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Numerical Simulation in Physics and Engineering: Trends and Applications

Lecture Notes of the XVIII 'Jacques-Louis Lions' Spanish-French School

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Editors

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Preface

"One looks back with appreciation to the brilliant teachers,
but with gratitude to those who touched our human feelings.
The curriculum is so much necessary raw material,
but warmth is the vital element for the growing plant
and for the soul of the child."
Carl Jung

This book contains the lecture notes of the XVIII Spanish-French School "Jacques Louis Lions" on Numerical Simulation in Physics and Engineering (http://ehf2018.iusiani.ulpgc.es), which took place from 25th to 29th June 2018 in the University of Las Palmas de Gran Canaria (ULPGC), organized by the Institute of Intelligent Systems and Numerical Applications in Engineering (SIANI), the Department of Mathematics, and the Department of Civil Engineering of ULPGC, and the Research Group in Numerical Simulation and Scientific Calculus (SINUMCC) of University of Salamanca.

The Spanish-French Schools on Numerical Simulation in Physics and Engineering are held biennially since 1984, becoming a meeting point for professionals, researchers, and students in the field of numerical methods. These conferences are sponsored by the Spanish Society of Applied Mathematics (SEMA), actively involved in the organization of these schools that, together with the Congresses of Differential Equations and Applications (CEDYA)/Congresses of Applied Mathematics (CMA), constitute the two most important series of scientific meetings sponsored. They also have the sponsorship of the Société de Mathématiques Appliquées et Industrielles (SMAI) of France since 2008. The 17 previous editions were held in Santiago de Compostela (1984), Benalmádena (1986), Madrid (1988), Santiago de Compostela (1990), Benicàssim (1992), Sevilla (1994), Oviedo (1996), Córdoba (1998), Laredo (2000), Jaca (2002), Cádiz (2004), Castro Urdiales (2006), Valladolid (2008), A Coruña (2010), Torremolinos (2012), Pamplona (2014), and Gijón (2016).

Each edition is organized around several main courses delivered by renowned French and Spanish scientists. On this last occasion, there were four 4-h courses, for which the lecturers were Philippe Destuynder, Héctor Gómez, Frederic Hecht, and José Sarrate, together with three 1-h talks by Luis Ferragut, Raphaele Herbin, and Jacques Periaux. In addition, four 30-minute talks were given by: SeMA "Antonio Valle" Young Investigator Award 2017, Javier Gómez Serrano, best thesis 2017 selected for ECCOMAS award, Adrián Moure, best paper 2016 of SeMA Journal, Serge Nicaise, and best paper 2017 of SeMA Journal, Pablo Pedregal. Furthermore, the participants in the School had the opportunity to present their research work in two poster sessions, where a total of 24 posters were exposed. Finally, a series of ten brief lectures were given by some of the former students, or direct scientific disciples of them, of Professor Luis Ferragut Canals (University of Salamanca), who himself acted as chairman of the session, in recognition of its special influence in the formation of applied mathematics groups in Las Palmas de Gran Canaria, Madrid, and Salamanca.

The editors warmly thank all the speakers and participants for their contributions to the success of the School. In particular, we would like to acknowledge the efforts of all the lecturers (first and second chapters) and speakers (third and fourth chapters) who have contributed to this SEMA SIMAI Springer Series volume, which additionally contains the two "ex-aequo" awarded posters (fifth and sixth chapters). We are also grateful to the Organizing and Scientific Committees for their efforts in the preparation of the School. We extend our thanks and gratitude to all sponsors and supporting institutions for their valuable contributions: SEMA, SMAI, University of Las Palmas de Gran Canaria, and their Departments of Mathematics and Civil Engineering, as well as the Institute of Intelligent Systems and Numerical Applications in Engineering (SIANI). Finally, the editors acknowledge SEMA SIMAI Springer Series and their editors-in-chief Luca Formaggia and Pablo Pedregal for the interest to this series in publishing the most representative scientific and industrial material presented in the Spanish-French School 'Jacques Louis Lions'. Since its XV edition, these lecture notes have been continuously published, this being the fourth volume held in this series.

