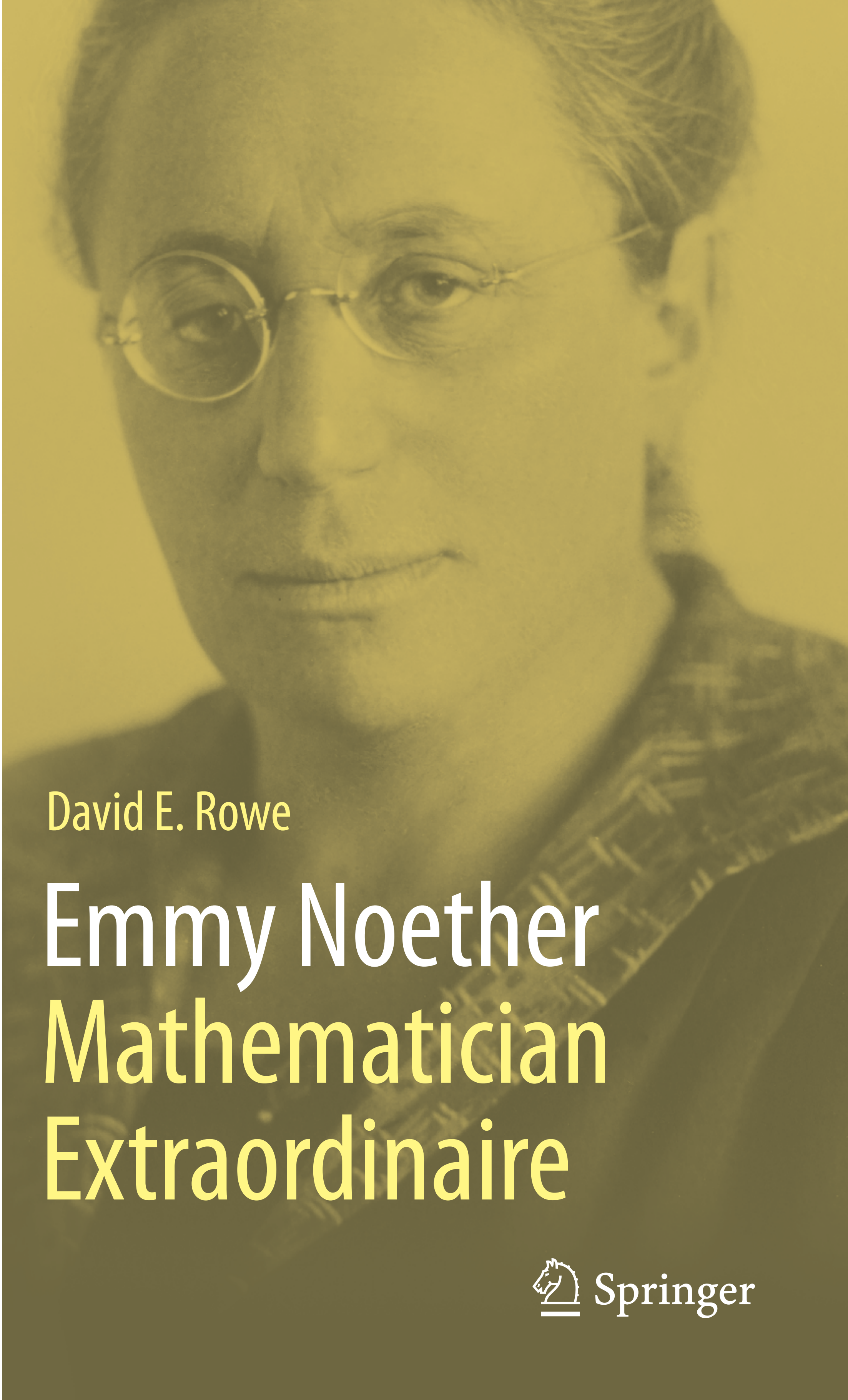

David E. Rowe

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Mathematician Extraordinaire

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Contents

Preface

Emmy Noether is today one of the most celebrated figures in the history of mathematics, universally recognized as a brilliant algebraist and familiar to many as an iconic personality for women in science. This recognition was long in coming, and even if she enjoyed a taste of it toward the end of her life, Noether would have probably felt puzzled and bemused by such boundless acclaim. That's pure speculation, of course, but then Emmy Noether enjoyed that sort of thing, especially when it involved mathematical fantasies. Otherwise, this extroverted woman was a rather private person who, for whatever reasons, tended not to talk about her worries and concerns, including those that eventually related to her own deteriorating health. Much about her life, which ended all too abruptly, we will never know and can only simply imagine.

This book arose out of a longstanding fascination with Emmy Noether's unique personality, but it never would have been written except for recent circumstances and events. In 2019, Mechthild Koreuber organized a major conference at the Freie Universität Berlin in cooperation with the Berlin Mathematics Research Center MATH+ and the Max Planck Institute for the History of Science to commemorate the hundredth anniversary of Noether's Habilitation in Göttingen.[1] This event, which took place on June 4, 1919, was not only a major milestone for Noether herself but also for women longing to pursue academic careers in Germany. As one of the highlights of the conference, the ensemble *portraittheater Vienna* presented the premiere performance of their play, "Mathematische Spaziergänge mit Emmy Noether" [Schüddekopf/Zieher 2019]. Both of us, as historians of mathematics, had been involved in its production, and we were delighted by the result. So the thought of adapting the script for an English-speaking audience occurred to us right away. It must be said, though, that we had no idea how Sandra Schüddekopf and Anita Zieher would manage to stage a play about a mathematician, one whose work even her peers found to be highly abstract. Nevertheless, they found a very elegant way to finesse that problem, and in a manner that would have appealed to Emmy Noether, whose personality shines through despite the handicap that most of her audience has absolutely no idea what she's really talking about. Noether was by no means a one-dimensional type who lived for mathematics and nothing else, and Anita Zieher truly brings her personality back to life on stage, now in the new adaptation of the original play, "Diving into Math with Emmy Noether" [Schüddekopf/Zieher 2020].[2]

Sensing that "Diving into Math" was an excellent vehicle for conveying the spirit of Emmy Noether's life, Mechthild Koreuber and I soon got the idea of writing a book that would expand on some of its themes. This we hope to have achieved in *Proving It Her Way: Emmy Noether, a Life in Mathematics*

[1]The interdisciplinary character of the conference – which brought together mathematicians, physicists, historians of science, gender researchers, and cultural historians – is reflected in the forthcoming conference volume [Koreuber 2021].

[2]For information or to book a performance, contact office@portraittheater.net.

[Rowe/Koreuber 2020], which aims to provide an overall picture of her life, but with only minimal attention paid to her mathematics. The present book has a similar structure and purpose, namely, to illuminate Noether's life by offering a full-blooded picture of her role in shaping the mathematical activity of her day and, as it happened, well beyond.[3] In places, however, it is more technically demanding. Thus, Chapter 3, "On Emmy Noether's Role in the Relativity Revolution," deals with a topic omitted from the smaller volume, since to appreciate what she accomplished in that context requires familiarity with Einstein's theory of general relativity. Likewise, the final Chapter 9, "Memories and Legacies of Emmy Noether," which reaches well beyond the events of her life, was left out of the shorter book. Elsewhere, as well, the reader will find many brief discussions of Noether's mathematics and related matters strewn throughout the pages of this volume. For those who wish to learn still more, this book contains numerous references to the works listed in its extensive bibliography, including all of Noether's own publications from her *Collected Papers* [Noether 1983].

Emmy Noether was not a particularly prolific mathematician, and while a few of her papers are now classics, most have long since been forgotten. Noether's fame and influence had much to with those well-known publications, of course, but one cannot really begin to grasp her importance merely by studying these published works. This would be to overlook her activities as a collaborator and critic, not to mention her role as referee for the journal *Mathematische Annalen*. Most of all, though, Noether's influence flowed through her role as leader of a dynamic new mathematical school, one in which she taught younger mathematicians how to exploit the new concepts and methods she promoted in her lectures and published work. Her approach aimed to strip mathematical objects down to their bare essentials in order to recognize deeper underlying relationships among them. Doing so, however, meant learning to think about mathematics on a higher abstract plane. Noether's enthusiasm was infectious, at least for those who entered her circle and caught the abstract algebra bug. By the mid-1920s, she was already riding a wave of modern methods that would eventually reshape major branches of mathematical knowledge.

