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Mathematical Modeling, Simulation and Optimization for Power Engineering and Management

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Editors

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Preface

The transformation of energy systems is the central part of Germany's High-Tech Strategy 2025. In the near future, a successful transformation guarantees a secure, economical, and environmentally compatible energy supply. A key aspect in the transition of energy systems is to enable green technologies and expand the range of renewable energies at the expense of fossil fuels. New storage concepts and intelligent energy networks are central hardware components also required in a transformed energy system. Alongside the expansion of renewable energy sources, a significant increase in energy efficiency as well as stability and resilience are further challenges.

Research and innovation are drivers of progress and in particular applied mathematics has been used in the past as methods-based, cross-sectional science that may enable novel results for sustainable and climate-friendly solutions also in the realm of energy systems. Aim of this book is, therefore, to show the relevance of mathematics in innovation by means of introducing a successful cooperation with industrial partners of the energy sector in order to show the wide range of applications. In many of the discussed problems, the use of modern mathematics in modeling, simulation, and optimization is shown to be a crucial factor for success. Besides novel approaches, the present book also shows the variety of mathematical techniques and disciplines involved in these activities. The selected scientific presentations highlight cooperation projects between mathematics and industry as a two-way transfer of technology and knowledge, providing the industry with applicable solutions and providing mathematics with novel research topics and inspiring new methodologies.

The starting point for this Springer publication was the KoMSO e.V. workshop on "Mathematical Modeling of Energy Systems" in March 2019 at the University of Mannheim. Partners from industry, universities, and research institutes discussed expected challenges and presented their new concepts for reliable energy predictions. We also acknowledged support by the Federal Ministry of Education and Research (BMBF) within the program "Mathematics for Innovations" (https://math4innovation.de).

The diversity of collaborations between mathematics and industry for different energy-related and economic applications is portrayed in the broad scope of topics discussed in this book, ranging from multi-energy systems to energy market models. These facets are presented in three chapters which are called **Economic Aspects, Technical Applications**, and **Energy Networks**. All contributions address the advancement of novel mathematical models, modern efficient numerical methods for simulation and optimization arising in energy systems and management, as well as the demands for future development of science and technology in the field of energy research.

Mannheim, Germany — Simone Göttlich
Aachen, Germany — Michael Herty

About KoMSO e.V.

KoMSO is the Committee for Mathematical Modeling, Simulation, and Optimization. It acts as a Germany-wide network of scientists from the field of mathematical modeling, simulation, and optimization (MSO) and potential users of this technology (https://www.komso.org).

KoMSO aims to promote scientific research and cooperation in the field of MSO between science, economy, society, and politics. In addition, the relevance of MSO for society and the economy is to be demonstrated through targeted public relations work.

KoMSO was set up in May 2011 as a result of the Mathematics 2020 Strategy Day initiated by the Federal Ministry of Education and Research (BMBF) as part of the German government's High-Tech Strategy 2020. Since March 2018 KoMSO e.V. is a non-profit registered association.

A central task of KoMSO is to initiate innovative scientific projects in the field of MSO—by bringing together representatives from science and industry at events such as the KoMSO Challenge Workshops. Over the years, numerous successful collaborations have been established, which are described in several KoMSO Success Stories.

KoMSO has access to a broad national and international network of partners from universities, research institutions, and industry, and is expanding this network through ongoing activities. The fields of expertise include aerospace, automotive industry, health care, pharmaceutical industry, industrial robotics, and many more.

On an international level, KoMSO acts as the German network node in the European Service Network of Mathematics for Industry and Innovation (EU-MATHS-IN), which consists of various national or multinational industrial mathematics networks. The declared goal of EU-MATHS-IN is to "leverage the impact of mathematics on innovations in key technologies [through] enhanced communication and information exchanges".

Heidelberg, Germany — Anja Milde

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Part I
Economic Aspects

Chapter 1
Modeling the Intraday Electricity Demand in Germany

Sema Coskun and Ralf Korn

Abstract Future electricity markets face new challenges such as increasing variation in supply due to the dominance of renewable energy providers or variation in demand due to the presence of price sensitive customers. In this contribution, we survey the first step to modeling the current demand process for electricity in Germany. Besides standard affine-linear diffusion processes, we aim to model the intraday electricity demand via a Jacobi process that has attractive properties for our applications. Further, we demonstrate the usefulness of the new models by conducting a comprehensive data analysis.

Keywords Electricity demand · Intraday market · Jacobi process · Stochastic differential equations

1.1 The ENets-Project—Modeling the Microstochastics of Intraday Electricity Demand and Intraday Electricity Prices

The BMBF-funded project ENets[1] has the aim to model the energy markets of the future and the corresponding supply networks. The main aspect of our part in this project is the task to model the micro-stochastics of the intraday demand and the electricity prices at the intraday market. Due to many diverse reasons such as e.g. the increased uncertainty about the production when renewable energy providers dominate the market, the occurrence of price sensitive demand caused by smart

[1] See https://math4innovation.de/index.php?id=15.

