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Chongjian Wu

Wave Propagation Approach for Structural Vibration

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Foreword

> *We must collect and sort the phenomena details until we find the truth from the scientific impression and find the laws from various phenomena while using these laws to deduce various phenomena and explain the future in turn [1].*
> W. R. Hamilton

The continuous improvement of structural dynamics is driven by the pursuit of engineering refinement. Generally, they can be separated into two categories: One is analytical methods, such as modal analysis method, transfer matrix method, modal truncation/synthesis method, modal impedance synthesis method, modal flexibility synthesis method, mobility method, dynamic stiffness synthesis method, etc., which are appropriate for basic theoretical analysis. The other is numerical methods, including the finite element method, boundary element method, and statistical energy method (SEA), which are appropriate for complex engineering structure calculation and system simulation. Unless with the special experience accumulation, researchers only rely on the exhaustion of numerical calculation, leading to difficulty in revealing the general law. Therefore, the analytical and numerical methods are still in further development.

This book introduces the equation derivation and the formation of the WPA intending to sort out its design. From the perspective of the structural wave, the author describes the input, propagation, and reflection of structural vibration energy and analyzes the waveform conversion. Through the simplification of the system and the clarification of the mechanism problems, designers can better find and summarize the mechanism causes of some frontier problems of such complex giant systems such as ships, connecting the model test and the real ships verification, and refining the general rule with conceptual, key, and common features while, therefore, guiding the technical research direction of the complex systems.

WPA Analysis

The analysis of structural dynamics is derived from the differential equation of vibration. In general, the method of separation of variables is used to establish characteristic functions: Time function is described by $\exp(j\omega t)$ while space function is described by the hyperbolic function. The WPA is also derived from the differential equation of vibration, with double-exponential function as its foundation. This book, by a unified framework, expresses simple systems, complex systems, and hybrid power systems consistently while forming unity and regularity.

The unity of the WPA is reflected in the regularity of equation derivation. The structural wave is used as the basic parameter to describe the structural vibration. For the wave being the ultimate parameter, it forms a direct causal relationship with the target control parameters, which is different from such indirect parameters as the acceleration of the structural vibration, insertion loss, or vibration level difference, although in many cases, they have similar or even identical mathematical descriptions. Therefore, in the event of differences between theoretical analysis and engineering measurement, designers should go back to the basics and rethink the wave in the structure.

The regularity of the WPA comes from the exponential function. The "noumenon" of the partial derivative of an exponential function $\exp(k_n x)$ always remains unchanged. Regardless of the times for taking the derivative, its derivative always remains a constant. Such a characteristic is described by Western researchers as being similar to cutting a watermelon, meaning that no matter how you cut a solid ball, its cross -section is always round. The Chinese explanation is more interesting: It is as if you cut off part of the Monkey King, which you think is just a small piece but essentially it is another Monkey King that is identical to the original.

Each instantaneous value of the solution is "forced" to coincide with the boundary conditions, enabling the WPA to meet the strict boundary conditions. The traditional analytical methods only have analytical solutions under a few constraint conditions, while the WPA is subject to relatively fewer restrictions. The convenience brought by the exponential function $\exp(k_n x)$ is genetic, which does not need to be expanded thoroughly. It is so consistent with simple harmonic motion that it demonstrates the WPA's magic for simplifying that is seemingly tedious but essentially neglected in a hidden way. After the introduction of symbolic operators, the derivation and deduction of mathematical equations show the rhythm of harmonic motion, which enjoys an inherent logic, complex representation, and unexpected unity and regularity between the harmonic excitation and simple harmonic motion with the hypothesis of dynamics. This is the characteristic of the WPA, which does not lose the original expression of the physical phenomena.

The Structural Wave

There are both simple structures, such as clamp-free beams or plates and complex structures, such as automobile axles, the stiffened cylindrical shell of a submarine, and a space platform composed of various base connections. These systems can be regarded as a waveguide collection with appropriate connections at the junction. The waveguide determines the energy of propagating the wave along the length direction, so we can study the power flow from a new perspective and observe how the structural wave relates to the target control parameters, such as structural-acoustic radiation. The physical information conveyed by the structure waveguide is much more complex than the recorded control parameters. Various sources gather on the radiation surface in the form of waves, some of which are transformed into air-borne noise or underwater sound. The role of studying the waves and analyzing the input, propagation, and attenuation of the vibration energy becomes increasingly important for the improved design of the carrier.

