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# Zhang-Gradient Control



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*To our parents and ancestors, as always*

# Preface

The tracking-control problems of nonlinear systems have been widely encountered in various applications, such as flight control, pendulum control, and robot control. For the purpose of tracking control, we need to design a controller in terms of control input for nonlinear systems such that the actual output can track the desired output. For solving the tracking-control problems of nonlinear systems, a number of methods have been presented and investigated, such as the input–output linearization (IOL) method, the optimal control method, and the backstepping method. However, most of the conventional control methods are relatively complex for their design procedures of controllers and practical implementations. Therefore, it is necessary and significant for practitioners to propose, develop, and investigate a simple and effective control method for the design of controllers.

From the viewpoint of time-varying (or say, dynamic) problem solving, the tracking control of nonlinear systems can be investigated in a unique manner. In recent years, a special class of neural dynamics has been exploited for the online solution of time-varying problems. As this neural-dynamic method is proposed by Zhang et al. and zeroes out each element of error function, it is called Zhang dynamics (also known as zeroing dynamics, ZD). Specifically, ZD is designed on the basis of an indefinite matrix-/vector-/scalar-valued error function (termed Zhang function, ZF) and takes full advantage of the time-derivative information of time-varying parameters. The ZD method is an error-based dynamic method, of which the core is the ZD design formula that forces each element of ZF to converge to zero exponentially. Such an idea can actually be found in the control field, i.e., forcing the error between the actual output and the desired output to be zero (or near zero in practice). Differing from the ZD, the conventional gradient dynamics (GD) is designed on the basis of a scalar-valued nonnegative error function (termed energy function, EF). The GD method is an energy-based minimization method, of which the core is the GD design formula such that the minimum point of the EF can be reached along the negative gradient direction. Besides, the GD method designed intrinsically for time-invariant (or say, static, constant) problem solving has been extended to solve time-varying problems. It is worth pointing out that such two methods both aim at forcing the error functions to be zero, which is essentially

consistent with the objective of tracking control. However, in the previous studies, the ZD method and the GD method are generally exploited for problem solving individually and comparatively, and other researchers rarely consider combining them to utilize the advantage of each method as well as the superiority of their combination.

In this book, by effectively combining the ZD and GD methods together, a simple and effective controller-design method is developed and presented, which is termed Zhang-gradient (ZG) method. Accordingly, based on the ZG method, a special kind of controllers termed ZG controllers are designed, developed, and investigated for tracking control of various nonlinear systems (including linear systems as a special case), i.e., chaotic systems, integrator systems, pendulum systems, affine-form nonlinear (AFN) systems, as well as time-varying linear and nonlinear systems. In general, under the framework of the ZG method, a ZG controller obtained by adopting the ZD method  $m$  times and the GD method  $n$  times is called a  $zmg_n$  controller. Specifically, the  $zmg_0$  controllers are designed by adopting the ZD method  $m$  times and without using the GD method, which can be viewed as a special case of ZG controllers and thus often termed ZD controllers directly for comparisons with the ZG controllers using the GD method; besides, the  $zmg_1$  controllers are designed by adopting the ZD method  $m$  times and the GD method 1 time. It is worth pointing out that, in most cases, the ZG controllers refer to the  $zmg_1$  controllers, which can elegantly conquer the knotty division-by-zero (DBZ) problem. In traditional investigations, the DBZ problem is rarely considered and studied since it is a knotty problem for conventional controller design. In the conventional controller design, the divisor of a controller is simply assumed to be nonzero at any time instant, which often leads to contradictions between theoretical investigations and practical applications. Note that the DBZ problem has existed for thirteen centuries. However, past efforts have been spent on studying the problem under a time-invariant premise, i.e., studying the division operation with fixed operands at a certain time instant. By contrast, this book mainly focuses on investigating the DBZ problem from the perspective of temporal evolution instead of under a time-invariant premise. The simple and effective ZG method presented in this book is capable of designing the ZG controllers in a division-free manner. That is, the ZG controllers get rid of the potential possibility of encountering the DBZ problem and thus remain valid at the DBZ points encountered during the tracking-control process of nonlinear systems. Through the related theoretical analyses, the ZD and ZG controllers (more specifically, the  $zmg_0$  and  $zmg_1$  controllers under the framework of the ZG method) both possess the global and exponential convergence performance, which theoretically guarantee the efficacy of controllers. Computer simulations with various illustrative examples are further performed to substantiate the feasibility and efficacy of the presented ZD and ZG controllers (as well as the ZG method) for tracking control of various nonlinear systems. More importantly, the superiority of ZG controllers in conquering the DBZ problem is also illustrated by comparative simulation results. In brief, the main highlights of this book can be listed as follows.