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grids, or the mainly non-storable character of electricity, there is still uncertainty about production and demand. This requires new stochastic models for electricity prices and the demand forecast.

This study lays the foundations of the main aims and introduces stochastic models for the current situation. Beneath standard Ornstein-Uhlenbeck (OU) type models we also consider the suitability of Cox-Ingersoll-Ross (CIR) type approaches (see Sect. 1.5) for the detailed introduction of these two process classes) and—as a new feature—Jacobi processes as stochastic models for the electricity demand.

1.2 Introduction—Demand and Electricity Prices

In economic markets commodity prices are often determined by the equilibrium of supply and demand. To a certain degree, this also holds for electricity markets. The following properties of electricity markets, particularly the German one, hint at the challenges of modeling electricity prices:

(1) The largest share of electricity is traded in auction-type markets. As a result of an auction, the price per unit of electricity (say, 1 GW) is then set to the price of the highest bid that is still needed to satisfy the total electricity demand.
(2) As the renewable energy law in Germany requires that all renewable energy produced has to enter the market first, the price is determined by the residual demand, i.e. the prices offered by non-renewable energy providers.
(3) Due to the uncertainty that comes with the production of wind or solar energy, we have an intrinsically stochastic component on the supply side.

As a consequence of the German energy transition to renewable resources (*Energiewende*[2]) the share of renewable energy production has already increased from 31.6 percent in 2016 to 36.2 percent in 2017 [25]. However, the renewable energy production results in a more volatile market environment due to forecast errors of the timing and amount of production. Together with the non-storable nature of electricity, this stochastic component on the supply side has already led to the introduction of the German intraday market.

Intraday markets allow the participants to react to the latest events such as weather changes or surprising demands. Typically, owners of renewable energy resources tend to trade in the intraday market [2]. In particular, in the German intraday market the trading continues up to 30 min before the delivery. So the market participants have the opportunity of reacting to the forecasted offer of renewable electricity production even closer to real-time, a very attractive feature. As a consequence, the traded volume

[2]The term Energiewende refers to the reforms caused by the German Renewable Energy Sources Act which are designed to gradually transform the energy production methods from the conventional fossil fuel methods to sustainable and renewable energy resources as e.g. wind and solar power.

in the German/Austrian[3] spot electricity market has increased from 41 TWh in 2016 to 47 TWh in 2017 [22].

As this development will continue in the future, it is necessary to introduce a model for both the spot price and the intraday electricity demand which closely captures the real dynamics of the German intraday market. To gain the necessary insights, we perform a time series analysis of the actual consumption data. By considering publicly available data we want to extract the stylized features of the electricity demand. A possible model framework for the intraday electricity demand V_t is

$$V_t = \Lambda_t + X_t \tag{1.1}$$

where Λ_t is a seasonality component and X_t a mean-reverting stochastic process that models the fluctuations around the seasonality part.

In the next sections, we introduce these components rigorously and judge their suitability on the basis of a detailed data analysis.

1.3 Basics on the Electricity Markets and Models

In this section we give a brief description of the German spot electricity markets and on our suggested stochastic process models for the electricity demand.

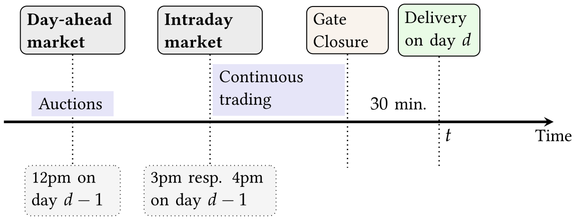
1.3.1 German Spot Electricity Markets

Let us briefly introduce to the spot electricity markets and their mechanism. In Germany, there are two main spot markets for trading electricity, namely the day-ahead and the intraday markets. Electricity trading in the German day-ahead markets is completely held by auctions. On the other hand, trading in the intraday market is mainly continuous. Recently, to enhance the flexibility in the German electricity market which is highly driven by renewables, the intraday auctions for 15-min periods are introduced. The particular reason behind this improvement is to provide a better balancing opportunity for the market participants against the solar ramps [21], i.e. the short-term influence of clouds on the solar energy production. The timing mechanism of electricity spot markets is summarized in Fig. 1.1 which is a modification of the figure given in [17].

As our main concern is the intraday electricity demand, we give further details about the German intraday market (see also [23]). In principle, electricity is traded

[3]The German and the Austrian electricity markets are considered as one bidding zone by the EPEX SPOT. Thus, the total traded volume is given as the sum of traded volumes in both countries. However, in autumn 2018, EPEX SPOT implemented the so-called split of the German-Austrian bidding zone, following a request of the regulators of these two countries [24].