There are different types of waves in the structure, including longitudinal waves, shearing waves, flexural waves, and their combination. Ship designers focus more on the flexural waves since it is directly coupled with the vibration of the surrounding acoustic medium to form the most effective radiation. This reminds us that the structural units can be understood as the waveguide, for example, a finite thin rod excited in the length direction will generate a longitudinal wave; if the thin rod is excited in the transverse direction, the generated wave will propagate in a semi-infinite body as if there is only one free surface. Only after some time will the wave meet other transverse surfaces and produce reflections. For a finite structure, the particle motion is a simple superposition of the initial wave with all reflected waves at certain nodes in the waveguide. However, the synthesis is very complex with some resonance peaks disappearing, and the phenomenon of “blanking” is observed but without correct prediction. The study of structural waves is beneficial to grasp basic control.

Beams and plates are common units in ships. The dimension of a beam in one direction is much larger than that in the other two directions, so the structural wave mostly propagates along the length direction while the dimension of the plate structure in two directions is much larger than that in the thickness direction, so the wave can propagate along the two directions. Plates usually contact with the surrounding fluid medium, allowing it to be the main radiator of underwater sound. To control the structural-acoustic radiation, it is necessary to store more vibration energy in the beams and less flexural waves in the plates, which is opposite to the design of the instrument!

Wave Propagation and Vibration

Designers pay attention to structural vibration, which enjoys a simple and intuitive expression. Essentially, wave and vibration sometimes have an identity and occasionally have a causal relationship, both of which are “languages” in two distinct and independent fields. This book tries to establish the relationship between the basic parameter waves and the target control parameters.

The concepts of the traveling wave and near-field wave must first be established to study the wave propagation. The near-field waves do not propagate far away, which only exist near the “discontinuity” and decay very quickly, so they are also called the decaying waves. However, this does not mean that we should only focus on the traveling wave since the near-field wave has an important function of waveform conversion. The conversion of the flexural wave to the longitudinal wave must be accompanied by the exchange and redistribution of vibration energy. The structure waves become so complex in the giant system that the sample study of the decaying waves shows great importance, influencing the structural sound radiation uniquely. The advantage of the WPA is to establish a direct correlation between structural waves and control parameters.

Different boundaries exist in engineering structures, so we can find a phenomenon: one of the main research contents of analytical methods is to continuously break through the influence of boundary and structural discontinuities on the analysis, such as studying the rational approximation of boundary conditions so that the mathematical solution of the system can be successfully "matched". The WPA is less affected by the boundary so it is relatively easy to obtain the solution of a complex boundary. The establishment of the concept of universal "discontinuity" can enable researchers to better understand the giant system.

The Theme and Outline

This book concentrates on the propagation of structural waves in beams, plates, distributed parameter systems, and hybrid power systems. The WPA is assumed to analyze the above dynamic problems in a consistent framework, such as the "mixing effect" of floating rafts. Although we find out through practice that it is difficult to express such a seemingly simple multi-input single-output dynamic system in a centralized and clear way. Without fail, the WPA is more suitable.

This book mainly studies the propagation of structural waves in a continuum, and their reflection and conversion at discontinuities such as the boundary, junction, and inflection point with a focus on their final reflection in engineering. The author aims to explore the dual integration of a giant system from "reductionism" to "holism" based on the simplification and abstraction of a giant system with mechanism research.