- (1) This book is the first book on the ZG method for controller design in connection with nonlinear/linear, time-varying/time-invariant, and multi-class or various systems.
- (2) This book overcomes the challenges of control singularity and system collapse posed by the DBZ problem.
- (3) This book provides detailed theoretical analyses, as well as abundant and comparative simulation results.

The idea for this book on neural dynamics and control was conceived during the classroom teaching as well as the research discussion in the laboratory and at international academic meetings. Most of the materials of this book are derived from the authors' papers published in journals and proceedings of the international conferences. In fact, since the early 1980s, the field of neural dynamics has undergone the phases of exponential growth, generating many new theoretical concepts and tools (including the authors' ones). At the same time, these theoretical results have been successfully applied to the solution of many practical problems. Our first priority is thus to cover each central topic in enough details to make the material clear and coherent; in other words, each part (and even each chapter) is written in a relatively self-contained manner.

In this book, Chap. 1 presents the introduction, concepts, and preliminaries, and the remainder contains 16 chapters that are classified into the following 5 parts:

- Part I: Chaotic Systems Using ZG Control (Chaps. 2–4);
- Part II: Integrator Systems Using ZG Control (Chaps. 5–7);
- Part III: Pendulum Systems Using ZG Control (Chaps. 8–10);
- Part IV: AFN Systems Using ZG Control (Chaps. 11–14);
- Part V: Time-Varying Systems Using ZG Control (Chaps. 15–17).

Chapter 2—In this chapter, we investigate the tracking-control problems of Lorenz, Chen, and Lu (also written as Lü) chaotic systems. By combining the ZD and GD methods together, a simple and effective controller-design method, termed ZG method, is presented for tracking control of the three chaotic systems. Both theoretical analyses and simulative verifications substantiate that the presented ZG controllers can achieve satisfactory tracking accuracy and successfully conquer the DBZ problem encountered during the tracking-control process.

Chapter 3—In this chapter, the ZG method is investigated for chaos synchronization with multiple inputs (i.e., three or two inputs). Based on the ZG method, the traditional three-input chaos synchronization problem can be successfully solved with desirable convergence rate and satisfactory accuracy. Besides, an important extension of the ZG method is investigated to solve the thorny two-input chaos synchronization problem. Simulation results illustrate that the controller groups designed by the ZG method not only achieve satisfactory synchronization accuracy and exponential convergence rate on the three-input chaos synchronization problem but also successfully solve the chaos synchronization problem with only two inputs.

Chapter 4—In this chapter, the ZG method is studied for solving the tracking-control problem of the modified Lorenz nonlinear system via additive input or mixed



inputs (i.e., the mixture of additive and multiplicative inputs). Both theoretical analyses and simulative verifications validate that the ZG controllers with additive input or mixed inputs not only achieve satisfactory tracking accuracy but also successfully conquer the DBZ problem encountered during the tracking-control process.

**Chapter 5**—In this chapter, we apply the ZG method to the tracking control of Brockett integrator. Based on the ZG method, different types of controller groups are designed for Brockett integrator. Both theoretical analyses and simulative verifications indicate that the tracking errors are bounded and exponentially convergent. More importantly, comparative simulation results illustrate that the ZG controller group is superior to the ZD controller group in conquering the DBZ problem encountered during the tracking-control process.

**Chapter 6**—In this chapter, the ZG controllers for explicit and implicit tracking control of a double-integrator (DI) system are designed and presented. In addition, we conduct the corresponding computer simulations with different values of the design parameter  $\lambda$  used to illustrate the efficacy of ZG controllers. However, different settings of simulation options in MATLAB ordinary differential equation (ODE) solvers may lead to different simulation results (e.g., failure and success). The successful and failed simulation results are both presented to remind us to pay more attention to MATLAB defaults and options during conducting such simulations.