Las Palmas de Gran Canaria, Spain — David Greiner
Salamanca, Spain — María Isabel Asensio
Las Palmas de Gran Canaria, Spain — Rafael Montenegro
September 2020

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About the Editors

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An Introduction to Quasi-Static Aeroelasticity

Philippe Destuynder and Caroline Fabre

Abstract The aeroelasticity is the science which models, analyses and describes the coupled movements between a flow and a flexible structure. The different phenomena encountered can be classified using three (at least) adimensional numbers: the Strouhal number, the Reynolds number and the reduce frequency number (which despite its name, has no dimension). For sake of clarity, let us just mention in this abstract, that the reduce frequency is the ratio between the time necessary to a flow particle for flying over a flexible structure and the fundamental period of oscillation of this structure.

In the framework of quasi-static aeroelasticity, it is always assumed that the reduce frequency is smaller than the unity. It enables one to define the flow fields (velocity, pressure) from a static position of the structure. The effect of its position with respect to the flow leads to a modification of the stiffness (added aerodynamic stiffness). Furthermore, the relative flow velocity (difference between the flow velocity and the one of the structure) leads to introduce damping due to the flow and therefore modifies the static analysis of stability into the dynamic stability study (aerodynamic damping).

Recently, engineers have upgraded this approach by introducing the added mass concept. This is a mechanical effect due to the fact that the inertia of the structure should take into account the mass of flow which is involved in a movement. This is performed using an incompressible and inviscid model which gives a retroaction effect on the structure proportionally to its velocity. The two first parts of this text are devoted to a formulation of this three effects which are necessary in the dynamic modeling of a flexible (or not) structure immersed in a flow (air or water for instance). Examples in civil engineering and aerodynamics are given in order to

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illustrate the theoretical formulation. Few control aspects in a dynamic behavior of the coupled fluid-structure modeling are also discussed in a section of this text.

Keywords Aeroelasticity · Flutter · Limit cycle of oscillation · Control of vibrations · Instabilities

1 Introduction

The aeroelasticity science is born with the first aircraft built by the Wright brothers in 1902 and also from the very interesting study performed by Otto Lilienthal since 1890 who unfortunately died in a crash of his wind glider. The work of Gustave Eiffel should also be mentioned even if he mainly focused his contribution to aerostatic. The static stability of aircrafts was a corner stone on which a lot of energy was spent by engineers, but the dynamic stability was only discussed after the second world war. Furthermore, the apparition of electronic computers enables one to develop new algorithms opening a new area in aeroelasticity. The physical understanding of the phenomena and their validations from the experiment are necessary. One could say that this science is relevant from scientific empiricism and is strongly based on the reasoning using mathematical models. Let us mention several references which have been helpful to us: A.V. Balakrishnan [2], R.L. Bisplinghoff, H. Ashley and H. Halfman [9], and R. H. Scanlan [29], H.C. Curtiss Jr., R.H. Scanlan, F. Sisto, E.H. Dowell, [13], J. R. Wright, and J.E. Cooper [32], M.E. Hoque [21] and Y.C. Fung [18]. Concerning general articles on aeroelasticity one can consult A.R. Collar [12], I.E. Garrick, and W.H. Reed [19]. Few extensions of this text can be found in Ph. Destuynder [16].

We start this course with the example of the Tacoma Narrows bridge. It is the opportunity to give a brief survey of the classical phenomena known in aeroelasticity. Sections 2.6–2.9 are devoted to the most recent analysis of the Tacoma bridge collapse which is now known as a stall flutter phenomenon. An other example implying a test model in a wind tunnel for which we have precise measurements, shows the similarity with the bridge. It is given in Sect. 8. Several control aspects are also discussed along the text but mainly in Sect. 5 in which we define most classical approaches. In case of an instability, an important problem for the engineers is to decide if there is or not a limit cycle of oscillations. If there is one, it is necessary to characterize its size. This enables one to decide if a fatigue phenomenon can occur or not. We explain how to characterize and to compute such limit cycles of oscillations (if there is one) in Sect. 6. But first of all, we give in the following section, a discussion of this Tacoma Narrows bridge failure which is sometimes described as the birth of modern quasi-static aeroelasticity.

