**Progress in Mathematical Physics**
Volume 47

# Einstein, 1905–2005

## Poincaré Seminar 2005

Thibault Damour
Olivier Darrigol
Bertrand Duplantier
Vincent Rivasseau
Editors

**Birkhäuser Verlag**
**Basel · Boston · Berlin**

Editors:

Thibault Damour
IHÉS
35, Route de Chartres,
91440 Bures-sur-Yvette
France
e-mail: damour@ihes.fr

Bertrand Duplantier
Service de Physique Théorique
Orme des Merisiers
CEA - Saclay
91191 Gif-sur-Yvette Cedex
France
e-mail: bertrand.duplantier@cea.fr

Olivier Darrigol
Université Denis Diderot (Paris 7)
2, place Jussieu
75251 Paris Cedex 05
France
e-mail: darrigol@paris7.jussieu.fr

Vincent Rivasseau
Laboratoire de Physique Théorique
Université Paris XI
91405 Orsay Cedex
France
e-mail: Vincent.Rivasseau@th.u-psud.fr

2000 Mathematics Subject Classification 83-03, 83A05, 83B05, 83Cxx, 60Hxx

A CIP catalogue record for this book is available from the Library of Congress, Washington D.C., USA

Bibliographic information published by Die Deutsche Bibliothek
Die Deutsche Bibliothek lists this publication in the Deutsche Nationalbibliografie;
detailed bibliographic data is available in the Internet at <http://dnb.ddb.de>.

ISBN 3-7643-7435-7 Birkhäuser Verlag, Basel – Boston – Berlin


Part of Springer Science+Business Media
Printed on acid-free paper produced of chlorine-free pulp. TCF ∞
Printed in Germany
ISBN-10: 3-7643-7435-7
ISBN-13: 978-3-7643-7435-8

9 8 7 6 5 4 3 2 1 www.birkhauser.ch

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# Foreword

This book is the fourth in a series of lectures of the *Séminaire Poincaré*, which is directed towards a large audience of physicists and of mathematicians.

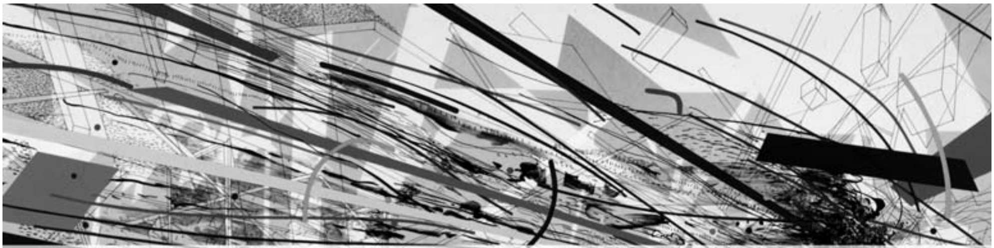
The goal of this seminar is to provide up-to-date information about general topics of great interest in physics. Both the theoretical and experimental aspects are covered, with some historical background. Inspired by the Bourbaki seminar in mathematics in its organization, hence nicknamed "Bourbaphi", the Poincaré Seminar is held twice a year at the Institut Henri Poincaré in Paris, with contributions prepared in advance. Particular care is devoted to the pedagogical nature of the presentations so as to fulfill the goal of being readable by a large audience of scientists.

This volume contains the seventh such Seminar, held in 2005. It is devoted to Einstein's 1905 papers and their legacy. After a presentation of Einstein's epistemological approach to physics, and the genesis of special relativity, a centenary perspective is offered. The geometry of relativistic spacetime is explained in detail. Single photon experiments are presented, as a spectacular realization of Einstein's light quanta hypothesis. A previously unpublished lecture by Einstein, which presents an illuminating point of view on statistical physics in 1910, at the dawn of quantum mechanics, is reproduced. The volume ends with an essay on the historical, physical and mathematical aspects of Brownian motion.

We hope that the publication of this series will serve the community of physicists and mathematicians at the graduate student or professional level.

We thank the Commissariat à l'Énergie Atomique (Division des Sciences de la Matière), the Centre National de la Recherche Scientifique (Sciences Physique et Mathématiques), and the Daniel Iagolnitzer Foundation for sponsoring the Seminar. Special thanks are due to Chantal Delongeas for the preparation of the manuscript.

Thibault Damour
Olivier Darrigol
Bertrand Duplantier
Vincent Rivasseau



Institut Henri Poincaré · Amphi Hermite
11, rue Pierre et Marie Curie · 75005 Paris

O. Darrigol : La genèse de la Relativité · 10h00
C. M. Will : Tests of Special Relativity · 11h00
B. Duplantier : Le mouvement brownien · 14h00
Ph. Grangier : Expériences à un seul photon · 15h00
T. Damour : Einstein épistémologue · 16h00

# Einstein, 1905-2005

Séminaire Poincaré
Samedi 9 avril 2005

Crédit photos :
**Julie Mehretu**, excerpt (suprematist evasion), 2003
ink and acrylic on canvas, 32"x54"
**Archives Lotte Jacobi**, Université du New Hampshire

www.lpthe.jussieu.fr/poincare

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Einstein, 1 – 31
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# The Genesis of the Theory of Relativity

Olivier Darrigol

The most famous of Albert Einstein's papers of 1905 is undoubtedly the one concerning the theory of relativity. Any modern physicist knows that this theory imposes a strict and general constraint on the laws of nature. Any curious layman wonders at the daring reform of our ancestral concepts of space and time. As often happens for great conceptual breakthroughs, the theory of relativity gave rise to founding myths whose charm the historian must resist.

The first of this myth is that Einstein discovered the theory of relativity in a single stroke of genius that defies any rational analysis. Some of Einstein's reminiscences favor this thesis, for instance his allusion to a conversation with Michele Besso in which he would have suddenly realized that a reform of the concept of time solved long standing paradoxes of electrodynamics. One could also argue that the historical explanation of a deep innovation is by definition impossible, since a radically new idea cannot be derived from received ideas. In the case of Einstein's relativity the rarity of pre-1905 sources further discourages historical reconstruction, and invites us to leave this momentous discovery in its shroud of mystery.

This romantic attitude does not appeal to teachers of physics. In order to convey some sort of logical necessity to relativity theory, they have constructed another myth following which a few experiments drove the conceptual revolution. In this empiricist view, the failure of ether-drift experiments led to the relativity principle; and the Michelson-Morley experiment led to the constancy of the velocity of light; Einstein only had to combine these two principles to derive relativity theory.

As a counterpoise to this myth, there is a third, idealist account in which Einstein is supposed to have reached his theory by a philosophical criticism of fundamental concepts in the spirit of David Hume and Ernst Mach, without even knowing about the Michelson-Morley experiment, and without worrying much about the technicalities of contemporary physics in general.

A conscientious historian cannot trust such myths, even though they may contain a grain of truth. He must reach his conclusions by reestablishing the contexts in which Einstein conducted his reflections, by taking into account his education and formation, by introducing the several actors who shared his interests, by identifying the difficulties they encountered and the steps they took to solve them. In this process, he must avoid the speculative filling of gaps in documentary sources. Instead of rigidifying any ill-founded interpretation, he should offer an open spectrum of interpretive possibilities. As I hope to show in this paper, this sober method allows a fair intelligence of the origins of relativity.

