, for any external gravitational field whatsoever, it is possible to locally "erase" the gravitational field by using an appropriate freely falling local reference frame and that, because of this, the non-gravitational physical laws apply within this local reference frame just as they would in an inertial reference frame (free of gravity) in special relativity. The second part of Einstein's equivalence principle says that, by starting from an inertial reference frame in special relativity (in the absence of any "true" gravitational field), one can create an apparent gravitational field in a local reference frame, if this reference frame is accelerated (be it in a straight line or through a rotation).

4. Gravitation and Space-Time Chrono-Geometry

Einstein was able (through an extraordinary intellectual journey that lasted eight years) to construct a new theory of gravitation, based on a rich generalization of the 1905 theory of relativity, starting just from the equivalence principle described above. The first step in this journey consisted in understanding that the principle of equivalence would suggest a profound modification of the chrono-geometric structure of Poincaré-Minkowski space-time recalled in Equation (1) above.

To illustrate, let X^α, $\alpha = 0, 1, 2, 3$, be the space-time coordinates in a local, freely-falling reference frame (or *locally inertial reference frame*). In such a reference frame, the laws of special relativity apply. In particular, the infinitesimal space-time interval $ds^2 = dL^2 - c^2\, dT^2$ between two neighboring events within such a reference frame X^α, $X'^\alpha = X^\alpha + dX^\alpha$ (close to the center of this reference frame) takes the form

$$ds^2 = dL^2 - c^2\, dT^2 = \eta_{\alpha\beta}\, dX^\alpha\, dX^\beta\,, \tag{5}$$

where we recall that the repeated indices α and β are summed over all of their values ($\alpha, \beta = 0, 1, 2, 3$). We also know that in special relativity the local energy and momentum densities and fluxes are collected into the ten components of the *energy-momentum tensor* $T^{\alpha\beta}$. (For example, the energy density per unit volume is equal to T^{00}, in the reference frame described by coordinates $X^\alpha = (X^0, X^i)$, $i = 1, 2, 3$.) The conservation of energy and momentum translates into the equation $\partial_\beta\, T^{\alpha\beta} = 0$, where $\partial_\beta = \partial/\partial\, X^\beta$.

The theory of special relativity tells us that we can change our locally inertial reference frame (while remaining in the neighborhood of a space-time point where one has "erased" gravity) through a Lorentz transformation, $X'^\alpha = \Lambda^\alpha_\beta\, X^\beta$. Under such a transformation, the infinitesimal interval ds^2, Equation (5), remains invariant and the ten components of the (symmetric) tensor $T^{\alpha\beta}$ are transformed according to $T'^{\alpha\beta} = \Lambda^\alpha_\gamma\, \Lambda^\beta_\delta\, T^{\gamma\delta}$. On the other hand, when we pass from a *locally* inertial reference frame (with coordinates X^α) to an *extended* non-inertial reference frame (with coordinates x^μ; $\mu = 0, 1, 2, 3$), the transformation connecting the X^α to the x^μ is no longer a *linear* transformation (like the Lorentz transformation) but becomes a *non-linear* transformation $X^\alpha = X^\alpha(x^\mu)$ that can take any form whatsoever. Because of this, the value of the infinitesimal interval ds^2, when expressed in a general, extended reference frame, will take a more complicated form than the very simple one given by Equation (5) that it had in a reference frame that was locally in free-fall. In fact, by differentiating the non-linear functions $X^\alpha = X^\alpha(x^\mu)$ we obtain the relation $dX^\alpha = \partial X^\alpha / \partial x^\mu \, dx^\mu$. By substituting this relation into (5) we then obtain

$$ds^2 = g_{\mu\nu}(x^\lambda) \, dx^\mu \, dx^\nu \,, \tag{6}$$

where the indices μ, ν are summed over $0, 1, 2, 3$ and where the ten functions $g_{\mu\nu}(x)$ (symmetric over the indices μ and ν) of the four variables x^λ are defined, point by point (meaning that for each point x^λ we consider a reference frame that is locally freely falling at x, with local coordinates X_x^α) by $g_{\mu\nu}(x) = \eta_{\alpha\beta} \, \partial X_x^\alpha(x) / \partial x^\mu \, \partial X_x^\beta(x) / \partial x^\nu$. Because of the nonlinearity of the functions $X^\alpha(x)$, the functions $g_{\mu\nu}(x)$ generally depend in a nontrivial way on the coordinates x^λ.

The local chrono-geometry of space-time thus appears to be given, not by the simple Minkowskian metric (2), with constant coefficients $\eta_{\mu\nu}$, but by a quadratic metric of a much more general type, Equation (6), with coefficients $g_{\mu\nu}(x)$ that vary from point to point. Such general metric spaces had been introduced and studied by Gauss and Riemann in the nineteenth century (in the case where the quadratic form (6) is positive definite). They carry the name *Riemannian spaces* or *curved spaces*. (In the case of interest for Einstein's theory, where the quadratic form (6) is not positive definite, one speaks of a pseudo-Riemannian metric.)

We do not have the space here to explain in detail the various geometric structures in a Riemannian space that are derivable from the data of the infinitesimal interval (6). Let us note simply that given Equation (6), which gives the distance ds between two infinitesimally separated points, we are able, through integration along a curve, to define the length of an arbitrary curve connecting two widely separated points A and B: $L_{AB} = \int_A^B ds$. One can then define the "straightest possible line" between two given points A and B to be the shortest line, in other words the curve that minimizes (or, more generally, extremizes) the integrated distance L_{AB}. These straightest possible lines are called *geodesic curves*. To give a simple example, the geodesics of a spherical surface (like the surface of the Earth) are the great circles (with radius equal to the radius of the sphere). If one mathematically writes the condition for a curve, as given by its parametric representation $x^\mu = x^\mu(s)$, where s is the length along the curve, to extremize the total length L_{AB} one finds that $x^\mu(s)$ must satisfy the following second-order differential equation:

$$\frac{d^2 x^\lambda}{ds^2} + \Gamma^\lambda_{\mu\nu}(x) \frac{dx^\mu}{ds} \frac{dx^\nu}{ds} = 0 \,, \tag{7}$$

where the quantities $\Gamma^\lambda_{\mu\nu}$, known as the *Christoffel coefficients* or *connection coefficients*, are calculated, at each point x, from the *metric components* $g_{\mu\nu}(x)$ by the equation

$$\Gamma^\lambda_{\mu\nu} \equiv \frac{1}{2} g^{\lambda\sigma} (\partial_\mu \, g_{\nu\sigma} + \partial_\nu \, g_{\mu\sigma} - \partial_\sigma \, g_{\mu\nu}) \,, \tag{8}$$