**Chapter 7**—In this chapter, the tracking-control problems of multiple-integrator (MI) systems are investigated by using the ZG method. Several types of ZD and ZG controllers are presented for tracking control of MI systems, e.g., triple-integrator (TI) systems. As an example, the design procedures of ZD and ZG controllers for TI systems with a linear output function (LOF) and a nonlinear output function (NOF) are presented. Corresponding theoretical analyses are given to guarantee the convergence performance of ZD and ZG controllers for TI systems. Computer simulations concerning the tracking control of MI systems with different types of output functions are further performed to substantiate the feasibility and efficacy of ZD and ZG controllers for tracking-control problem solving. Moreover, comparative simulation results for the tracking control of MI systems with NOFs substantiate that the ZG controllers can effectively conquer the DBZ problem.

**Chapter 8**—In this chapter, we firstly design ZD controllers for the explicit and implicit tracking control of a simple pendulum system. For achieving the DBZ-containing implicit tracking control, ZG controllers are further designed for conquering the DBZ problem. Computer simulations with an explicit tracking example and two implicit tracking examples are conducted. Comparative simulation results have substantiated the superiority of the ZG controllers for the DBZ-containing implicit tracking control of simple pendulum system.

**Chapter 9**—In this chapter, the cart path tracking control of an inverted-pendulum-on-a-cart (IPC) system is considered and investigated. Based on the ZG method, several types of ZD and ZG controllers are developed to achieve the tracking-control purpose. Besides, theoretical analyses are presented to guarantee the global and exponential convergence performance of both ZD and ZG controllers.

Computer simulations are further performed to illustrate the feasibility and efficacy of both ZD and ZG controllers. More importantly, comparative simulation results indicate that ZG controllers can effectively conquer the DBZ problem.

**Chapter 10**—In this chapter, two tracking controllers based on the ZG method are designed for the IPC system. Importantly, the presented ZG controller not only realizes the simultaneous control of pendulum swinging up and pendulum angle tracking but also conquers the DBZ problem elegantly without using any switching strategy. Besides, corresponding theoretical analyses on the convergence performance of both ZD and ZG controllers are provided. Computer simulations with three illustrative examples are further conducted to show the efficacy of both ZD and ZG controllers for the pendulum tracking control of the IPC system. In particular, comparative simulation results substantiate the superiority of the z2g1 controller for the control of pendulum tracking (including swinging up) of the IPC system in conquering the DBZ problem.

**Chapter 11**—In this chapter, we incorporate the GD into IOL, which leads to the GD-aided IOL method for conquering the DBZ problem encountered in the AFN system, with the proposition of the loose condition on relative degree. Corresponding theoretical analyses on tracking-error bound and convergence performance of the GD-aided IOL controller are provided. Moreover, comparative simulation results further substantiate that the GD-aided IOL controller is capable of fulfilling the tracking-control task with the DBZ problem conquered.

**Chapter 12**—In this chapter, a classic nonlinear system of Van der Pol oscillator in the affine-control form is investigated. By applying the ZG method, a ZG controller is designed for trajectory generation of the aforementioned nonlinear oscillator. Simulation results illustrate the feasibility and efficacy of the ZG controller with the DBZ problem conquered. In addition, the effects of ZD and GD design parameters on the performance of ZG controller are further studied.

**Chapter 13**—In this chapter, by following the ZG method, a ZD controller and a ZG controller are presented for tracking control of AFN system, which may encounter the DBZ problem. For comparison, the conventional IOL controller is also presented. The ZD, ZG, and IOL controllers are compared in different relative-degree cases, i.e., the standard relative-degree case, the pseudo-DBZ (PDBZ) relative-degree case, and the true-DBZ (TDBZ) relative-degree case. In addition, the theoretical analyses on ZD and ZG controllers are provided. Corresponding computer simulations are further performed to illustrate the tracking performance of the ZD, ZG, and IOL controllers, as well as to show the superiority of the ZG controller in conquering the TDBZ problem for tracking control of AFN system.

**Chapter 14**—In this chapter, according to the impact of DBZ points on the state variables of the controlled nonlinear system, the concepts of the PDBZ problem and the TDBZ problem are presented. Besides, the two classes of DBZ problems are solved under the framework of the ZG method. Specific examples are investigated to illustrate such two concepts and the efficacy of the ZG controllers in conquering PDBZ and TDBZ problems. The practical application to a two-wheeled mobile robot further substantiates the efficacy of the ZG method for tracking control of nonlinear system with physical meaning while conquering the TDBZ problem.