A first indication of the primary context of the early theory of relativity is found in the very title of Einstein's founding paper: "On the electrodynamics of moving bodies." This title choice may seem bizarre to the modern reader, who defines relativity theory as a theory of space and time. In conformity with the latter view, the first section of Einstein's paper deals with a new kinematics meant to apply to any kind of physical phenomenon. Much of the paper nonetheless deals with the application of this kinematics to the electrodynamics and optics of moving bodies. Clearly, Einstein wanted to solve difficulties he had encountered in this domain of physics. A survey of physics literature in the years 1895-1905 shows that the electrodynamics of moving bodies then was a widely discussed topic. Little before the publication of Einstein's paper, several studies with similar titles appeared in German journals. Much experimental and theoretical work was being done in this context. The greatest physicists of the time were involved. They found contradictions between theory and experience or within theory, offered mutually incompatible solutions, and sometimes diagnosed a serious crisis in this domain of physics.

Since Heinrich Hertz's experiments of 1887-8 on the electric production of electromagnetic waves, Maxwell's field theory was the natural frame for discussing both the electrodynamics and the optics of moving bodies. In order to understand the evolution of this subject, one must first realize that the theory that Maxwell offered in his treatise of 1873 widely differed from what is now meant by "Maxwell's theory."

## 1 Maxwell's theory as it was

Like most of his contemporaries, Maxwell regarded the existence of the ether as a fundamental and undeniable fact of physics. He held this medium responsible for the propagation of electromagnetic actions, which included optical phenomena in his view. His theory was a phenomenological theory concerned with the macroscopic states of a continuous medium, the ether, which could combine with matter and share its velocity $\mathbf{v}$. These states were defined by four vectors $\mathbf{E}$, $\mathbf{D}$, $\mathbf{H}$, $\mathbf{B}$ that obeyed a few general partial differential equations as well as some relations depending on the intrinsic properties of the medium. In the most complete and concise form later given by Oliver Heaviside and Heinrich Hertz, the fundamental equations read

$$\begin{aligned}&\nabla\times\mathbf{E}=-D\mathbf{B}/Dt\ ,\quad \nabla\times\mathbf{H}=\mathbf{j}+D\mathbf{D}/Dt\\&\nabla\cdot\mathbf{D}=\rho\ ,\nabla\cdot\mathbf{B}=0\ ,\end{aligned}\tag{1}$$

where $\mathbf{j}$ is the conduction current and $D/Dt$ is the convective derivative defined by

$$D/Dt=\partial/\partial t-\nabla\times(\mathbf{v}\times\ )+\mathbf{v}(\nabla\cdot\ ).\tag{2}$$

In a linear medium, the "forces" $\mathbf{E}$ and $\mathbf{H}$ were related to the "polarizations" $\mathbf{D}$ and $\mathbf{B}$ by the relations $\mathbf{D} = \epsilon\mathbf{E}$ and $\mathbf{B} = \mu\mathbf{H}$, and the energy density $(1/2)(\epsilon E^2 + \mu H^2)$ of the medium had the form of an elastic energy. For Maxwell and his followers, the charge density and the conduction current $\mathbf{j}$ were not primitive concepts: the former corresponded to the longitudinal gradient of the polarization or "displacement" $\mathbf{D}$, and the latter to the dissipative relaxation of this polarization in a conducting medium. The variation $D\mathbf{D}/Dt$ of the displacement constituted another form of current. Following Michael Faraday, Maxwell and his disciples regarded the electric fluids of earlier theories as a naïvely substantialist notion.[1]

The appearance of the convective derivative $D/Dt$ in Maxwell's theory derives from his understanding of the polarizations $\mathbf{D}$ and $\mathbf{B}$ as states of a single medium made of ether and matter and moving with a well-defined velocity $\mathbf{v}$ (that may vary from place to place): the time derivatives in the fundamental equations must be taken along the trajectory of a given particle of the moving medium. The resulting law of electromagnetic induction,

$$\nabla \times \mathbf{E} = -D\mathbf{B}/Dt = -\partial\mathbf{B}/\partial t + \nabla \times (\mathbf{v} \times \mathbf{B}) \ , \tag{3}$$

contains the $(\mathbf{v} \times \mathbf{B})$ contribution to the electric field in moving matter. By integration around a circuit and through the Kelvin-Stokes theorem, it leads to the expression

$$\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \iint \mathbf{B} \cdot d\mathbf{S} \tag{4}$$

of Faraday's law of induction, wherein the integration surface moves together with the bordering circuit. When the magnetic field is caused by a magnet, the magnetic flux only depends on the relative position of the magnet and the circuit so that the induced current only depends on their relative motion.

In sum, the conceptual basis of Maxwell's original theory widely differed from what today's physicists would expect. Electricity and magnetism were field-derived concept, whereas modern electromagnetism treats them as separate entities. A quasi-material ether was assumed. The fundamental equations (1) only correspond to our "Maxwell equations" in the case of bodies at rest, for which the velocity $\mathbf{v}$ is zero and the convective derivative $D/Dt$ reduces to the partial derivative $\partial/\partial t$. One thing has not changed, however: the theory's ability to unify electromagnetism and optics. In a homogenous insulator at rest, Maxwell's equations imply the existence of transverse waves propagating at the velocity $c = 1/\sqrt{\epsilon\mu}$. Having found this electromagnetic constant to be very close to the velocity of light, Maxwell identified these waves with light waves. The resulting theory automatically excludes the longitudinal vibrations that haunted the earlier, elastic-solid theories of optics.

Within a few years after Maxwell's death (in 1879), a growing number of British physicists saluted this achievement and came to regard Maxwell's theory as

[1] J.C. Maxwell, *A treatise on electricity and magnetism*, 2 vols. (Oxford, 1973); H. Hertz, "Über die Grundgleichungen der Elektrodynamik für bewegte Körper," *Annalen der Physik*, 41 (1890), 369-399.

philosophically and practically superior to earlier theories. The Germans had their own theories of electricity and magnetism, based on electric and magnetic fluids (or Amperean currents) directly acting at a distance. They mostly ignored Maxwell's theory until in 1888 Heinrich Hertz demonstrated the emission of electromagnetic waves by a high-frequency electric oscillator. After this spectacular discovery was confirmed, a growing number of physicists adopted Maxwell's theory in a more or less modified form.

Yet this theory was not without difficulties. Maxwell had himself noted that his phenomenological approach led to wrong predictions when applied to optical dispersion, to magneto-optics, and to the optics of moving bodies. In these cases he suspected that the molecular structure of matter had to be taken into account.

## 2 Flashback: The optics of moving bodies

Maxwell's idea of a single medium made of ether and matter implied that the ether was fully dragged by moving matter, even for dilute matter. Whereas this conception worked very well when applied to moving circuits and magnets, it was problematic in the realm of optics. The first difficulty concerned the aberration of stars, discovered by the British astronomer James Bradley in 1728: the direction of observation of a fixed star appears to vary periodically in the course of a year, by an amount of the same order as the ratio ($10^{-4}$) of the orbital velocity of the earth to the velocity of light.[2]

The old corpuscular theory of light simply explained this effect by the fact that the apparent velocity of a light particle is the vector sum of its true velocity and the velocity of the earth (see Fig. 1). In the early nineteenth century, the founders of the wave theory of light Thomas Young and Augustin Fresnel saved this explanation by assuming that the ether was completely undisturbed by the motion of the earth through it. Indeed, rectilinear propagation at constant velocity is all that is needed for the proof.[3]

Fresnel's assumption implied an ether wind of the order of 30km/s on the earth's surface, from which a minute modification of the laws of optical refraction ought to follow. As Fresnel knew, an earlier experiment of his friend François Arago had shown that refraction by a prism was in fact unaffected by the earth's annual motion. Whether or not Arago had reached the necessary precision of $10^{-4}$, Fresnel took this result seriously and accounted for it by means of a partial dragging of the ether within matter. His theory can be explained as follows.