Noether lived during the pre-Bourbaki era, a time when modern forms of collaboration were only emerging. André Weil, the unofficial leader of the group that wrote under the pseudonym Nicolas Bourbaki, remembered the atmosphere in Noether's Göttingen circle during the mid-1920s as very different from the one he encountered when talking with those in Richard Courant's group, from whom he learned very little. Nearly every time he got into a conversation with one of the latter's students, the exchange would end rather abruptly with a remark like, "sorry, I have to go write a chapter for Courant's book" [Weil 1992, 51]. This

[3] Both books, it should be noted, draw heavily on the information concerning Noether's school as well as the interpretation of its impact found in [Koreuber 2015].

"publish or perish" mentality predominated in Courant's circle, whereas Emmy Noether felt no such urgency to rush her work into print.[4]

Weil recalled conversations with Pavel Alexandrov in Noether's cramped little attic apartment. Its ceiling was so angular that Edmund Landau – who lived in a veritable palace by comparison – wondered out loud whether Euler's polyhedral formula still applied to her living room. Here and elsewhere, Weil saw how

> Emmy Noether good-naturedly played the role of mother hen and guardian angel, constantly clucking away in the midst of a group from which van der Waerden and Grell stood out. Her courses would have been more useful had they been less chaotic, but nevertheless it was in this setting, and in conversations with her entourage, that I was initiated into what was beginning to be called "modern algebra" and, more specifically, into the theory of ideals in polynomial rings. [Weil 1992, 51]

Many who heard Noether's lectures reacted similarly, like the young Carl Ludwig Siegel, who remembered them as badly prepared. In one of her courses, which ended at 1 o'clock, he scribbled in the margin of a notebook: "It's 12:50, thank God!" [Dick 1970/1981, 1981: 37]. Siegel much preferred lectures in the style of his mentor Landau, who prided himself on presenting polished lectures already nearly ripe for publication. Landau's teaching style, which Siegel largely emulated, aimed to convey the formalized end products of mathematical activity. For Noether, on the other hand, the excitement came when she was still searching, groping along halfway in the dark. She gradually developed a teaching style in which oral communication, dialogue, and collaboration dominated. Doing mathematics meant, for her, engaging with all facets of the process, and in this way she came to embody the oral component in Göttingen's vibrant mathematical culture.[5]

Her approach, however, should by no means be understood as one that neglected the importance of formal rigor in published communications. Indeed, the relative sparsity of her own published work reflects the fact that she always resisted putting less than perfect texts into print. Moreover, Noether's letters and postcards – in particular those she sent to Helmut Hasse, published in [Lemmermeyer/Roquette 2006] – reveal very clearly that she always upheld the highest standards for mathematical publications. "Pauca sed matura" (few but ripe), the famous watchword of Carl Friedrich Gauss, applies just as well to Emmy Noether. Yet Gauss, who was anything but generous when it came to communicating his unripe ideas with others, stands in this respect in complete opposition to Noether, whose success and influence had much to do with her un-

[4] Weil was especially struck by the radically different atmosphere in Frankfurt, where Max Dehn and Carl Ludwig Siegel cultivated mathematics as an art form, in conscious opposition to the factory-like production facilities in Courant's Göttingen [Weil 1992, 52–53].

[5] On the importance of the oral dimension in Göttingen, see [Rowe 2004]. For an analysis of Noether's oral style as part of her conceptual approach to mathematics, see [Koreuber 2015, Kap. 2].

selfish generosity. Indeed, despite her unorthodox teaching style, she made her greatest impact as a teacher and through her influence as leader of what came to be called the “Noether school” (or “Noether community” or sometimes “Noether family”).

For Emmy Noether, just as for Emil Artin, mathematicians were in the first instance artists, not scientists.[6] This rather contentious position was apparently a favorite topic of friendly disputes between Emmy and her brother Fritz, a leading applied mathematician who worked on problems like modeling turbulence in continuum mechanics. Their father, Max Noether, was a different type of mathematician still, as will be seen in Chapter 1, which deals with their years together in Erlangen. Mathematical talent ran through the Noether family, leading the Göttingen number-theorist Edmund Landau to liken Emmy Noether’s kinfolk with a coordinate system in which she occupied the origin [Dick 1970/1981, 1981: 95]. Had Landau lived longer, he would have needed to imagine a new coordinate axis for Fritz Noether’s younger son Gottfried, who became a leading authority in the field of non-parametric statistics (see Section 9.2).

Labels are often misleading, and in the case of mathematical elites like the Noethers lumping them together as “mathematicians” simply overlooks the wide range of different intellectual pursuits in which these researchers were engaged. In the case of Emmy Noether, this is a crucially important point; her way of thinking about mathematics using abstract concepts, rather than concrete objects, was by no means new. She became, however, the foremost exponent of this approach to mathematical theorizing, which she promoted in a radical manner, a style quite unlike that of any other contemporary figure. She and Artin also both believed that all truly deep mathematical truths must be beautiful. One can only begin to understand what that means, of course, by delving deeply into the mathematical world they shared, as many famous figures who came after them did.

Quite apart from her accomplishments as a mathematician, Emmy Noether was also a singularly remarkable representative of that famous group of German Jewish intellectuals who fled from Nazi Germany. How this tragedy unfolded in Göttingen is described in Chapter 7, which provides a fairly detailed account of the events that led to the destruction of its star-studded mathematical faculty. Much has been written about “Hitler’s gift” to the Western democracies and how Weimar’s exiled intellectuals enriched cultural life in the United States. Emmy Noether’s name would surely have appeared in many more of those studies had she only lived longer. Instead, her tragic and wholly unanticipated death in April 1935 prevented her from importing her distinctive style for doing abstract algebra to the United States, even though others, in particular Artin and Richard Brauer, promoted similar ideas soon afterward.

Still, we can easily imagine how that story might have unfolded at Bryn Mawr College, but especially at Princeton’s Institute for Advanced Study. Her “girls” at

[6] On the broader context of earlier debates over the status of mathematics as art or science, see [Rowe 2018a, 401–411].

Bryn Mawr (graduate students and post-docs) were among the first to sink their teeth into "German algebra" by reading B.L. van der Waerden's new textbook *Moderne Alegbra* [van der Waerden 1930/31] and by studying the *Ausarbeitung* of Helmut Hasse's Marburg lectures on class field theory. Noether taught them in English, but with such large dosages of German jargon that this became part of their natural vocabulary, and so they spoke to each other about this "new math" in a kind of pidgin German. Noether's favorite pupil at Bryn Mawr, Ruth Stauffer, later recalled how "it was very easy for us to simply accept the German technical terms and to think about the concepts behind the terminology. Thus from the beginning we discussed our ideas and our difficulties in a strange language composed of some German and some English" [Quinn et al. 1983, 142].

Although Emmy Noether is justly famous as the "mother of modern algebra," it is important to understand what she meant by "doing algebra." Her vision of its role in mathematics did not seek to erect clear disciplinary boundaries setting algebraic investigations off from those in other fields. On the contrary, her work was closely tied to an older trend that aimed to algebraicize other fields, from complex functions and number theory to topology, i.e. major parts of all mathematical knowledge. In this respect, she took inspiration from earlier studies by Richard Dedekind, Heinrich Weber, Ernst Steinitz, and David Hilbert. Moreover, she clearly identified with words she once cited from Leopold Kronecker's 1861 inaugural address, when he was inducted into the Berlin Academy: "algebra is not actually a discipline in itself but rather the foundation and tool of all mathematics."[7] Not that many of Noether's contemporaries shared this view; far from it. Nor should we imagine that Noether meant this literally; she was well aware of the vast fields in analysis and applied mathematics that lie well beyond the realm of even her all-embracing view of algebraic research. Still, she was without doubt the leading spokesperson of her generation for this position, one that many of her contemporaries found extreme.