White initiated the theory of vibrational power flow. Professor Mead conducted considerable pioneering research on the analysis of structural waves. The outstanding research conducted by Pinnington and Langley supplemented the wave propagation. Wu Chongjian and White also worked to develop the WPA as an independent analytical method. Chapter 1 of this book makes a brief introduction to the basic theory of vibration and noise, discusses the common functional analytical methods for a differential equation of vibration, which are used to analyze the vibration modes of beams and plates, and outlines the Continuous Fourier Transform and the spectral analysis to examine the sound pressure, sound power, and sound radiation efficiency. Chapter 2 describes the use of the WPA to solve differential equations, explains the same-origin relations between the WPA and the traditional analytical methods in detail, and provides two key concepts in the analysis process: One is to retain the spectrum relationship (the specific relationship between frequency and wave number) while the other is to express the space-time of the characteristic function with the double-exponential function. For these concepts, the constraints are automatically relieved when solving practical boundary value problems, even if all solutions are not strictly defined mathematically. In Chap. 3, the WPA is used to examine the plate structure and calculate the flow mode of vibration energy in the plate. Chapter 4 studies the parameter system of complex dynamics, and the unified response expression and spectrum relation are obtained by the WPA. Relying on the idea from mechanism analysis to practical engineering application and based on the finite arbitrary multi-supported beams, the specific engineering cases are analyzed. Chapter 5 analyzes the hybrid power system by the WPA, for example, the dynamic model of the multi-supported mast with heavy objects mounted on top and the dynamic characteristics of multi-supported beams with TMD; the interaction and mechanical coupling of structural waves and structural discontinuity are also reviewed. The applicability of the WPA in engineering is demonstrated by the analysis of the submarine mast, concluding that "there is more static stiffness and less dynamic stiffness". The mechanism study points out that the engineering choice is to change the dynamic design of the mast instead of the high-strength material. In Chap. 6, the WPA is used to analyze the structural response under distributed force excitation. Chapter 7 explores the engineering application of MTMD in vibration isolation of the main motor. Chapter 8 makes an analysis of the floating rafts by the WPA. The force basis, as a "signal generator", is embedded into the analytical equation, while some of the research is not demonstrated here for the reason of inconvenience. Chapter 9 describes the power flow carried by the structure waves by the WPA. The structural sound intensity shares some similarity with the (air) sound intensity but the waveguide analysis is more complicated. The early experimental research results are also provided in this chapter.

The use of structural waves to describe the vibration and noise can be found in all chapters of this book. Readers are expected to understand the control parameters from the perspective of the waves and "observe" the wave process. Another analysis subject includes "universal discontinuity". Using the discontinuity and inflection point as the starting point for analysis, researchers analyze the multiple effects of structural discontinuity on wave propagation, and understand how discontinuity can change the wave propagation and attenuation in a complex system fundamentally due to a quantitative change. Relying on the deep understanding of the simple structure, engineering designers can establish the thinking of "holism" for complex systems and better cope with increasingly complex engineering projects.

The WPA is particularly suitable for dynamics analysis of beam structures, such as tricky finite quasi-periodic structure and hybrid power systems. Considering that any method has its limitations, the author has added the relevant reference documents to the corresponding chapter in as much detail as possible for theoretical development and application explanation, on which further research and discussions can be based.

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Preface

The vibration and acoustic performance of the carrier are not only linked to the comfort of the staff but are also closely linked to its comprehensive operational capability. Presently, the acoustic power radiated by an international advanced submarine is already less than 0.3 mW. Therefore, the acoustic energy radiated by a submarine in seawater is smaller than the screen-on power consumption of a smartphone. So, the vibration and noise reduction of submarines drive the structural dynamics into an era of refining development, which is considered as tackling a key cutting-edge technical issue among the core technological secrets of world powers.

Behind this silent contest are the continuous breakthrough of engineering ability and basic theory! From components and equipment to systems, structural dynamics, and structure-borne noise to power flow theory, new theories are emerging and applied continuously, which is the result of the profound integration of theoretical methods and engineering practice, as well as the concentrated embodiment of decades of continuous improvement.

The Wave Propagation Approach (WPA) is a supplement of the analytical method, which arises in response to the need for refinement. It provides a microscopic perspective and analysis means for new thinking. In short, WPA has the following characteristics:

I. The WPA focuses on the study of structural waves. Structural inputs are transformed into forces or structural waves, which form a direct causal relationship with such target control parameters as a vibration level, quantity of vibration isolation, and acoustic radiation. In terms of the wave propagation, reflection and attenuation, and the waveform conversion of longitudinal waves and flexural waves, WPA has advantages in reducing misjudgment in combination with the power flow analysis, resulting in the easy formation of internal relations between parameters, featuring relatively simple mathematical description, which will affect the ways of thinking and design concepts.

