MATRIX Book Series 4

David R. Wood *Editor-in-Chief*
Jan de Gier
Cheryl E. Praeger
Terence Tao *Editors*

2019-20 MATRIX Annals

MATRI

Springer

MATRIX Book Series 4

MATRIX is Australia's international and residential mathematical research institute. It facilitates new collaborations and mathematical advances through intensive residential research programs, each lasting 1–4 weeks.

More information about this series at https://link.springer.com/bookseries/15890

David R. Wood
Editor-in-Chief

Jan de Gier · Cheryl E. Praeger · Terence Tao
Editors

2019-20 MATRIX Annals

ISSN 2523-3041 ISSN 2523-305X (electronic)
MATRIX Book Series
ISBN 978-3-030-62496-5 ISBN 978-3-030-62497-2 (eBook)
https://doi.org/10.1007/978-3-030-62497-2

Mathematics Subject Classification: 00-XX

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Preface

MATRIX is Australia's international and residential mathematical research institute. It was established in 2015 and launched in 2016 as a joint partnership between Monash University and The University of Melbourne, with seed funding from the ARC Centre of Excellence for Mathematical and Statistical Frontiers. In 2020, The Australian National University joined MATRIX in a three-way partnership. The purpose of MATRIX is to facilitate new collaborations and mathematical advances through intensive residential research programs, which are currently held in Creswick, a small town nestled in the beautiful forests of the Macedon Ranges, 130km west of Melbourne.

This book is a scientific record of the ten programs held at MATRIX in 2019 and two programs held in January 2020:

- *Topology of Manifolds: Interactions Between High and Low Dimensions*
  Guest editors: Diarmuid Crowley, Stefan Friedl, Stephan Tillmann
- *Australian-German Workshop on Differential Geometry in the Large*
- *Aperiodic Order meets Number Theory*
  Guest editors: Michael Baake and Uwe Grimm
- *Ergodic Theory, Diophantine Approximation and Related Topics*
  Guest editor: Mumtaz Hussain
- *Influencing Public Health Policy with Data-informed Mathematical Models of Infectious Diseases*
  Guest editor: Jennifer Flegg
- *International Workshop on Spatial Statistics*
  Guest editor: Pavel Krupskiy
- *Mathematics of Physiological Rhythms*
  Guest editor: Maia Angelova
- *Conservation Laws, Interfaces and Mixing*
  Guest editors: Snezhana I. Abarzhi, Alexander Nepomnyashchy, Anthony J. Roberts, Joseph Klewicki
- *Structural Graph Theory Downunder*
  Guest editor: Anita Liebenau
- *Tropical Geometry and Mirror Symmetry*
  Guest editor: Mandy Cheung
- *Early Career Researchers Workshop on Geometric Analysis and PDEs*
  Guest editor: Paul Bryan
- *Harmonic Analysis and Dispersive PDEs: Problems and Progress*
  Guest editor: Kenji Nakanishi

The MATRIX Scientific Committee selected these programs based on scientific excellence and the participation rate of high-profile international participants. This committee consists of: David Wood (Monash Uni., Chair), Ben Andrews (Australian National Uni.), Santiago Badia (Monash Uni.), Peter Bouwknegt (Australian National Uni.), Peter Bühlmann (ETH Zurich), Alison Etheridge (Uni. Oxford), Jan de Gier (Uni. Melbourne), Cecilia González Tokman (Uni. Queensland), Frances Kuo

(UNSW Sydney), Joshua Ross (Uni. Adelaide), Terence Tao (Uni. California, Los Angeles), Ole Warnaar (Uni. Queensland), and Geordie Williamson (Uni. Sydney).

These programs involved organisers from a variety of Australian universities, including Adelaide, Deakin, LaTrobe, Macquarie, Monash, Melbourne, Newcastle, UNSW, Sydney, Western Australia, along with international organisers and participants.

Each program lasted 1–4 weeks, and included ample unstructured time to encourage collaborative research. Some of the longer programs had an embedded conference or lecture series. All participants were encouraged to submit articles to the MATRIX Annals.

The articles were grouped into refereed contributions and other contributions. Refereed articles contain original results or reviews on a topic related to the MATRIX program. The other contributions are typically lecture notes or short articles based on talks or activities at MATRIX. A guest editor organised appropriate refereeing and ensured the scientific quality of submitted articles arising from each program. The Editors (Jan de Gier, Cheryl E. Praeger, Terence Tao and myself) finally evaluated and approved the papers.

Many thanks to the authors and to the guest editors for their wonderful work.

MATRIX is hosting 12 programs in 2021, with more to come beyond that; see `www.matrix-inst.org.au`. Our goal is to facilitate collaboration between researchers in universities and industry, and increase the international impact of Australian research in the mathematical sciences.