2 The Collapse of Tacoma Narrows Bridge

In this section, we use Den Hartog's theory [14] (which describe the preliminary movement of the bridge before the collapse). Then, the explanation suggested by Y. Billah and R. Scanlan which led to the final torsion movement is given [8]. This is also the opportunity for us, to give a summary of phenomena involved in aeroelasticity problems (Fig. 1).

2.1 What Has Been Observed

The story happened on November 11th 1940 at Tacoma Narrows. A nice suspended new bridge collapsed down after 40 min of oscillations. There was a wind which velocity was approximately 25 m/s and the period of oscillations was about 5 s. The width of the cross section of the bridge was about 10 m (see Fig. 2); hence the reduce frequency was approximately:

$$f_r = \frac{10\ \text{m}}{(25\ \text{m/s}) \times 5\ \text{s}} = 0.08 << 1, \tag{1}$$

which enables to ensure that quasi-static aeroelasticity can be apply.

The Reynolds number representing a ratio between the energy transferred and the diffused one is given by:

$$R_e = \frac{UL}{\nu}, \tag{2}$$

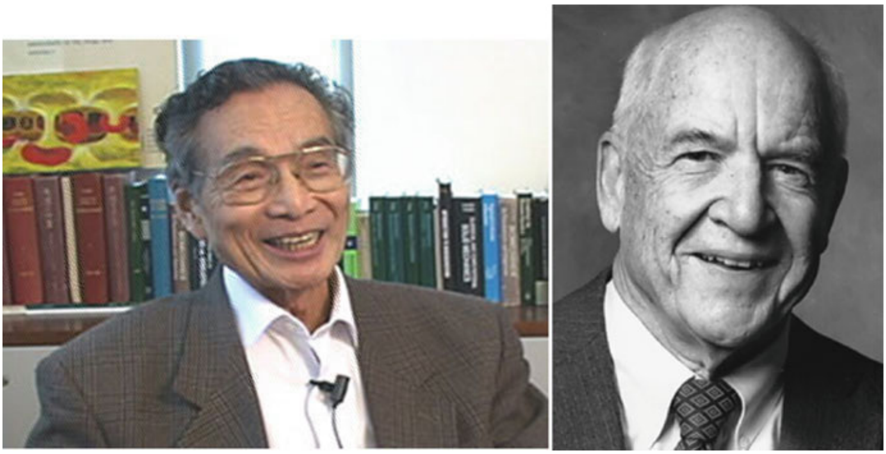

Fig. 1 Two professors who contribute to the Aeroelasticity science: Y. Fung and R. Scanlan

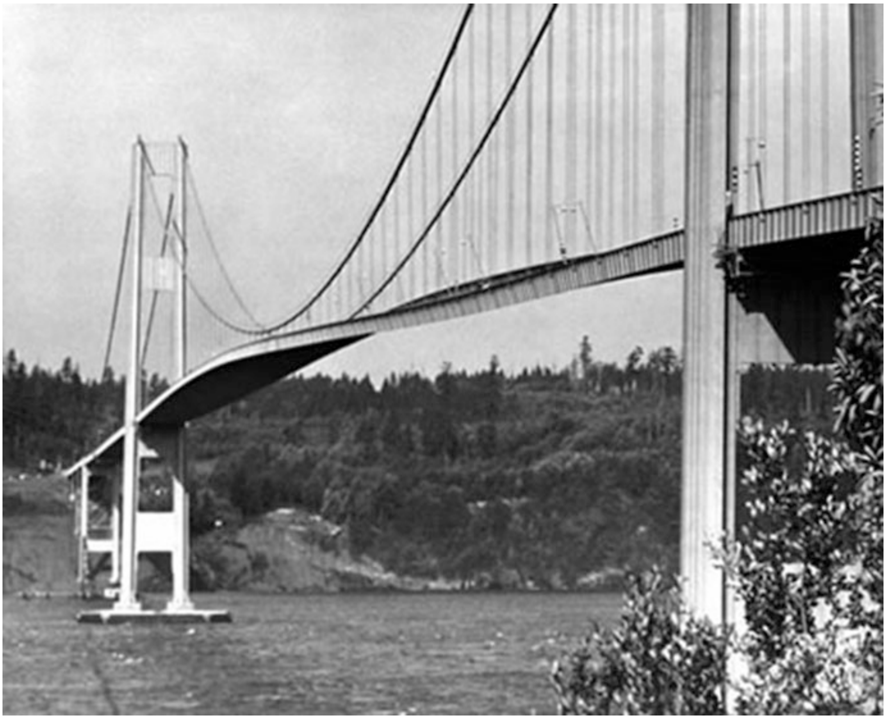

Fig. 2 The Tacoma bridge which collapsed down on the 11-7-1940 four months after its inauguration