One of Noether's closest collaborators, Helmut Hasse, clearly recognized the import of Noether's message, but he also sensed the need to spread the word. In a lecture on "The modern algebraic Method," he made this mission clear:

> The aim of my talk is to promote modern algebra among non-specialists instead of preaching to the choir. It is not my intention to lure anyone from his field of specialization to become an algebraist. I see my task, rather, as laying the groundwork for a favorable understanding of modern algebra, helping to establish its methods – insofar as they are of general importance, and integrating these methods into the common knowledge of contemporary mathematicians. [Hasse 1930, 22]

Noether's former Göttingen colleague, Hermann Weyl, on the other hand, had deep misgivings, though not so much with regard to abstract algebra per se. What concerned Weyl was the general trend toward abstraction in mathematical research,

[7] [Noether 1932c]; the relevance of this citation is discussed in [Koreuber 2015, 225] as well as in [Merzbach 1983, 161].

a tendency he felt could easily lead to artificiality and superficial results. He mentioned these concerns in his famous memorial address [Weyl 1935], delivered at Bryn Mawr College. In this respect, Hermann Weyl was far closer to his former mentor, David Hilbert, than he was to Emmy Noether, much as he admired her brilliance.[8]

Beyond these matters, some of Weyl's remarks reflect a rather condescending view of Emmy Noether and her family. This attitude seems particularly striking when he writes about her background and life in Erlangen, a time and place Weyl could only imagine. He portrayed the Noethers as impressive intellectuals, even drawing a parallel between their family, with its three distinguished mathematicians, and the Bernoullis, whose Huguenot ancestors fled Spanish repression in the Netherlands to settle in Basel. But he also saw the Noether family as representatives of a shallow bourgeois society, with "their sentimentality, their Wagnerism, and their plush sofas" [Weyl 1935, 430]. Perhaps Weyl also felt unnerved by Emmy's apparent inability to grasp evil in the world. She had lived her whole life as a fully integrated German Jew, which meant of course that antisemitism was no stranger to her, but when the barbarians came to power and threatened to sweep away everything she loved, she reacted not only with restraint but with an almost super-human equanimity. Those lonely months during the spring of 1933 – the time when they were last together in Göttingen – no doubt profoundly shaped Weyl's view of her, yet his opinion seems to have wavered between two extremes: Emmy was either a tower of moral strength or she was simply naive.

Although Hermann Weyl knew Wilhelmian Germany exceedingly well, he may never have met Max Noether. He did know Fritz Noether, who came to Göttingen as a post-doctoral researcher when Weyl was teaching there as a *Privatdozent*. As for Emmy, he could easily have met her at the annual meetings of the German Mathematical Society, but he probably only got to know her well in 1927 when he spent a semester as a guest professor in Göttingen. Notwithstanding the importance of his testimony, Weyl's personal opinions need not be taken as authoritative, particularly since his impressions in 1935 were colored by the turbulent events he experienced during the previous two years. Moreover, he did not have access to most of the contemporary documentary sources that form the basis for the present study and which inform its interpretation of Emmy Noether's social and intellectual background. This book also consciously avoids certain stereotypic themes found in much of the secondary literature dealing woth Noether. Many standard studies of women in the history of mathematics have chosen to follow Weyl's lead by comparing Noether with the internationally renowned Russian mathematician Sofia Kovalevskaya, who also appears in several places below as well. These comparisons, to be sure, rarely have anything to do with serious interest in what these women accomplished as mathematicians. Very often, they are coupled together as two trailblazers in a field then totally dominated by men, even though neither really saw herself in such a role. Talk of glass ceilings, after

[8] For an analysis of Weyl's scientific work and views, see [Scholz 2001].

all, was yet to come, whereas gender roles in that era were exceedingly constrained. Earlier commentators usually could not get past the notion that a "lady mathematician" was a freak of nature, a view clearly supported by the scarce number of these creatures then walking the earth. How that has changed! Contemporary opinions of Emmy Noether – and these were quite mixed – clearly have considerable importance for understanding the context in which she lived. Even more important – especially for the present undertaking – are those sources that tell us how she thought about herself and the world around her, and especially how she expressed those thoughts. Others occasionally compared her as a mathematician with Richard Dedekind (no one writing about her mathematics would have imagined a comparison with Sofia Kovalevskaya), but she quite rightly said about herself "I always went my own way."[9]

Hermann Weyl's account of Emmy Noether's intellectual development has, in one sense, been very influential. Many subsequent commentators have, in fact, adopted his tripartite division of her career:[10] (1) the period as a post-doc, 1907–1919, followed by (2) her work on the general theory of ideals, 1920–1926, and then (3) her contributions to non-commutative algebras with applications to commutative number fields, 1927–1935 [Weyl 1935, 439]. This periodization is certainly apt and even quite useful to a point, but it can also easily lead to quite misleading impressions. Those who have adopted it have tended to underplay the significance of the first period, while overlooking some of the threads that ran through all three phases of Noether's career.[11]

Emmy Noether was nearly forty years old when she began publishing the papers on modern algebra that made her famous. By the mid-1920s, she had become the leader of an international school that would soon thereafter exert a deep and lasting influence on mathematical research. All her most familiar and significant work was thus undertaken during the latter two periods, when many of her ideas and findings quickly propagated through the network of the Noether school. Little wonder, then, that this success story has completely dominated nearly all the accounts of Noether's life. Moreover, as Uta Merzbach has stressed, one of the great ironies behind her success was that part of it stemmed from never having gained a regular professorial appointment at Göttingen. This "allowed her to organize her algebraic research as single-mindedly as she did, to display that generosity to her followers to which van der Waerden, Alexandroff, and others have given eloquent testimony, and to engage so fully in the editing of Dedekind's

[9]See the opening of Chapter 2 in *Proving it Her Way: Emmy Noether, a Life in Mathematics*.

[10]Weyl's obituary of Hilbert was somewhat similar; there he discerned that the master's work fell into five periods [Weyl 1944, 4: 135].

[11]Pavel Alexandrov fully appreciated the importance of Noether's early work on finiteness results, but wrote that she herself was partly responsible for the fact that this work had been unjustly neglected, since she "considered those results to have been a diversion from the main path of her research, which had been the creation of a general, abstract algebra" [Alexandroff 1935, 2]. Since Alexandrov and Urysohn pioneered the theory of general compact spaces (in which every open covering has a finite subcovering), one can easily imagine their affinity for finiteness results in algebra.

works and selected correspondence" [Merzbach 1983, 169–170]. A similar view was expressed by Emmy Noether's first biographer, Auguste Dick, who wrote that she preferred the position of *Dozentin* because it gave her freedom. As an *Ordinarius* "she would have been obliged to teach basic courses and exercises for which she was not well suited. Much of her time would have been absorbed by preparations for classes, and her own research would have suffered" [Dick 1970/1981, 1981: 72–73].

Most of Noether's publications from the first period, on the other hand, received little attention during her lifetime. This applies even to her famous paper "Invariant Variational Problems" [Noether 1918b], which today is perhaps her best-known single work. As documented in [Kosmann-Schwarzbach 2006/2011], this paper was rarely ever cited, much less carefully read, until many years after Noether's death. No doubt Weyl's periodization of her research interests offers a useful schematic, so long as we are not misled into thinking that Emmy Noether's earlier work had little to do with her publications from the 1920s. As Merzbach noted, a great deal of her work had clearly identifiable classical roots:

> Her deep knowledge of the literature and her ability to recognize and bring to the fore those concepts that would prove most fruitful prepared her . . . to undertake her grand synthesis. If one examines her work after 1910, one finds continual growth, but little change in methodological pattern. [Merzbach 1983, 169][12]

This should come as no surprise if we remember that Emmy Noether had a thorough knowledge of the mathematical literature of her time; she was also well-versed in major works from the latter half of the nineteenth century.