David R. Wood
MATRIX Annals Editor-in-Chief

Topology of Manifolds: Interactions Between High and Low Dimensions

7 – 18 January 2019

Organisers

Jonathan Bowden
Uni. Regensburg

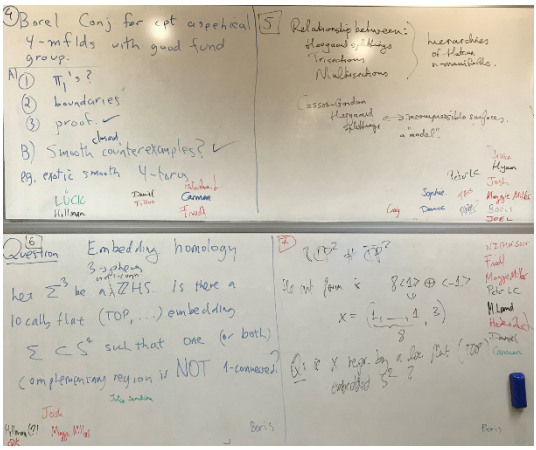
Diarmuid Crowley
Uni. Melbourne

Stefan Friedl
Uni. Regensburg

Stephan Tillmann
Uni. Sydney

Jim Davis
Indiana Uni.

Carmen Rovi
Uni. Heidelberg



Participants

Jonathan Bowden (Monash Uni.), Diarmuid Crowley (Uni. Melbourne), Jim Davis (Indiana Uni.), Stefan Friedl (Uni. Regensburg), Carmen Rovi (Indiana Uni.), Stephan Tillmann (Uni. Sydney), Wolfgang Lück (Uni. Bonn, Germany), Andras Stipsicz (Hungarian Acad. Sci.), Bea Bleile (Uni. New England), Jessica Purcell (Monash Uni.), Jae Choon Cha (POSTECH), Abby Thompson (Uni. California, Davis), Ana Lecuona (Uni. Glasgow), Daniel Kasprowski (Uni. Bonn), Imi Bokor, Jonathan Hillman (Uni. Sydney), Craig Hodgson (Uni. Melbourne), Hyam Rubinstein (Uni. Melbourne), Markus Land (Uni. Regensburg), Boris Lishak (Uni. Sydney), Fabian Hebestreit (Uni. Bonn), Ruth Kellerhals (Uni. Fribourg), Joel Hass (Uni. California, Davis), Julia Semikina (Uni. Bonn), Christoph Winges (Uni. Bonn), Fabian Henneke (Uni. Bonn), Fabio Gironella (Alfred Renyi Inst.), Josh Howie (Monash Uni.), Anthony Conway (Uni. Durham), Sylvain Cappell (Courant Inst.), Irving Dai (Princeton Uni.), Jen Hom (Georgia Tech.), Qayum Khan (Saint Louis Uni.), Peter Lambert-Cole (Georgia Tech.), Adam Levine (Duke Uni.), Duncan McCoy (Uni. Texas Austin), Maggie Miller (Princeton Uni.), Kent Orr (Indiana Uni.), Lisa Piccirillo (Uni. Texas Austin), Linh Truong (Columbia Uni.), Min Hoon Kim (Korea Inst. Advanced Study), Csaba Nagy (Uni. Melbourne), Johanna Meumertzheim (Uni. Regensburg), Dominic Tate (Uni. Sydney), Huijun Yang (Henan Uni.), Johnny Nicholson (Uni. College London), Sophie Ham (Monash Uni.), Kevin Yin (Courant Inst.)

This workshop explored connections between the study of manifolds in high and low dimensions, via the comparison of phenomena and methods across dimensions and via analysing higher dimensional spaces in terms of lower-dimensional subspaces.

Low-dimensional spaces ($n \leqslant 4$) appear naturally in physics, for example as the dimensions of space and space-time, and exhibit unique phenomena. Higher dimensional spaces ($n > 4$) arise as the parameters spaces of complex systems. The areas of low-dimensional topology and high-dimensional topology have developed rather independently since the days of Milnor and Smale, reflecting the differing nature of problems in dimensions three and four and in higher dimensions. In dimension three Thurston's geometrisation program led to the possibility of a complete classification of 3–manifolds. Dimension four is marked by the failure of the Whitney trick and is intermediate between high and low dimensions. In dimensions five and higher, surgery theory and smoothing theory provide powerful tools for analysing manifolds.

The workshop was organised around three key elements, listed here in the order in which they made their first entrance during the two-week program:

- **Lecture Series** designed to provide bridges between the different areas represented at the workshop,
- **Problem Sessions and Working Groups** designed to stimulate interaction and collaboration between researchers from different areas, and
- **Research Talks** addressed at a wide audience.

Lecture Series

Three lecture series were given in the mornings of the first week of the workshop, and were supported by discussion and exercise sessions in the afternoons.

Surgery: high-dimensional methods in low dimensions
by Diarmuid Crowley, Jim Davis and Kent Orr

This lecture series gave an introduction to topological 4–manifolds, normal maps and the surgery obstruction, reviewing the work of Wall and Cappell-Shaneson and the stable s/h cobordism theorem, as well as Kreck's surgery machine for classification. This led to the stable classification of $2q$–manifolds and in particular 4–manifolds. Further topics included the Q–form conjecture, application of the surgery machine in low dimensions and an overview of the current state of knowledge concerning topological concordance of classical knots.