where U is the average velocity of the flow, L the length of the structure swept by the flow and ν the kinematical viscosity. Obviously it is only an approximation of physical phenomena, but it enables one to classify the type of flow. In a standard way one refers to Fig. 3 for the different types of flow with respect to the value of the Reynolds number. In the case of Tacoma Narrows bridge the Reynolds number is approximately 10^7. The period of oscillations observed from a movie taken by a man who was in a car stopped on the bridge and who escape from the accident, was about 5 s. No body died excepted the dog of the man which refused to get out of the car.

2.2 The Strouhal Instability

When the Reynolds number is small enough (less than 2000) some particular instabilities can appear in the boundary layer and vortices are shedded into the wake. They are called Strouhal instabilities and have been widely studied by Th. von Karman. The characteristic number used is the so-called Strouhal number and

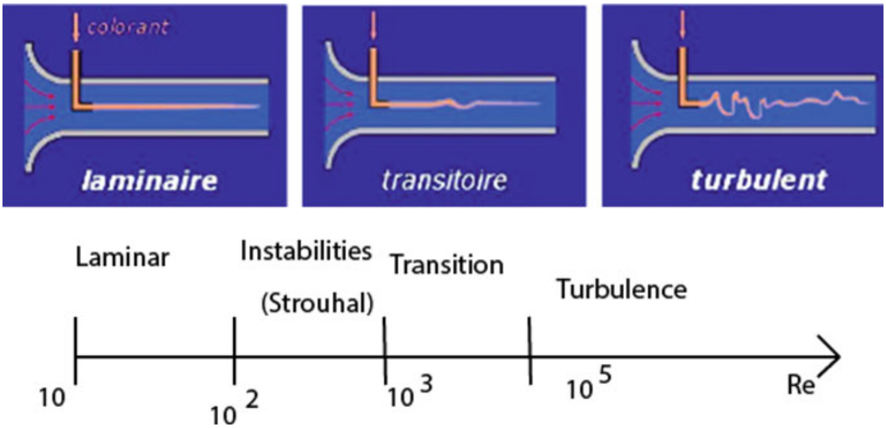

Fig. 3 Classification of flows versus the Reynolds number

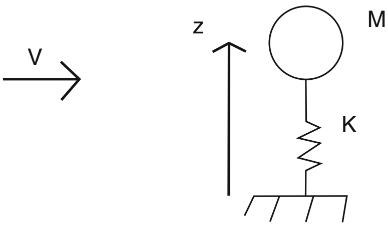

Fig. 4 The simple model discussed for the Strouhal effect

is commonly defined by:

$$S = \frac{D}{U} f_s, \tag{3}$$

where U is the velocity of the flow, f_s is the frequency of the vortices shedding and D the dimension of the obstacle transversally to the flow direction. In the case of a cylinder—which main axis is transversal to the flow velocity—this Strouhal number is measured experimentally and is equal to .2 for this shape (a cylinder). Let us explain how this phenomenon can induce instabilities which could be assimilated to a resonance. This justifies why this terminology is correct.

Let us consider a cylinder of mass M as shown on Fig. 4.

