The Physics of the Manhattan Project

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Springer

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ISBN 978-3-642-14708-1 e-ISBN 978-3-642-14709-8
DOI 10.1007/978-3-642-14709-8
Springer Heidelberg Dordrecht London New York

Library of Congress Control Number: 2010937572

Cover illustration: Mushroom cloud of the Trinity test, Monday, July 16, 1945. The yield of this implosion-triggered plutonium fission bomb is estimated at 21 kilotons. This is the only color photograph taken of the Trinity test. Photo by Jack Aeby courtesy of the Los Alamos National Laboratory.

Cover design: eStudio Calamar S.L.

Printed on acid-free paper

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This work is dedicated to my wife Laurie,
whose love knows no half-life.

Preface

The scientific, social, political, and military implications of the development of nuclear weapons under the auspices of the United States Army's "Manhattan Project" in World War II drove much of world geopolitical strategy for the last half of the twentieth century. These implications remain with us today in the form of ongoing concerns and debates regarding issues such as weapons stockpiles and deployments, proliferation, fissile material security and test-ban treaties. For better or worse, the historical legacy of Los Alamos, Oak Ridge, Hanford, *Trinity*, *Little Boy*, *Fat Man*, Hiroshima and Nagasaki will influence events for decades to come even as the number of nuclear weapons in the world continues to decline.

While even a casual observer of the world situation cannot help but be aware that the idea of terrorists or unstable international players being able to acquire enough "fissile material" to assemble the "critical mass" necessary to construct a nuclear weapon is of concern, popular understanding of the history and science of nuclear weapons is extremely limited. Even most physics and engineering graduates probably have no deeper appreciation of the science underlying these weapons than a typical high-school student. Why is there is such a thing as a critical mass in the first place, and how can one determine it? How does a reactor differ from a weapon? Why can't a nuclear weapon be made with a common metal such as aluminum or iron as its "active ingredient"? How did the properties of various uranium and plutonium isotopes lead in World War II to the development of "gun-type" and "implosion" weapons? How can one estimate the energy yield of these devices? How does one arrange to assemble the critical mass at just the time when a bomb is to be detonated?

This book is an effort to address such questions. It covers, at about the level of a junior-year undergraduate physics major, the basic physics underlying fission weapons as they were developed during the Manhattan Project.

This work has grown out of three courses that I have taught at Alma College. One of these is a conventional undergraduate sophomore-level "modern physics" class for physics majors which contains a unit on nuclear physics, the second is an algebra-level general-education class on the history of the making of atomic bombs

in World War II, and the third a junior-level topics class for physics majors that uses the present volume as its text. My motivation in preparing this book was that there seemed to be no one source available for a reader with a college-level background in physics who desired to learn something of the technical aspects of the Manhattan Project in more detail than is typically presented in conventional modern/nuclear texts or popular histories. Readers are often left wondering about the details of questions such as outlined above. As my own knowledge of these issues grew, I began assembling an informal collection of derivations and results to share with my students and which have evolved into the present volume. I hope that readers will discover, as I did, that studying the physics of nuclear weapons is not only fascinating in its own right but also an excellent vehicle for reinforcing understanding of foundational physical principles such as energy, electromagnetism, dynamics, statistical mechanics, modern physics, and of course nuclear physics.

This book is consequently neither a conventional text nor a work of history. I assume that readers are already familiar with the basic history of some of the physics that led to the Manhattan Project and how the project itself was organized (Fig. 1). Excellent background sources are Richard Rhodes' masterful *The Making*