The (stable) Cannon Conjecture
by Wolfgang Lück

Starting with an introduction of 3–manifold theory and properties of hyperbolic groups, this lecture series centred around the statement of Cannon's conjecture that a torsionfree hyperbolic group has the 2–sphere as its boundary if and only if it is the fundamental group of a closed hyperbolic 3–manifold. After a discussion of

topological rigidity and L^2–invariants, the lectures culminated in a sketch of the recent proof by Ferry, Lück and Weinberger of the Stable Cannon Conjecture.

Invariants of knots from Heegaard Floer homology
by András Stipsicz

The third lecture series moved from the theory of Heegaard diagrams of 3–manifolds to the definitions and properties of Floer homology theory and Knot Floer homology theory. With these tools in hand, the invariant Υ_K of a knot K invented by Ozsváth, Stipsicz, and Szabó was defined, and numerous applications of this concordance invariant were given.

Problem Sessions and Working Groups

Key elements to our workshop were the organised problem sessions and working groups. We had invited participants working in different areas of topology, and many of them had never met before, let alone glanced at each other's work. Before the workshop, we encouraged participants to submit difficult problems that they feel cannot be tackled from one viewpoint alone, or which aim to translate methods or insights from one area to another.

During the lunch break before our first Open Problem Session on the Monday afternoon of the first week, we asked participants to write their problems on the boards in the main lecture hall. During the session, they then had five minutes (or thereabouts) to explain their problem and answer questions. The session concluded with each participant writing their name next to every problem they were interested in. An example of this is shown in the image above.

The organisers then looked at which problems made sense to run concurrently and allocated time and space for groups to meet and work on a subset of the problems. A deciding factor was to create diverse groups, bringing together researchers from different areas and career stages. The groups would meet each day of the workshop, and we also had regular sessions with all participants in which the working groups reported on progress, asked for input and received feedback. This gave the opportunity to shift focus (for instance, after declaring victory or defeat on a problem), to move to other problems that were initially posed or to formulate new ones.

MATRIX house, which allowed participants to wander from one working group to another, provided an ideal environment for this flexible and collaborative approach.

The problems and progress reports were collected on the online platform Manifold Atlas, `http://www.map.mpim-bonn.mpg.de/`, where we expect to keep track of these and related problems.

Several new participants joined in the second week of the workshop, which therefore included another Open Problem Session to expand and continue the work done in the first week. The papers and the problem list published in this book, as well as additional publications, and the progress reported in the Manifold Atlas are testament to the fruitful interactions at the workshop and indicate that there is scope for deeper synergy between these areas.

Research Talks

The second week featured research talks by invited speakers in the mornings, ranging from graduate students to seasoned and established experts, and covering all aspects of this program.

Bea Bleile (Uni. New England)
Homotopy Types of Poincare Duality Complexes
Sylvain Cappell (NYU Courant Institute)
Using Atiyah-Bott classes to produce polynomial invariants of 3–manifolds
Jae Choon Cha (Postech)
Freely slicing good boundary links with a homotopically trivial plus property
Jen Hom (Georgia Tech)
An infinite-rank summand of the homology cobordism group
Qayum Khan (Saint Louis Uni.)
Stable existence of incompressible 3–manifolds in 4–manifolds
Daniel Kasprowski (Uni. Bonn)
$\mathbb{C}P^2$*–stable diffeomorphism of 4–manifolds*
Peter Lambert-Cole (Georgia Tech.)
Bridge trisections and the Thom conjecture
Markus Land (Uni. Regensburg)
A vanishing theorem for tautological classes of aspherical manifolds
Ana Lecuona (Uni. Glasgow)
Torus knots and rational homology balls
Adam S. Levine (Duke Uni.)
Simply-connected, spineless 4–manifolds
Maggie Miller (Princeton Uni.)
Extending fibrations from knot complements to ribbon disk complements
Csaba Nagy (Uni. Melbourne)
The Q–form conjecture for some 1–connected manifolds
Lisa Piccirillo (Uni. Texas Austin)
The Conway knot is not slice
Jessica Purcell (Monash Uni.)
Combinatorial criteria to determine whether a state surface is a fiber
Hyam Rubinstein (Uni. Melbourne)
Multisections of PL manifolds
Abigail Thompson (Uni. California Davis)
Trisections and surgery questions on links in 3–manifolds
Christoph Winges (Uni. Bonn)
Mapping class groups of high-dimensional, aspherical manifolds

Conclusion

The papers collected in this volume give evidence that this workshop did indeed achieve its aim of stimulating new work through the interaction of topologists working in different subfields that do not usually meet. The organisers expect further work to be submitted elsewhere to materialise in the near future.

We are writing this document in August 2020, a time when most conferences and workshops planned for the current year have been cancelled or been moved to on-line formats. The energetic discussions in Creswick often lasted over many hours spent at blackboards with intermittent walks through the bush. They resulted in knowledge transfer and progress in research that would have otherwise not seemed possible. Such intensive interaction is difficult to accomplished via on-line solutions with the currently available technology. We hope that what now feels like a distant past will become a (virtual?) reality in the not so distant future.