The only movement authorized is the vertical displacement denoted by z. Because of the spring which stiffness is K, the equation of the movement is:

$$M\ddot{z} + Kz = f(t),\ z(0) = 0,\ \dot{z}(0) = 0, \tag{4}$$

where $f(t)$ is the reaction force applied to the cylinder and is due to the separation of the vortices from the boundary layers around the cylinder (see Fig. 5). The function

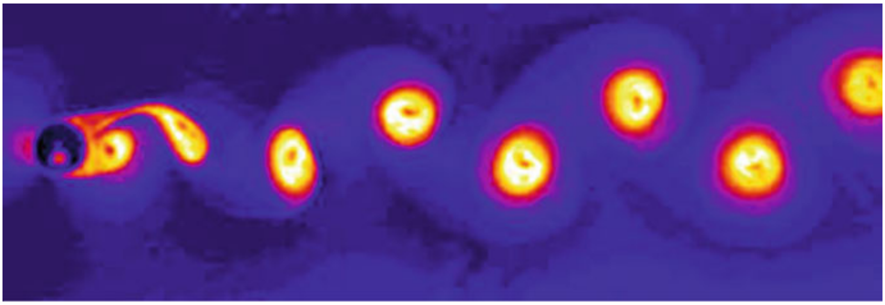

Fig. 5 The von Karman instabilities

f is periodic with a period $T_s = 1/f_s$ as far as there is a Strouhal instability of the boundary layers and one can consider for instance that:

$$f(t) = F \sin(2\pi f_s t), \tag{5}$$

Setting $\omega = \sqrt{\frac{K}{M}}$, the solution is:

$$\begin{aligned} z(t) &= \frac{F}{M\omega} \int_0^t \sin(2\pi f_s s) \sin(\omega(t-s))ds \\ &= \frac{F}{M\omega((2\pi f_s)^2 - \omega^2)} [\omega \sin(2\pi f_s t) - (2\pi f_s) \sin(\omega t)]. \end{aligned} \tag{6}$$

When $\omega \simeq 2\pi f_s$, one has:

$$z(t) \simeq \frac{F}{2M\omega^2} [\sin(\omega t) - t\omega \cos(\omega t)]. \tag{7}$$

The curves representing the solution z are plotted on Fig. 6 for different values of ω close to $2\pi f_s$. The frequency of vibration which was measured from the movie taken by the owner of the car left on the bridge, was about .2 Hz and the Strouhal frequency of a H-shaped cross section as the one of Tacoma bridge (see Fig. 9), is about $.1 \times \frac{D}{U} = .08$ Hz (the coefficient .1 is the Strouhal number and was measured in several wind tunnels) and $D \simeq 2\ m$ is the height of the girders. Therefore, there is a large difference between this two values. Anyway the forces implied in a Strouhal instability are small and can be cancelled by a small damping of the mechanical system. In addition -and mainly- the phenomenon occurs for very low Reynolds numbers (less than 10^4) and in the Tacoma Narrows bridge case the Reynolds number was about $\frac{U \times L}{\nu} \simeq 10^7$. This is why this hypothesis has not been kept, even if it was strongly supported by Th. von Karman. In order to try to converge to an agreement between all the members of the aeroelasticity community,