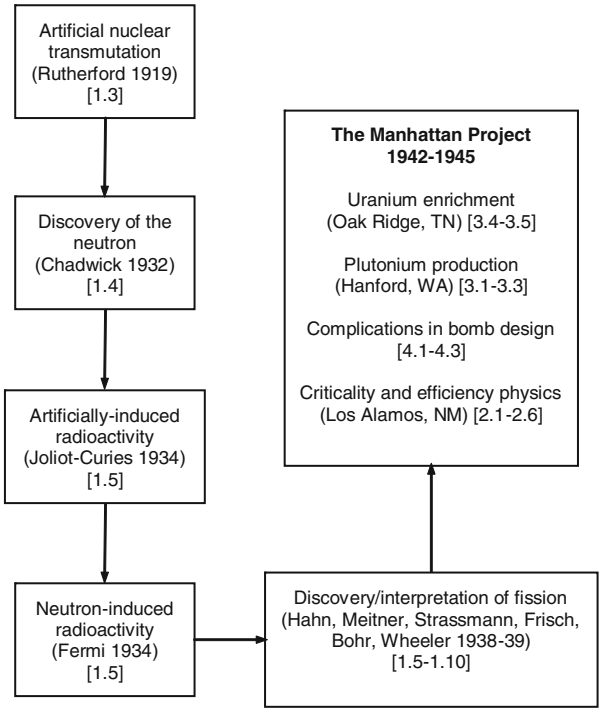

Fig. 1 Concept map of important discoveries in nuclear physics and the organization of the Manhattan Project. Numbers in *square brackets* indicate sections in this book where given topics are discussed

of the Atomic Bomb (1986) and F. G. Gosling's *The Manhattan Project: Making the Atomic Bomb* (1999). While I include some background material for sake of a reasonably self-contained treatment, it is assumed that within the area of nuclear physics readers will be familiar with concepts such as reactions, alpha and beta decay, Q-values, fission, isotopes, binding energy, the semi-empirical mass formula, cross-sections, and the concept of the "Coulomb barrier." Familiarity with multivariable calculus and simple differential equations is also assumed. In reflection of my own interests (and understanding), the treatment here is restricted to World War II-era fission bombs. As I am neither a professional nuclear physicist nor a weapons designer, readers seeking information on postwar advances in bomb and reactor design and related issues such as isotope separation techniques will have to look elsewhere; a good source is Garwin and Charpak (2001). Similarly, this book does not treat the *effects* of nuclear weapons, for which authoritative official analyses are available (Glasstone and Dolan 1977). For readers seeking more extensive references, an annotated bibliography appears in Appendix I of the present book.

This book comprises 27 sections within five chapters. Chapter 1 examines some of the history of the discovery of the remarkable energy release in nuclear reactions, the discovery of the neutron, and characteristics of the fission process. Chapter 2 details how one can estimate both the critical mass of fissile material necessary for a fission weapon and the efficiency one might expect of a weapon that utilizes a given number of critical masses of such material. Aspects of producing the fissile material by separating uranium isotopes and synthesizing plutonium are taken up in Chap. 3. Chapter 4 examines some complicating factors that weapons engineers need to be aware of. Some miscellaneous calculations comprise Chap. 5. Useful data are summarized in Appendices A and B. Some background derivations are gathered in Appendices C–G. For readers wishing to try their own hand at calculations, Appendix H offers a number of questions, with brief answers provided. A bibliography for further reading is offered in Appendix I, and some useful constants and conversion factors appear in Appendix J. The order of the main chapters, and particularly the individual sections within them, proceeds in such a way that understanding of later ones sometimes depends on knowledge of earlier ones.

It should be emphasized that there is no material in the present work that cannot be gleaned from publicly-available texts, journals, and websites: I have no access to classified material.

I have developed spreadsheets for carrying out a number of the calculations described in this work, particularly those in Sects. 1.4, 1.7, 1.10, 2.2–2.5, 4.1, 4.2, and 5.3. These are freely available at a companion website, http://www.manhattanphysics.com. When spreadsheets are discussed in the text they are referred to in **bold** type. Users are encouraged to download these, check calculations for themselves, and run their own computations for different choices of parameters. A number of the problems in Appendix H are predicated on using these spreadsheets.

This book is the second edition of this work. The first edition was self-published with Trafford Publishing, and I am grateful for their very professional work. The present edition includes a number of new and revised sections. A discussion of

numerically estimating bomb yield and efficiency (Sect. 2.5), an analysis of Rudolf Peierls' criticality parameter (Sect. 2.6), development of a model for estimating Pu-240 production in a reactor (Sect. 5.3), and a formal derivation of the Bohr–Wheeler spontaneous fission limit (Appendix E) are completely new, as is a bibliography of books, articles, and websites dealing with the Manhattan Project (Appendix I). The discussion of predetonation probability as a consequence of spontaneous fission (Sect. 4.2) has been significantly upgraded, the analysis of estimating the average neutron escape probability from within a sphere has been revised (Appendix D), and some corrections have been made to the discussion of analytically estimating bomb efficiency (Sect. 2.4).