As noted in [Koreuber 2015, 5], Weyl's tripartite framework is highly problematic if one hopes to gain a deeper understanding of Emmy Noether's intellectual growth. To gain a more balanced picture requires recognizing, first of all, the critical importance of the first period in her career. Those years form part of the larger context taken up in Chapter 1, which deals with her life in the mid-size university city of Erlangen, where she grew up as the daughter of the eminent mathematician Max Noether. The Noether family – Max and Ida and their four children – were members of the local Jewish community, which numbered around 200 persons during Emmy's childhood, less than 2% of the city's population. More important still, all of them were recent arrivals, as before 1861 Jews were not permitted to live within the city limits. Max Noether and his older colleague Paul Gordan, who was also Jewish by birth, were the only mathematicians on the faculty, a highly unusual situation, especially given the small number of university professorships that existed throughout Germany. Young women had virtually no chances of even studying at a university, let alone dreaming of a teaching career at one of these institutions. That Emmy Noether dreamed of such a life at an early age probably cannot be documented, but clearly she did, and the fact that she longed to follow

[12] A more recent study that argues for a similar view is [McLarty 2017].

in her father's footsteps gives us the first key to understanding how such a thing could even happen.

When Emmy Noether finally left Erlangen in 1915, she did so with the hope of joining the faculty in Göttingen, a plan supported by its two senior mathematicians, Felix Klein and David Hilbert. Their efforts, however, at first failed, and as recounted in Chapter 2, it took four long years before Noether was allowed to habilitate in Göttingen. This matter, which hinged entirely on the fundamental question of whether qualified women were entitled to become members of a university faculty, led to a dramatic clash of opinions within Göttingen's highly polarized philosophical faculty. Indeed, the Noether affair was perhaps the most infamous in a series of running battles which would eventually lead to a complete cessation of relations between its two departments, comprised of natural scientists in one division, and humanists in the other. In 1922, the Ministry finally approved a proposal, put forth by the latter group (members of the historical-philological department), that called for the formation of two wholly distinct faculties. In that same year, the newly established faculty of mathematics and natural sciences appointed Emmy Noether as an honorary associate professor, a title normally bestowed only six years after habilitation. In recommending her for this honor, the faculty noted that she had been unjustly denied the right to habilitate in 1915.[13]

During the war years, both Hilbert and Klein had become deeply immersed in mathematical problems connected with Albert Einstein's novel approach to gravitation, the general theory of relativity. Working first with Hilbert and then with Klein, Noether ultimately unraveled one of the major mathematical mysteries that they and Einstein had struggled to solve, namely, the role of energy conservation in physical theories based on variational principles. Chapter 3 provides a fairly detailed account of Emmy Noether's role in that particular phase of the relativity revolution. This story culminates with the publication of the "Noether theorem" (actually two theorems) in "Invariant Variational Problems" [Noether 1918b], a result nearly every physicist today is familiar with in some guise. The story of how she actually came to write that paper, however, has rarely been told and surely deserves to be better known, despite the technical complexities involved. Those who are unfamiliar with mathematical methods in general relativity can skip this chapter without losing the main threads that tie Emmy Noether's first creative period with her work from the early 1920s.

Noether's most influential papers stem from her second period, when she made major contributions to ideal theory. She was almost 40 when she published "Ideal Theory in Ring Domains" [Noether 1921b], one of her most famous algebraic works. Here she introduced the general concept of rings satisfying the ascending chain condition, familiar today as Noetherian rings. Soon afterward, her reputation as a leading algebraist began to spread beyond Göttingen, leading to her fame as "der Noether." To appreciate the importance of this work, described briefly in Chapter 4, one needs to understand its role in the general shift from classical to

[13]Universitätsarchiv Göttingen, Personalakte Emmy Noether, UAG.Math.Nat.Pers.9.

modern algebra. This is easily illustrated by comparing Noether's approach with Richard Dedekind's earlier theory of ideals in number fields.[14]

Noether's second major contribution to ideal theory was [Noether 1927a] ("Abstract Structure of Ideal Theory in Algebraic Number and Function Fields"). Here she followed Dedekind, who had proved a fundamental decomposition theorem for the ideals of a number ring. Noether was able to prove an analogous, but much more general theorem, valid for all commutative rings that satisfied the five axioms for a Dedekind ring – and vice versa – which means this theorem characterized Dedekind rings.[15] In this theory, the prime ideals play the same role as the prime numbers in elementary number theory. So her theorem was a fundamental structure theorem for ideal theory – which is now understood as part of the broader discipline of commutative algebra.[16]

Noether's other great achievement came in her earlier paper [Noether 1921b]. Here she was able to place Emanuel Lasker's decomposition theorem for ideals in a ring of polynomials on a much broader and clearer basis. The building blocks in this case were the primary ideals introduced by Lasker, but instead of five axioms Noether essentially only needed one restriction, namely, that the ring does not contain an infinitely ascending chain of ideals. This property was not new, but Emmy Noether was the first to recognize its central importance. This is why rings that satisfy the ascending chain condition (acc) are today called Noetherian rings. She later made this acc condition the first of her five axioms in [Noether 1927a]. Noether's structure theorem for polynomial rings was of great importance for the algebraization of algebraic geometry. Her father had proved a fundamental theorem for this discipline in 1871, which later served as the foundation for the work of the "Italian school." However, his daughter took up earlier results of Hilbert and Lasker in order to lay the foundation for a new and far more general direction in algebraic geometry based on polynomial ideals. Yet even more important than these results were Noether's methods, which clearly revealed the strength of her conceptual arguments compared with earlier more computational methods. Her goal throughout was to make everything as transparent as possible, and her most important works can still be read today with interest and understanding, a rare achievement in mathematics.

After this brief excursion into Noether's work on ideal theory, the focus in Chapters 4, 5, and 6 shifts to her relationships with the four other mathematicians who appear in "Diving into Math with Emmy Noether": Bartel L. van der Waerden, Pavel Alexandrov, Helmut Hasse, and Olga Taussky. While none of these four took a doctoral degree under Noether, all were closely connected with her school in one way or another. Each, in fact, represents a strand of influence that ran through the Noether school, thereby contributing to its diverse and eclectic character. When

[14] For a detailed comparison, see [Corry 2017].

[15] For a sketch of the steps in her proof, see Jacobson's introduction in [Noether 1983, 14].

[16] She exploited this new theory immediately afterward in [Noether 1927b] by proving a generalization of Dedekind's discriminant theorem that applies to arbitrary orders in a number field.

van der Waerden arrived in Göttingen from Amsterdam, his principal interests were closely related to Max Noether's work in algebraic geometry. After studying under Noether's daughter and then under Emil Artin in Hamburg, he published his classic two-volume textbook *Moderne Algebra* [van der Waerden 1930/31], which for decades afterward served as the standard introduction to the subject.

The Russian topologist Pavel Alexandrov was a regular visitor in Göttingen during the summer months. As one of Emmy Noether's closest friends, he spent countless hours "talking mathematics" with her, eventually joined by another topologist, Heinz Hopf. These conversations proved of vital importance for the emergence of modern topology, a field that began to take on clear form in their textbook [Alexandroff/Hopf 1935]. Both van der Waerden and Alexandrov very consciously adopted Noether's conceptual approach in writing these two seminal works, which distilled and synthesized essential knowledge in two fundamentally new disciplines: abstract algebra and algebraic topology.

During the final phase of Noether's career, Helmut Hasse was her closest collaborator. As a student of Kurt Hensel in Marburg, Hasse developed a new local-global principle, based on Hensel's p-adic numbers, that proved highly fertile for research in algebraic number theory. As he began to explore a new research agenda for class field theory, Emmy Noether pointed out the relevance of ongoing work on hypercomplex number systems (i.e., non-commutative algebras) for generalizing the number-theoretic investigations of Hasse and Artin. Thanks to the carefully edited publication of her letters to Hasse, published in [Lemmermeyer/Roquette 2006], one can easily recognize how Noether's ideas had a catalytic effect on Hasse's work after 1927. Her constant, unrelenting prodding, mixed with praise and encouragement, played a major part in their symbiotic relationship, underscoring the importance of purely human factors in mathematical research. Noether's parallel collaboration with Richard Brauer soon led to a threesome, who together succeeded in proving the Brauer-Hasse-Noether theorem.

