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Preface

Predicting wave-induced ship motions and ensuing loads is often part of the design of ships, their structure, and possibly their operation. When, instead of predictions, rules are used, these are based on numerical predictions, which in turn have been verified by model experiments.

Knowledge of a ship's seakeeping is required to assess the safety of the people on board, the feasibility and efficiency of the ship for its intended purpose, and the integrity of its cargo. Today, numerical methods play a crucial role in predicting the seakeeping of ships.

Ship motions cannot be determined without accurate knowledge of the water flow around the hull. To determine this flow, most methods are based on potential theory or solving the Reynolds-averaged Navier-Stokes equations for incompressible flow. In each of these two groups of methods, the procedures differ in many details of discretization, simplifying assumptions, and implementation.

The book presents a range of numerical methods of increasing complexity. In Chap. 5, a linear (with respect to wave amplitude) strip method is presented: an efficient and widely used technique accurate enough for many practical problems. In Chaps. 6 and 7, this is followed by linear three-dimensional boundary element methods dealing with ships in regular waves. The first group of methods is based on Green functions without forward speed and includes also a nonlinear correction; the second uses Rankine sources. The latter group considers also the interaction of the stationary flow due to the ship's speed with the periodical flow in regular waves.

To further improve the accuracy in steep regular or irregular waves, effects depending nonlinearly on wave amplitude must be taken into account. The relevant published literature includes several approaches based on potential flow, which include certain, but not all nonlinear effects. For example, some include only the nonlinear Froude-Krylov force, i.e., the force produced by the wave pressure, unchanged by the body, acting up to the actual waterline. Fully nonlinear boundary element methods that account for the influence of forward speed and all substantial nonlinear effects of the waves are hardly found. In Chap. 8, the authors present such a method.

Then an overview of common methods is provided that solve the mass conservation equation and the Navier-Stokes equations for the fluid surrounding the ship (Chap. 9). The chapter includes a guidance as to the choice of approximations appropriate for predicting ship motions and loads. In addition, the associated numerical errors are elaborated, and common procedures are presented to estimate the discretization errors. Also, different methods for generating waves and for avoiding wave reflections at the outer boundary of the computational grid are discussed and illustrated.

Several additional chapters deal with empirical formulas for the effect of fins, bilge keels, sails, etc. (Chaps. 11 and 12), and, besides others, with determining hull pressure and loads in transverse cross sections (Chap. 13), and with second-order drift forces (Chap. 14). Of the latter, especially the added resistance in waves is of practical interest.

An additional Chap. 15 compares ship motions in regular waves and linear and nonlinear loads in transverse sections, determined in model experiments and computed by various numerical methods. Another Chap. 10 deals with vibrations of the elastic ship structure generated by waves, excited either continually (springing) or induced by slams of the water surface against the hull (whipping).

The book also contains a description of the basic equations for incompressible flows of Newtonian fluids and of the motion of rigid bodies (Chap. 2). In addition, there are chapters on methods for computing incompressible potential flows (Chap. 3), on wind waves and the seaway (Chap. 4), and on statistical methods for dealing with Gaussian and non-Gaussian stochastic processes (Chap. 16), which model ship responses in a natural seaway. A final Chap. 17 sketches special topics: fast roll simulation methods for intact and damaged displacement ships and for planing boats; a method aimed to simplify second-order response calculations, and a comparison of motions between common and staggered-hull catamarans (Weinblums).

The book's material is based on many years of experience in the development and application of numerical methods for computing wave-induced motions of and loads acting on ships. Part of these methods, developed by one of us (Söding), resulted in codes PDStrip, GLRANKINE, SIS, and ROLLS. These codes are just examples of several currently applied programs used to deal with practical design-related problems. In addition, the text is based on lectures at the Technical University Hamburg and the University of Duisburg-Essen given by the authors. Of same importance are the many years of professional experience of the authors' affiliation with a classification society, and in providing services to the maritime industry.

The book is aimed at master's and doctoral students and engineers in practice. The authors thank Dr. Udo Lantermann, Guillermo Chillcce, and Dirk Hünniger for their support in generating figures.

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Contents

Chapter 1
Introduction

Abstract This chapter shows that total losses of seagoing ships decrease worldwide at an impressive rate: from about 150 to roughly 50 per year within the last 15 years. However, the share of losses caused by adverse weather conditions rises slowly and amounts now to about 50%, thus showing the relevance of this book's topic.

The primary purpose of computing motions and loads of ships in a seaway is to assure the safety of persons on board, the integrity of the ship and, if present, its cargo, and to improve its performance and efficiency. Excessive motions may not only shift cargo or cause damage from loosened deck containers or equipment, but may even cause dangerously large heel angles and capsizing. Furthermore, ship motions affect the comfort of persons on board, leading to sea sickness or, in extreme cases, to render it impossible for the crew to accomplish the ship's mission. Knowledge of wave-induced loads is necessary to assess the integrity of the ship's structure. Most important for this are vertical and horizontal bending moments, torsional moments, and sometimes shear forces in transverse sections of the hull girder. Wave-induced local pressure acting on the hull determines the necessary strength of plates, stiffeners, and web frames. Furthermore, steady wave- and wind-induced forces and moments should not prevent the ship from arbitrary course changes and from making some speed ahead.

Due to continuous effort for improving ship safety, the number of total losses of ships per year is decreasing (Fig. 1.1) in spite of the increase in the number of ships worldwide. The decreasing trend is present for centuries. However, according to the International Union of Marine Insurance [1], the percentage of ship losses due to severe weather conditions is increasing and is, at present, approximately 50% (Fig. 1.2). In this figure, also part of the losses due to hull damage and perhaps grounding are caused by heavy seaway.