Over several years now, I have benefitted from discussions on this material with Gene Deci, Jeremy Bernstein, Harry Lustig, Carey Sublette, and Peter Zimmerman, and am grateful for their time and patience. I am grateful to John Coster-Mullen for permission to reproduce his beautiful cross-section diagrams of *Little Boy* and *Fat Man* that appear in Chaps. 2 and 4. Students in the first version of my topics class – Charles Cook, Reid Cuddy, David Jack and Adam Sypniewski – served as guinea pigs for these notes and pointed out a number of confusing statements. I owe a great debt of gratitude to Alma College for various forms of professional development support extending over many years.

Finally, I am grateful to the staff of Springer for helping to bring this project to fruition. Their efficiency and professionalism are nothing short of outstanding. Naturally, I claim exclusive ownership of any errors that remain.

Suggestions for corrections and additional material will be gratefully received. I can be reached at: Department of Physics, Alma College, Alma, MI 48801.

Alma, MI, USA B. Cameron Reed
May 17, 2010

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Contents

Chapter 1
Energy Release in Nuclear Reactions, Neutrons, Fission, and Characteristics of Fission

Abstract This introductory chapter covers the background nuclear physics necessary for understanding later calculations of critical mass, nuclear weapon efficiency and yield, and how fissile materials are produced. It describes how the energy released in nuclear reactions can be calculated, how artificially-produced nuclear transmutations were discovered, the discovery of the neutron, artificially-produced radioactivity, the discovery and interpretation of neutron-induced nuclear fission, why only certain isotopes of uranium and plutonium are feasible for use in nuclear weapons, and how nuclear reactors differ from nuclear weapons.

While this book is not intended to be a history of nuclear physics, it will be helpful to set the stage by briefly reviewing some historically relevant discoveries. To this end, we first explore the discovery of the enormous energy release characteristic of nuclear reactions, work that goes back to Ernest Rutherford and his collaborators at the opening of the twentieth century; this is covered in Sect. 1.2. Rutherford also achieved, in 1919, the first artificial transmutation of an element (as opposed to this happening naturally, such as in an alpha-decay), an issue we examine in Sect. 1.3. Nuclear reactors and weapons cannot function without neutrons, so we devote Sect. 1.4 to a fairly detailed examination of James Chadwick's 1932 discovery of this fundamental constituent of nature. The neutron had almost been discovered by Irène and Frédéric Joliot–Curie, who misinterpreted their own experiments. They did, however, achieve the first instance of artificially inducing radioactive decay, a situation we examine in Sect. 1.5, which also contains a brief summary of events leading to the discovery of fission. In Sects. 1.6–1.10 we examine the release of energy and neutrons in fission, some theoretical aspects of fission, and delve into why only certain isotopes of heavy elements are suitable for use in fission weapons. Before doing any of these things, however, it is important to understand how physicists notate and calculate the energy liberated in nuclear reactions. This is the topic of Sect. 1.1.

B.C. Reed, *The Physics of the Manhattan Project*,
DOI 10.1007/978-3-642-14709-8_1,

1.1 Notational Conventions for Mass Excess and Q-Values

On many occasions we will need to compute the energy liberated in a nuclear reaction. Such energies are known as Q-values; this section develops convenient notational and computational conventions for dealing with such calculations.

Any reaction will involve *input* and *output* reactants. The total energy of any particular reactant is the sum of its kinetic energy and its relativistic mass-energy, mc^2. Since total mass-energy must be conserved, we can write

$$\sum KE_{input} + \sum m_{input}c^2 = \sum KE_{output} + \sum m_{output}c^2, \tag{1.1}$$

where the sums are over the reactants; the masses are the *rest masses* of the reactants. The Q-value of a reaction is defined as the difference between the output and input kinetic energies:

$$Q = \sum KE_{output} - \sum KE_{input} = \left(\sum m_{input} - \sum m_{output}\right) c^2. \tag{1.2}$$

If $Q > 0$, then the reaction liberates energy, whereas if $Q < 0$ the reaction demands a *threshold* energy to cause it to happen.