Diarmuid Crowley, Stefan Friedl, Stephan Tillmann
Guest editors

Australian-German Workshop on Differential Geometry in the Large

4 – 15 February 2019

Organisers

Owen Dearricott
Uni. Melbourne

Diarmuid Crowley
Uni. Melbourne

Thomas Leistner
Uni. Adelaide

Yuri Nikolayevsky
LaTrobe Uni.

Wilderich Tuschmann
Karlsruhe Uni.

Katrin Wendland
Freiburg Uni.

Participants

Diarmuid Crowley (Uni. Melbourne), Owen Dearricott (Uni. Melbourne), Thomas Leistner (Uni. Adelaide), Wilderich Tuschmann (Karlsruhe Institute of Technology), Yuri Nikolayevsky (La Trobe Uni.), Ben Andrews (Australian National Uni.), Burkhard Wilking (Uni. Muenster), Christoph Böhm (Uni. Muenster), Claude LeBrun (Stony Brook Uni.), Thomas Farrell (Tsinghua Uni.), Frances Kirwan (Oxford Uni.), Fuquan Fang (Capital Normal Uni.), Guofang Wei (Uni. California Santa Barbara), Neil Trudinger (Australian National Uni.), Peter Petersen (Uni. California Los Angeles), Robert Bryant (Duke Uni.), Rod Gover (Uni. Auckland), Ramiro Lafuente (Uni. Queensland), Karsten Grove (Uni. Notre Dame), Sebastian Goette (Uni. Freiburg), Lashi Bandara (Uni. Potsdam), Katharina Neusser (Masaryk Uni.), Artem Pulemotov (Uni. Queensland), Jesse Gell-Redman (Uni. Melbourne), Lee Kennard (Syracuse Uni.), Haotian Wu (Uni. Sydney), Paul Bryan (Macquarie Uni.), Julian Scheuer (Uni. Freiburg), Krishnan Shankar (Uni. Oklahoma), Xianzhe Dai (Uni. California Santa Barbara), Fernando Galaz-Garcia (Karlsruhe Institute of Technology), Valentina Wheeler (Uni. Wollongong), Julie Clutterbuck (Monash Uni.), Martin Kerin (Uni. Muenster), Fred Wilhelm (Uni. California Riverside), Catherine Searle (Wichita State Uni.), Mathew Langford (Uni. Tennessee, Knoxville), Uwe Semmelmann (Uni. Stuttgart), Joseph Wolf (Uni. California Berkeley), Tracy Payne (Idaho State Uni.), Boris Vertman (Uni. Oldenburg), Pedro Solarzano (UNAM-CONACYT Oaxaca), Jim Davis (Indiana Uni.), Lorenz Schwachhoefer (TU Dortmund), Stephan Klaus (MFO Oberwolfach/Uni. Mainz), Klaus Kröncke (Uni. Hamburg), Matthias Ludewig (Uni. Adelaide), Vicente Cortes (Uni. Hamburg), Vladimir

Matveev (Uni. Jena), Charles Boyer (Uni. New Mexico), Vincent Pencastaing (Uni. Luxembourg), Fernando Cortes Kuehnast (TU Berlin), William Campbell Wylie (Syracuse Uni.), Franziska Beitz (WWU Münster), James McCoy (Uni. Newcastle), Megan Kerr (Wellesley College), Adam Moreno (Uni. Notre Dame), Anusha Krishnan (Uni. Pennsylvania), Curtis Porter (North Carolina State Uni.), Romina Arroyo (Uni. Queensland), Gerd Schmalz (Uni. New England), Nan Li (City Uni. New York), Zheting Dong (Oregon State Uni.), Changwei Xiong (Australian National Uni.), Xianfeng Wang (Australian National Uni.), Yuhan Wu (Uni. Wollongong), Brett Parker (Monash Uni.), Jian He (Monash Uni.)

The first week of this program took the form of an international conference with several prominent keynote speakers. These included Ben Andrews and Neil Trudinger from Australia; Rod Gover from New Zealand; Christoph Böhm and Burkhard Wilking from Germany; Robert Bryant, Karsten Grove, Claude LeBrun, Peter Petersen and Guofang Wei from the United States; Dame Frances Kirwan from the United Kingdom; and Tom Farrell and Fuquan Fang from China. Additional contributed talks were delivered in topics across differential geometry and geometric analysis.

In the second week, the meeting was less formal with specialised talks in parallel sessions in the mornings and free time for discussion and research in the remainder of the day. The parallel sessions were organised around themes which included:

- geometric evolutions equations and curvature flow,
- structures on manifolds and mathematical physics,
- higher invariants and positive scalar curvature, and
- recent developments in non-negative sectional curvature.

The organisers wish to thank MATRIX for hosting the event; and the Australian Mathematical Sciences Institute, the Australian Mathematical Society, DFG national priority research scheme “Geometry at Infinity, SPP2026”, the National Science Foundation, the University of Melbourne International Research and Research Training Fund, La Trobe University, and the Ian Potter Foundation for their financial support.