The marine sector is unique in organizing the safety of ships and marine structures on a private basis by classification societies. They edit and supervise rules ships and marine structures have to fulfill to be 'classed'. Traditionally, the rules are the result of experiences with casualties as well as theoretical and experimental studies including full-scale measurements. These rules are adequate for most ships. Since about 60 years, computational methods have been used to improve and extend the rules related

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Fig. 1.1 Worldwide total losses of seagoing ships over 100 gross tons per year. *Source* Statista

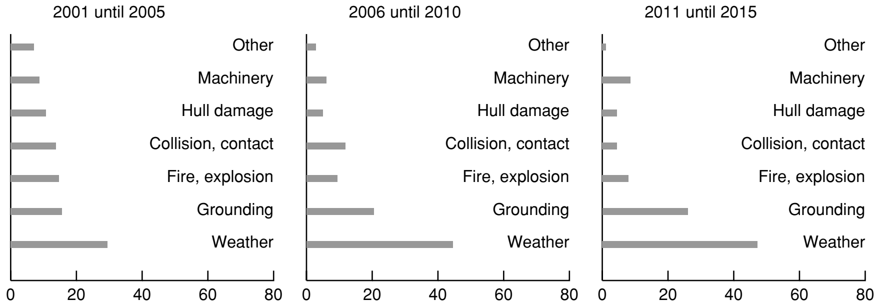

Fig. 1.2 Percentage of total losses of ships exceeding 500 GT by different causes between 2001 and 2015. *Source* [1]

to wave loads and seakeeping, and to investigate wave responses for newbuilds that differ substantially from those for which the rules were prepared.

Reliable load predictions enable reducing the allowances specified to account for uncertainties of loads and structural strength. It may be advantageous to apply advanced, possibly costly computations to reduce a ship's scantlings or the probability of structural failures. Regarding ship motions, numerical simulations may help to estimate the probability of excessive motions and accelerations. This may help to extend the safe limits of metacentric height.

In principle, wave-induced motions and loads can be assessed by using full-scale measurements, model tests, and numerical methods. Full-scale measurements are possible only if the ship, or a similar one, has been built already. They are expensive; the wave conditions cannot be controlled; and assessing the wave conditions during the measurements with the required accuracy is usually impossible. Model test results can be converted to full-scale data except for the influence of viscosity, which is small in most cases. More important is the limited size of the model basin, the degree of sophistication of the equipment of the test facility, and cost and time to perform such experiments. In irregular seaways, long test runs are required to obtain representative results. Thus, for seakeeping model, experiments are used today mostly to validate numerical methods. An exception is the sloshing of fluids in tanks, where small-scale effects like wave breaking and the collapse of bubbles may be important for practical questions, but are difficult or impossible to simulate accurately.

Currently, in designing and approving new ships, often a linear strip or panel method is used to determine motions and structural hull girder loads for a ship advancing at constant forward speed in small amplitude regular waves under various combinations of wave frequency and heading. For any seaway described by a wave spectrum, the results are combined to obtain root mean square values of loads extrapolated linearly over wave amplitude. Results for different seaway conditions are then combined to a long-term probability distribution of loads. For suitably selected design conditions, nonlinear corrections to the linear loads can be applied. If more accuracy is required, solvers for Navier-Stokes or Euler equations may be applied, which take into account the water/air interface. Today, using such a code is the obvious choice to compute free-surface waves around the ship including breaking waves, sprays, and air trapping: phenomena that should be considered to predict slamming loads in severe seas accurately.

A full understanding and an accurate prediction of hydrodynamic wave body interactions is challenging. The associated nonlinear effects become critical when large-amplitude body motions and/or high surface waves are involved. Indeed, no numerical method is yet capable to perform accurate, fully nonlinear seakeeping calculations during the required long-time simulations. The objective of this book is to present the current computational methods for seakeeping problems suitable for numerical predictions of acceptable accuracy. Among these methods is a new, fully nonlinear Rankine panel method as well as an advanced technique for avoiding wave reflections for arbitrary wave encounter angles when using a Navier-Stokes equations solver.

Reference

1. IUMI. International Union of Marine Insurance (2016), http://www.iumi.com/

Chapter 2
Fundamental Governing Equations

Abstract In this chapter, the conservation equations for mass and momentum are derived to describe the flow of Newtonian fluids. Subsequently, the basic equations for incompressible potential flows, namely, the continuity equation (in the form of the Laplace equation) and the Bernoulli equation, are described. Finally, the equations of motion for rigid bodies are derived.

2.1 Governing Equations of Fluid Flow

Fluids are substances that do not resist external shear forces. This definition includes not only liquids, but also gases. Fluids are treated here as continua, neglecting the motions of their molecules. Furthermore, we confine ourselves to Newtonian fluids, i.e., we consider only mechanical forces and assume that the shear stress in the fluid is proportional to the shear rate.

Flows of Newtonian fluids can be described by four fields (physical quantities depending on space): density ρ, velocity $\mathbf{v}$, pressure p, and temperature T. Three equations that govern the flow are obtained from conservation principles:

- Continuity equation (mass conservation)
- Navier-Stokes equation (momentum conservation)
- Heat capacity equation (energy conservation)

Furthermore, there is an equation of state, which relates pressure, density, and temperature to each other and which is specific for the fluid material.

The heat capacity equation and the equation of state are only required when computing the flow of compressible fluids. Because these occur only exceptionally in ship technical flows, they will not be discussed here.

2.1.1 Conservation Principles

Extensive quantities for which a conservation equation exists, here mass, momentum, and energy, depend on the amount of matter, whereas the intensive quantities density

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(ρ), velocity (v), and temperature (T) are the corresponding extensive quantities per volume or mass. If Φ is an extensive quantity, and ϕ is the corresponding intensive quantity per mass, we have

$$\Phi(t) = \int_{V_{\mathrm{CM}}} \rho(\mathbf{r}, t)\, \phi(\mathbf{r}, t) dV, \tag{2.1}$$

where $\mathbf{r}$ is the position vector to a point in the fluid, and V_{CM} is the volume occupied by the 'control mass' CM, i.e., an arbitrarily selected subset of the fluid particles. The boundary of V_{CM} is moving with the local velocity $\mathbf{v}(\mathbf{r}, t)$ of the fluid.