Fig. 1.1 Timing of spot electricity markets



depending on the delivery time which in spot markets usually is the next day (e.g. day d in Fig. 1.1). The delivery time (e.g. time t in Fig. 1.1) can be a full hour, a 15-min period or a block of hours. In the German intraday market, the market participants can trade all of these delivery time alternatives. Moreover, in the German intraday market each hour, 15-min periods or block of hours can be continuously traded until 30 min before the delivery begins. This leads to the possibility of face-to-face-trades comparable to the situation on stock exchanges. This possibility will be used in the final part of the ENets project when we include price sensitive traders and demand sensitive pricing. Trading of full hours of the following day (e.g. day d in Fig. 1.1) starts at 3pm on the current day (e.g. day $d-1$ in Fig. 1.1). Furthermore, starting at 4pm on the current day all 15-min periods of the following day can be traded in the German intraday market. Moreover, as opposed to the stock markets the trading continues for 7 days a week and 24 hours a day, i.e. electricity is traded also on the weekends.

1.3.2 Structural Models for Electricity Prices

In nearly all markets, prices and the relation between supply and demand are closely connected. Electricity price models which are based on the equilibrium between supply and demand are known as *structural* or *supply/demand* models. Barlow [4] models the electricity demand by an Ornstein-Uhlenbeck (OU) process. In combination with a deterministic supply function it is considered as the only driver of the electricity prices. However, considering the demand as the only driver of electricity prices might be insufficient. To overcome this, many researchers considered additional underlying factors. For instance, in [6] electricity demand and capacity have been presented as the drivers of the spot electricity prices. Also, in [1] demand is combined with the prices of different fuels to introduce a structural spot price model. Coulon and Howison [8] adopt a parametric approach to the bid stack function, i.e. the marginal cost of electricity supply, by allowing multiple fuel prices as underlying driving factors. They further consider the capacity or margin issues such as outages in the electricity supply. Especially, in the German market with its high part of renewable energy providers, a sophisticated modeling and prediction of the electricity demand is indispensable. For that reason, Wagner [27] proposes a comprehensive model for residual demand which is obtained by subtracting the renewable infeed generated by solar and wind power plants from the total demand.

1.3.3 The Jacobi Process as a New Modeling Ingredient

While popular processes for demand modeling such as the OU process are analytically very tractable, they do not consider practical bounds on demand such as its non-negativity or the installed capacity of electricity production. A stochastic process that is tailored to the situation of a bounded state space is the Jacobi process.

The Jacobi diffusion belongs to the class of Pearson diffusions. In its general form, a Pearson diffusion is a stationary solution to the following stochastic differential equation (SDE)

$$dX_t = -\kappa(X_t - \theta)dt + \sqrt{2\kappa(aX_t^2 + bX_t + c)}dW_t \tag{1.2}$$

where $\kappa > 0$ and the coefficients a, b, c ensure that the square root is well defined when X_t is in its state space [12]. The parameters also determine the state space of the diffusion as well as the shape of the invariant distribution. The Jacobi diffusion is defined as the stationary solution to the SDE

$$dJ_t = \kappa(\theta - J_t)dt + \sigma\sqrt{J_t(1 - J_t)}dW_t \tag{1.3}$$

where the parameters κ, θ and σ ensure existence of the stationary distribution which indeed is the Beta distribution. Conditions for existence and uniqueness of the solution to Eq. (1.3) are given in [12], see section "Appendix: The Jacobi Process". Obviously, J_t is only defined for values in (0, 1). If we want to use the process for modeling electricity demand D_t that is known to be in the interval (D_m, D_M), then the transformation

$$D_t = D_m + (D_M - D_m)J_t \tag{1.4}$$

that leads to the SDE

$$dD_t = \kappa(D_\mu - D_t)dt + \sigma\sqrt{(D_t - D_m)(D_M - D_t)}dW_t \tag{1.5}$$

is the appropriate rescaling. Note that now D_μ has the role of the mean demand level, D_m of the minimum and D_M of the maximal demand level, while κ is the mean reversion speed of demand. This transformation is motivated by an application in interest rate modeling by Delbaen and Shirakawa [11]. For modeling (the logarithm of) exchange rates that should be kept in a target zone, a similarly transformed Jacobi diffusion is used by [16, 18]. In a slightly different framework, a recent study by [3] utilizes a Jacobi-type process in order to introduce a probabilistic day-ahead forecasting model for the solar irradiation. More technical details on the properties of the Jacobi process are provided in the appendix.