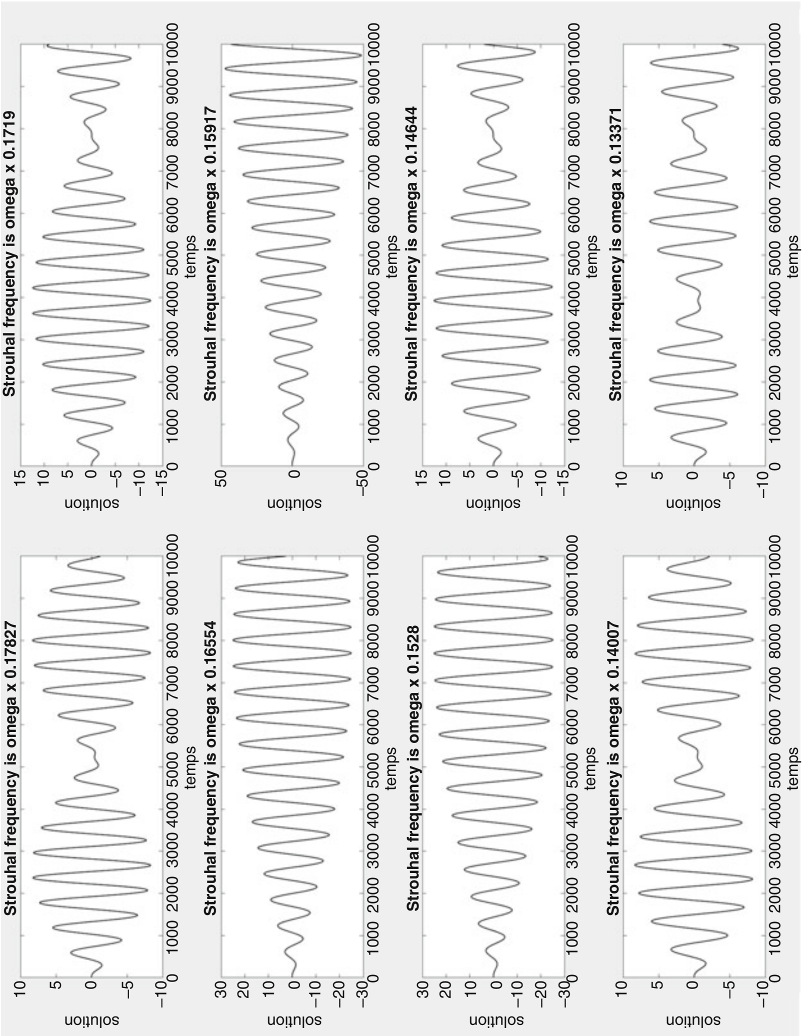

Fig. 6 Solutions of Eq. (4) for different values of f_s. In this example the resonance occurs for $f_s = \frac{\omega}{2\pi} \simeq 0.15917$ Hz

one could say that *may be the Strouhal instability can be at the very origin of the phenomenon observed when the wind velocity was about* 1 m/s.

2.3 The Resonance of a Bridge

In 1850, in the city of Angers in France (see Fig. 7), a bridge over the river *La Loire* collapsed due the passage of soldiers walking at pace. Unfortunately the frequency