For numerical calculations of fluid flow, it is preferred to have 'control volumes' CV, the boundaries of which can move with arbitrary velocity $\mathbf{v}_b$ (including $\mathbf{v}_b = 0$ for stationary volumes). In the following, it is assumed that, for each CM, we have a corresponding CV, and that at time t both volumes are the same. Then, for any conserved quantity Φ, the following relation holds between integrals over CM and CV:

$$\frac{d\Phi}{dt} = \underbrace{\frac{d}{dt}\int_{V_{\mathrm{CM}}} \rho\phi dV}_{\text{Rate of Change of } \Phi \text{ in CM}} = \underbrace{\frac{d}{dt}\int_{V_{\mathrm{CV}}} \rho\phi dV}_{\text{Rate of Change in CV}} + \underbrace{\int_{S_{\mathrm{CV}}} \rho\phi\, (\mathbf{v} - \mathbf{v}_b) \cdot \mathbf{n}\, dS}_{\text{Net flux of } \Phi \text{ through } S_{\mathrm{CV}}}, \tag{2.2}$$

where
$V_{\mathrm{CV}} = V$ = volume of the CV,
$S_{\mathrm{CV}} = S$ = surface enclosing the CV, and
$\mathbf{n}$ = unit vector $\perp$ to S directed outwards.
The index CV will be omitted in the following.

2.1.2 Mass Conservation (Continuity Equation)

Using $\phi \equiv 1$ and $\mathbf{v}_b \equiv 0$ in (2.1) and (2.2) gives an equation for conservation of fluid mass M within V_{CM}:

$$\frac{dM}{dt} = \frac{d}{dt}\int_{V_{\mathrm{CM}}} \rho\, dV = \frac{\partial}{\partial t}\int_V \rho\, dV + \int_S \rho\mathbf{v} \cdot \mathbf{n}\, dS = 0. \tag{2.3}$$

We apply the Gauss theorem

$$\int_S f(\mathbf{r})\mathbf{n}\, dS = \int_V \nabla f(\mathbf{r}) dV, \tag{2.4}$$

which holds for continuously differentiable scalar functions f, to the surface integral in (2.3), using each component of $\mathbf{v}$ as f and adding the result. This transforms the mass conservation equation into

$$\frac{\partial}{\partial t}\int_V \rho\, dV + \int_V \nabla\cdot(\rho\mathbf{v})\, dV = 0. \tag{2.5}$$

Because V is fixed in space, this can be written as

$$\int_V \left(\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{v})\right) dV = 0.$$

In the limit of V shrinking to a point, this results in the continuity equation (mass conservation) as partial differential equation:

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0. \tag{2.6}$$

2.1.3 Momentum Conservation (Navier-Stokes Equation)

Using for ϕ the components of the velocity $\mathbf{v}$, we obtain from (2.1) the definition of the momentum

$$\mathbf{P} = \int_{V_{\mathrm{CM}}} \rho\mathbf{v}\, dV. \tag{2.7}$$

Newton's second law states

$$\frac{d}{dt}\mathbf{P} = \sum_i \mathbf{F}_i,$$

where the $\mathbf{F}_i$ are all external forces acting on the CM. Applying (2.2) with $\mathbf{v}_b \equiv 0$ for each component v_j of $\mathbf{v}$ gives

$$\frac{\partial}{\partial t}\int_V \rho\mathbf{v}\, dV + \int_S \rho\, \underbrace{(\mathbf{v}\mathbf{v})\cdot\mathbf{n}}_{v_j v_k n_k}\, dS = \sum_i \mathbf{F}_i. \tag{2.8}$$

The term $v_j v_k = \mathbf{v}\mathbf{v}$ is the outer product of the velocity $\mathbf{v} = (v_1, v_2, v_3)^T$ with itself:

$$v_j v_k = \mathbf{v}\mathbf{v} = \begin{pmatrix} v_1\, v_1 & v_1\, v_2 & v_1\, v_3 \\ v_2\, v_1 & v_2\, v_2 & v_2\, v_3 \\ v_3\, v_1 & v_3\, v_2 & v_3\, v_3 \end{pmatrix}. \tag{2.9}$$

The external force $\sum_i \mathbf{F}_i$ contains surface forces, which act on S, and mass forces acting on the fluid inside the CV.

Surface forces are forces, which must be applied at the imagined dissection of the fluid inside the CV from that outside of it, to take account of the internal stress tensor $\mathbf{T}$:

$$\mathbf{F}_S = \int_S \mathbf{T} \cdot \mathbf{n}\, dS.$$

For Newtonian fluids, the stress tensor depends on the pressure p and on the velocity field $\mathbf{v}$:

$$\mathbf{T} = -\mathbf{E}\, p + 2\,\mu\,\mathbf{D} + \frac{2}{3}\,\mu\,(\nabla \cdot \mathbf{v})\mathbf{E} \quad \text{with} \quad \mathbf{D} = \frac{1}{2}\left[\nabla \mathbf{v} + (\nabla \mathbf{v})^{\mathsf{T}}\right], \tag{2.10}$$

Where $\mathbf{D}$ denotes the rate of the strain tensor, $\mathbf{E}$ the unit tensor, p the pressure, and μ the dynamic viscosity.

In index notation using Cartesian coordinates, this is

$$T_{ij} = -p\delta_{ij} + 2\mu D_{ij} + \frac{2}{3}\mu\delta_{ij}\frac{\partial v_k}{\partial x_k} \quad \text{with} \quad D_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right),$$

where δ_{ij} is the Kronecker tensor: $\delta_{ij} = 1$ for $i = j$, otherwise 0.