II. The WPA uses "structural discontinuity" to divide the units, which is different from the "geometric division" of the finite element method. In the complex giant system, the discontinuity is in the same category as structural damping and has a “universal correlation” to structural vibration. The new division method is adopted to analyze the complex systems, making the main structure and dominant characteristics clearer.

III. The WPA has more relaxed constraint conditions. Many boundary conditions cannot be examined because function types cannot fulfill the mathematical matching. The WPA uses the linear superposition of the general solution of the infinite system with the particular solution of the finite system to define structural vibration, allowing a computer to complete the tedious complex matrix operations and “forcing” the instantaneous value of the solution to coincide with the boundary conditions and compatibility conditions, thereby resulting in fewer constraints and consequently leading to wider boundary adaptability.

Large-scale complex projects, such as ships, aviation, aerospace, and bridge construction have experienced decades of rapid development and technological accumulation. While enjoying the results, researchers face the new stage of returning to the mechanisms, clarifying the details, and completing the increase. Designers having a good command of the basic theoretical methods, and the achievements are of great benefit to promote professional integration. The WPA may help you better understand structural waves and apply it.

At present, we have entered the critical period of detail determining the success or failure of the carrier development, and structural dynamics analysis is confronted with the fundamental theoretical challenges of development: certain seemingly insignificant parameters become the main causes, with a high contribution rate to the system while certain influential factors that seem to possess great weight are either overcome or there are deviations from past understanding, leading them to make little actual contribution. Designers should continuously achieve simplicity and optimization schemes.

Wave is the microscopic trace of structural vibration and transmission and it is the criterion of vibration source and mechanism formation when rising to the macro levels, so the improvement direction established and the conclusion reached are comprehensive and refined. The differences between theoretical calculations and data measured on real ships often make designers despondent and frustrated. Structural waves provide a new perspective for the interpretation of these problems, such as the phenomenon of the “blanking” or misplacing of the modal test peaks of complex systems, as explained by D. J. Ewins of the Imperial College London, UK.

Combined with the power flow, the WPA, from basic units such as beams, power sources, discontinuity to coupled systems, and hybrid power systems, establishes the concept of discontinuity to help us better understand important issues, such as whether the “breathing mode” of a submarine radiates a strong voiceprint. Many papers have been published on that respect at home and abroad. The WPA adds an argument for independent judgment in advance that some radiated energy of a submarine as a complex system is consumed by numerous discontinuities instead of viscoelastic damping or other ways.

Numerical analyses can complete the complex engineering dynamics analysis, with three issues encountered: First, the homogenization competition of commercial software is intensified while the personalized application and improvement are insufficient. Second, the needs of mechanism research are emerging constantly. There are some open loops in the simulation of complex systems while the numerical calculation is only a special one among exhaustive cases rather than a general rule. Third, the interface between analytical methods and numerical methods is becoming increasingly blurred, with the most important constructs depending on mechanism research.

We need to admit that many dynamic types of research of complex systems are far from the unified theoretical analysis framework, and cognitive shortcomings persist. The author has attempted to maintain the integrity of this book as much as possible and has tried his best not to affect the extended understanding of the readers. The limitations of personal ability, shortcomings of WPA, and popular research topics' transition may lead to unbalanced contents of the book, for example, research on longitudinal waves of rods is not yet included and some parts are limited by the content, so only the basic conclusions are given without the process. Moreover, as the WPA is still in the process of development and perfection, the author sincerely hopes that readers can get to grips with the characteristics and shortcomings of the WPA through the application of different scenarios. My email address is cjw2018WPA@163.com and I look forward to your comments.