According to an extension of Fermat's principle, the trajectory that light takes to travel between two fixed points (with respect to the earth) is that for

[2] J. Bradley, "A new apparent motion discovered in the fixed stars; its cause assigned; the velocity and equable motion of light deduced," Royal Society of London, *Proceedings*, 35 (1728), 308-321.

[3] A. Fresnel, "Lettre d'Augustin Fresnel à François Arago sur l'influence du mouvement terrestre dans quelques phénomènes d'optique," *Annales de chimie et de physique*, 9(1818), also in *Oeuvres complètes*, Paris (1868), vol. 2, 627-636.

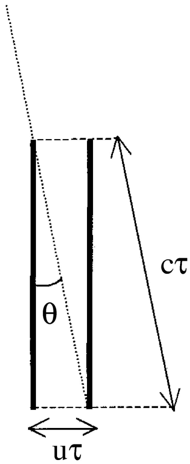


Figure 1: *Stellar aberration.* Suppose that the position of a fixed star in the sky is judged by the orientation of a narrow straight tube through which it can be seen. If the earth is moving with respect to the fixed stars at the velocity $\mathbf{u}$, the latter sweeps the distance $u\tau$ during the time $\tau$ that the light from the star takes to travel from the beginning to the end of the tube. Therefore, the true light path makes a small angle with the direction of the tube. When the velocity of the earth is perpendicular to the tube, this angle is $\theta \approx \tan\theta = u/c$. Owing to the annual motion of the earth, the apparent position of the star varies with a period of one year.

which the traveling time is a minimum, whether the medium of propagation is at rest or not. The velocity of light with respect to the ether in a substance of optical index $n$ is $c/n$, if $c$ denotes the velocity of light. The absolute velocity of the ether across this substance is $\alpha\mathbf{u}$, where $\alpha$ is the dragging coefficient and $\mathbf{u}$ is the absolute velocity of the substance (the absolute velocity being that with respect to the remote, undisturbed parts of the ether). Therefore, the velocity of light along the element $d\mathbf{l}$ of an arbitrary trajectory is $c/n + (\alpha - 1)\mathbf{u} \cdot d\mathbf{l}/ds$ with respect to the substance (with $ds = \|d\mathbf{l}\|$). To first order in $u/c$, the time taken by light during this elementary travel is

$$dt = (n/c)ds + (n^2/c^2)(1 - \alpha)\mathbf{u} \cdot d\mathbf{l} \ . \tag{5}$$

Note that the index $n$ and the dragging coefficient in general vary along the path, whereas the velocity $\mathbf{u}$ has the same value (the velocity of the earth) for the whole optical setting. The choice $\alpha = 1$ (complete drag) leaves the time $dt$ and the trajectory of minimum time invariant, as should obviously be the case. Fresnel's choice,

$$\alpha = 1 - 1/n^2 \tag{6}$$

yields

$$dt = (n/c)ds + (1/c^2)\mathbf{u} \cdot d\mathbf{l}\ , \tag{7}$$

so that the time taken by light to travel between two fixed points of the optical setting differs only by a constant from the time it would take if the earth were not moving. Therefore, the laws of refraction are unaffected (to first order) under Fresnel's assumption.[4]

In 1846, the Cambridge professor George Gabriel Stokes criticized Fresnel's theory for making the fantastic assumption that the huge mass of the earth was completely transparent to the ether wind. In Stokes' view, the ether was a jelly-like substance that behaved as an incompressible fluid under the slow motion of immersed bodies but had rigidity under the very fast vibrations implied in the propagation of light. In particular, he identified the motion of the ether around the earth with that of a perfect liquid. From Lagrange, he knew that the flow induced by a moving solid (starting from rest) in a perfect liquid is such that a potential exists for the velocity field. From his recent derivation of the Navier-Stokes equation, he also knew that this property was equivalent to the absence of instantaneous rotation of the fluid elements. Consequently, the propagation of light remains rectilinear in the flowing ether, and the apparent position of stars in the sky is that given by the usual theory of aberration.[5]

In order to account for the absence of effects of the earth's motion on terrestrial optics, Stokes further assumed that the ether adhered to the earth and had a negligible relative velocity at reasonable distances from the ground.

To sum up, before the middle of the century, there were two competing theories of the optics of moving bodies that both accounted for stellar aberration and for the absence of effects of the earth's motion on terrestrial optics. Fresnel's theory assumed the stationary character of the ether everywhere except in moving refractive media, in which a partial drag occurred. Stokes' theory assumed complete ether drag around the earth and irrotational flow at higher distances from the earth.

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[4]Cf. E. Mascart, *Traité d'optique*, 3 vols. (Paris, 1893), vol. 3, chap. 15. Fresnel justified the value $1 - 1/n^2$ of the dragging coefficient by making the density of the ether inversely proportional to the square of the propagation velocity $c/n$ (as should be in an elastic solid of constant elasticity) and requiring the flux of the ether to be conserved. As Mascart noted in the 1870s, this justification fails when double refraction and dispersion are taken into account.

[5]G.G. Stokes, "On the aberration of light," *Philosophical magazine*, 27(1845), 9-55; "On Fresnel's theory of the aberration of light," ibid., 28 (1846), 76-81; "On the constitution of the luminiferous ether, viewed with reference to the phenomenon of the aberration of light," ibid., 29 (1846), 6-10. This result can be obtained from Fermat's principle, by noting that to first order the time taken by light to travel along the element of length $d\mathbf{l}$ has the form $dt = (1/c)ds - (1/c^2)\mathbf{v} \cdot d\mathbf{l}$ ($\mathbf{v}$ denoting the velocity of the ether), so that its integral differs only by a constant (the difference of the velocity potentials at the end points) from the value it would have in a stationary ether.

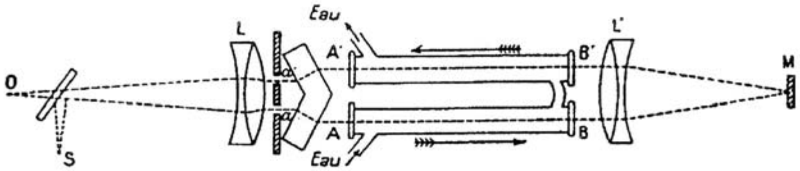


Figure 2: *Fizeau's experiment.* After reflection on a semi-reflecting blade, the light from the source S is divided into two beams. The upper beam travels against the water stream in A′B′, crosses the lens L′, is reflected on the mirror M, crosses L′ again, travels *against* the water stream in AB, and returns to the semi-reflecting blade. The lower beam does the symmetrical trip, which runs twice along the water stream. The phase difference between the two beams is judged from the interference pattern in O.

In 1850 Hippolyte Fizeau performed an experiment in which he split a light beam into two beams, had them travel through water moving in opposite directions, and measured their phase difference by interference (see fig. 2). The result confirmed the partial drag of light waves predicted by Fresnel. Maxwell knew about Fizeau's result, and, for a while, wrongly believed that it implied an alteration of the laws of refraction by the earth's motion through the ether. In 1864, he performed an experiment to test this modification. The negative result confirmed Arago's earlier finding with improved precision. As Stokes explained to Maxwell, this result pleaded for, rather than contradicted the Fresnel drag. Yet Maxwell remained skeptical about the validity of Fizeau's experiment. In 1867 he wrote:

> This experiment seems rather to verify Fresnel's theory of the ether; but the whole question of the state of the luminiferous medium near the earth, and of its connexion with gross matter, is very far as yet from being settled by experiment.