If the masses in (1.2) are in kg and c is in m/s, Q will emerge in Joules. However, rest masses are usually tabulated in atomic mass units (abbreviation: amu or simply u). If f is the number of kg in one amu, then we can put

$$Q = \left(\sum m_{input}^{(amu)} - \sum m_{output}^{(amu)}\right) fc^2. \tag{1.3}$$

Q-values are conventionally quoted in MeV. If g is the number of MeV in 1 J, then Q in MeV for masses given in amu will be given by

$$Q = \left(\sum m_{input}^{(amu)} - \sum m_{output}^{(amu)}\right) \left(gfc^2\right). \tag{1.4}$$

Define $\varepsilon = gfc^2$. Recalling that $1\ \text{MeV} = 1.602176462 \times 10^{-13}$ J, then $g = 6.24150974 \times 10^{12}$ MeV/J. Putting in the numbers gives

$$\begin{aligned} \varepsilon = gfc^2 = &\left(6.24150974 \times 10^{12} \frac{\text{MeV}}{\text{J}}\right) \times \left(1.66053873 \times 10^{-27} \frac{\text{kg}}{\text{amu}}\right) \\ &\times \left(2.99792458 \times 10^8 \frac{\text{m}}{\text{s}}\right)^2 = 931.494 \frac{MeV}{\text{amu}}. \end{aligned} \tag{1.5}$$

More precisely, this number is 931.494013. Thus, we can write (1.4) as

$$Q = \left(\sum m_{input}^{(amu)} - \sum m_{output}^{(amu)} \right) \varepsilon, \tag{1.6}$$

where $\varepsilon = 931.494$ MeV/amu. Equation (1.6) will give Q-values in MeV when the masses are in amu.

Now consider an individual reactant of mass number A. The *mass excess* μ of this species is defined as the number of amu that has to be added to A amu (as an integer) to give the actual mass (in amu) of the species:

$$m^{(amu)} = A + \mu. \tag{1.7}$$

Substituting this into (1.6) gives

$$Q = \left(\sum \left[A_{input} + \mu_{input} \right] - \sum \left[A_{output} + \mu_{output} \right] \right) \varepsilon. \tag{1.8}$$

Nucleon number is always conserved, $\Sigma A_{input} = \Sigma A_{output}$, which reduces (1.8) to

$$Q = \left(\sum \mu_{input} - \sum \mu_{output} \right) \varepsilon. \tag{1.9}$$

The product $\mu\varepsilon$ is conventionally designated as Δ:

$$Q = \left(\sum \Delta_{input} - \sum \Delta_{output} \right). \tag{1.10}$$

Δ-values for various nuclides are tabulated in a number of texts and references and are usually given in units of MeV. The most extensive such listing is published as the *Nuclear Wallet Cards* and is available from Brookhaven National Laboratory at http://www.nndc.bnl.gov; a list of selected values appears in Appendix A. The value of quoting mass excesses as Δ-values is that the Q-value of any reaction can be quickly computed via (1.10) without having to worry about factors of c^2 or 931.494. Various examples of Δ-value calculations appear in the following sections.

For a nuclide of given Δ-value, its mass in atomic mass units is given by

$$m^{(amu)} = A + \frac{\Delta}{\varepsilon}. \tag{1.11}$$

1.2 Rutherford and the Energy Release in Radium Decay

The energy released in nuclear reactions is on the order of a million times or more than that typical of chemical reactions. This vast energy was first quantified by Rutherford and Soddy (1903) in a paper titled “Radioactive Change”. In this paper they wrote: “It may therefore be stated that the total energy of radiation during the

disintegration of one gram of radium cannot be less than 10^8 g-cal and may be between 10^9 and 10^{10} g-cal. The union of hydrogen and oxygen liberates approximately 4×10^3 g-cal per gram of water produced, and this reaction sets free more energy for a given weight than any other chemical change known. The energy of radioactive change must therefore be at least 20,000 times, and may be a million times, as great as the energy of any molecular change".

Let us have a look at the situation using modern numbers. ^{226}Ra has an approximately 1,600-year half-life for alpha decay:

$$^{226}_{88}\mathrm{Ra} \rightarrow {}^{222}_{86}\mathrm{Rn} + {}^{4}_{2}\mathrm{He}. \tag{1.12}$$

The delta-values here are, in MeV,

$$\begin{cases} \Delta\left({}^{226}_{88}\mathrm{Ra}\right) = 23.669 \\ \Delta\left({}^{222}_{86}\mathrm{Rn}\right) = 16.374 \\ \Delta\left({}^{4}_{2}\mathrm{He}\right) \ = 2.425. \end{cases} \tag{1.13}$$

These give $Q = 4.87$ MeV in contrast to the *few eV* typically released in chemical reactions.