The weight of the fluid causes the mass force

$$\mathbf{F}_g = \int_V \rho\,\mathbf{g}\, dV,$$

where $\mathbf{g}$ is the gravity acceleration vector. In rotating or accelerating reference systems, further mass forces must be added. In the following, an inertial reference system is presumed.

Taking together the above momentum equations give

$$\frac{\partial}{\partial t}\int_V \rho\mathbf{v}\, dV + \underbrace{\int_S \rho\mathbf{v}\mathbf{v} \cdot \mathbf{n}\, dS}_{\text{convective flux of momentum}} = \underbrace{\int_S \mathbf{T} \cdot \mathbf{n}\, dS}_{\text{diffusive flux of momentum}} + \underbrace{\int_V \rho\mathbf{g}\, dV}_{\text{source term}}, \tag{2.11}$$

The surface integrals can be transformed to volume integrals using the Gauss theorem (2.4) for each tensor component:

$$\frac{\partial}{\partial t}\int_V \rho\mathbf{v}\, dV + \underbrace{\int_V \nabla \cdot (\rho\mathbf{v}\mathbf{v})\; dV}_{\text{convective flux of momentum}} = \underbrace{\int_V \nabla \cdot \mathbf{T}\, dV}_{\text{diffusive flux of momentum}} + \underbrace{\int_V \rho\mathbf{g}\, dV}_{\text{source term}}. \tag{2.12}$$

Because V does not move, the differential operator and the integral may be interchanged in the first term. Taking again the limit of V shrinking into a point results in the Navier-Stokes equation in inertial coordinate systems:

$$\frac{\partial}{\partial t}(\rho\mathbf{v}) + \nabla \cdot (\rho\mathbf{v}\mathbf{v}) = \nabla \cdot \mathbf{T} + \rho\mathbf{g}. \tag{2.13}$$

In some cases, the viscosity of the fluid can be neglected. Then the stress tensor $\mathbf{T}$ in (2.10) simplifies to

$$\mathbf{T} = -\mathbf{E}p, \tag{2.14}$$

and

$$\nabla \cdot \mathbf{T} = -\nabla p. \tag{2.15}$$

If this simplified $\mathbf{T}$ is used in (2.13), the equation is called Euler equation.

2.1.4 Flow of Incompressible Fluids

Fluids can be approximated as incompressible if the maximum flow velocity is less than about 0.3 times the velocity of sound waves. That condition is usually satisfied in the flow of air and water around ships; an exception may be water-air mixtures treated as a single substance.

If the fluid density ρ is constant over time and space, the continuity Eq. (5.1) becomes

$$\nabla \cdot \mathbf{v} = 0, \tag{2.16}$$

and the Navier-Stokes equation (2.13) simplifies to

$$\frac{\partial \mathbf{v}}{\partial t} + \nabla \cdot (\mathbf{v}\mathbf{v}) = \nabla \cdot \mathbf{T}/\rho + \mathbf{g}. \tag{2.17}$$

2.1.5 Potential Flow

The rotation of the velocity field is defined as

$$\text{rot } \mathbf{v} = \nabla \times \mathbf{v} = (v_{3y} - v_{2z},\ v_{1z} - v_{3x},\ v_{2x} - v_{1y}), \tag{2.18}$$

where indices x, y, z designate partial derivatives with respect to the Cartesian space coordinates x, y, z. If an inviscid fluid motion starts from rest, rot $\mathbf{v}$ will remain zero. Therefore, in many real flows rot $\mathbf{v} = 0$ is a reasonable approximation. Flows with rot $\mathbf{v} = 0$ everywhere within the flow field are called potential flows because their velocity field $\mathbf{v}(x, y, z, t)$ can be expressed as the gradient of a scalar function $\phi(x, y, z, t)$ called flow potential:

$$\mathbf{v} = \nabla \phi. \tag{2.19}$$

If ϕ is twofold continuously differentiable, the condition rot $\mathbf{v} \equiv 0$ is satisfied identically:

$$\text{rot } \mathbf{v} = (\phi_{zy} - \phi_{yz},\ \phi_{xz} - \phi_{zx},\ \phi_{yx} - \phi_{xy}) = (0, 0, 0). \tag{2.20}$$

For a potential flow of an incompressible fluid, the continuity equation follows from (2.16) as

$$\nabla \cdot (\nabla\phi) = (\nabla \cdot \nabla)\phi = \Delta\phi = \phi_{xx} + \phi_{yy} + \phi_{zz} = 0. \tag{2.21}$$

This second-order linear differential equation for ϕ is called the Laplace equation.

For a potential flow of an incompressible fluid, also the momentum conservation (2.17) with (2.15),

$$\frac{\partial \mathbf{v}}{\partial t} + \nabla \cdot (\mathbf{v}\mathbf{v}) = -\nabla p/\rho + \mathbf{g}, \tag{2.22}$$

can be simplified: If $\dot{\phi}$ is the partial time derivative of the potential, g is the scalar gravity acceleration, and the z coordinate is directed downward, one obtains

$$\nabla\dot{\phi} + \nabla \cdot (\mathbf{v}\mathbf{v}) = -(\nabla p)/\rho + \nabla(gz). \tag{2.23}$$