In this situation, it was too early to worry about an incompatibility between the electromagnetic theory of light and the optics of moving bodies. In 1878, one year before his death, Maxwell still judged Stokes' theory "very probable."[6]

[6] H. Fizeau, "Sur les hypothèses relatives à l'éther lumineux, et sur une expérience qui paraît démontrer que le mouvement des corps change la vitesse avec laquelle la lumière se propage dans leur intérieur," Académie des Sciences, *Comptes-rendus*, 33 (1851), 349-355; J.C. Maxwell, "On an experiment to determine whether the motion of the earth influences the refraction of light," unpub. MS, in Maxwell, *The scientific letters and papers*, ed. Peter Harman, vol. 2 (Cambridge, 1995), 148-153; Maxwell to Huggins, 10 Jun 1867, ibid., 306-311; "Ether," article for the *Encyclopedia Britannica* (1878), reproduced ibid., 763-775.

In the 1870s a multitude of experiments confirmed the absence of effect of the earth's motion on terrestrial optics. In 1874, the author of the best of those, Eleuthère Mascart, concluded:

> The translational motion of the earth has no appreciable influence on optical phenomena produced by a terrestrial source, or light from the sun, so these phenomena do not provide us with a means of determining the absolute motion of a body, and relative motions are the only ones that we are able to determine.

Mascart and other continental experts interpreted this finding by means of Fresnel's theory. British physicists mostly disagreed, as can be judged from a British Association report of 1885 in which a disciple of Maxwell criticized "Fresnel's somewhat violent assumptions on the relation between the ether within and without a transparent body."[7]

In 1881 the great American experimenter Albert Michelson conceived a way to decide between Fresnel's and Stokes' competing theories. Through an interferometer of his own, he compared the time that light took to travel the same length in orthogonal directions (see fig. 3). If the ether was stationary, he reasoned, the duration of a round trip of the light in the arm parallel to the earth's motion was increased by a factor $[l/(c-u)+l/(c+u)]/(2l/c)$, which is equal to $1/(1-u^2/c^2)$. The corresponding fringe shift was about twice what his interferometer could detect. From the null result, Michelson concluded that Fresnel's theory had to be abandoned.[8]

A French professor at the Ecole Polytechnique, Alfred Potier, told Michelson that he had overlooked the increase of the light trip by $1/\sqrt{1-u^2/c^2}$ in the perpendicular arm of his interferometer. With this correction, the experiment became inconclusive. Following William Thomson's and Lord Rayleigh's advice and with Edward Morley's help, Michelson first decided to repeat Fizeau's experiment with his powerful interferometric technique. In 1886 he thus confirmed the Fresnel dragging coefficient with greatly improved precision.[9]

At this critical stage, the Dutch theorist Hendrik Lorentz entered the discussion. He first blasted Stokes' theory by noting that the irrotational motion of an incompressible fluid around a sphere necessarily involves a finite slip on its surface.[10] The theory could still be saved by integrating Fresnel's partial drag, but only at the price of making it globally more complicated than Fresnel's. Lorentz therefore favored Fresnel's theory, and called for a repetition of Michelson's experiment

[7] E. Mascart, "Sur les modifications qu'éprouve la lumière par suite du mouvement de la source et du mouvement de l'observateur," *Annales de l'Ecole Normale*, 3 (1874), 363-420, on 420; R.T. Glazebrook, Report on "optical theories," British Association for the Advancement of Science, *Report* (1885), 157-261.

[8] A. Michelson, "The relative motion of the earth and the luminiferous ether," *American journal of science*, 22 (1881), 120-129.

[9] A. Michelson and E. Morley, "Influence of the motion of the medium on the velocity of light", *American journal of science*, 31 (1886), 377-386.

[10] It seems dubious that Stokes, as an expert on potential theory in fluid mechanics, could have overlooked this point. More likely, his jelly-like ether permitted temporary departures from irrotationality.

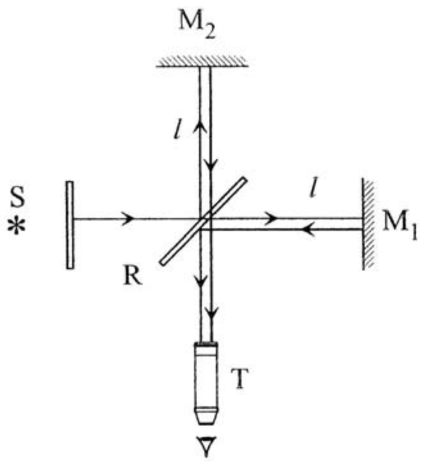


Figure 3: *The Michelson-Morley experiment.* The light from the source **S** is divided into two beams by the semi-reflecting blade **R**. After reflection on the mirrors $\mathbf{M}_1$ and $\mathbf{M}_2$, the two beams return to **R**. Their interference pattern is observed through the telescope **T**.

of 1881 after noting the error already spotted by Potier. Michelson and Morley fulfilled this wish in 1887 with an improved interferometer. The result was again negative, to every expert's puzzlement: while Fizeau's experiment confirmed Fresnel's theory, the new experiment contradicted it.[11]

## 3 Lorentz's theory

When in the early 1890s Hertz and Heaviside perfected Maxwell's electrodynamics of moving bodies, they noted that it was incompatible with Fresnel's theory of aberration, but decided to postpone further study of the relation between ether and matter. Unknown to them, Lorentz had long ago reflected on this relation and reached conclusions that sharply departed from Maxwell's original ideas. Unlike Maxwell's British disciples, Lorentz learned Maxwell's theory in a reinterpretation by Hermann Helmholtz that accommodated the continental interpretation of charge, current, and polarization in terms of the accumulation, flow, and

[11] H.A. Lorentz, "De l'influence du mouvement de la terre sur les phénomènes lumineux," *Archives néerlandaises* (1887), also in *Collected papers*, 9 vols. (The hague, 1934-1936), vol. 4, 153-214; Michelson and Morley, "On the relative motion of the earth and the luminiferous ether," *American journal of science*, 34 (1887), 333-345.

displacement of electric particles. In 1878 he gave a molecular theory of optical dispersion based on the idea of elastically bound charged particles or "ions" that vibrated under the action of an incoming electromagnetic wave and thus generated a secondary wave. For the sake of simplicity, he assumed that the ether around the molecules and ions had exactly the same properties as the ether in a vacuum. He could thus treat the interactions between ions and electromagnetic radiation through Maxwell's equations in a vacuum supplemented with the so-called Lorentz force.[12]

Using lower-case letters for the microscopic fields and Hertzian units, these equations read

$$\begin{aligned} &\nabla \times \mathbf{e} = -c^{-1}\partial \mathbf{b}/\partial t \ , \quad \nabla \times \mathbf{b} = c^{-1}[\rho_m \mathbf{v} + \partial \mathbf{e}/\partial t] \ , \\ &\nabla \cdot \mathbf{e} = \rho_m \ , \quad \nabla \cdot \mathbf{b} = 0 \ , \\ &\mathbf{f} = \rho_m[\mathbf{e} + c^{-1}\mathbf{v} \times \mathbf{b}] \ , \end{aligned} \tag{8}$$

where $\rho_m$ denotes the microscopic charge density (confined to the ions) and $\mathbf{f}$ denotes the density of the force acting on the ions. Note that there are only two independent fields $\mathbf{e}$ and $\mathbf{b}$ since the constants $\epsilon$ and $\mu$ are set to their vacuum value. Although from a formal point of view these equations can be seen as a particular case of the Maxwell-Hertz equations (1), they were unthinkable to true Maxwellians who regarded the concepts of electric charge and polarization as emergent macroscopic concepts and believed the molecular level to be directly ruled by the laws of mechanics.