Emmy Noether's relationship with Olga Taussky was unlike any other, not least because Taussky, too, was a woman with a mind of her own. Their first lengthier interactions took place during the academic year 1931/32 when Taussky came to Göttingen as a young Viennese post-doctoral student, having been hired by Richard Courant to lend help in editing Hilbert's early works on number theory. She was highly qualified to do so, having studied under Phillip Furtwängler, a leading expert on class field theory. After returning to Vienna for two years, Taussky rejoined Emmy Noether at Bryn Mawr College in 1934, a difficult time in the lives of both women, as Taussky would recall late in her life. Olga Taussky never became an enthusiast for Noether's abstract style of mathematics, and yet her encounters with Emmy Noether, particularly during the last year of her life, proved to be of great importance for the young woman's career. Indeed, none of these four mathematicians – van der Waerden, Alexandrov, Hasse, and Taussky – who went on to write hundreds of papers and produce dozens of Ph.D.s in the course of their careers, can really be called a disciple of Emmy Noether, even though all of them were inspired by her ideas and personality in significant ways.

During the 1920s, Richard Courant and Emmy Noether actively promoted the trend toward internationalization that became a hallmark of Göttingen mathematics during this period. Their efforts received a major boost from American philanthropy and the vision of Wickliffe Rose, who founded the International Education Board (IEB) in 1923, backed by financial support from John D. Rockefeller, jun.[17] Several of those who visited Göttingen during these years were IEB Fellows. Two who came from France were André Weil and Paul Dubreil; both attended Noether's lectures, as did another co-founder of the Bourbaki group, Claude Chevalley. The Norwegian Øystein Ore visited Göttingen twice, the second time as an IEB fellow working under Noether. He was afterward recruited by James Pierpont, who invited him to join the faculty at Yale University, where he would remain throughout his career. He also joined Emmy Noether and Robert Fricke in editing the collected works of Richard Dedekind, [Dedekind 1930–32] (see Section 6.4).

As Hermann Weyl emphasized in his memorial address, Noether stood at the very heart of mathematical life in Göttingen, just as its larger scientific community was a manifestation of Weimar Germany's vibrant cultural life.[18] As one of Weimar culture's leading representatives, Albert Einstein later wrote about Emmy Noether's highly significant role in this ultimately tragic story.[19] Chapter 7 describes the traumatic events of 1933 that dramatically ended that life, as Noether had known it. She and Richard Courant, the director of the Mathematics Institute, were both forced to take refuge in the United States. Helmut Hasse would ultimately be appointed to Courant's chair, but while still in Marburg he initiated a campaign to maintain Noether's modest position in Göttingen. Predictably, this effort failed, though through the intercession of friends in the United States Emmy Noether gained a temporary appointment at Bryn Mawr College, a distinguished institution of higher learning for women.

Chapter 8 briefly recounts Bryn Mawr's importance for the history of mathematics before relating various events and circumstances connected with Noether's association with the college. During her 18 months there, she also began to spread the gospel of modern algebra in weekly lectures at Princeton's Institute for Advanced Study, where her seminar attracted a number of prominent, as well as up and coming mathematicians. Her collaborator from Germany, Richard Brauer, attended regularly, as did Nathan Jacobson. The latter filled in for Emmy Noether at Bryn Mawr the following year, and he would later edit her *Collected Papers* [Noether 1983]. Her sudden death on 14 April 1935, following an operation, came as a huge shock to everyone, perhaps most of all to her brother Fritz, who also had been forced to leave Germany with his two sons. Emmy had tried to find work for

[17]For a detailed account of the IEB's impact on mathematics, especially in Western Europe after World War I, see [Siegmund-Schultze 2001].

[18]This interpretation of Göttingen mathematics as a phenomenon within the larger context of Weimar culture is addressed in [Rowe 1986].

[19]Einstein's obituary of Noether, which appeared in the *New York Times*, is discussed in Chapter 9; it was first analyzed in [Siegmund-Schultze 2007].

him in the United States (see Section 8.3), but after her efforts failed he took a position at Tomsk Polytechnic University in Western Siberia. She was spared by her premature death from learning about the tragic events that afterward befell her brother and his family, described in Section 9.2.

Emmy Noether's last two years in the United States were filled with all kinds of worries, few of which she spoke about even with her closest friends. One of these was Anna Pell Wheeler, chair of the Mathematics Department at Bryn Mawr College, who in many ways helped her to adjust to life in the United States. Chapter 9 recounts some of the memories other friends of Emmy Noether shared with each other as well as with the Bryn Mawr community. Many sensed the grandeur of her intellectual legacy, but it would take some time to recognize clearly her importance for subsequent mathematical developments. This closing chapter cannot, of course, do justice to Noether's legacy; nevertheless it seems appropriate to end with some reflections on her place in the mathematics of the last century. A contemporary mathematician once told David Hilbert, the man who first brought Noether to Göttingen, "You have made us all think only that which you would have us think."[20] Those very same words could just as aptly have been said about Emmy Noether, whose ambitions for directing and shaping mathematical research spring to life in her correspondence, but also from later recollections written by contemporaries who knew her very well. By the end of her life, she had many admirers who recognized in her unique personality and boundless vitality the marks of a genial mathematician.

As pointed out in Chapters 7 and 9, even some of those who knew Emmy Noether's work very well considered it somehow "Hebraic," and hence foreign to what they imagined to be good, sound "German" mathematics. Faced with seeing their teacher banned from the German universities, Noether's faithful students tried to counter this by underscoring how her research was rooted in the tradition of Richard Dedekind, one of the great German mathematicians of the nineteenth century. Perhaps this was simply a matter of political expediency, but more likely it reflected a genuinely felt conviction that Emmy Noether's mathematics was truly "Germanic" and was therefore not be be conflated with a "Jewish style." A standard stereotype presumed that Jews had a distinctly different way of thinking about mathematics stemming from a Talmudic tradition that favored abstract theorizing, while neglecting fields with close ties to the physical sciences. Such stereotypes were particularly widespread in the German mathematical community during the period considered here, and yet for every Emmy Noether representing the first tendency, there was a Fritz Noether practicing the second.

This larger point was brought out forcefully in the traveling exhibition "Transcending Tradition: Jewish Mathematicians in German-Speaking Academic Culture," which presented a wide array of books and articles written by German-Jewish scholars. These impressive works completely refute the claim that there

[20]From Constantin Carathéodory's funeral speech for Hilbert, Hilbert Nachlass, SUB Göttingen, 750.

was a "typical form of 'Jewish mathematics', remote from geometrical intuition or from applications" [Bergmann/Epple/Ungar 2012, 134].[21] "Transcending Tradition" grew out of an earlier effort that went on display at the Poppelsdorf Palace in Bonn in September 2006.[22] This soon led to a broader undertaking, organized by Moritz Epple at Frankfurt University, that produced a traveling version of the original German-language exhibition. The latter was shown in a number of cities during 2008, the "Year of Mathematics" in Germany, and aroused considerable interest among the public at large. As a result, support was sought and obtained from governmental agencies for an English-language traveling exhibition that went on view in cities throughout Israel, the United States, and Australia. Naturally, Emmy Noether was accorded a prominent place in it, as was the Bonn mathematician Felix Hausdorff (see [Bergmann/Epple/Ungar 2012, 83–85; 94–104]), both of whom contributed in quite different ways to shaping the face of modern mathematics. In fact, the original impetus behind the 2006 exhibition arose from then ongoing work on the multi-volume Hausdorff edition, a highly ambitious project that was only completed quite recently.

Although the names Emmy Noether and Felix Hausdorff are famous in the annals of mathematics, they are rarely mentioned together. And, in fact, it would be hard to imagine two mathematicians whose works, personalities, and influence differed so sharply. Nor does it appear that they had more than perhaps fleeting contacts with one another, since Hausdorff rarely attended the annual meetings of the German Mathematical Society, an event Noether rarely missed. Nevertheless, one merely needs to open [Hausdorff 2012], the correspondence volume in the Hausdorff edition, to recognize that Pavel Alexandrov, the great Russian topologist, acted as a kind of mediator between these two eminent figures.[23] Indeed, his letters to Hausdorff clearly reflect the paths Alexandrov followed in an effort to link point set topology in the style of Felix Hausdorff with the then emergent algebraic topology promoted by Emmy Noether. This is but one of the many currents that ran through Noether's life's work. Here, as elsewhere in this book, effort has been made to illuminate her career through new findings based on recent research. Much more can be found in the many works listed in the bibliography, especially for those who read German.