The second term on the left-hand side is

$$\nabla \cdot (\mathbf{v}\mathbf{v}) = \nabla \cdot \begin{pmatrix} \phi_x\phi_x & \phi_x\phi_y & \phi_x\phi_z \\ \phi_y\phi_x & \phi_y\phi_y & \phi_y\phi_z \\ \phi_z\phi_x & \phi_z\phi_y & \phi_z\phi_z \end{pmatrix} \tag{2.24}$$

$$\begin{aligned} = \ & (\phi_{xx}\phi_x + \phi_{yy}\phi_x + \phi_{zz}\phi_x,\ \phi_{xx}\phi_y + \phi_{yy}\phi_y + \phi_{zz}\phi_y,\ \phi_{xx}\phi_z + \phi_{yy}\phi_z + \phi_{zz}\phi_z) \\ & +(\phi_x\phi_{xx} + \phi_y\phi_{xy} + \phi_z\phi_{xz},\ \phi_x\phi_{yx} + \phi_y\phi_{yy} + \phi_z\phi_{yz},\ \phi_x\phi_{xz} + \phi_y\phi_{yz} + \phi_z\phi_{zz}) \end{aligned} \tag{2.25}$$

The upper line of (2.25) is zero because of the continuity Eq. (2.21), while the lower line is one half of the expression

$$\begin{aligned} \nabla(\mathbf{v} \cdot \mathbf{v}) &= \nabla(\phi_x^2 + \phi_y^2 + \phi_z^2) \\ &= 2\phi_x\phi_{xx} + 2\phi_y\phi_{xy} + 2\phi_z\phi_{xz},\ 2\phi_x\phi_{xy} + 2\phi_y\phi_{yy} + 2\phi_z\phi_{yz}, \\ &\quad 2\phi_x\phi_{xz} + 2\phi_y\phi_{yz} + 2\phi_z\phi_{zz}. \end{aligned} \tag{2.26}$$

Thus

$$\nabla \cdot (\mathbf{v}\mathbf{v}) = \frac{1}{2}\nabla(\mathbf{v} \cdot \mathbf{v}) = \frac{1}{2}\nabla|\mathbf{v}|^2. \tag{2.27}$$

Inserting this into (2.23) gives

$$\nabla\left(\dot{\phi} + \frac{1}{2}|\mathbf{v}|^2 + \frac{p}{\rho} - gz\right) = 0. \tag{2.28}$$

From this follows the Bernoulli equation for incompressible potential flows:

$$\dot{\phi} + \frac{1}{2}|\mathbf{v}|^2 + \frac{p}{\rho} - gz = C(t), \tag{2.29}$$

in which C may depend on time, but not on position within the fluid. In potential flows, this scalar equation substitutes the vector equation for momentum conservation in more general flows.

2.2 Rigid Body Motions

Ships are elastic bodies. Their flexibility has a considerable influence on the stresses caused by wave loads. To compute these stresses, ship motions are superimposed from rigid body motions and elastic deformations. The latter are discussed in Chap. 10; here, the equations for rigid body motions are elaborated.

2.2.1 Coordinate Systems

We introduce an inertial Cartesian right-handed coordinate system $\mathbf{O}(x, y, z)$ to describe the motions of the ship and the fluid surrounding it. Its z-axis is directed downward. Position vectors expressed in inertial coordinates are $\mathbf{x}$; especially the center of gravity G of the ship is located at $\mathbf{x}_G = (x_G, y_G, z_G)^T$.

Sometimes, especially for ships sailing in regular waves, an inertial coordinate system is chosen that follows the mean forward speed of the ship. Then $\mathbf{x}_G$ specifies the translational motion of the ship (at its center of gravity), excluding the steady forward speed. The longitudinal motion $x_G(t)$ is then called surge, the transverse motion $y_G(t)$ is called sway, and the vertical motion $z_G(t)$ is called heave; however, sometimes the ship-fixed reference point for these motions is chosen differently from G. On the other hand, to describe maneuvering motions, the inertial system should be either fixed to the earth, or it should follow the constant stream of water not disturbed by the ship.

A second Cartesian coordinate system $\mathbf{o}(\underline{x}, \underline{y}, \underline{z})$ is fixed to the ship's body. Its $\underline{x}$-axis is directed forward, its $\underline{y}$-axis to starboard, and its $\underline{z}$-axis normal to the plane decks downward. Its origin is at the center of gravity G (see Fig. 2.1). It is used to describe the hull shape and other quantities (for instance, the mass distribution) which are constant in ship-fixed, but not in inertial coordinates.

Position vectors of the same point, expressed (a) in ship-fixed coordinates as $\underline{\mathbf{x}}$ and (b) in inertial coordinates as $\mathbf{x}$, are related by

$$\mathbf{x} = \mathbf{x}_G + \mathbf{T}\underline{\mathbf{x}}. \tag{2.30}$$

Here, the 3 by 3 matrix $\mathbf{T}$ takes account of the different directions of the inertial and the ship-fixed coordinates.

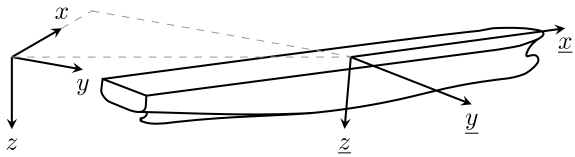

Fig. 2.1 Coordinate system used to describe ship motions

The ship's inclination (compared to the upright floating condition) about the longitudinal axis is the heel or roll angle φ; it is defined positive for a right-hand rotation around $\underline{x}$. Correspondingly, the trim or pitch angle θ and the course or yaw angle ψ describe the rotations about the transverse and vertical axes, respectively. In case of a combined rotation around all three axes, the ship is imagined to be starting from the initial orientation $\varphi = \theta = \psi = 0$, then rotating about the x-axis by φ, then about the inertial y-axis by θ, and at last about the inertial z-axis by ψ. This sequence of rotations allows determine the matrix $\mathbf{T}$ in (2.30): The heel rotation corresponds to a left multiplication of $\underline{\mathbf{x}}$ by the matrix

$$\mathbf{T}_\varphi = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{pmatrix}, \tag{2.31}$$

and the trim and course rotations, correspondingly, by left multiplications by

$$\mathbf{T}_\theta = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} \quad \text{and} \quad \mathbf{T}_\psi = \begin{pmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{2.32}$$

respectively. Thus, the matrix $\mathbf{T}$ combining the three rotations is

$$\begin{aligned} \mathbf{T} &= \mathbf{T}_\psi \mathbf{T}_\theta \mathbf{T}_\varphi \\ &= \begin{pmatrix} \cos\psi\cos\theta & \cos\psi\sin\theta\sin\varphi - \sin\psi\cos\varphi & \cos\psi\sin\theta\cos\varphi + \sin\psi\sin\varphi \\ \sin\psi\cos\theta & \sin\psi\sin\theta\sin\varphi + \cos\psi\cos\varphi & \sin\psi\sin\theta\cos\varphi - \cos\psi\sin\varphi \\ -\sin\theta & \cos\theta\sin\varphi & \cos\theta\cos\varphi \end{pmatrix} \end{aligned} \tag{2.33}$$