Using his equations and averaging over a macroscopic volume element, Lorentz obtained the first electromagnetic theory of dispersion. In 1892, he realized that he could perform similar calculations when the dielectric globally moved through the ether at the velocity $\mathbf{u}$ of the earth. He only had to assume that the ions and molecules moved through the ether without disturbing it. Superposing the incoming wave and the secondary waves emitted by the moving ions, he found that the resulting wave traveled at the velocity predicted by Fresnel's theory. The partial ether drag imagined by Fresnel was thus reduced to molecular interference in a perfectly stationary ether.[13]

Notwithstanding with their global intricacy, Lorentz's original calculations contained an interesting subterfuge. In order to solve equations that involved the wave operator $\partial^2/\partial x^2 - c^{-2}(\partial/\partial t - u\partial/\partial x)^2$ in a reference frame bound to the transparent body, Lorentz introduced the auxiliary variables

$$x' = \gamma x \ , \quad t' = \gamma^{-1}t - \gamma ux/c^2 \tag{9}$$

[12] Lorentz, "Over het verband tusschen de voortplantings sneldheit en samestelling der midden stofen," Koninklijke Akademie van Wetenschappen, *Verslagen* (1878), transl. as "Concerning the relation between the velocity of propagation of light and the density and composition of media" in *Collected papers* (ref. 11), vol. 2, 3-119.

[13] Lorentz, "La théorie électromagnétique de Maxwell et son application aux corps mouvants," *Archives néerlandaises* (1892), also in *Collected papers* (ref. 11), vol. 2, 164-321.

that restored the form of the operator in the ether-bound frame for

$$\gamma = 1/\sqrt{1-u^2/c^2}\ . \tag{10}$$

He thus discovered the Lorentz transformation for coordinates (up to the Galilean transformation $x = \bar{x} - ut$, where $\bar{x}$ is the abscissa in the ether frame).[14]

A few months later, Lorentz similarly realized that to first order in $u/c$ the field equations in a reference frame bound to the earth could be brought back to the form they have in the ether frame through the transformations

$$t' = t - ux/c^2\ , \quad \mathbf{e}' = \mathbf{e} + c^{-1}\mathbf{u}\times\mathbf{b}\ , \quad \mathbf{b}' = \mathbf{b} - c^{-1}\mathbf{u}\times\mathbf{e}\ . \tag{11}$$

In other words, the combination of these transformations with the Galilean transformation $x = \bar{x} - ut$ leaves the Maxwell-Lorentz equations invariant to first order. Lorentz used this remarkable property to ease his derivation of the Fizeau coefficient and to give a general proof that to first order optical phenomena were unaffected by the earth's motion through the ether.[15]

It is important to understand that for Lorentz the transformed coordinates and fields were mathematical aids with no direct physical significance. They were only introduced to facilitate the solution of complicated differential equations. The "local time" $t'$ was only called so because it depended on the abscissa. The true physical quantities were the absolute time $t$ and the fields $\mathbf{e}$ and $\mathbf{b}$ representing the states of the ether. In order to prove the first-order invariance of optical phenomena, Lorentz considered two systems of bodies of identical constitution, one at rest in the ether, the other drifting at the velocity $\mathbf{u}$. He first noted that to a field pattern $\mathbf{e}_0 = F(x,y,z,t),\ \mathbf{b}_0 = G(x,y,z,t)$ for the system at rest corresponded a field pattern $\mathbf{e},\mathbf{b}$ for the drifting system such that $\mathbf{e}' = F(x,y,z,t'),\ \mathbf{b}' = G(x,y,z,t')$ (the abscissa $x$ being measured in a frame bound to the system). He then noted that $\mathbf{e}'$ and $\mathbf{b}'$ vanished simultaneously if and only if $\mathbf{e}$ and $\mathbf{b}$ did so. Consequently, the borders of a ray of light or the dark fringes of an interference pattern have the same locations in the system at rest and in the drifting system. The change of the time variable is irrelevant, since the patterns observed in optical experiments are stationary. We may conclude that Lorentz's use of the Lorentz invariance was quite indirect and subtle.

There remained a last challenge for Lorentz: to account for the negative result of the Michelson-Morley experiment of 1887. As George Francis FitzGerald had already done, Lorentz noted that the fringe shift expected in a stationary ether theory disappeared if the longitudinal arm of the interferometer underwent a contraction by the amount $\gamma^{-1} = \sqrt{1-u^2/c^2}$ when moving through the ether. In order to justify this hypothesis, Lorentz first noted that in the case of electrostatics the field equations in a frame bound to the drifting body could be brought back to those for a body at rest through the transformation $x' = \gamma x$. He further assumed that the equilibrium length or a rigid rod was determined by the value

[14]Ibid. : 297

[15]Lorentz, "On the reflexion of light by moving bodies," Koninklijke Akademie van Wetenschappen, *Verslagen* (1892), also in *Collected papers* (ref.11), vol. 4, 215-218.

of intermolecular forces and that these forces all behaved like electrostatic forces when the rod drifted through the ether. Then the fictitious rod obtained by applying the dilation $x' = \gamma x$ to a longitudinally drifting rod must have the length that this rod would have if it were at rest. Consequently, the moving rod contracts by the amount $\gamma^{-1}$. The Lorentz contraction thus appears to result from a postulated similarity between molecular forces of cohesion and electrostatic forces.[16]

Fully explained in the *Versuch* of 1895, Lorentz's theory gained broad recognition before the end of the century. Two other physicists, Joseph Larmor of Cambridge and Emil Wiechert of Königsberg, proposed similar theories in the same period. In the three theories, the basic idea was to hybridize Maxwell's theory with the corpuscular concept of electricity and to reduce every optic and electromagnetic phenomenon to the interactions between electric particles through a stationary ether. Besides the optics of moving bodies, these theories explained a variety of magnetic and magneto-optic phenomena, and of course retrieved the confirmed predictions of Maxwell's theory. They benefited from the contemporary rise of an experimental microphysics, including the discoveries of x-rays (1895), radioactivity (1896), and the electron (1897). In 1896, the Dutch experimenter Pieter Zeeman revealed the magnetic splitting of spectral lines, which Lorentz immediately explained through the precession of the orbiting charged particles responsible for the lines. Being much lighter than hydrogen, these particles were soon identified to the corpuscle discovered in cathode rays by Emil Wiechert and Joseph John Thomson. Following Larmor's terminology, this corpuscle became known as the electron and replaced the ions in Lorentz's theory.[17]

## 4 Poincaré's criticism

In France, the mathematician Henri Poincaré had been teaching electrodynamics at the Sorbonne for several years. After reviewing the theories of Maxwell, Helmholtz, Hertz, Larmor, and Lorentz, he judged that the latter was the one that best accounted for the whole range of optic and electromagnetic phenomena. Yet he was not entirely satisfied with Lorentz's theory, because he believed it contradicted fundamental principles of physics. In general, Poincaré perceived an evolution of physics from the search of ultimate mechanisms to a "physique des principes" in which a few general principles served as guides in the formation of theories. Among these principles were three general principles of mechanics: the

[16] Lorentz, "De relative beweging van der aarde en den aether," Koninklijke Akademie van Wetenschappen, *Verslagen* (1892), transl. as "The relative motion of the earth and the ether" in *Collected papers* (ref. 11), vol. 4, 220-223.