The author would like to thank Sir Bao Yugang Foundation for the scholarship that allowed me to complete the relevant research at the Institute of Sound and Vibration Research of the University of Southampton in the UK, laying the foundation for the WPA. I would also like to extend thanks to my tutor, Prof. R. G. White, Academician Yang Shuzi, Academician Zhu Yingfu, Prof. LuoDongping, and researcher Ma Yunyi, as it was their continuous encouragement that gave me the courage to finish this monograph. I would also like to thank Prof. D. J. Mead, Prof. F. Fahy, and Dr. R. J. Pinnington for their knowledge, experience, moral character, and academic discussions from which I greatly benefited. Academician Hu Haiyan read the first draft and provided many useful suggestions. I am also thankful to Academician Yang Wei, Academician Wu Yousheng, Academician Zhang Qingjie, and Academician He Yaling for their encouragement and guidance.

Xiong Jishi and Chen Zhigang assisted in the writing of Chaps. 3 and 8. Doctoral students Lei Zhiyang, Zhang Shiyang, and Yan Xiaojie conducted the programming and assisted in drawing a large number of calculation charts. Professor Li Tianyun and Associate Prof. Zhu Xiang provided many useful suggestions for the revision of the manuscript. Many seminar-style discussions were also held with Du Kun, Xu Xintong, Chen Lejia, Qiu Changlin, Wang Chunxu, and others, yielding fruitful results. An academic monograph such as this cannot be completed without the selfless help of many people, in particular, Lu Xiaohui, Cai Daming, Yi Jisheng, Hu Wenli, Zhang Zhipeng, Zhu Xianming, Xue Bing, Wang Yan, Yang Yuting, Fan Yongjiang, Xia Guihua, Zhang Ling, and Xue Li, whom I thank for their sincere support and guidance, as well as their selfless contribution.

Wuhan, PR China
August 2018

Chongjian Wu

Introduction

Based on fourth-order differential equations, the Wave Propagation Approach (WPA) continually utilizes the time-domain spatial exponential function in the equation derivation and reconstruction to create a unified framework for analyzing structural vibration. Focusing on wave propagation, reflection, attenuation, and waveform conversion combined with the power flow research, this book aims to examine the finite beam structures, periodic/quasi-periodic structures, coupling structures, and the offsetting mechanism of multiple disturbance sources from the microscopic perspective of structural waves, beginning with the basic elements such as beams and TMD.

WPA is a supplementary analytical method. The readers include undergraduate students, graduate students, and engineering designers that wish to deepen their studies on structure-borne noise, power flow theory, fine control of vibration noise, etc. To read this book, one needs to have a certain basic theoretical knowledge of vibration and noise control and preferably engineering practice.

Contents

About the Author

Chongjian Wu born in October 1960, is a Doctoral Tutor and the Chief Technical Expert in the field of vibration and noise reduction for China State Shipbuilding Corporation Limited, the Chief Designer of key types of submarines, and a Guest Director of the Chinese Society of Theoretical and Applied Mechanics. He has once worked as a Visiting Scholar/Assistant Researcher in ISVR of the University of Southampton in the UK, enjoying the experience of in-depth research on the basic theory and engineering application of vibration and noise reduction. He has won a Special Award and the First Prize for the National Science and Technology Progress Award, respectively, and was awarded the “Ship Design Master”.

Symbols

a_n, b_n, c_n, d_n	The coefficients response function of an infinite structure
A	Coefficient
A_n, B_n, C_n, D_n	Coefficient related to the frequency
b	Thickness, depth
c_g	Group velocity
c_b	Flexural wave velocity
c_l	Longitudinal wave velocity, $\sqrt{EA/\rho S}$
D	Plate stiffness, $Eh^3/[12(1-\nu^2)]$
E	Young's modulus
EI	The flexural rigidity of a beam
F	Axial force
G	Shear modulus, single-side frequency function
h	The height of a beam or rod, the thickness of a plate
i, Z	Integer
I	The second moment of a section, rectangular beam $I = bh^3/12$
j	Imaginary number, $j = \sqrt{-1}$
k, k_1, k_2, k_3, k_4	Wave number
k_x, k_y, k_z	Two-dimensional wave number
K	Spring stiffness
$[k]$, $[K]$	Stiffness matrix
L	Beam length, distance from a boundary
M, M_x	(Bending) moment
n	Frequency count
N	Integer
$\tilde{p}_o(t), \hat{p}(\omega)$	Transverse force (harmonic force), sound pressure
$\tilde{P}(t), \hat{P}(\omega)$	Power/energy flow in the time domain/frequency domain
q	Distributed load
r	Radial coordinate
R	External radius, the correlation function
t	Time