Table 1.1 Descriptive statistics of the ex-post consumption data in MWh per 15 min on a yearly basis and for 2015–2018. ♯ and NaN denote the number of observations and the number of missing values, respectively

Year	♯(NaN)	Mean	Std	Med	Min	Max
2015	35032 (8)	13,640.34	2,502.70	13,543.37	7,458.50	19,159.00
2016	35128 (8)	13,693.03	2,453.53	13,582.37	7,824.50	18,952.75
2017	35032 (8)	14,078.24	2,572.08	14,022.00	7,363.25	19,870.25
2018	35023 (17)	14,516.28	2,479.28	14,419.25	8,025.75	19,698.25
	140215 (41)	13,981.74	2,526.82	13,906.00	7,363.25	19,870.25

1.4 Data Analysis—Stylized Facts of German Electricity Demand

In this section, we present the results of our detailed analysis of the electricity consumption per quarter hour (i.e. we have a 15 min resolution) for the time span from 01.01.2015 to 31.12.2018. The data set for Germany is provided by Smard.[4] As the data correspond to the actually realized amounts of intraday electricity consumption, they are called ex-post data.

Missing values in Table 1.1 (and duplicate values) in years 2015, 2016 and 2017 are only due to daylight saving practice in Germany, i.e. the change from winter to summer time. As the proportion of missing values in the whole data set is fairly low ($<0.03\%$), we proceed with our analysis by sampling out the missing values.

There is a slight upward movement in the quarter hourly electricity consumption as the mean value has increased almost 900 MWh in four years, see also Fig. 1.2. Next we are going to present the main characteristics of the electricity demand in Germany, its so-called *stylized facts*.

Weekend Effect For German day-ahead electricity prices, a weekend effect (i.e. prices are lower on the weekend) is shown in [15]. Given the mechanism for the German electricity prices, the natural reason for this is a smaller demand on the weekend. We have discovered such a weekend effect for the demand (i.e. lower demand on weekends) and have illustrated it in Fig. 1.3.

The reasons for this are mainly stopped industry and business processes. Outlier values on weekends (i.e. surprisingly high 15-min demands) may be due to some nationwide specific events such as football matches, etc. The assertions on the differing demands for weekends and weekdays are also confirmed in Table 1.2.

Seasonality It is well documented in the literature that electricity price series exhibit strong seasonality which varies depending on the electricity consumption with different time scales. For a general overview of the seasonality of the electricity prices and different functions used to model this behavior we refer to [15, 29]. For the electricity demand, we first focus on the yearly seasonality. Geman and Roncoroni

[4]This platform is operated by the Federal Network Agency (Bundesnetzagentur) and the data is obtained directly from the European Transmission System Operators Association (ENTSO-E).

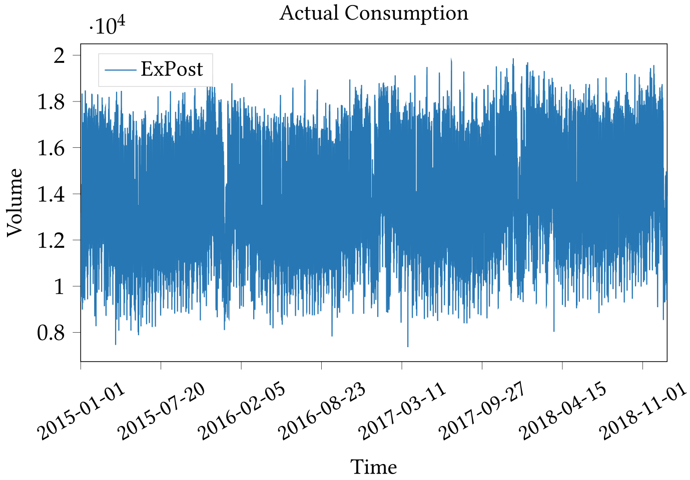
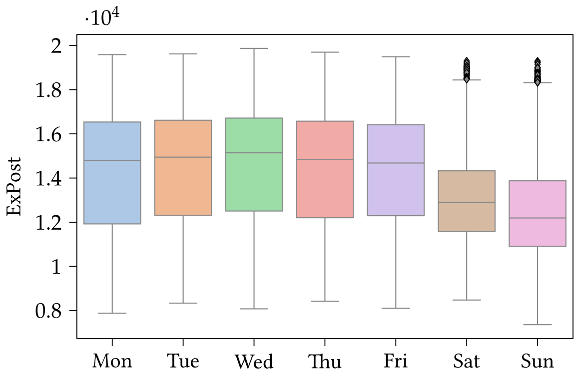


Fig. 1.2 Electricity consumption data in Germany for 15-min slots (2015–2018)

Fig. 1.3 Summary of the data set with respect to days of a week



[13] argue that electricity prices typically follow a periodic path having two maxima per year of possibly different magnitude, i.e. a 12-month and a 6-month periodicity which roughly account for winter and summer peaks in demand. Furthermore, they also integrate a linear trend function into the seasonality component.