[17] Lorentz, *Versuch einer Theorie der elektrischen un optischen Erscheinungen in bewegten Körpern* (Leiden, 1895), also in *Collected papers* (ref. 11), vol.5, 1-139; "Optische verschinitjnelsen die met de lading en de massa der ionen in verband staan," Koninklijke Akademie van Wetenschappen, Verslagen (1898), transl. as "Optical phenomena connected with the charge and mass of ions" in *Collected papers* (ref. 11) vol. 3, 17-39.

principle of relativity, the principle of reaction, and the principle of least action.[18]

For any believer in the mechanical nature of the electromagnetic ether, it was obvious that these three principles applied to electrodynamics, since ether and matter were together regarded as a complex mechanical system. In particular, it was clear that electromagnetic phenomena would be the same if the same uniform boost was applied to the ether and all material objects. It the boost was applied to matter only, effects of this boost were expected to occur. For instance, Maxwell believed that the force between two electric charges moving together uniformly on parallel lines had to vanish when their velocity reached the velocity of light. Poincaré thought differently. In his view, the ether only was a convenient convention suggested by the analogy between the propagation of sound and the propagation of light. In the foreword of his lectures of his lecture of 1887/8 on the mathematical theories of light, he wrote:[19]

> It matters little whether the ether really exists: that is the affair of the metaphysicians. The essential thing for us is that everything happens as if it existed, and that this hypothesis is convenient for us for the explanation of the phenomena. After all, have we any other reason to believe in the existence of material objects? That too, is only a convenient hypothesis; only this will never cease to do so, whereas, no doubt, some day the ether will be thrown aside as useless.

As we will see, Poincaré actually never abandoned the ether. But he refused to regard it as an ordinary kind of matter whose motion could affect observed phenomena. In his view, the principle of reaction and the principle of relativity had to apply to matter alone. In his lectures of 1899 on Lorentz's theory, he wrote: I consider it very probable that optical phenomena depend only on the relative motion of the material bodies present –light sources and apparatus– and this not only to first or second order but exactly.

It must be emphasized that at that time no other physicist believed in this acceptance of the relativity principle. Most physicists conceived the ether as a physical entity whose wind should have physical effects, even though the precision needed to test this consequence was not yet available. The few physicists, such as Paul Drude or Emil Cohn, who questioned the mechanical ether, felt free to violate principles of mechanics, including the relativity principle.[20]

Lorentz's theory satisfied Poincaré's relativity principle only approximately and did so through what Poincaré called two "coups de pouce": the local time and the Lorentz contraction. Moreover, it violated Poincaré's reaction principle, since Lorentz's equations implied that the net force acting on all the ions or electrons should be the space integral of $\partial(\mathbf{e} \times \mathbf{b})/c\partial t$, which does not vanish in general. In his contribution to Lorentz's jubilee of 1900, Poincaré iterated this criticism and further discussed the nature and impact of the violation of the reaction principle. In the course of this argument, about which more details will be given in

[18]H. Poincaré, *Electricité et optique. La Lumière et les théories électrodynamiques* [Sorbonne lectures of 1888, 1890 and 1899)], ed. J. Blondin and E. Néculcéa, (Paris, 1901).

[19]Poincaré, *Théorie mathématique de la lumière* (Sorbonne lectures, 1887-88), ed. J. Blondin (Paris, 1889), I.

[20]Poincaré, ref. 18, 536.

a moment, he relied on Lorentz's transformations (11) to compute the energy of a pulse of electromagnetic radiation from the standpoint of a moving observer. The transformed fields $\mathbf{e}'$ and $\mathbf{b}'$, he noted, are the fields measured by a moving observer. Indeed the force acting on a test unit charge moving with the velocity $\mathbf{u}$ is $\mathbf{e} + c^{-1}\mathbf{u} \times \mathbf{b} = \mathbf{e}'$ according to the Lorentz force formula. Poincaré went on noting that the local time $t' = t - ux/c^2$ was that measured by moving observers if they synchronized their clocks in the following manner:[21]

> I suppose that observers placed in different points set their watches by means of optical signals; that they try to correct these signals by the transmission time, but that, ignoring their translational motion and thus believing that the signals travel at the same speed in both directions, they content themselves with crossing the observations, by sending one signal from A to B, then another from B to A.

Poincaré only made this remark *en passant*, gave no proof, and did as if it had already been on Lorentz's mind. The proof goes as follows. When B receives the signal from A, he sets his watch to zero (for example), and immediately sends back a signal to A. When A receives the latter signal, he notes the time $\tau$ that has elapsed since he sent his own signal, and sets his watch to the time $\tau/2$. By doing so he commits an error $\tau/2 - t_-$, where $t_-$ is the time that light really takes to travel from B to A. This time, and that of the reciprocal travel are given by $t_- = \mathrm{AB}/(c+u)$ and $t_+ = \mathrm{AB}/(c-u)$, since the velocity of light is $c$ with respect to the ether (see fig. 4). The time $\tau$ is the sum of these two traveling times. Therefore, to first order in $u/c$ the error committed in setting the watch A is $\tau/2 - t_- = (t_+ - t_-)/2 = u\mathrm{AB}/c^2$. At a given instant of the true time, the times indicated by the two clocks differ by $u\mathrm{AB}/c^2$, in conformity with Lorentz' expression of the local time.

Poincaré transposed this synchronization procedure from an earlier discussion on the measurement of time, published in 1898. There he noted that the dating of astronomical events was based on the implicit postulate "that light has a constant velocity, and in particular that its velocity is the same in all directions." He also explained the optical synchronization of clocks at rest, and mentioned its similarity with the telegraphic synchronization that was then being developed for the purpose of longitude measurement. As a member of the Bureau des Longitudes, Poincaré naturally sought an interpretation of Lorentz's local time in terms of cross-signaling. As a believer in the relativity principle, he understood that moving observers would never know their motion through the ether and therefore could only do as if these signals propagated isotropically.[22]

Poincaré thus provided a physical interpretation of the transformed time $t'$ and the transformed fields $\mathbf{e}'$ and $\mathbf{b}'$, which only referred to a fictitious system for Lorentz. This interpretation greatly eased the use of this transformation, for it made the (first-order) invariance of optical phenomena a direct consequence of

[21] Poincaré, "La théorie de Lorentz et le principe de la réaction." In Recueil de travaux offerts par les auteurs à H.A. Lorentz à l'occasion du 25ème anniversaire de son doctorat le 11 décembre 1900, *Archives néerlandaises*, 5 (1900), 252-278, on 272.

[22] Poincaré, "La mesure du temps", *Revue de métaphysique et de morale*, 6 (1898), 371-384.

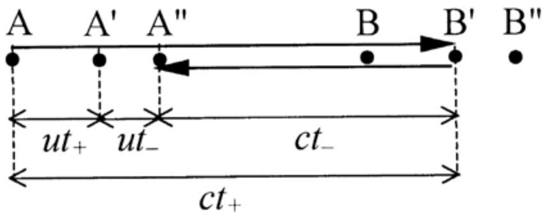


Figure 4: Cross-signaling between two observers moving at the velocity u through the ether. The points **A**, **A**′, **A**″, **B**, **B**′, **B**″ represent the successive positions of the observers in the ether when the first observer sends a light signal, when the second observer receives this signal and sends back another signal, and when the first observer receives the latter signal.

the formal invariance of the Maxwell-Lorentz equations. Yet it would be a mistake to believe that Poincaré thereby redefined the concepts of space and time. In his terms, the Lorentz-transformed quantities referred to the apparent states of the field for a moving observer. The true states remained those defined with respect to the ether. As we will see, Poincaré never gave up this view.