S	Cross-sectional area, cross-correlation density function
T	Time window, temperature
$u(t)$	The response, velocity, strain, etc.
u, v, w	Displacement response, spatial variable function
V	Velocity response of the structure
W	Spatial transformation window
x, y, z	Rectangular space coordinates

Greek Alphabet

α	Coefficient
β	Loss factor, viscoelastic damping coefficient
$\delta_{i,j}$	Kronecker symbol
Δ	Determinant
η	The damping loss factor of a structure
θ	Angular coordinate
v	Poisson's ratio
μ	Ratio
λ	Wavelength
ρ	Density
σ, ε	Stress, strain
ξ	Viscous damping coefficient
$\Phi(t)$	Time variable function
ψ	Lateral contraction
ω, ω_r	Angular frequency and resonance frequency
ω_c	Cutoff, coincidence, and critical frequency

Specific Symbols

$\Re_n$	Stochastic noise
Π	Radiated sound power
∇^2	Laplace operator, $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$
$[\,], \boldsymbol{M}$	Matrix, or $\boldsymbol{M}$ in black
$\{\,\}, \boldsymbol{X}$	Vector, or $\boldsymbol{X}$ in black

Subscripts

a, u, m	Input power, the power generated by shearing force and bending moment
d	Dynamic vibration absorber, TMD (tuned mass damper)
nf, ff	Near field, far field
$+, -$	Negative and positive waves along the coordinates
n	Integer, counting unit
r	Resonance frequency
$1, 2$	Sensor, mode label
B, L	Flexural wave and longitudinal wave

Superscripts

$*$	Complex conjugate
$-$	Average value
.	Time derivative
$\wedge$	Quantity in the frequency domain (after the conversion)
$\sim$	Quantity in the time domain
'	Derivation of the parameters
T	Matrix transpose
H	Matrix conjugate transpose

Chapter 1
The Basic Theory of Structure–Borne Noise

Depth determines breadth!
—Qian Xuesen

Before we discuss the WPA method, it is necessary to examine the basic theory of structural vibration noise, including basic parameters such as wave number, wavelength, and lateral displacement. The examination of continuous systems such as the bending vibration of a beam and a plate is the focus of this chapter. The vibration modes and natural frequencies of beams and plates are discussed and then the sound pressure, sound power, and sound radiation efficiency of simple structures are analyzed and discussed.

1.1 The Vibration Modes of Beams

1.1.1 Basic Equations

The structural wave is the basic parameter of structural vibration and acoustic radiation, which is directly linked to the target control parameters. The theory of bending vibration of beams and plates is derived from the fourth-order differential equation [1]:

$$\left.\begin{array}{c} \nabla^2\left(\nabla^2\tilde{w}\right) + \rho S\frac{\partial^2\tilde{w}}{\partial t^2} = \tilde{p}_o \\ \tilde{w} = \tilde{w}(x, y, z, t) \\ \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \end{array}\right\} \tag{1.1}$$

where

∇^2	Laplace operator;
$\tilde{w}$	Lateral displacement of the structure;
ρ	Material density of the beam;

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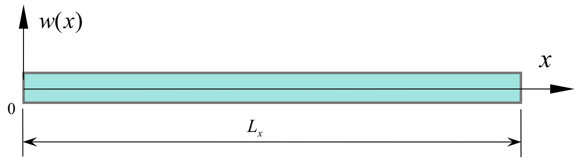

Fig. 1.1 A schematic diagram of a vibrating beam

S	Cross-sectional area of the structure;
$\tilde{p}_0(x, y, t)$	External excitation harmonic force

The displacement $\tilde{w}$ is linked not only to the spatial coordinate of the particle (x, y, z) but also to the time. It is important to calculate the structural mode and modal frequency when conducting a structural analysis. The excitation features and the generation of the state matrix are based on the structure mode and modal frequency.