A separate proceedings volume for this meeting will be published as “Differential Geometry in the Large”, London Mathematical Society Lecture Note Series (463), Cambridge University Press.

Owen Dearricott, Diarmuid Crowley
for the organisers


AUSTRALIAN-GERMAN
WORKSHOP ON
DIFFERENTIAL
GEOMETRY IN THE LARGE
CRESWICK
MATRIX
MATRIX

Aperiodic Order meets Number Theory

25 February – 1 March 2019

Organisers

Michael Baake
Bielefeld Uni.

Michael Coons
Uni. Newcastle

Uwe Grimm
Open U.

John A. G. Roberts
UNSW Sydney

Reem Yassawi
Uni. Claude Bernard Lyon 1

Participants

Shigeki Akiyama (Uni. Tsukuba, Japan), Michael Baake (Bielefeld Uni.), Valérie Berthé (Uni. Paris Diderot), Yann Bugeaud (Uni. Strasbourg), Álvaro Bustos (Uni. Chile), Michael Coons (Uni. Newcastle), María-Isabel Cortez (Uni. Santiago Chile), Karma Dajani (Utrecht Uni.), David Damanik (Rice Uni.), Robbert Fokkink (TU Delft), Franz Gähler (Bielefeld Uni.), Amy Glen (Murdoch Uni.), Uwe Grimm (Open Uni.), Mumtaz Hussain (LaTrobe Uni.), Jeffrey C. Lagarias (Uni. Michigan), Dong-il Lee (Seoul Women's Uni.), Jeong-Yup Lee (Kwandong Uni.), Mariusz Lemańczyk (Nicolaus Copernicus Uni.), Manuel J. C. Loquias (Uni. Philippines), Michael Mampusti (Uni. Wollongong), Neil Mañibo (Bielefeld Uni.), Robert V. Moody (Uni. Victoria, Canada), John A. G. Roberts (UNSW Sydney), Tanja Schindler (Australian National Uni.), Bernd Sing (Uni. West Indies), Nicolae Strungaru (MacEwan Uni.), Venta Terauds (Uni. Tasmania), Franco Vivaldi (Queen Mary Uni. London), Peter Zeiner (Xiamen Uni. Malaysia)

This workshop benefited from the participation of a diverse group of 29 mathematicians ranging from world-experts and rising stars to eager new doctoral students. Our common thread was a desire to understand the connections between aperiodic order and number theory and to consider the further development of those connections.

During the week of our workshop, we had about four talks a day, two of which formed a pair of shorter talks on a coordinated theme. They covered topics from harmonic analysis, dynamical systems, ergodic theory, discrete geometry, number theory, topological dynamics, spectral theory, algebra and invariants. Most topics had connections to number theory, which occurred on various levels. At present, the majority of connections are of the form that known results from elementary, algebraic and analytic number theory are helping to answer questions in aperiodic order.

However, there is an increasing activity on open problems in number theory such as the Möbius disjointness conjecture or connections to the Riemann hypothesis.

Even though aperiodic order at present is profiting more from number theory than the other way round, it became clear that there is an increasing potential for the reverse direction. This view was strengthened by conversations with number theorists in attendance including Yann Bugeaud, Jeffrey Lagarias and Michael Coons. Each of these number theorists has interests in integer sequences and, in particular, the statistical properties of base expansions of integers. Questions in this area are in a unique position to be considered in the context of aperiodic order, and it is our hope that results in aperiodic order can lead to new number theoretic results. What is interesting is that this connection is not new — indeed it goes back to a near collaboration between the famous American mathematician Norbert Wiener and the famous German-Australian number theorist Kurt Mahler.

In 1926, Norbert Wiener received a Guggenheim fellowship to work with Max Born in Göttingen and then to travel on to work with Niels Bohr in Copenhagen. In that year, Born's assistant was Werner Heisenberg, who would follow Wiener to Copenhagen and develop what would later become his famous uncertainty principle. It is in this setting that, while in Göttingen, Wiener was given an (unpaid) assistant — the young Kurt Mahler! Collectively, Wiener and Mahler produced a two-part series of papers entitled, "The spectrum of an array and its application to the study of the translational properties of a simple class of arithmetical functions." As Wiener introduces his part, he writes

> "The purpose of the present paper is to extend the spectrum theory already developed by the author in a series of papers to the harmonic analysis of functions only defined for a denumerable set of arguments — *arrays*, as we shall call them — and **the application of this theory to the study of certain power series admitting the unit circle as an essential boundary**." (Boldness added by author.)