## 5 The Lorentz invariance

Strangely, Lorentz overlooked Poincaré's reinterpretation of his transformations, and kept reasoning in terms of a fictitious system brought to rest. So did other experts on the electrodynamics of moving bodies until at least 1904. Nevertheless, Lorentz took some of Poincaré's criticism seriously. In 1904, he offered a new version of his theory in which the invariance of optical phenomena held at every order in $u/c$, without the "coups de pouce" reproached by Poincaré. He knew since 1899 that the homogenous field equations for a system bound to the earth could be brought to the form they have for a system at rest in the ether through the transformations

$$\begin{aligned} &x' = \gamma\epsilon x, \quad y' = \epsilon y, \quad z' = \epsilon z, \quad t' = \epsilon(\gamma^{-1}t - \gamma u x c^{-2}) \\ &\mathbf{e}' = \epsilon^{-2}(1,\gamma)(\mathbf{e} + c^{-1}\mathbf{u}\times\mathbf{b}), \quad \mathbf{b}' = \epsilon^{-2}(1,\gamma)(\mathbf{b} - c^{-1}\mathbf{u}\times\mathbf{e}) \ , \end{aligned} \tag{12}$$

where $\epsilon$ is an undetermined constant (for a given value of $u$) and the factor $(1,\gamma)$ means a multiplication by 1 of the component of the following vector parallel to $\mathbf{u}$ and a multiplication by $\gamma$ of the component perpendicular to $\mathbf{u}$. In 1904, he generalized this result to the coupling between electrons and field.[23]

[23] Lorentz, "Electromagnetic phenomena in a system moving with any velocity smaller than light," Royal Academy of Amsterdam, *Proceedings* (1904), also in *Collected papers* (ref. 11), vol. 5, 172-197.

Specifically, Lorentz realized that for a spherical electron subjected to the Lorentz contraction and carrying the electromagnetic momentum

$$\mathbf{p} = c^{-1} \int (\mathbf{e} \times \mathbf{b}) d\tau \ , \tag{13}$$

his transformations brought back the equation of motion

$$d\mathbf{p}/dt = e[\mathbf{e} + c^{-1}(\mathbf{u} + \mathbf{v}) \times \mathbf{b}] \tag{14}$$

of an electron of charge $e$ to the form it has for a system at rest ($u = 0$), if and only if the constant $\epsilon$ had the value 1. On his way to this result, he derived the expression $\mathbf{p} = m_0 \gamma \mathbf{v}$ of the momentum, where $m_0 = e^2/6\pi R c^2$ is the electromagnetic mass of a spherical-shell electron of radius $R$. Lastly, Lorentz gave expressions of the transformed source terms of the field equations such that dipolar emission in the moving system transformed into dipolar emission in the system at rest. Combining all these results, he could assert that optical phenomena in a moving system were the same as in a system at rest.

This result only held in the dipolar approximation, because Lorentz's expression of the transformed source terms was not the one today regarded to be correct. Lorentz also neglected the spinning motion of the electrons, and overlooked the cohesive forces that the stability of his contractile electron required. His derivation of the invariance of optical phenomena was complex and indirect, for it involved a double-step transformation, the fictitious system at rest, and comparison between the states of this system and those of the real system. For other phenomena, there is no doubt that Lorentz still believed that motion with respect to the ether could in principle be detected.

Poincaré reacted enthusiastically to Lorentz memoir, because he saw in it an opportunity to satisfy the relativity principle in a complete and exact manner. He published the results of the ensuing reflections under the title "Sur la dynamique de l'électron," first as a short note of 5 June 1905 in the *Comptes rendus*, and as a bulky memoir in the *Rendiconti* of the Circolo matematico di Palermo for the following year. He first defined the "relativity postulate" as follows:

> It seems that the impossibility of experimentally detecting the absolute motion of the earth is a general law of nature; we naturally incline to assume this law, which we shall call the Postulate of Relativity, and to do so without any restriction.

Correcting Lorentz's expression of the transformed source terms, he then showed that "the Lorentz transformations"

$$\begin{aligned} &x' = \gamma(x - ut), \quad y' = y, \quad z' = z, \quad t' = \gamma(t - uxc^{-2}) \ , \\ &\mathbf{e}' = (1, \gamma)(\mathbf{e} + c^{-1}\mathbf{u} \times \mathbf{b}), \quad \mathbf{b}' = (1, \gamma)(\mathbf{b} - c^{-1}\mathbf{u} \times \mathbf{e}) \ , \end{aligned} \tag{15}$$

left the Maxwell-Lorentz equations invariant. These transformations are obtained by combining the transformations (12), which Lorentz used, with the Galilean transformation $x' = x - ut$. Poincaré showed that they formed a group, and used this property to determine the global scaling factor $\epsilon$. He noted that the coordinate transformations left the quadratic form $x^2 + y^2 + z^2 - c^2t^2$ invariant and could thus be regarded as rotations in a four-dimensional space with an imaginary fourth coordinate. He obtained the relativistic law for the "addition" of velocities, for which the combined velocity always remains inferior to the limit $c$.[24]

Next, Poincaré showed that a model of the contractile electron could be conceived in which the cohesive forces (the so-called Poincaré tension) preserved the Lorentz invariance. He thus retrieved Lorentz's expression $\mathbf{p} = m_0\gamma\mathbf{v}$ for the momentum of the electron. Lastly, he argued that in order to be compatible with the postulate of relativity, gravitational interactions should propagate at the velocity of light; and he proposed modifications of Newton's law of gravitation that made it compatible with Lorentz invariance.

Thus, there is no doubt that Poincaré regarded Lorentz invariance as a general requirement for the laws of physics, and that he identified this formal condition with the principle of relativity. On the latter point, his only comment was:

> The reason why we can, without modifying any apparent phenomenon, confer to the whole system a common translation, is that the equations of an electromagnetic medium are not changed under certain transformations which I shall call the Lorentz transformations; two systems, one at rest, the other in translation, thus become exact images of one another.

The Palermo memoir, long and thorough as it was, said nothing on the interpretation to be given to the transformed coordinates and fields. Perhaps Poincaré believed this should not be the main point. Perhaps he had not yet been able to provide an operational understanding of Lorentz's local time at any order in $u/c$. There is no doubt, however, that he regarded the transformed fields and coordinates as the ones measured by moving observers. At the Saint-Louis conference of 1904, he repeated (and attributed to Lorentz!) his definition of the local time by optical cross-signaling. In his Sorbonne lectures of 1906, he proved that this definition remained valid at any order in $u/c$, and he characterized the Lorentz transformations as the ones giving the "apparent space and time coordinates."[25]

The same lectures and later talks on the "mécanique nouvelle" show that Poincaré nonetheless maintained the ether and the ordinary concepts of space and time. In his view, the clocks bound to the ether frame gave the true time, for it was only in this frame that the true velocity of light was $c$. The clocks of a moving frame only gave the apparent time. For those who would think that the difference with Einstein's theory of relativity is merely verbal, it is instructive to look at an argument Poincaré repeatedly gave to justify optical synchronization.[26]

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[24] Poincaré, "Sur la dynamique de l'électron," Académie des Sciences, *Comptes-rendus*, 140, (1905), 1504-1508; "Sur la dynamique de l'électron," *Rendiconti del Circolo matematico di Palermo* (1906), also in Poincaré, *Oeuvres* (Paris, 1954), vol. 9, 494-550, on 495.

[25] Poincaré, ibid., 495; "L'état actuel et l'avenir de la physique mathématique" (Saint-Louis lecture), *Bulletin des sciences mathématiques*, 28 (1904), 302-324, transl. in Poincaré, *The foundations of science* (New York, 1929); "Les limites de la loi de Newton," Sorbonne lectures (1906-1907) ed. by H. Vergne in *Bulletin astronomique publié par l'observatoire de Paris*, 17 (1953), 121-365, chap. 11.

[26] Poincaré, ibid., 218-220; "La dynamique de l'électron," *Revue générale des sciences pures et appliquées* (1908), also in *Oeuvres* (ref. 24), vol. 9, 551-586.