The rows of matrix $\mathbf{T}$ are pairwise orthogonal; thus, the inverse of $\mathbf{T}$ is its transpose:

$$\mathbf{T}^{-1} = \mathbf{T}^T. \tag{2.34}$$

Table 2.1 summarizes the motion components of ships.

Table 2.1 Definition of ship motions

DOF	Name	Description	Velocity	Position, Euler angle
1	Surge	Translation in x direction	u	x
2	Sway	Translation in y direction	v	y
3	Heave	Translation in z direction	w	z
4	Roll	Rotation around x-axis	p	φ
5	Pitch	Rotation around y-axis	q	θ
6	Yaw	Rotation around z-axis	r	ψ

2.2.2 Kinematics

To simplify the following, the so-called Euler angles φ, θ, and ψ are combined to a pseudo-vector $\boldsymbol{\alpha} = (\varphi, \theta, \psi)^T$. Its time derivative $\dot{\boldsymbol{\alpha}}$ is related to the angular velocity $\boldsymbol{\omega}$ expressed in inertial coordinates:

$$\boldsymbol{\omega} = \mathbf{R}\dot{\boldsymbol{\alpha}}. \tag{2.35}$$

The 3 by 3 matrix $\mathbf{R}$ follows from the relation (2.30) applied to a body rotating about G with angular velocity $\boldsymbol{\omega}$. Ship-fixed points move then with velocity

$$\dot{\mathbf{x}} = \boldsymbol{\omega} \times \underbrace{(\mathbf{x} - \mathbf{x}_G)}_{\mathbf{T}\underline{\mathbf{x}}} = \dot{\mathbf{T}}\underline{\mathbf{x}}. \tag{2.36}$$

With (2.35) follows

$$\mathbf{R}\dot{\boldsymbol{\alpha}} \times \mathbf{T}\underline{\mathbf{x}} = \dot{\mathbf{T}}\underline{\mathbf{x}} \tag{2.37}$$

for arbitrary vectors $\underline{\mathbf{x}}$. Using (2.33), it is easily verified for three linearly independent vectors $\underline{\mathbf{x}}$, e.g., for $(1, 0, 0)^T$, $(0, 1, 0)^T$, and $(0, 0, 1)^T$, that (2.37) is satisfied if

$$\mathbf{R} = \begin{pmatrix} \cos\psi\cos\theta & -\sin\psi & 0 \\ \sin\psi\cos\theta & \cos\psi & 0 \\ -\sin\theta & 0 & 1 \end{pmatrix}. \tag{2.38}$$

To determine $\dot{\boldsymbol{\alpha}}$ from $\boldsymbol{\omega}$, we need the inverse of $\mathbf{R}$:

$$\mathbf{R}^{-1} = \begin{pmatrix} \cos\psi/\cos\theta & \sin\psi/\cos\theta & 0 \\ -\sin\psi & \cos\psi & 0 \\ \tan\theta\cos\psi & \tan\theta\sin\psi & 1 \end{pmatrix}. \tag{2.39}$$

It exists because, in practice, $|\theta|$ will be less than 90°.

2.2.3 Motion Equations

Rigid body translations are governed by Newton's second law

$$m\ddot{\mathbf{x}}_G = \mathbf{F}, \tag{2.40}$$

where m is ship mass, and $\mathbf{F}$ is the total force acting on the ship. Further terms would occur if the body motion were defined not as that of the center of gravity $\mathbf{x}_G$, but for any other point. Here, we prefer to use G as a reference point; after translations at G and rotations have been determined, the motion of any other point P is easily determined from (2.30).

Rigid body rotations are governed by the equation

$$\frac{d}{dt}(\mathbf{I}\boldsymbol{\omega}) = \mathbf{I}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \mathbf{I}\boldsymbol{\omega} = \mathbf{M}, \tag{2.41}$$

where $\mathbf{I}\boldsymbol{\omega}$ is the angular momentum of the body. Here, $\mathbf{M}$ is the external moment about G acting on the body and $\mathbf{I}$ is its inertia matrix referring to G, both expressed in inertial coordinates. The inertia matrix is constant in ship-fixed coordinates if all mass items m_i belonging to the ship and its load do not move relative to the ship:

$$\underline{\mathbf{I}} = \begin{pmatrix} \underline{I}_{xx} & -\underline{I}_{xy} & -\underline{I}_{xz} \\ -\underline{I}_{yx} & \underline{I}_{yy} & -\underline{I}_{yz} \\ -\underline{I}_{zx} & -\underline{I}_{zy} & \underline{I}_{zz} \end{pmatrix} = \begin{pmatrix} \sum_i m_i(\underline{y}_i^2 + \underline{z}_i^2) & -\sum_i m_i \underline{x}_i \underline{y}_i & -\sum_i m_i \underline{x}_i \underline{z}_i \\ -\sum_i m_i \underline{y}_i \underline{x}_i & \sum_i m_i(\underline{x}_i^2 + \underline{z}_i^2) & -\sum_i m_i \underline{y}_i \underline{z}_i \\ -\sum_i m_i \underline{z}_i \underline{x}_i & -\sum_i m_i \underline{z}_i \underline{y}_i & \sum_i m_i(\underline{x}_i^2 + \underline{y}_i^2) \end{pmatrix}. \tag{2.42}$$