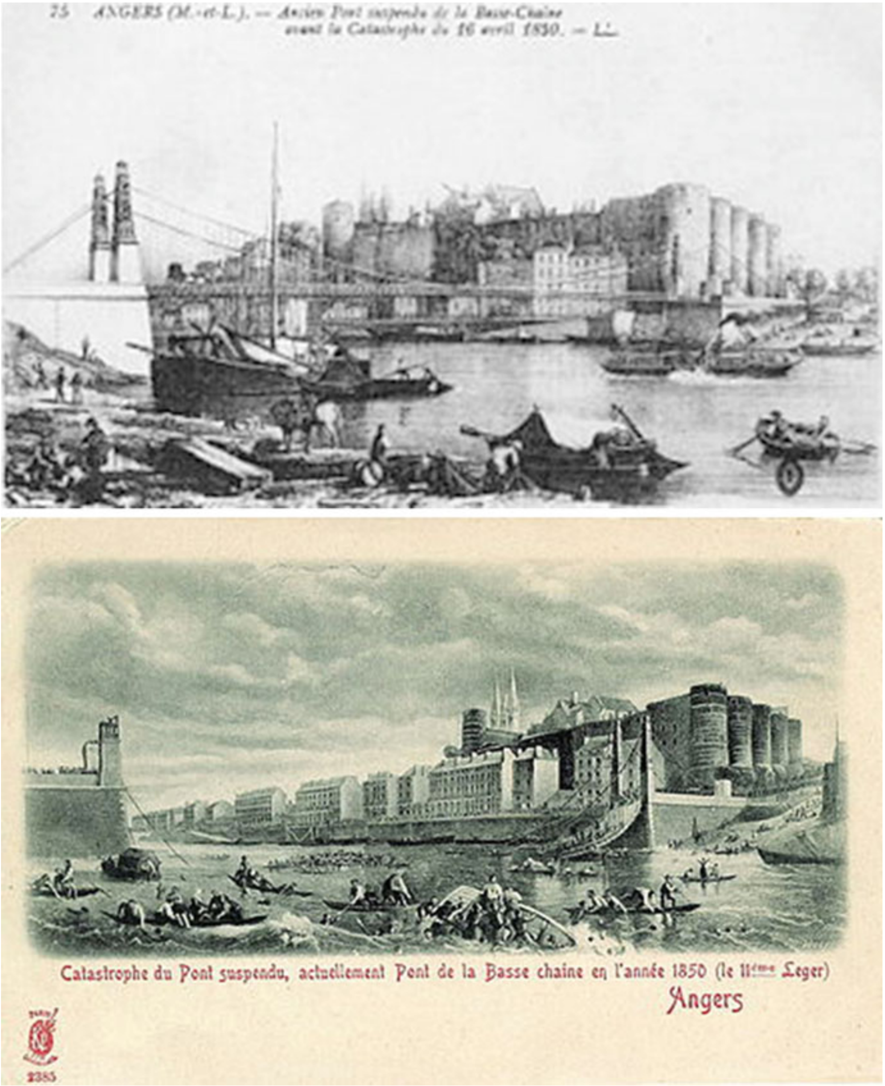

Fig. 7 The bridge over the river *La Loire* in France which collapse in 1850: before (above) and after (below)

of the pace was one of the eigenfrequency of the bridge which enter in resonance and collapsed. Since that date, it is forbidden to walk at pace on any bridge in the world. In the case of Tacoma Narrows bridge the frequencies have been computed very accurately using finite element codes by different research teams in the world. The one observed in the final movement before the destruction seems to be different and the spectrum of the turbulent flow observed in this district (near Tacoma in the south part of Seattle which is in the state of Washington) is not very stable in time as usual in such a phenomenon. Therefore the resonance, which is a linearly increasing instability as shown on Fig. 6, can not be triggered by this turbulent flow. Hence, this explanation is not strongly propped and was given up by most of the engineers (but not by several journalists who found this simple—but false—theory very attractive).

2.4 The Bufffting Phenomenon

During the second world war, the military aircrafts met several strange phenomena. One was the bufffting (mainly observed on the so called Wildcat aircrafts). It is due to vortices shedding from the main wings on the rear wings which induces a periodic excitation. The movement of the rear wing can be modeled in a first step, by a Hill's equation (see for instance the book by M. Roseau [28]) as follows:

$$\ddot{\alpha} + \omega^2(1 + a(t))\alpha = a_0, \ \alpha(0) = 0, \ \dot{\alpha}(0) = 0, \tag{8}$$

where α is the amplitude of an eigenmode of the rear wing. One can prove that if a is a periodic function which period is close to $\frac{2\pi}{\omega}$ (and not necessarily exactly equal), then the solution α increase exponentially with respect to the time. In fact non linear terms can reduce the magnitude of α. This is the bufffting phenomenon. In the case of the Tacoma Narrows bridge, it would have been necessary that the energy spectrum of the wind turbulence contains meaningful terms at the frequency corresponding the movement observed on the movie. This is a quite low frequency which should correspond to large vortices in the flow. This phenomenon has not been registered by the weather forecasting station in the Tacoma valley and therefore it seems difficult to imply this bufffting instability in the collapse of the bridge.

2.5 Flutter Induced by a Coupling Between Two Eigenmodes

The classical flutter mechanism is more complex and has been detected since the beginning of aeronautic but its understanding is more recent. One can refer to an history of flutter by I.E. Garrick and W.H. Reed [19]. It is due to a coupling between two eigenmodes of a flexible structure. In fact, this coupling appears when a double eigenfrequency situation occurs. It depends obviously on the conception of the structure itself, but also on the velocity of the flow and on its evolution with respect to the position of the structure with respect to the main flow direction. When a double eigenfrequency appears, it is possible that an imaginary eigenvalue is developed and leads to an exponential instability. In this case, the eigensubspace is two dimensional. One eigenmode capture energy from the flow and transfers it to the other one which stores it creating the instability. Let us give a simple and classical example.

2.5.1 A Short Description of Flutter Phenomenon

Let us consider a two dimensional airfoil as shown on Fig. 8. It is fixed by two springs: one is a traction spring and the second one is a torsion spring. The stiffness are respectively k and c. Two movements are possible. One is the heaving, denoted by z and the other one is the pitching denoted by α. The linearized equations of the model around $z = \dot{z} = \alpha = \dot{\alpha} = 0$, are the following one and can be easily derived from the expression of the Lagrangian. Let us introduce few notations.