With consideration of the Bernoulli–Euler beam as shown in Fig. 1.1, the vibration equation of Eq. (1.1) degenerates to free vibration [2]:

$$\frac{\partial^4 \tilde{w}(x, t)}{\partial x^4} + \frac{\rho S}{EI} \cdot \frac{\partial^2 \tilde{w}(x, t)}{\partial t^2} = 0 \tag{1.2}$$

where

EI	Bending stiffness of the beam;
E	Young's modulus of the material;
$I = bh^3/12$	Cross-sectional moment of inertia of the beam, where b is the width and h is the thickness of the beam.

For harmonic vibration, the displacement response can be divided into two parts: the space function and time function according to the process of separating variables:

$$\tilde{w}(x, t) = w(x) \cdot \Phi(t) \tag{1.3}$$

where

$w(x)$	Structure mode shape function;
$\Phi(t)$	Time correlation function

When substituting Eq. (1.3) into Eq. (1.2) and dividing the variable for time t and space x, two ordinary differential equations are attained as

$$\frac{\partial^4 w(x)}{\partial x^4} - \frac{\rho S \omega^2}{EI} w(x) = 0 \tag{1.4}$$

$$\frac{\partial^2 \Phi(t)}{\partial t^2} + \omega^2 \Phi(t) = 0 \tag{1.5}$$

where ω is the circular frequency.

Set

$$k^4 = \frac{\rho S \omega^2}{EI} \tag{1.6}$$

Therefore, Eq. (1.4) can be rewritten as

$$\frac{\partial^4 w(x)}{\partial x^4} - k_n^4 w(x) = 0 \tag{1.7}$$

where k_n is the complex wave number of the beam's bending wave, $n = 1, 2, 3, 4$.

For beam-type structures, the general boundary conditions are as follows:

(1) Simply supported boundary condition (S-S beam):

$$\left.\begin{array}{ll} \tilde{w}(0,t) = 0, & \frac{\partial^2 \tilde{w}(0,t)}{\partial x^2} = 0 \\ \tilde{w}(L_x,t) = 0, & \frac{\partial^2 \tilde{w}(L_x,t)}{\partial x^2} = 0 \end{array}\right\} \tag{1.8}$$

(2) Clamped boundary condition (C-C beam):

$$\left.\begin{array}{ll} \tilde{w}(0,t) = 0, & \frac{\partial \tilde{w}(0,t)}{\partial x} = 0 \\ \tilde{w}(L_x,t) = 0, & \frac{\partial \tilde{w}(L_x,t)}{\partial x} = 0 \end{array}\right\} \tag{1.9}$$

(3) Free boundary condition (F-F beam):

$$\left.\begin{array}{ll} \frac{\partial^2 \tilde{w}(0,t)}{\partial x^2} = 0, & \frac{\partial^3 \tilde{w}(0,t)}{\partial x^3} = 0 \\ \frac{\partial^2 \tilde{w}(L_x,t)}{\partial x^2} = 0, & \frac{\partial^3 \tilde{w}(L_x,t)}{\partial x^3} = 0 \end{array}\right\} \tag{1.10}$$

The general solution to the differential equation Eq. (1.7) is

$$w(x) = A\sin(kx) + B\cos(kx) + C\sinh(kx) + D\cosh(kx) \tag{1.11}$$

where A, B, C, and D are unknown coefficients, respectively, $k^4 = \rho S \omega^2 / EI$.

Using simply supported beams as an example and substituting the displacement of the beam into the boundary conditions, it can be used to solve the unknowns A, B, C, and D.

At the left end $x = 0$, substituting Eq. (1.11) into Eq. (1.8), we get

$$w(0) = B + D = 0 \tag{1.12}$$

$$\frac{\partial^2 w(0)}{\partial x^2} = k^2(-B + D) = 0 \tag{1.13}$$

Thus, $B = D = 0$. At the right end $x = L_x$, we get

$$w(L_x) = A\sin(kL_x) + C\sinh(kL_x) = 0 \tag{1.14}$$

$$\frac{\partial^2 w(L_x)}{\partial x^2} = k^2[-A\sin(kL_x) + C\sinh(kL_x)] = 0 \tag{1.15}$$

From Eqs. (1.14) and (1.15), we get

$$A\sin(kL_x) + C\sinh(kL_x) = 0 \tag{1.16}$$

Since $\sinh(kL_x) \neq 0$, provide $kL_x \neq 0$, and therefore,

$$C = 0,\ k = \tfrac{n\pi}{L_x},\ A = \sqrt{\tfrac{2}{mL_x}} \tag{1.17}$$

where m is beam mass per unit length.