To apply this ship-fixed inertia matrix in (2.41), we change its right-hand factors in (2.41) to ship-fixed coordinates by left multiplication with $\mathbf{T}^{-1} = \mathbf{T}^T$, then multiply with $\underline{\mathbf{I}}$, and then change the result back to inertial coordinates by left multiplication with $\mathbf{T}$. From the associative law for matrix products follows that this procedure is equivalent to using the relation

$$\mathbf{I} = \mathbf{T}\underline{\mathbf{I}}\mathbf{T}^T. \tag{2.43}$$

Equations (2.40) and (2.41) are used to determine the linear and angular accelerations:

$$\ddot{\mathbf{x}}_G = \mathbf{F}/m \quad \text{and} \quad \dot{\boldsymbol{\omega}} = \mathbf{I}^{-1}(\mathbf{M} - \boldsymbol{\omega} \times \mathbf{I}\boldsymbol{\omega}). \tag{2.44}$$

To compute the body motions, the linear and angular accelerations $\ddot{\mathbf{x}}_G$ and $\dot{\boldsymbol{\omega}}$ are determined from $\mathbf{F}$ and $\mathbf{M}$ using (2.44). Any method of integrating ordinary differential equations may then be used to determine $\dot{\mathbf{x}}_G$ and $\boldsymbol{\omega}$. The latter quantity is transformed to $\dot{\boldsymbol{\alpha}} = \mathbf{R}^{-1}\boldsymbol{\omega}$ using (2.39). $\dot{\mathbf{x}}_G$ and $\dot{\boldsymbol{\alpha}}$ are again numerically integrated to obtain $\mathbf{x}_G$ and $\boldsymbol{\alpha}$.

In bodies which are partly or fully immersed in a fluid, the fluid force depends on the initially unknown linear and angular acceleration of the body. For one-dimensional motion, the derivative $-dF/d\ddot{x}$ is called added mass; for six-dimensional motion (three components each of translation and rotation), there is, instead, a 6 by 6 added mass matrix **A** that depends on the actual immersion and on the body orientation:

$$\begin{pmatrix} \mathbf{F} \\ \mathbf{M} \end{pmatrix} = -\mathbf{A} \begin{pmatrix} \ddot{\mathbf{x}}_G \\ \dot{\boldsymbol{\omega}} \end{pmatrix} + \begin{pmatrix} \mathbf{F}_r \\ \mathbf{M}_r \end{pmatrix}, \tag{2.45}$$

where $\mathbf{F}_r$ and $\mathbf{M}_r$ are the rest force and rest moment, respectively, after subtracting the acceleration-dependent fluid force and moment contributions.

Various methods may be applied to resolve the difficulty of **F** and **M** depending on the initially unknown acceleration:

- Equation (2.44) is solved by iteration using under-relaxation, starting with the accelerations found in the previous time step. This is often done if an iteration is required anyway to determine **F** and **M**; both iterations should be included in the same loop. However, erratic acceleration and force histories may result from this procedure, and for light bodies in heavy fluids (for instance, airplanes ditching in water in case of emergency), small under-relaxation factors are required to enforce convergence. That may increase the required number of iterations.
- If the added mass matrix **A** is known, (2.40) and (2.41) should be combined to the form

$$\left[\begin{pmatrix} m\mathbf{E} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{pmatrix} + \mathbf{A}\right] \begin{pmatrix} \ddot{\mathbf{x}}_G \\ \dot{\boldsymbol{\omega}} \end{pmatrix} = \begin{pmatrix} \mathbf{F}_r \\ \mathbf{M}_r - \boldsymbol{\omega} \times \mathbf{I}\boldsymbol{\omega} \end{pmatrix}, \tag{2.46}$$

  where **E** is the 3 by 3 unit matrix. The accelerations follow from solving this 6 by 6 linear equation system.
- Söding [1] describes a method to approximate the added mass matrix **A** from the accelerations found in previous iterations and previous time steps; this method may then be used in (2.46).

2.2.4 Linearized Equations of Motion

If the translation amplitudes are small compared to the ship's draft d (say, $<d/10$), and if the body's rotation amplitudes are small compared to unity (say, <0.1 rad), nonlinear terms of the motion equations can be neglected. Then cosine functions of rotations are approximated as unity and sine functions are substituted by their argument. Thus,

$$\mathbf{T} = \begin{pmatrix} 1 & -\psi & \theta \\ \psi & 1 & -\varphi \\ -\theta & \varphi & 1 \end{pmatrix} \tag{2.47}$$

and

$$\boldsymbol{\omega} = \dot{\boldsymbol{\alpha}}. \tag{2.48}$$

In the motion equations, the gyroscopic term $\boldsymbol{\omega} \times I\boldsymbol{\omega}$ is omitted because it is proportional to $|\boldsymbol{\omega}|^2$.

Reference

1. H. Söding, How to integrate free motions of solids in fluids, in *Numerical Towing Tank Symposium NuTTS, Hamburg* (2001)

Chapter 3
Numerical Methods to Compute Incompressible Potential Flows

Abstract This chapter illustrates the principles of computing potential flows in an unbounded fluid (without a free surface). It applies a strongly simplified source-sink method to the non-lifting flow around a body of arbitrary shape, and the steady lifting flow around an airfoil, both in two space dimensions. In the latter example, the patch method is explained: a variant of the usual panel method, if uses aims simpler formulas and yields better accuracy of the pressure force.