Figure 1.4 illustrates the monthly fluctuations in the electricity demand separately for the years in the data set. Yearly seasonality in electricity consumption occurs

Table 1.2 Comparison of the 15-min electricity demand in Germany on weekdays and weekends

	Mean	Std	Min	Med	Max	Skewness	Kurtosis
Weekdays	14,446.53	2,501.21	7,878.75	14,876.75	19,870.25	−0.2598	1.9339
Weekends	12,821.09	2,195.46	7,363.25	12,557.37	19,299.25	0.4437	2.6516

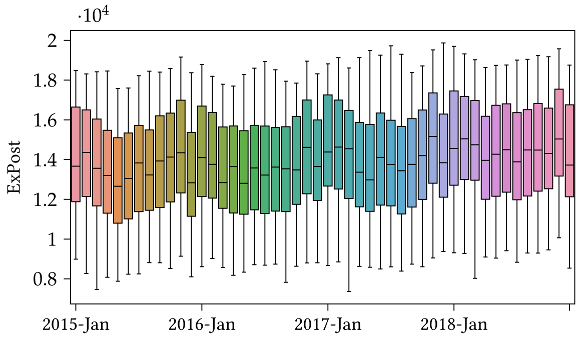


Fig. 1.4 Monthly deviation of the German 15-min electricity consumption

Table 1.3 Yearly seasonality coefficients determined via the ordinary least square (OLS) regression fit leading to an adjusted $R^2 = 0.072$ for 2015–2018

	a	b	c_1	c_2
Coefficients	13,950	0.0116	798.0500	153.7067
Std error	15.325	0.000	10.816	10.816

mainly due to climate and extreme weather conditions. Keeping this in mind, we see in Fig. 1.4 that although there exists a certain sinusoidal behavior in the data set, the yearly seasonality has a moderate level. We employ the following function—a modified version of the function given in [13] and similar to the one given in [20]—to capture the yearly seasonality in our data set

$$g(t) = a + bt + c_1 \cos(2\pi t) + c_2 \cos(4\pi t)\,. \tag{1.6}$$

Here, the linear trend is also contained in the function.

The results given in Table 1.3 confirm the observation of [20] regarding the poor contribution of the yearly seasonality to the overall variability of the electricity prices. Although we consider the electricity consumption data, this observation is still valid and in particular confirmed by the very small value of the adjusted R^2 that indicates nearly no (linear) predictability of demand by yearly seasonality.

Figure 1.5 contains the fitted yearly seasonality function with coefficients from Table 1.3. Besides this yearly periodic behavior, electricity demand also exhibits a weekly seasonal pattern, see for example Fig. 1.3. As we dropped the weekends in our analysis, we eliminated the weekly seasonality.

The most interesting seasonality behavior for our analysis is the intraday cyclic behavior of electricity consumption. It is usually associated with the working hours during a day. This pattern is also realized in the German intraday market by distinguishing the peak hours which cover the time range from 9 to 20 o'clock and the remaining off-peak hours. Meanwhile, the base load covers the hours from 1 to 24 o'clock in the German intraday market. To capture the hourly effect, we use dummy

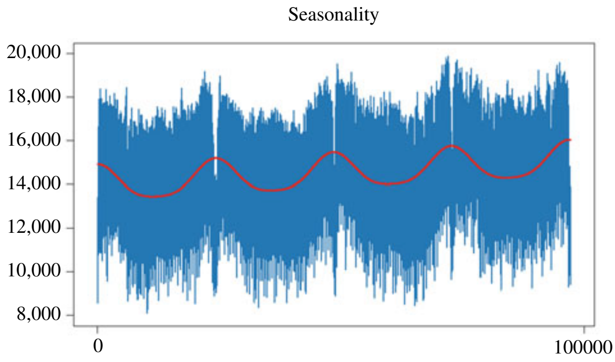


Fig. 1.5 Yearly seasonality (red line) of the German 15-min electricity consumption in 2015–2018 with coefficients from Table 1.3

Table 1.4 Intraday seasonality coefficients determined via OLS regression fit leading to an adjusted $R^2 = 0.697$ for 2015–2018