A book such as this one could obviously not have been written without the efforts of many others, including those whose names appear in the many works cited throughout. Rather than making this preface any longer than it is already, though, let me first extend thanks to all the unnamed individuals who have contributed

[21] David Hilbert was one of the few who forthrightly claimed that "mathematics knows no races" (see [Siegmund-Schultze 2016]). His colleague, Felix Klein, thought that geometrical intuition (*Anschauung*) was deeply rooted in the Teutonic race. Yet as Klein and Max Noether well knew, after 1890 this impulse lost ground in Germany just as it was being taken up by leading Italian geometers: Corrado Segre, Guido Castelnuovo, and Federigo Enriques, all of whom were of Jewish descent.

[22] On this prehistory, see Moritz Epple's remarks in [Bergmann/Epple/Ungar 2012, 7–8].

[23] Alexandrov's letters to Emmy Noether, cited hereinafter from [Tobies 2003], are part of the rich correspondence found in the Hochschularchiv der ETH Zürich, Hs 160.

directly or indirectly to making this book possible. My inspiration to write about Emmy Noether came about through a most pleasant and fruitful collaboration with Mechthild Koreuber, whose passionate interest in this phenomenal figure quickly rubbed off on me. The same can be said for Sandra Schüddekopf and Anita Zieher of *portraittheater Vienna*, whose creative efforts provided another vital source of inspiration. As noted in our book *Proving it Her Way: Emmy Noether, a Life in Mathematics*, Mechthild and I are grateful for the cooperation we received from a number of institutions in the course of our work on both books. I would like once again to express appreciation for the help we received from archivists at the Austrian Academy of Sciences, Bryn Mawr College, Caltech, Göttingen State and University Library, Hebrew University, and Oberwolfach Research Institute for Mathematics. I am especially grateful to the grandchildren of Fritz Noether – Monica Noether, Margaret Noether Stevens, and Evelyn Noether Stokvis – for sharing records and documents in their family archives. Special thanks also go to Qinna Shen, Professor of German Studies at Bryn Mawr College, for her efforts in supporting this project, as well as to Ayse Gökmenoglu for the care she took in producing the photos included in this book. I also benefited from the helpful advice of Catriona Byrne and Rémi Lodh at Springer Nature, who both supported this venture from the outset.

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David E. Rowe

Chapter 1

Max and Emmy Noether: Mathematics in Erlangen

Until 1933, most of Emmy Noether's life was spent in two middle-sized cities: Erlangen, her birthplace, and Göttingen, where she began her mathematical career. Noether was already thirty-three when she left Erlangen for Göttingen in 1915. Although her brilliant career as an algebraist only began after her habilitation in 1919, one can trace many roots of her later mathematical activity and the work that would later make her famous back to Erlangen. The university's mathematical faculty, one of the smallest in Germany, had only two members. Both also happened to be of Jewish descent: Emmy's doctoral advisor, Paul Gordan, and her father, Max Noether, a leading algebraic geometer. These circumstances were highly unusual, making Erlangen an important locale for gauging the careers of Jewish mathematicians, as will be seen in this chapter.

In Erlangen, but also during her first years in Göttingen, Emmy Noether was primarily known as the daughter of Max Noether. Today he is mainly known as the father of the famous "mother of modern algebra." Aside from this not uninteresting observation, Max and Emmy Noether have seldom been compared, even though there are plenty of indications that she studied her father's works in detail. Moreover, careful examination of her earlier work clearly reveals streams of thought from her Erlangen period that flowed into her later work in Göttingen. Like her father, Emmy was an impressive scholar, a mathematician whose work evinced broad and detailed knowledge of the mathematical literature. In this respect, both were outstanding representatives of Germany's high mathematical culture, to which they made fundamental contributions. Yet Max Noether has rarely received serious attention in the by now quite extensive literature devoted to his daughter Emmy. Not only in this chapter but elsewhere in this book, Max Noether's name comes up often, and this for good reason: he was most definitely a major formative influence on Emmy Noether's life. This chapter thus aims, among

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other things, to shed a small beam of light on the relationship between these two great mathematicians, who in several respects belong together.

1.1 Max Noether's early Career

To gain a sense of Emmy's early development, one must go back to her years in Erlangen (Fig. 1.2), beginning with her home life there as the oldest of four children and the only daughter of Max and Ida Noether. Emmy Noether's mother grew up in a large and very wealthy family from Cologne; she was one of eleven children of Markus Kaufmann and his wife Frederike Kaufmann née Scheuer. Two of Ida Kaufmann's brothers assisted Emmy Noether financially after the death of her parents. These were her two uncles in Berlin: Paul, a wholesale merchant, and Wilhelm, a university professor who specialized in international economics [Dick 1970/1981, 1981: 8].

After her father's death in 1866, Ida moved with her mother to Wiesbaden, a city known for its spas and aristocratic culture. Up until that year, Wiesbaden had been the capital of the Duchy of Nassau, but having sided with the Austrians in the Austro-Prussian War it fell into the hands of the Hohenzollern monarchy. During the era of the Kaiserreich, the emperors began making annual summer trips to Wiesbaden, which led to a construction boom that continued up until the First World War. It was in this glamorous city in 1880 that Max Noether married Ida Kaufmann, who would spend the remainder of her life looking after their household in Erlangen. Although little is known about Emmy Noether's mother, Auguste Dick reported that she enjoyed playing the piano all her life, a talent she tried to pass on to her daughter, but without success [Dick 1970/1981, 1981: 9–10]. How she and her husband first met is also unknown; since marriages were still quite often arranged during this era, the couple may have barely known one another when they wed.[1] Ida Noether's family no doubt offered a substantial dowry at the time, which surely made life in Erlangen for the young family more comfortable. Max Noether's salary as an associate professor was considerably less than that of a full professor (*Ordinarius*), and he would only gain that coveted title eight years later.

Beginning with the nineteenth century, the city of Erlangen belonged to Bavaria. Its citizenry was fairly equally divided between Catholics and Protestants, whereas Jews were only allowed to settle in the city after 1861. Until then, fairly large Jewish communities existed in outlying villages, where life was hard and poverty widespread. A decade later, after the unification of Germany under the domination of Prussia, 65 Jews were living in Erlangen, a city of some 12,500 inhabitants. That number steadily rose to around 240 in 1890, which was roughly 1.5% of the total population. In the meantime, Jewish life in the outlying villages nearly disappeared as the flight to larger cities took place throughout large parts

[1] Marion A. Kaplan describes the era of the Kaiserreich as a transitional period for Jewish families, as they began to allow young couples limited freedom in choosing a partner [Kaplan 1991].

of Germany.[2] Here real economic opportunities awaited them, and the German Jews contributed greatly to the modernization of urban centers in nearly all parts of the German Empire. When economic misfortune struck, on the other hand, as happened in the early 1870s, the blame often fell on Jewish financiers. This was a new form of antisemitism, a hatred tinged by envy rather than the loathing of Christian society.[3]

To what extent Max and Ida Noether's children were exposed to milder forms of prejudice against Jews no one will likely know. They belonged to a special elite, as the offspring of a university professor, and their parents may well have avoided talking about antisemitism in their presence. Although Ida Noether was eight years younger than her husband, she died six years before him on 9 May 1915. As the oldest of the children, Emmy thereafter took on major responsibility for running household affairs. Up until her father's death on 13 December 1921, she often left Göttingen to care for him in Erlangen. One year before his passing, she officially left the city's Jewish community, though her family's religious orientation had probably never been strong.[4] She and her father were not only personally close; Emmy also developed a deep appreciation for Max Noether's place in the mathematics of his time.