Potential flows are flows in which the rotation of the velocity vector $\text{rot}\mathbf{u} = \nabla \times \mathbf{u}$ is zero in the whole fluid domain Ω. As shear stresses, but not the pressure p, change $\text{rot}\mathbf{u}$ (especially near walls), potential flows occur only in inviscid fluids. In many cases, especially for highly accelerated flows (ship in waves), the influence of viscosity is small. Then potential flow is a suitable approximation for $\mathbf{u}$ outside the boundary layer and, for pressure and the resulting normal forces, often also on the body surface.

Viscous flow simulations require a higher effort in grid generation and computing time than potential flows. Therefore, potential flow simulations are still widely used and the preferred choice in industry except for ship resistance.

In the following, an inertial (not accelerated, not rotating) reference system is assumed, where x, y, z are Cartesian space coordinates; z is pointing downward. t designates time. For potential flow, the gradient of the potential $\phi(x, y, z, t)$ gives the velocity:

$$\mathbf{u} = \nabla\phi. \tag{3.1}$$

This assures that $\text{rot}\mathbf{u} = \nabla \times \nabla\phi = 0$. The pressure follows from the Bernoulli equation

$$p + \frac{1}{2}\rho\mathbf{u}^2 + \rho\phi_t - \rho g z = \text{ constant}, \tag{3.2}$$

where g is the gravity acceleration, ρ the fluid density. In our applications, the pressure is meant to be the difference to air pressure, which is approximated as constant. Then the ‘Bernoulli constant’ on the right-hand side is constant in space and time.

Furthermore, we presuppose that the flow speed is always much less than the speed of sound, which is about 1480 m/s in bubble-free saltwater; thus, the water

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can be assumed incompressible. An incompressible inviscid fluid is called an ideal fluid. In chapters dealing with potential flow, we always treat the water as an ideal fluid.

In an incompressible fluid, conservation of mass is expressed by the Laplace equation:

$$u_x + v_y + w_z = \phi_{xx} + \phi_{yy} + \phi_{zz} = \Delta\phi = 0. \tag{3.3}$$

We do not need equations for conservation of momentum, as the flow is determined solely by the Laplace equation and the boundary conditions.

3.1 Source-Sink Method

The Laplace equation is linear in ϕ. Thus, it is possible to approximate an unknown potential $\phi(x, y, z, t)$ as superposition of elementary solutions $\varphi_i(x, y, z)$ which all fulfill the Laplace equation in Ω:

$$\phi(x, y, z, t) = \sum_{i=1}^{n} a_i(t)\varphi_i(x, y, z). \tag{3.4}$$

Then ϕ also fulfills the Laplace equation for arbitrary a_i. The functions $a_i(t)$ are chosen such that the boundary conditions are fulfilled, at least approximately. The most important elementary solution functions φ for this purpose are:
Parallel flow: e.g.,

$$\varphi = -Ux \tag{3.5}$$

for a parallel flow in negative x-direction with speed U.
Three-dimensional sink located at (ξ, η, ζ):

$$\varphi = 1/\sqrt{(x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2}. \tag{3.6}$$

The streamlines of this flow are straight lines of arbitrary direction through (ξ, η, ζ). The sink takes in a volume per time of 4π.
Two-dimensional source located at (ξ, η):

$$\varphi = \log[(x-\xi)^2 + (y-\eta)^2]. \tag{3.7}$$

The source ejects an area per time of 4π (if log is the natural logarithm, as is meant also hereafter).

These functions φ fulfill the (2D and 3D) Laplace equation everywhere except at the source or sink point. Therefore, sources or sinks should be arranged only outside the fluid domain Ω. At the location of the source or sink, φ becomes singular. Therefore, methods based on (3.4) with singular elementary solutions φ (source, dipole, vortex) are called singularity methods.

3.2 Example: Two-Dimensional Flow Around Smooth Body Without Lift

We consider the example of a two-dimensional flow of an ideal unbounded fluid around a rigid body with a smooth contour. Far away from the body, the fluid is at rest. The body moves with speed U in x-direction. U may be time-dependent; this plays no role as neither differential equation nor boundary conditions contain time derivatives. The flow is assumed free of circulation; this excludes flows around wings and other lifting surfaces for now.

The body contour is described by

$$r(x, y, t) = r(x - Ut, y, 0) = 0. \tag{3.8}$$

The condition for $r = 0$ is that a fluid particle on the body contour at time t (i.e., at $r = 0$) is also on the contour at time $t + dt$. Let D/Dt denote the substantial derivative (i.e., the time derivative at a point moving as a fluid particle with speed (u, v)). Then we have the following boundary condition at the body contour:

$$\frac{Dr}{Dt} = r_t + ur_x + vr_y = 0. \tag{3.9}$$

With $u = \phi_x$, $v = \phi_y$ and (3.8), we have:

$$\phi_x r_x + \phi_y r_y = Ur_x. \tag{3.10}$$

Far away from the body, the fluid is at rest:

$$\lim_{x^2+y^2\to\infty} \nabla\phi = 0. \tag{3.11}$$

This condition is automatically fulfilled if we use for φ_i only source potentials (at locations ξ_i, η_i inside the body).

The initially unknown coefficients a_i are chosen such that the body boundary condition (3.10) is fulfilled at n points on the body contour, the so-called collocation or control points $(x_k,\ y_k)$, $k = 1$ to n. Such a method is called collocation method.

The function $r(x, y, 0)$ describing the body contour (at time $t = 0$) is approximated here in the vicinity of each collocation point by the equation of a straight line through the point and parallel to the straight line through the neighboring collocation points:

$$y = y_k + \frac{x - x_k}{x_{k+1} - x_{k-1}}(y_{k+1} - y_{k-1}). \tag{3.12}$$

We assume here that the collocation points 1–n circumscribe the body contour (almost) equidistantly. Then (3.12) converges for $n \to \infty$ to the tangent at the collocation point, provided the body has no sharp corners. Sharp corners require a special