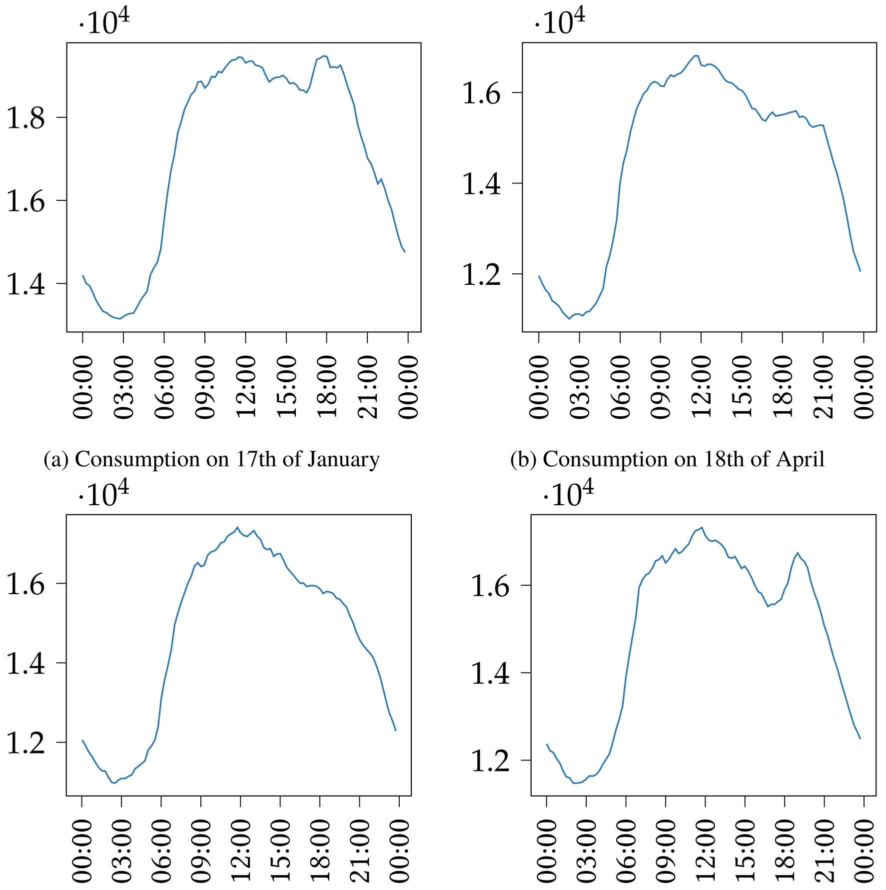
	Coefficients		Coefficients		Coefficients		Coefficients
D_0	−2722.19	D_6	−633.02	D_{12}	2195.77	D_{18}	1430.58
D_1	−3183.48	D_7	743.53	D_{13}	1961.50	D_{19}	1282.63
D_2	−3356.97	D_8	1448.52	D_{14}	1639.27	D_{20}	617.16
D_3	−3413.75	D_9	1707.18	D_{15}	1407.19	D_{21}	−87.52
D_4	−3069.97	D_{10}	2034.13	D_{16}	1217.14	D_{22}	−775.24
D_5	−2335.91	D_{11}	2321.63	D_{17}	1372.58	D_{23}	−1800.40

variables that assign an indicator function for each of the different time points as follows:

$$h(t) = \sum_{i=0}^{23} \alpha_i D_i(t) \tag{1.7}$$

Here, $D_i(t)$ is the indicator function for each hour and α_i, $i = 0, \ldots, 23$ are the coefficients which have to be estimated. Using OLS regression, we fit the dummy variables to the data, where we have eliminated the yearly seasonality with the sinusoidal function. The results are given in Table 1.4. From Table 1.4, it can be seen that starting from 7 am the electricity consumption gets higher which refers to the peak hours classification. Moreover, after 6 pm there seems to be a slight increase in the electricity consumption.

It can also be seen from Fig. 1.6 that there are usually two peak times in the intraday electricity demand, one is reached around noon and the second one occurs mostly after 6 pm. This might be related to the demand of the households after the working hours on a day. As a result, we conclude that our seasonality function consists of two parts, $g(t)$, $h(t)$ given by



(a) Consumption on 17th of January

(b) Consumption on 18th of April

(c) Consumption on 18th of July

(d) Consumption on 17th of October

Fig. 1.6 Intraday deviation of the electricity consumption on several days in 2018

$$\Lambda_t = g(t) + h(t) = a + bt + c_1 \cos(2\pi t) + c_2 \cos(4\pi t) + \sum_{i=0}^{23} \alpha_i D_i(t) \, . \tag{1.8}$$

To analyze the remaining part of the series after removing the trend and seasonality of time series, the histogram of the residuals is presented in Fig. 1.7. Obviously, the residuals are not normally distributed. When we empirically fitted several distributions to the residuals, the best empirical fit is achieved by a beta distribution.

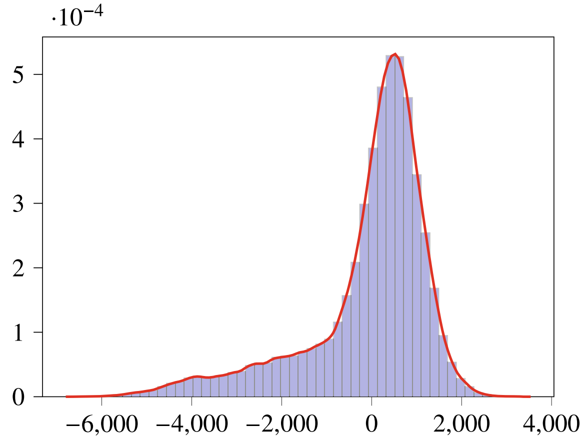


Fig. 1.7 Histogram and fitted empirical kernel density estimation for residuals

The method of moments [5] resulted in a beta distribution on $[-6515, 3592]$ with parameters $(p, q) = (6.955, 3.592)$.