Emmy grew up with three younger brothers: Alfred, Fritz, and Gustav Robert. By all reports, she enjoyed a happy childhood, but her mother's life was hardly carefree, as two of her sons had serious health problems. Alfred, the eldest, had a weak constitution and died near the end of the war at age 35. Robert, the youngest, was mentally handicapped and spent his last years in a sanatorium; he died before reaching the age of 40. Fritz, on the other hand, was healthy and robust. He and his sister were very close all their lives, though temperamentally they differed quite strikingly. Fritz was more serious and sober-minded, whereas Emmy had a fun-loving spirit. As a professor's daughter, she looked forward to dancing parties at the houses of Max Noether's colleagues [Dick 1970/1981, 1981: 11]. Her easy-going manner no doubt led people to overlook that she was also ambitious and self-disciplined; already as a teenager she knew that she wanted to study mathematics, perhaps even follow in her father's footsteps [Tollmien 2016a]. When she first had such a dream is impossible to say, but since her brother Fritz had similar thoughts, it seems more than likely that both talked about such plans for the future. Moreover, in one sense they were in a privileged situation. How many teenagers could even imagine the kind of life their father led, constantly steeped in thought about matters neither they nor their mother could comprehend? Yet this was a natural part of their home life, and so they grew up knowing

[2] A very similar pattern can be seen in the case of Göttingen, where the Jewish population nearly tripled between 1867 and 1885; for a detailed study, see [Wilhelm 1979].

[3] The distinction between modern antisemitism and traditional religious forms was made by Hannah Arendt in the first essay in her study *The Origins of Totalitarianism* (1951).

[4] According to an article written by Ilse Sponsel to commemorate the fiftieth anniversary of Emmy Noether's death, she resigned from the Erlangen Jewish community on 29 December 1920 (*Erlanger Tageblatt*, 12 April 1985).

instinctively how mathematicians think and talk, and also how they demand peace and quiet to concentrate on their work.

Very few published sources contain information about Max Noether's early life, and those that happen to report on his youth invariably obtained those facts from his daughter.[5] Max Noether (Fig. 1.1) was born in Mannheim on September 24, 1844 as the third of five siblings. His father and an uncle ran a well-established wholesale iron business that provided their families with financial stability. According to Emmy Noether, her father was very close with his mother, although she knew this only through him; Emmy's grandmother died long before she was born. Max was an ambitious child and went straight into the third grade of the Gymnasium after primary school. At the age of 14, however, he contracted polio, which resulted in paralysis in one of his legs. For the next few years he could barely walk at all. He took private lessons during this time, but as Emmy reported, he also spent many hours reading. It was thus through self-initiative that "he laid the foundation for a very extensive literary and historical education. At home he took it upon himself to work through the usual university curriculum in mathematics" [Brill 1923, 212–213]. Under very different circumstances, Emmy Noether would later do the same when preparing to take the examination required for admission to a university.

Max Noether's first love was astronomy, which he pursued at the local Mannheim observatory. His first publication was a short paper on the paths of comets; it appeared in *Astronomische Nachrichten* in 1867 when he was still a student at Heidelberg University. More than twenty years later, having long since made a name for himself in algebraic geometry, he published a lengthy review of Henri Poincaré's famous prize-winning study of the 3-body problem [Barrow-Green 1997]. In Heidelberg, Noether mainly studied theoretical physics under Gustav Kirchhoff. Emmy Noether commented briefly on how Kirchhoff indirectly kindled her father's early mathematical interests by way of mapping problems in theoretical physics. These led him to Riemann's works and then to the geometric theory of algebraic functions, which he learned by reading Riemann as well as the monograph by Clebsch and Gordan [Brill 1923, 213]. Noether needed only three semesters to complete his doctorate in Heidelberg. At that time, a dissertation was not even required, but he nevertheless submitted his astronomical work as a doctoral thesis, only to have it returned to him. In the end, Noether merely had to endure an easy "oral examination in the dean's apartment, for which the doctoral student was obligated to supply the wine" [Brill 1923, 213]. These details we owe to Emmy Noether's recollections of her father's early life.

In Heidelberg Max Noether also befriended Jakob Lüroth, who habilitated there after studying under Alfred Clebsch in Giessen. On Lüroth's advice, Noether left for Giessen in 1868, a decision that would decisively influence the course of his subsequent career. Five years earlier, Clebsch had published a paper that gave a new impulse to algebraic geometry, connecting it with Riemann's theory of al-

[5] This applies to [Brill 1923] as well as for [Castelnuovo/Enriques/Severi 1925].

gebraic functions, while exploiting the notion of the *genus* of an algebraic curve. This concept was closely connected with Abel's Theorem as well as Riemann's central result, which established that the connectivity of a Riemann surface was given by the number of everywhere bounded integrals the surface supported. Clebsch identified this number as a birational invariant of the corresponding algebraic curve, which opened the way to develop a purely algebraic approach to this theory, thereby evading some problematic aspects in Riemann's geometric approach to function theory. Clebsch pursued that goal together with Paul Gordan, who had briefly studied with Riemann in Göttingen. Their joint work led to the treatise *Theorie der Abelschen Funktionen*, published in 1866. Clebsch had invited Gordan to habilitate in Giessen, where he taught as a *Privatdozent* until his promotion to associate professor in 1865. Three years later, Clebsch assumed Riemann's chair in Göttingen, and in 1869 Gordan married Sophie Deurer, the daughter of a professor of law in Giessen.

Noether was by now strongly drawn to Clebsch, so he left Giessen to continue working under him in Göttingen. In the meantime, Clebsch had found a way to extend the notion of genus for algebraic curves to surfaces. He published this new birational invariant – later dubbed the "geometric genus" of a surface – in the *Comptes Rendus* of the French Academy in 1868. Originally this invariant was only defined for surfaces whose singularities were double and cuspidal curves, but Noether showed that Clebsch's theorem on the birational invariance of the geometric genus could be extended to surfaces with more general singularities. In a letter from Göttingen, written to his future collaborator Alexander Brill on July 7, 1869, Noether soberly noted: "The work I hereby send to you, as you will see, stems from the sphere of Clebsch's findings, though I claim for myself the ideas developed and hinted at therein" [Brill 1923, 214]. Clebsch was much impressed by Noether's new results; he later told Brill, he would have been even happier had he found them himself [Brill 1923, 215]. Around this same time, Felix Klein came to Göttingen from Bonn to study with Clebsch, who was by now the head of a prominent mathematical school [Tobies 2019, 37–48]. One year earlier, Clebsch and Carl Neumann founded the journal *Mathematische Annalen*, which later would become the main publishing organ for mathematicians with close ties to the Göttingen network.

Noether and Klein soon became close friends – adopting the more intimate "du" form when they addressed each other – a friendship they maintained up until Noether's death in 1921. Although their time together in Göttingen was brief, it was also very significant for both of them. Klein left for Berlin in the fall of 1869, and then in the spring of 1870 he went to Paris, where he joined his new-found Norwegian friend Sophus Lie. Their stay, however, ended abruptly, when in mid-July France declared war on Prussia. Klein returned home quickly, joined a crew of emergency volunteers, and returned to France, where he witnessed the battle sites around Metz and Sedan, before falling ill. After spending several weeks recovering from gastric fever at his family's home in Düsseldorf, he habilitated in January 1871 in Göttingen, under the watchful eye of Clebsch. By now, however, Noether was already back in Heidelberg, where he habilitated in the winter semester 1870/71. During all this time, Klein and Noether corresponded regularly, not least because their mutual mathematical interests were very close during these years.