- M is the mass of the airfoil;
- J_0 its inertia around point O, J_G is the inertia around point G, the center of mass;
- V is the flow velocity far away from the structure;
- a is the algebraic distance between O and G;
- c_x, c_z are respectively the drag and the lift coefficients. The corresponding aerodynamical forces are $\frac{\rho S V^2}{2} c_x$ and $\frac{\rho S V^2}{2} c_z$, S is a cross section used as a reference surface;

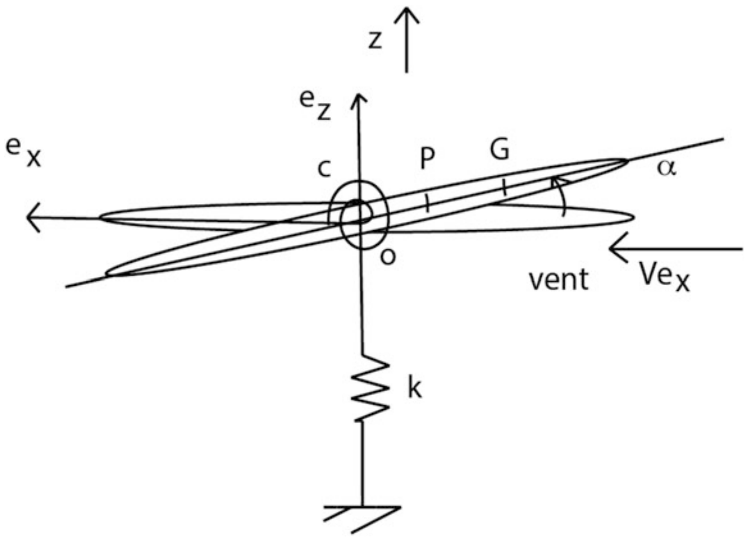

Fig. 8 The system considered

- c_m is the pitching coefficient and the pitching moment is $\frac{\rho SLV^2}{2}c_m$, L is a characteristic length.

All the aerodynamic coefficients are assumed to be known in this presentation. They can be obtained from a wind tunnel test or from a computer code. In fact, there are six coefficients (the three other are c_y for the sheering, c_r for the rolling and c_l for the yawing). All these coefficients are functions of the angles characterizing the position of the structure: α for the pitch, β for the yaw and γ for the roll. But we essentially use c_x, c_z and c_m in this text.

The lagrangian of the mechanical system is with (the springs are assumed to be at rest at the origin):

$$\begin{cases} L(\dot{z}, z, \dot{\alpha}, \alpha) = E_c - E_p, \\ Ec = \frac{1}{2}(M[\dot{z}^2 + 2a\dot{\alpha}\dot{z}\cos(\alpha)] + J_0\dot{\alpha}^2) \\ E_p = \frac{1}{2}(kz^2 + c\alpha^2) + Mg(z + a\alpha). \end{cases}$$

Therefore the dynamical model consists in finding z and α such that:

$$\begin{cases} \frac{d}{dt}(\frac{\partial L}{\partial \dot{z}}) - \frac{\partial L}{\partial z} = \frac{\rho SV^2}{2}c_z(\alpha) - Mg, \\ \frac{d}{dt}(\frac{\partial L}{\partial \dot{\alpha}}) - \frac{\partial L}{\partial \alpha} = \frac{\rho SLV^2}{2}c_m(\alpha) - aMg. \end{cases} \tag{9}$$

or else from a simple computation:

$$\begin{cases} M\ddot{z} + a\cos(\alpha)M\ddot{\alpha} - a\dot{\alpha}^2\sin(\alpha)M + kz = \frac{\rho SV^2}{2}c_z(\alpha) - Mg, \\ J_0\ddot{\alpha} + a\cos(\alpha)M\ddot{z} + c\alpha = \frac{\rho SLV^2}{2}c_m(\alpha) - aMg. \end{cases} \tag{10}$$

And after a linearization near $\alpha = 0$:

$$\begin{cases} M\ddot{z} + aM\ddot{\alpha} + kz = \frac{\rho SV^2}{2}[c_z(0) + \frac{\partial c_z}{\partial \alpha}(0)\alpha] - Mg, \\ J_0\ddot{\alpha} + aM\ddot{z} + c\alpha = \frac{\rho SLV^2}{2}[c_m(0) + \frac{\partial c_m}{\partial \alpha}(0)\alpha] - aMg. \end{cases} \tag{11}$$