Concerning the actual contribution, given a sequence A, Wiener describes a method to construct a monotone non-decreasing function $A(x)$, which he calls the *spectral function of* A. By a result of Fréchet, $A(x)$ may contain three possible additive parts: a monotone step function, a function which is the integral of its derivative, and a continuous function which has almost everywhere a zero derivative. In modern terminology, what Wiener is describing is how one can associate a measure to the sequence A. The three possible parts of the measure are then described by the Lebesgue decomposition theorem: *Any regular Borel measure* μ *on* $\mathbb{R}^d$ *has a unique decomposition* $\mu = \mu_{\mathrm{pp}} + \mu_{\mathrm{ac}} + \mu_{\mathrm{sc}}$ *where* $\mu_{\mathrm{pp}} \perp \mu_{\mathrm{ac}} \perp \mu_{\mathrm{sc}} \perp \mu_{\mathrm{pp}}$ *and also* $|\mu| = |\mu_{\mathrm{pp}}| + |\mu_{\mathrm{ac}}| + |\mu_{\mathrm{sc}}|$. Here, μ_{pp} is a pure point measure corresponding to the monotone step function, μ_{ac} is an absolutely continuous measure corresponding to the function that is the integral of its derivative, and μ_{sc} is a singular continuous measure corresponding to the continuous function which has almost everywhere a zero derivative. Wiener provided two examples giving pure point measures and absolutely continuous measures, respectively, and an 'almost all' result for examples having a singular continuous measure. As it turns out, periodic sequences give pure point measures. Wiener's example giving an absolutely continuous measure is reminiscent of the sequence of digits of Champernowne's number. Mahler's contribution

is to find a piece of hay in the haystack — an example of a sequence whose associated measure is singular continuous. His example, the Thue–Morse sequence, is paradigmatic and started an area of transcendence theory now called Mahler's method. The Thue–Morse sequence $\{t(n)\}_{n\geqslant 0}$ is defined by $t(0) = 1$, $t(1) = -1$, $t(2n) = t(n)$ and $t(2n+1) = -t(n)$. This sequence is now ubiquitous in the areas of theoretical computer science and symbolic dynamics.

Two areas emerged, then diverged, from these two related papers. Therein lies what the participants of this conference intend to do: *to bring back together these areas and to use the results of aperiodic order to address fundamental questions in number theory, such as those concerning power series that have the unit circle as a natural boundary*; that is, to address Wiener's original purpose in studying the harmonic analysis of functions on countable sets!

The first paper arising from this program discusses the origin and structure of the field of aperiodic order. The other 18 papers are extended abstracts of the presented talks.

The Guest Editors would like to thank Michael Coons who co-authored this summary.

Michael Baake and Uwe Grimm
Guest editors

Ergodic Theory, Diophantine Approximation and Related Topics

17 – 28 June 2019

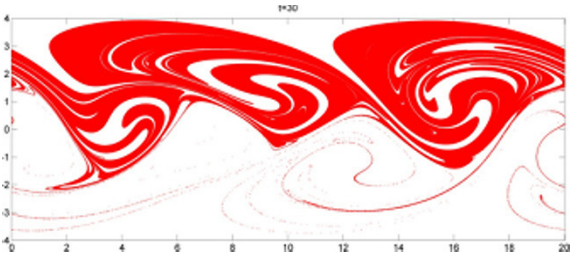
Organisers

Dzmitry Badziahin
Uni. Sydney

Alexander Fish
Uni. Sydney

Mumtaz Hussain
La Trobe Uni.

Bao-Wei Wang
Huazhong Uni.



Participants

Jinpeng An (Peking Uni.), Dzmitry Badziahin (Uni. Sydney), Michael Bjorklund (Chalmers Uni), Michael Coons (Uni. Newcastle), Alexander Fish (Uni. Sydney), Alexander Gorodnik (Uni. Zürich), Mumtaz Hussain (LaTrobe Uni.), Dmitry Kleinbock (Brandeis Uni.), Bing Li (South China Uni. Tech.), Nikolay Moshchevitin (Moscow State Uni.), Johannes Schleischitz (Middle East Cyprus Uni.), Lovy Singhal (Peking Uni.), Sanju Velani (Uni. York), Oleg German (Moscow State Uni.), Sam Chow (Uni. Oxford), Changhao Chen (UNSW Sydney), Ayreena Bakhtawar (LaTrobe Uni.)

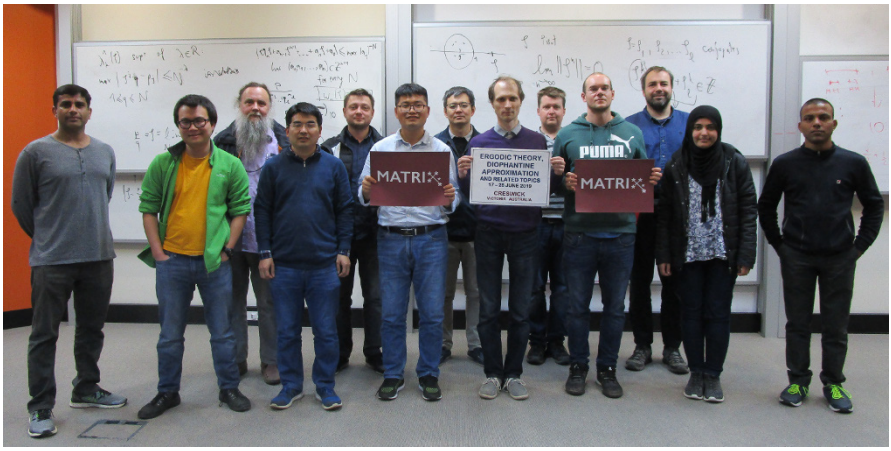
This two-week research workshop was a continuation of the conference "Dynamics and number theory" held at the University of Sydney (11–14 June 2019). The workshop was on interconnected topics in Ergodic Theory and Analytical Number Theory with the focus on Diophantine Approximation. Progress on cutting edge problems in these fields were presented and discussed in a free flowing manner. The focus was on methods and techniques that could lead to the resolution of some long standing open problems such as the Littlewood Conjecture (1930), Wirsing's problem (1961), and Generalised Baker-Schmidt Problem (1970), etc. During the workshop we learned that Koukoulopoulos and Maynard had resolved the Duffin-Schaeffer Conjecture; the official announcement followed a few days later.