The notation used here to designate nuclides, ${}^{A}_{Z}X$, is standard in the field of nuclear physics. X denotes the symbol for the element, Z its atomic number (= number of protons) and A its nucleon number (= number of neutrons plus number of protons, also known as the atomic weight and the mass number). The number of neutrons N is given by $N = A - Z$.

Rutherford and Soddy expressed their results in gram-calories, which means the number of calories liberated per gram of material. Since $1\ \mathrm{eV} = 1.602 \times 10^{-19}$ J, $4.87\ \mathrm{MeV} = 7.80 \times 10^{-13}$ J. One calorie is equivalent to 4.186 J, so the Q-value of this reaction is 1.864×10^{-13} cal. One mole of ^{226}Ra has a mass of 226 g, so a single atom has a mass of 3.75×10^{-22} g. Hence the energy release per gram is about 4.97×10^8 cal, in line with their estimate of 10^8–10^{10}. The modern figure for the heat of formation of water is 3,790 cal/g; gram-for-gram, therefore, radium decay releases about 131,000 times as much energy as the formation of water from hydrogen and oxygen. We are assuming here that the entire gram of radium is decaying in computing the figure of 5×10^8 cal; in reality, this would take an infinite amount of time and cannot be altered by any human intervention. But the important fact is that individual alpha decays release *millions* of electron-Volts of energy, a fantastic number compared to any chemical reaction.

Another notational convention can be introduced at this point. In this book, reactions will usually be written out in detail as above, but some sources express them in a more compact notation. As an example, in the next section we will encounter a reaction where alpha-particles (helium nuclei) bombard nitrogen nuclei to produce protons and oxygen:

$$^{4}_{2}\mathrm{He} + {}^{14}_{7}\mathrm{N} \rightarrow {}^{1}_{1}\mathrm{H} + {}^{17}_{8}\mathrm{O}. \tag{1.14}$$

This can be written more compactly as

$$^{14}_{7}\mathrm{N}\left(^{4}_{2}\mathrm{He},\ ^{1}_{1}\mathrm{H}\right)\ ^{17}_{8}\mathrm{O}. \tag{1.15}$$

In this notation, convention is to have the target nucleus as the first term, the bombarding particle as the first term within the brackets, the lighter product nucleus as the second term within the brackets, and finally the heavier product nucleus outside the right bracket.

1.3 Rutherford's First Artificial Nuclear Transmutation

The discovery that nitrogen could be transformed into oxygen under the action of alpha-particle bombardment marked the first time that a nuclear transmutation had been deliberately achieved (Rutherford 1919). This work had its beginnings in experiments conducted by Ernest Marsden in 1915.

In Rutherford's experiment, alpha particles emitted by radium bombard nitrogen, producing hydrogen and oxygen in the reaction:

$$^{4}_{2}\mathrm{He} + {}^{14}_{7}\mathrm{N} \rightarrow {}^{1}_{1}\mathrm{H} + {}^{17}_{8}\mathrm{O}. \tag{1.16}$$

The hydrogen nuclei (protons) are detected via the scintillations they produce when they strike a fluorescent screen. The Δ values for this reaction are:

$$\begin{cases} \Delta\left(^{4}_{2}\mathrm{He}\right) = 2.425 \\ \Delta\left(^{14}_{7}\mathrm{N}\right) = 2.863 \\ \Delta\left(^{1}_{1}\mathrm{H}\right) = 7.289 \\ \Delta\left(^{17}_{8}\mathrm{O}\right) = -0.809. \end{cases} \tag{1.17}$$

The Q-value of this reaction is -1.19 MeV. That Q is negative means that this process has a *threshold* of 1.19 MeV, that is, the bombarding alpha must possess at least this much kinetic energy to cause the reaction to happen. This energy emerges from the spontaneous decay of radium which gives rise to the alphas. We saw in the preceding section that decay of ^{226}Ra liberates some 4.87 MeV of energy, more than enough to power the nitrogen-bombardment reaction.

The conditions of energy and momentum conservation relevant to "two body" reactions of the general form A + B → C + D are detailed in Appendix C. A companion spreadsheet, **TwoBody.xls**[1], allows a user to input nucleon numbers and Δ-values for all four nuclides, along with an input kinetic energy for reactant A; nucleus B is presumed to be stationary when struck by A. The spreadsheet then

[1]All Excel sheets are available at http://www.manhattanphysics.com