When substituting Eq. (1.17) into Eq. (1.11), the nth mode shape function for a simply supported beam can be attained as

$$w_n(x) = A\sin\left(\frac{n\pi}{L_x}x\right) \tag{1.18}$$

When substituting Eq. (1.17) into Eq. (1.6), the corresponding natural frequencies can be written as

$$\omega_n = \sqrt{\frac{EI}{m}} \cdot \left(\frac{n\pi}{L_x}\right)^2 \tag{1.19}$$

The mode shapes are orthogonal with respect to the mass and stiffness distribution [1, 3]:

$$\int_0^{L_x} m w_j(x) w_k(x)\mathrm{d}x = \mu_j \delta_{jk} \tag{1.20}$$

$$\int_0^{L_x} EI \frac{\partial^2 w_j(x)}{\partial x^2} \cdot \frac{\partial^2 w_k(x)}{\partial x^2}\mathrm{d}x = \mu_j \omega_j^2 \delta_{jk} \tag{1.21}$$

$$\delta_{jk} = \begin{cases} 1 & j = k \\ 0 & j \neq k \end{cases}$$

where

δ_{jk} Kronecker delta symbol;
μ_j Modal mass of the nth mode.

The generalized mass corresponding to the mode shapes in Eq. (1.18) is $mL_x/2$.

Table 1.1 The structural mode shapes and natural frequencies

Boundary conditions	Structural mode shape functions	Natural frequencies
Simply supported	$w_n(x) = \sqrt{2/\rho S L_x} \cdot \sin(k_n x)$	$k_n = n\pi/L_x$
Clamped–clamped	$w_n(x) = \cosh(k_n x) - \cos(k_n x) -$ $\beta_n[\sinh(k_n L_x) - \sin(k_n L_x)]$ $\beta_n = \frac{\cosh(k_n L_x) - \cos(k_n L_x)}{\sinh(k_n L_x) - \sin(k_n L_x)}$	$\cos(k_n L_x) \cdot \cosh(k_n L_x) - 1 = 0$
Clamped-free	$w_n(x) = \cosh(k_n x) - \cos(k_n x) -$ $\beta_n[\sinh(k_n L_x) - \sin(k_n L_x)]$ $\beta_n = \frac{\cosh(k_n L_x) - \cos(k_n L_x)}{\sinh(k_n L_x) - \sin(k_n L_x)}$	$\cos(k_n L_x) \cdot \cosh(k_n L_x) + 1 = 0$
Clamped–simply supported	$w_n(x) = \cosh(k_n x) - \cos(k_n x) -$ $\beta_n[\sinh(k_n L_x) - \sin(k_n L_x)]$ $\beta_n = \frac{\cosh(k_n L_x) - \cos(k_n L_x)}{\sinh(k_n L_x) - \sin(k_n L_x)}$	$\tan(k_n L_x) \cdot \tanh(k_n L_x) + 1 = 0$

As a result of the mode shapes being orthogonal to each other, the response of the beam can be expressed at any arbitrary point as a linear combination of these mode shape functions. This is known as the mode superposition method. The WPA method selects different technical paths, as shown in Sect. 2.4 of Chap. 2.

$$\tilde{w}(x, t) = \sum_{n=1}^{\infty} w_n(x) \cdot \Phi_n(t) \tag{1.22}$$

As a result of the infinite modes in a continuous system, it is necessary to intercept a finite number of modes, such as N. In this way, the analysis of the complex system is simplified, and analytical accuracy is certain.

For other boundary conditions, the structural mode shapes and natural frequencies are listed in Table 1.1.

1.1.2 MATLAB Examples

Consider equations in Table 1.1 for this example. Using the MATLAB program to calculate the first 5th mode shapes of the beam structure, we can get the structure modal shape functions corresponding to different boundary conditions listed in the Table, as shown in Figs. 1.2, 1.3, 1.4 and 1.5.