Mean Reversion The mean reverting behavior of a stochastic process implies that it tends to revert to a certain constant or time varying mean level. In [30] a Hurst analysis shows that the return of electricity prices exhibits a mean-reverting property. To check the validity of the mean reversion assumption, we tested the stationarity of the time series in our data set [5]. For this, we used the Augmented Dickey-Fuller (ADF) test for unit roots.[6] The result of the ADF test indicates that the deseasonalized consumption data set exhibits stationary behavior. We can reject the null hypothesis at the 5% significance levels (Test statistic $= -22.5427$, $p = 0.0$, critical value for 5% is -2.8615). We also conducted the ADF test for the non-deseasonalized time series and can again confidently reject the null hypothesis. Hence, our time series can be assumed to be stationary which addresses the mean reverting behavior.

Spikes While electricity price spikes (i.e. sudden upward or downward jumps of a large magnitude) have received a high attention in electricity price modeling (see e.g. [20]), they play no huge role in the electricity demand. To show this, we checked the existence of consumption values which are higher/lower than three times the inter quartile range. Although, there exist values larger than the 1.5 inter quartiles, they still remain in the range of three times the inter quartiles for the complete data set. As these values occur specifically on public holidays, we removed the holidays from our data set.

Distribution and Tail Behavior The descriptive statistics given in Table 1.5 regarding the skewness and kurtosis of the electricity consumption data set imply that the distribution is not Gaussian. Although the skewness of our data set is fairly close to

[5] I.e. we choose the parameters p and q of the beta distribution with support $[a, b] = [-6515, 3592]$ such that $E(X) = (b-a)p/(p+q) + a$, $V(X) = (b-a)^2 pq/((p+q+1)(p+q)^2)$ is satisfied where the mean and the variance are estimated by their empirical counter parts.

[6] The null hypothesis of the Augmented Dickey-Fuller is that there is a unit root (i.e. the time series is non-stationary), with the alternative that there is no unit root (i.e. the time series is stationary).

Table 1.5 Skewness and Kurtosis of the German 15-min electricity consumption

Year	2015	2016	2017	2018	All
Skewness	−0.0298	−0.0271	−0.0273	−0.0596	−0.0314
Kurtosis	1.8453	1.8399	1.9172	1.8454	1.9014

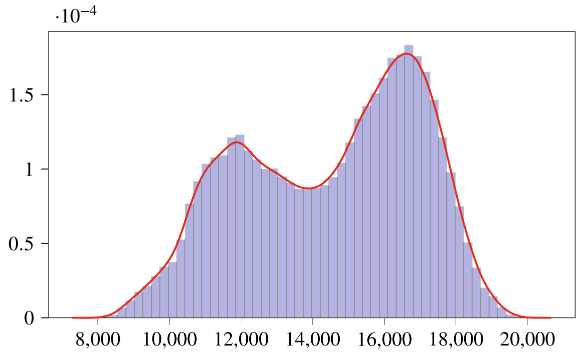


Fig. 1.8 Histogram and fitted empirical kernel density estimation for the 15-min electricity consumption

0, the kurtosis statistics is clearly negative. This implies that our data set might have a light tailed distribution.

We further checked the histogram and the corresponding kernel density estimation of the German 15-min electricity consumption. Figure 1.8 shows that its distribution exhibits two peaks.

Autocorrelation In the analysis of electricity price time series, the exponential decay coefficient of the autocorrelation function provides an estimation of the mean reversion speed of the stochastic process. Meyer-Brandis and Tankov [20] argue that the following sum of two exponentials describes quite precisely the observed structure of electricity price series:

$$\rho(t) = w_1 e^{-t/\lambda_1} + w_2 e^{-t/\lambda_2}.$$

In the light of this approach, we fit the following exponential function to our electricity consumption data

$$h(t) = w_1 e^{-t/\kappa} \tag{1.9}$$

with κ being the mean reversion speed of the mean reverting process. The result of the exponential fit implies that the mean reversion speed κ is approximately 82 and the multiplier w_1 is equal to 0.95. We illustrate the exponential fit by Fig. 1.9.