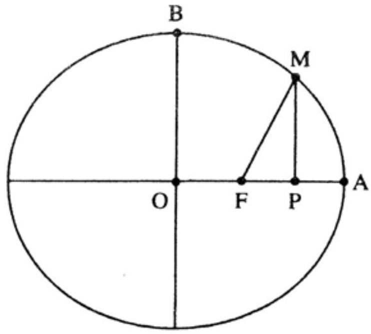


Figure 5: *Poincaré's light ellipsoid* $(a = OA,\ b = OB,\ f = OF)$.

Simultaneity should be transitive, namely: if the clock A is synchronized with the clock B, and if the clock B is synchronized with the clock C, then the clock A should be synchronized with clock C for any given choice of the positions of the three clocks. Indeed, any breakdown of transitivity could be used to detect motion through the ether and thus to violate the relativity principle. Now consider an observer moving with the constant velocity $\mathbf{u}$ through the ether and emitting a flash of light at time zero. At the value $t$ of the true time, this light is located on a sphere of radius $ct$ centered at the emission point. Poincaré next considered the appearance of this light shell for a moving observer, the rulers of which are subjected to the Lorentz contraction. The result is an ellipsoid of revolution, the half-axes of which have the values $a = \gamma ct$ and $b = ct$ (see fig. 5). As the eccentricity is $e = \sqrt{1 - b^2/a^2} = u/c$, the focal distance $f = ea = \gamma ut$ is equal to the apparent distance traveled by the observer during the time $t$. Therefore, the Lorentz contraction is the contraction for which the position of the observer at time $t$ coincides with the focus F of the light ellipsoid he has emitted.

Now consider a second observer traveling with the same velocity $\mathbf{u}$ and receiving the flash of light at the time $t_+$. The position M of this observer belongs to the ellipsoid $t = t_+$, and the distance $FM$ represents the apparent distance between the two observers, which is invariable. According to a well-known property of ellipses, we have

$$FM + eFP = b^2/a \ , \tag{16}$$

where P denotes the projection of M on the larger axis. The length $FP$ being equal to the difference $x'$ of the apparent abscissas of the two observers, this implies

$$t_+ = \gamma FM/c + \gamma ux'/c^2 \ . \tag{17}$$

Suppose that the two observers synchronize their clocks by cross-signaling. The traveling time of the reverse signal is

$$t_- = \gamma FM/c - \gamma ux'/c^2 \tag{18}$$

Therefore, two events are judged simultaneous by these observers if and only if their true times differ by

$$(t_+ - t_-)/2 = \gamma u x'/c^2 \ . \tag{19}$$

This condition is obviously transitive.[27]

For any one familiar with Einstein's theory of relativity, this reasoning seems very odd. Indeed, the light ellipsoid corresponds to a fixed value of the time $t$ in one reference frame and to space measured in another frame. In general, Poincaré's theory allows for the use of the "true time" in any reference system, whereas our relativity theory regards this sort of mixed reference as a mathematical fiction. This means that the conceptual basis of Poincaré's theory is not compatible with Einstein's, even though both theories are internally consistent and have the same empirical predictions (for the electrodynamics of moving bodies).[28]

Another oddity of Poincaré's theory is his naming the Lorentz contraction "a hypothesis." As we just saw, Poincaré showed that the contraction was necessary to the transitivity of optical synchronization, which itself derives from the relativity principle. He nonetheless spoke of a hypothesis, probably because he did not quite trust the implicit conventions made in this reasoning. In the Palermo memoir, he clearly indicated his dissatisfaction with the present state of the theory:

> We cannot content ourselves with simply juxtaposed formulas that would agree only by some happy coincidence; the formulas should, so to say, penetrate each other. Our mind will be satisfied only when we believe that we perceive the reason of this agreement, so that we may fancy that we have predicted it.

Poincaré meant that the Lorentz covariance of all forces in nature, including gravitation, could not be regarded as a mere consequence of the principle of relativity. He believed this symmetry also implied more arbitrary assumptions, such as the similarity between electromagnetic and other forces and the universality of the velocity of light as a propagation velocity.[29]

As Poincaré reminded his reader, one way to justify these assumptions was the electromagnetic view of nature, according to which electromagnetism should be the ultimate basis of all physics. More appealing to him was the following suggestion:

> The common part of all physical phenomena would only be an appearance, something that would pertain to our methods of measurement. How do we perform our measurements? By superposing objects that are regarded as rigid bodies, would be one first answer; but this is no longer true in

[27] Although Poincaré did not do so much, the expression of the local time $t'$ can simply be obtained by requiring the apparent velocity of light to be equal to $c$. This condition implies $FM = ct'$, and $t = \gamma(t' + ux'/c^2)$. Calling $x$ the true abscissa of the second observer at time $t$ (with respect to the emission point of the flash), we also have $x' = \gamma(x - ut)$. Consequently, $t' = \gamma(t - ux/c^2)$, in conformity with the Lorentz transformations.

[28] The empirical equivalence of the two theories simply results from the fact that any valid reasoning of Einstein's theory can be translated into a valid reasoning of Poincaré's theory by arbitrarily calling the time, space, and fields measured in one given frame the true ones, and calling all other determinations apparent.

[29] Poincaré, ref. 24 (1906), 497.

the present theory, if one assumes the Lorentz contraction. In this theory, two equal lengths are, by definition, two lengths which light takes equal time to travel through. Perhaps it would be sufficient to renounce this definition so that Lorentz's theory would be as completely overturned [bouleversée] as Ptolemy's system was through Copernicus' intervention.

Lorentz's explanation of the null result of the Michelson-Morley experiment, Poincaré reasoned, implicitly rested on the convention that two lengths (sharing the same motion) are equal if and only if light takes the same (true) time to travel through them. What he meant by dropping this convention is not clear. Some commentators have speculated that he meant a revision of the concept of time, in Einstein's manner. This is not very likely, because the context of Poincaré's suggestion was length measurement instead of time measurement, and also because he ignored Einstein's point of view to the end of his life. More likely he was alluding to a suggestion he had earlier made at the Saint-Louis conference: "that the ether is modified when it moves relative to the medium which penetrates it."[30]

To sum up, in 1905/6 Poincaré obtained a version of the theory of relativity based on the principle of relativity and the Lorentz group. He believed this symmetry should apply to all forces in nature. He exploited it to derive the dynamics of the electron on a specific model and to suggest a modification of the law of gravitation. He nevertheless maintained the ether as the medium in which light truly propagated at the constant velocity c and clocks indicated the true time. He regarded the quantities measured in moving frames as only apparent, although the principle of relativity forbade any observational distinction between a moving frame and the ether frame. He understood the compatibility of the Lorentz transformations of coordinates with the optical synchronization of clocks and the invariance of the *apparent* velocity of light, but hesitated on the physical significance of the Lorentz contraction and never discussed the dilation of time.

## 6 Einstein's theory

Albert Einstein had an early interest in electrodynamics, if only because his family owned a small electrotechnical company. At age sixteen, he wrote a little essay on the state of the ether in an electromagnetic field. If we believe a late reminiscence, he also wondered about the appearance of a light wave for an observer traveling along with it. In 1896 he entered the Zürich Polytechnikum, where he learned electrodynamics in the standard continental style. Two years later he studied Maxwell's theory by himself from Drude's *Physik des Aethers*. Drude was a sympathizer of Ernst Mach's philosophy, and belonged to a tradition of German physics that favored phenomenological theories over mechanistic assumptions. In his rendering of Maxwell's theory, he avoided any picture of ether processes and propounded to redefine the ether as space endowed with special physical properties.[31]

[30] Poincaré, ref. 24 (1906), 498; ref. 25 (1904), 315 (Foundations).

[31] A. Einstein, "Über die Untersuchung des Aetherzustandes im magnetischen Felde," in John Stachel et al. (eds), *The collected papers of Albert Einstein*, vol. 1 (Princeton, 1987), 6-9; P.