During the workshop several new collaborations emerged (as detailed below) and progress was made on several long standing open problems such as the Wirsing Problem. There were two expository talks every day followed by several hours of research collaboration time. The covered topics included,

- **Diophantine exponents:** This topic was specifically discussed by Badziahin, Moshchevitin, German, Schleischitz, and Chow. In particular, Badziahin and Schleischitz improved bounds on Wirsing's problem during this workshop (https://arxiv.org/abs/1912.09013). This paper is now published in Transactions of the American Mathematical Society (https://doi.org/10.1090/tran/8245).

- **Diophantine approximations on fractal sets:** Schleischitz and Singhal discussed various problems concerning Diophantine approximation on fractal sets.
- **Metric recurrence and shrinking target problems**: Hussain and Li worked on this problem and made some progress in establishing the metrical theory for shrinking target and recurrence problems for dynamical systems satisfying some natural conditions. The systems include the continued fractions, beta dynamical systems, and homogeneous self-similar sets.
- **Singular vectors on manifolds and fractals:** Kleinbock and Moshchevitin worked on proving the existence of totally irrational vectors and linear forms with large uniform Diophantine exponents; see https://arxiv.org/abs/1912.13070.
- **Generalised Baker-Schmidt problem on manifolds:** Badziahin, Hussain, and Schleschitz discussed Diophantine approximation problems on manifolds especially the generalised Baker-Schmidt problem. In particular, Hussain and Schleischitz made progress in settling this problem for all non-degenerate co-dimensional two manifolds not only for the Euclidean setting but also for p-adics.
- **Uniform Diophantine approximation:** Hussain and Kleinbock discussed improvements to Dirichlet's theorem. An article is in preparation on this topic.
- **Multiplicative Diophantine approximation:** Gorodnik, Badziahin, Fish, Chow, Moshchevitin, and German discussed problems within the theory of multiplicative Diophantine approximation such as the well-known Littlewood conjecture (1930). In particular, Chow presented his results using Bohr sets and German by using the parametric geometry of numbers.
- **Central limit theorems and Diophantine approximation:** Bjorklund and Gorodnik discussed this topic.
- **Hitting probabilities and shrinking targets:** Li and Velani initiated a collaboration on hitting probabilities within the shrinking target settings of dynamical systems.

Mumtaz Hussain
Guest editor



Influencing Public Health Policy with Data-informed Mathematical Models of Infectious Diseases

1 – 12 July 2019

Organisers

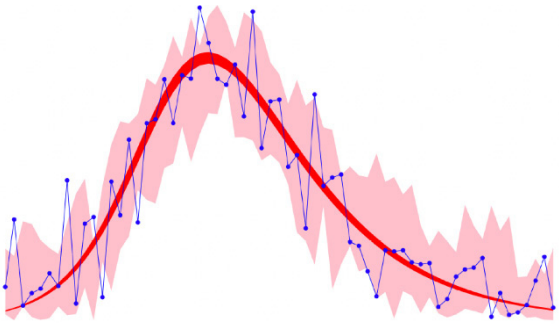
Jennifer Flegg
Uni. Melbourne

James McCaw
Uni. Melbourne

Joshua Ross
Uni. Adelaide

Thomas House
Uni. Manchester

Ben Cooper
Mahidol, Uni. Oxford



Participants

Jennifer Flegg (Melbourne), James McCaw (Melbourne), Joshua Ross (Adelaide), Thomas House (Manchester), Jonathan Keith (Monash), Lisa White (Oxford), Nick Golding (Melbourne), Deborah Cromer (UNSW), Andrew Black (Adelaide), Ada Yan (Imperial College), Jason Whyte (Melbourne), Sai Thein Than Tun (Oxford), Carla Ewels (JCU), Alex Zarebski (Oxford), James Walker (Adelaide), Amani Alahmadi (Monash), Sarah Belet (Monash), Nurul Anwar (Melbourne), Freya Shearer (Melbourne), Pavithra Jayasundara (UNSW), Hom Nath Dhungana (UTS), Zari Dzalilov (Federation), Rob Moss (Melbourne)

With the ever-growing emphasis on the importance of sound evidence in healthcare decision-making and policy, the power of data-informed mathematical models to provide much needed insight is substantial. In order for conclusions drawn from a mathematical model to be reliable, it is essential for unknown model parameters to be estimated from data in a statistically sound manner and to account for uncertainty in the parameter values. Our MATRIX workshop brought together local and international experts in this area to discuss the use of existing statistical methods and showcase new methods for parameter estimation in models of infectious diseases.