**Chapter 15**—In this chapter, the output tracking of time-varying linear (TVL) system is investigated. For solving such an output-tracking problem, three different types of controllers are presented, i.e., the conventional controller, ZD controller, and ZG controller. Simulation results with two illustrative examples show that such three types of controllers are feasible and effective for output-tracking problem solving. Especially, the presented ZG controller is capable of conquering the DBZ problem of TVL system.

**Chapter 16**—In this chapter, the stabilization of TVL system is investigated with PDBZ phenomenon shown. Based on the ZG method, a ZD stabilization controller and a ZG stabilization controller are designed. Simulation results indicate that the ZD stabilization controller is able to realize the stabilization of the TVL system in spite of the controller itself containing DBZ points, and that the ZG stabilization controller not only realizes the stabilization of the TVL system but also solves the PDBZ problem contained in the ZD stabilization controller.

**Chapter 17**—In this chapter, the ZG method is utilized to design ZD and ZG controllers for the output tracking of TVL and time-varying nonlinear (TVN) systems. Particularly, the investigated TVL and TVN systems may both have PDBZ phenomena. From the simulation results, although the presented ZD and ZG controllers fulfill well the output tracking of TVL and TVN systems, the infinite value of the former and the finite value of the latter at DBZ time instants indicate that the ZG controller is more effective in dealing with the PDBZ problem.

In summary, this book presents a simple and effective ZG method for solving the tracking-control problems of various nonlinear systems in the control field and further applies such a method to the tracking control of practical systems, e.g., IPC system and two-wheeled mobile robot (showing its application prospect). This book is written for undergraduate and postgraduate students as well as academic and industrial researchers studying in the developing fields of neural dynamics/neural networks, nonlinear control, computer mathematics, time-varying problem solving, modeling and simulation, analog hardware, and robotics. It provides a comprehensive view of the combined research of these fields, in addition to its accomplishments, potentials, and perspectives. We do hope that this book will generate curiosity and also happiness to its readers for learning more in the fields and the research, and that it will provide new challenges to seek new theoretical tools and practical applications.

At the end of this preface, it is worth pointing out that, in this book, a new and inspiring direction on the control method is provided for the design of controllers, together with the notorious DBZ problem conquered effectively, which has existed and has been investigated for more than 1300 years in academia and has stood in the tracking-control area of nonlinear systems for several decades (specifically, since the work of Alberto Isidori in 1985). This completely opens the door to the theoretical researches, simulative verifications, and practical/industrial applications of the DBZ-conquering ZG controllers designed by the ZG method, as the knotty DBZ problem has now been solved truly, systematically, and methodologically. It may promise to become a major inspiration for studies and researches in neural dynamics/neural networks, nonlinear control, computer mathematics, time-

varying problem solving, modeling and simulation, analog hardware, and robotics. Without doubt, this book can be extended. Any comments or suggestions are welcome. The authors can be contacted via e-mails: [zhynong@mail.sysu.edu.cn](mailto:zhynong@mail.sysu.edu.cn), [qiubb6@mail.sysu.edu.cn](mailto:qiubb6@mail.sysu.edu.cn), and [lixd@mail.sysu.edu.cn](mailto:lixd@mail.sysu.edu.cn). The web page of Yunong Zhang is available at <http://sdc.sysu.edu.cn/content/2477>.

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# Acronyms

AFN	Affine-form nonlinear
DBZ	Division-by-zero
DI	Double-integrator
EF	Energy function
GD	Gradient dynamics
IOL	Input-output linearization
IPC	Inverted-pendulum-on-a-cart
LOF	Linear output function
MI	Multiple-integrator
MIMO	Multiple-input multiple-output
NOF	Nonlinear output function
ODE	Ordinary differential equation
PDBZ	Pseudo-DBZ
TDBZ	True-DBZ
TI	Triple-integrator
TVL	Time-varying linear
TVN	Time-varying nonlinear
UBIBS	Uniformly bounded-input bounded-state
ZD	Zhang dynamics
ZF	Zhang function
ZG	Zhang-gradient
ZNN	Zhang neural network

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