The friendship that developed between Klein and Noether clearly had much to do with the fact that both enjoyed close ties with Clebsch. Three years later, in November 1872, both were deeply shocked when they learned that their revered master, who was not yet 40 years old, had suddenly died from an attack of diphtheria. Only a short time before his death, Clebsch had paved the way for Klein – who was then only 23 years old – to be appointed as the new professor of mathematics in Erlangen. In so doing, Clebsch passed over two far older candidates from his school, namely, Gordan and Noether. Klein remained in Erlangen for only three years, yet his name remains prominently associated with this university owing to his famous Erlangen Program [Klein 1872], which he published in 1872 when he joined its philosophical faculty. He was then its only mathematician, but in 1874, the year before he succeeded Otto Hesse in Munich, Klein managed to gain a second position for Erlangen. He also arranged for Paul Gordan, Emmy Noether's future doctoral supervisor, to fill this associate professorship. This enabled Gordan to assume Klein's chair one year later, thereby opening the door for Max Noether to fill Gordan's post as associate professor. It was a classical case of networking, but with long-term significance, since these arrangements helped to stabilize the precarious state of the Clebsch school and its journal, *Mathematische Annalen*. Klein and Gordan continued to collaborate during the years that followed, often meeting in the small town of Eichstätt, which was conveniently located halfway between Erlangen and Munich. Later, and up through the final phase of Klein's highly successful career in Göttingen, he continued to cultivate close relations with his longtime allies in Erlangen.[6]

1.2 Academic Antisemitism

These events from the early 1870s led to an unusual situation in Erlangen. During an era when very few German Jews could hope to attain a professorship in Germany, both mathematicians on the small faculty at Erlangen University were of Jewish background. This unusual circumstance certainly did not go unnoticed at the time, and the present section attempts to gauge the effects of academic antisemitism on their careers. Unlike Max Noether, who remained a non-practicing Jew all his life, Paul Gordan converted to Christianity at the age of 18.[7] Still, in the eyes of many, a baptized Jew was not to be confused with a "real German."

[6] Over the course of their friendship, Klein and Noether exchanged some 340 letters, from which 280 are still extant in Klein's estate (SUB Göttingen).

[7] A finding due to Cordula Tollmien, who kindly sent me a copy of Gordan's baptismal certificate dated July 1857. Tollmien points out that Gordan's baptism took place before he began his academic studies, though nothing is known about his motives at this time.

Figure 1.1: Max Noether (Auguste Dick Papers, 13-1, Austrian Academy of Sciences, Vienna)

This pervasive attitude surely helps to explain why both Gordan and Noether never had a chance to leave Erlangen. In Gordan's case, he may have been quite content to stay in Erlangen since he was already a full professor, but Noether, as an associate professor, could hardly feel the same way. Yet he was passed over on numerous occasions; in some cases Klein informed him in advance that certain localities were simply opposed to any and all Jewish candidates. Over time, Noether came to realize that his best chance for promotion would likely come if Gordan were to receive an outside offer; that was Klein's frank opinion, too. When a mathematics professorship opened in Tübingen in 1885, Noether hoped this might indeed transpire.[8] A short time before, Max Noether's friend and collaborator, Alexander Brill, was appointed to a newly established second chair there, a situation that lifted Noether's hopes Gordan might well be chosen.

[8]This was the position formerly occupied by Paul Du Bois-Reymond, who one year earlier accepted an offer from the Technische Hochschule in Berlin. Hermann von Stahl from Aachen Institute of Technology was eventually appointed his successor in Tübingen.

Instead, however, Gordan's candidacy received no serious consideration at all, as Brill informed him in a letter from July 1, 1885:

> You should know that it was not the faculty or the Senate that blocked Gordan's candidacy, nor was it the chancellor nor the government: the entire country [meaning the state of Württemberg] is currently of such a mind that a professor of Jewish origin in Tübingen is impossible. This can and will change, but as a newcomer I am unable to make the first breach in this prejudice. [Seidl, et al. 2018, 23]

Brill gave no clear indications as to what was behind this disaffection for Jewish candidates. He merely stated that this specific appointment had caused a great deal of controversy because of differences between Paul Du Bois-Reymond, the previous chair holder, and the faculty, a circumstance that obviously had no bearing on the issue of antisemitism. More than likely, Brill alluded to this merely in order to explain why he, a newcomer, had only limited influence on the faculty's decision. As a matter of fact, before this time only one mathematician of Jewish origin, Sigmund Gundelfinger, had ever been a member of the Tübingen faculty.[9] It should be noted that Brill's prediction, according to which future prospects for Jewish mathematicians would improve in Tübingen, never materialized. Although his friendship with Max Noether apparently remained firm over the years, his general view of German Jewry became increasingly hostile, reflecting opinions held by conventional antisemites. On January 5, 1914, not long before the outbreak of World War I, Brill wrote this entry in his diary:

> The effect of the Jews on Germanic peoples is like alcohol on the individual! In small doses they are stimulating and invigorating, but in large quantities devastating like poison. The organism of our people requires time to assimilate them. Therefore they should be warded off because otherwise the flood from the east threatens to destroy the body of the people, like aphids attacking a plant, which will then perish. Fend them off! They know nonetheless how to smuggle themselves in. [Seidl, et al. 2018, 23-24]

This theme of Germania as the victim of merciless and conspiring Jews, who threatened to invade the young nation from the East, would become a standard trope in the period after the monarchy fell in November 1918. The fact that Brill had already adopted this viewpoint even before the outbreak of the Great War suggests how deep-rooted these types of fears must have been among Germany's educated classes.

Conditions in Erlangen during the Wilhelmian age may have been more liberal, at least in some academic circles, but German Jews who managed to attain

[9] Coincidentally, Gundelfinger had studied under Clebsch and Gordan in Giessen and, like them, he worked mainly on invariant theory and its application to algebraic curves. After taking his doctorate in Giessen in 1867, he habilitated two years later in Tübingen, where he was appointed associate professor in 1873. Six years later he joined the faculty at the Technical University in Darmstadt as a full professor.

professorships were acutely aware that their presence on university faculties was rarely welcomed [Kaplan 1991, 137–150]. In some disciplines, classical philology being a noteworthy example, scholars of Jewish origin had virtually no chance of advancement. Mathematics, on the other hand, was long seen as a field in which high-quality research was recognized objectively and judged accordingly. If that was the ideal, then the reality was very different indeed.

In today's universities, mathematics is strongly allied with the natural sciences, in part due to the current importance of applied mathematics. Historically, however, these relationships were by no means self-evident. During the nineteenth century, the ties between mathematics and the human sciences were, in some ways, the stronger ones. First, it should be remembered that the research interests of most mathematicians at the German universities were devoted to some branch of pure mathematics. It was not until the advent of the twentieth century that applied fields began to receive strong attention. Second, throughout most of the nineteenth century, humanists and natural scientists were colleagues in a single philosophical faculty. Mathematicians could therefore interact just as easily with philologists and philosophers as with their colleagues in astronomy and physics. Third, and perhaps most important, it was mainly the humanists who set the tone at faculty meetings and in broader forums outside the university proper. The most prominent among them spoke as *Kulturträger*, an elite class of intellectuals often called "Mandarins" (*Bonzen*). This group reached its zenith during the last decades of the Wilhelminian era. Its demise began with the fall of the German Reich, accelerating as Germany descended into Nazism; this familiar story is described and documented in detail in [Ringer 1969]. Looking backward to the early decades of the nineteenth century, mathematicians often had a stronger affinity for idealistic philosophy than for the materialism many identified with the natural sciences. The latter fields had, in any case, a lower status than the human sciences, and since mathematicians saw themselves as purveyors of pure knowledge they naturally followed the lead of their colleagues in classical philology, who were the first to establish research-oriented seminars.

One seminar that was particularly influential for physics and mathematics was founded in 1834 in Königsberg [Olesko 1991]. Initially, this seminar was under the direction of the physicist Franz Neumann and the Jewish mathematician Carl Gustav Jacob ("Jacques") Jacobi, who later went to Berlin in 1844. Jacobi, the son of a Potsdam banker, became a model figure for numerous Jewish mathematicians who pursued careers in Germany after him, one of whom was Leo Koenigsberger, Jacobi's biographer [Koenigsberger 1904]. Koenigsberger delivered a speech in honor of his hero at the Third International Congress of Mathematicians, which was held in Heidelberg in 1904, one hundred years after Jacobi's birth. He ended this oration by proclaiming: "We are all students of Jacobi." This, of course, was an exaggeration, and a well-known German mathematician[10] wrote to Felix Klein,

[10] He was Klein's former student Walther von Dyck; see [Hashagen 2003]