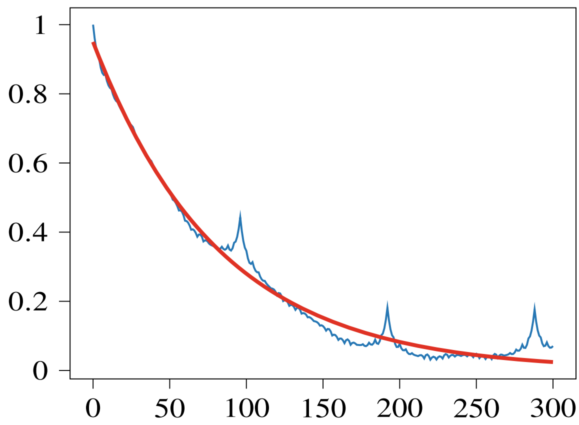


Fig. 1.9 Autocorrelation function of the consumption data and exponential fit

1.5 Case Study: Modeling the Intraday Electricity Demand

In the preceding section, we have seen that the residuals of the electricity consumption became stationary after the introduction of suitable dummy variables. Also, the so modified data exhibited a mean-reverting behaviour.

Consequently, we now consider several mean-reverting polynomial processes as suitable stochastic models for the intraday electricity demand modified by the dummy variables. Subsequently, we calibrate these models to the intraday electricity consumption data as being the indicator of the demand.

Ornstein-Uhlenbeck Process. We start with the simple, but still widely used Ornstein-Uhlenbeck (OU) process. It is a mean reverting stochastic process driven by the SDE

$$dV_t = \kappa(\theta - V_t)dt + \sigma dW_t \tag{1.10}$$

where κ, θ and σ are constant parameters that have to be estimated and further, W_t is a one-dimensional Brownian motion. Here—and also in the processes below—the parameters κ, θ and σ refer to the mean reversion speed, the long term mean reversion level and the volatility of the process, respectively. The explicit form of V_t is given as

$$V_t = \theta\left(1 - e^{-\kappa(t-s)}\right) + V_s e^{-\kappa(t-s)} + \sigma e^{-\kappa t} \int_s^t e^{\kappa u} dW_u \tag{1.11}$$

between any two time instants s and t. Its discrete time correspondence is the autoregressive AR(1) model which reads as

$$V(t_i) = c + bV(t_{i-1}) + \delta\epsilon(t_i) \tag{1.12}$$

with $\Delta t = t_i - t_{i-1}$ an equidistant, constant time step and $\epsilon(t)$ a Gaussian white noise (i.e. $\epsilon \sim \mathcal{N}(0, 1)$). By matching the Eqs. (1.11) and (1.12) and further using the Itô isometry for the volatility parameter, one obtains the following equations

$$c = \theta\left(1 - e^{-\kappa\Delta t}\right), \quad b = e^{-\kappa\Delta t}, \quad \delta = \sigma\sqrt{\left(1 - e^{-2\kappa\Delta t}\right)/2\kappa}.$$

For a time series $V(t_i)$, we calibrate the parameters given above using the ordinary least squares method (OLS). From the parameters $c,\ b$ and δ, we obtain the parameters for the OU process $\kappa,\ \theta$ and σ as:

$$\kappa = \frac{-\ln(b)}{\Delta t}, \quad \theta = \frac{c}{(1-b)}, \quad \sigma = \frac{\delta}{\sqrt{\frac{(b^2-1)\Delta t}{2\ln(b)}}}. \tag{1.13}$$

For our data set, the calibration results of the AR(1) estimation yield

$$\kappa = 83.37, \quad \theta = 14515.29, \quad \sigma = 3266.76$$

Furthermore, we estimated the parameters by using the maximum likelihood estimation (MLE) which is given in [5] by the following estimators

$$\hat{b} = \frac{N\sum_{i=1}^N V_i V_{i-1} - \sum_{i=1}^N V_i \sum_{i=1}^N V_{i-1}}{N\sum_{i=1}^N V_{i-1}^2 \left(\sum_{i=1}^N V_{i-1}\right)^2}$$
$$\hat{\theta} = \frac{\sum_{i=1}^N \left[V_i - \hat{b}V_{i-1}\right]}{N(1-\hat{b})}$$
$$\hat{\delta} = \frac{1}{N}\sum_{i=1}^N \left[V_i - \hat{b}V_{i-1} - \hat{\theta}(1-\hat{b})\right]^2.$$

Here, N is number of observations in the data set, V_i denotes the observation $V(t_i)$ at time t_i. We obtain the required parameters κ and σ using the equations in (1.13). The result of the MLE application for the OU model calibration is very close to that from the AR(1) calibration and reads as

$$\kappa = 83.35, \quad \theta = 14515.14, \quad \sigma = 3266.76$$

CIR Process. The CIR process, which is initially used to model the short rate in [9], has its dynamics driven by the following SDE

$$dV_t = \kappa(\theta - V_t)dt + \sigma\sqrt{V_t}dW_t \tag{1.14}$$

with the constant parameters $\kappa,\ \theta$ and σ and W_t a one-dimensional Brownian motion. Under the Feller condition, i.e. $2\kappa\theta > \sigma^2$, the CIR process remains strictly positive,