During the program there were three groups, each working on a focus problem:

- using prior knowledge to improve inference and forecasting of infectious disease transmission;
- integrating multiple data sources in infectious disease modelling;
- fitting complex models: identifying the problems and the solutions.

The first week of the program saw the introduction of the three focus problems, collaborative time on the focus problems, a software demonstration (GRETA) and

scientific talks. The second week of the program was focussed around more collaborative time but also saw a software demonstration (SHINY) and more scientific talks.

The third focus group, which focused on issues of parameter identifiability, soon found that there was significant theory underlying the topic. One of the focus group leaders, Dr Jason Whyte, has put together a review for this book which is entitled "Model structures and structural identifiability: What? Why? How?". This paper provides an overview of the importance of structural global identifiability in dynamical systems models, details some essential theory and distinctions, and demonstrates these by some key examples.

Jennifer Flegg
Guest editor

International Workshop on Spatial Statistics

15 – 19 July 2019

Organisers

Tingjin Chu
Uni. Melbourne

Davide Ferrari
Free Uni. Bozen-Bolzano

Wei Lin
Peking Uni.

Aihua Xia
Uni. Melbourne

Sanming Zhou
Uni. Melbourne

Lawrence Reeves
Uni. Melbourne

Participants

Tingjin Chu (Uni. Melbourne), Pavel Krupskiy (Uni. Melbourne), Guoqi Qian (Uni. Melbourne), Noel Cressie (Uni. Wollongong), Jun Zhu (Uni. Wisconsin-Madison), Erin Peterson (Queensland Uni. Technology), Le Chang (The Australian National Uni.), Andriy Olenko (La Trobe Uni.), Yan Wang (RMIT Uni.), Wei Lin (Peking University, China), Wenlin Dai (Renmin Uni. China), Xianbin Cao (Uni. Melbourne), Illia Donhauzer (La Trobe Uni.), Ravindi Nanayakkara (La Trobe Uni.), Nan Zheng (Monash Uni.), Jianzhong Qi (Uni. Melbourne), Jiabao Li (Leon), (Uni. Melbourne), Benjamin Hines (Uni. Melbourne), Manling Qian (Uni. Melbourne), Paul Keeler (Uni. Melbourne), Alan Pearse (Queensland Uni. Technology)

The program brought together internationally recognised experts in spatial statistics as well as early career researchers to discuss statistical modelling and applications. Scholars from fields outside of statistics also attended the workshop, and interdisciplinary research topics were discussed. The focus of this workshop was to explore new methodology and develop computational tools to make challenging problems in spatial statistics more tractable.

During the program, the following topics were discussed in small groups:

- New methodology for spatial data with complex dependence structures including models for non-Gaussian spatial processes, spherical data and categorical/proportional spatial multivariate data.
- Models and inference for high-dimensional data with temporal dependence dynamics and spatial covariates with applications in climatology, geology and environmental science. Outlier detection for multivariate functional data.

- Spatial data reconstruction tools and construction of spatial maps for sparse data. Uncertainty quantification for data coming from different sources.

Young researchers had opportunity to interact with senior academics, and several discussion groups were organized to discuss challenging problems in spatial statistics and to create new collaboration opportunities. This lead to the paper in this volume by B. Hines, Y. Kuleshov and G. Qian "Spatial modelling of linear regression coefficients for gauge measurements against satellite estimates," which studies the problem of predicting rainfall in remote areas of Australia using satellite estimates.

Pavel Krupskiy
Guest editor

Mathematics of Physiological Rhythms

9 – 13 September 2019

Organisers

Maia Angelova
Deakin Uni.

James Sneyd
Uni. Auckland

Aneta Stefanovska
Lancaster Uni.

Plamen Ivanov
Uni. Boston, Uni. Harvard

Participants

Maia Angelova (Deakin Uni.), Aneta Stefanovska (Lancaster Uni.), Plamen Ch. Ivanov (Boston Uni.), Anne Skeldon (Uni. Surrey), Krasimira Tsaneva-Atanassova (Uni. Exeter), Adelle Coster (UNSW Sydney), Andrew Phillips (Monash Uni.), David Liley (Uni. Melbourne), Ruben Fossion (National Autonomous Uni. Mexico), Chandan Karmakar (Deakin Uni.), Ye Zhu (Deakin Uni.), Sutharshan Rajasegarar (Deakin Uni.), Christopher Stephens (National Autonomous Uni. Mexico), Sergiy Shelyag (Deakin Uni.), Shitanshu Kusmakar (Deakin Uni.), Jyotheesh Gaddam (Deakin Uni.), Mohammad Abdul Motin (Uni. Melbourne), Emerson Keenan (Uni. Melbourne), Shreyasi Datta (Uni. Melbourne), Md Ahsan Habib (Deakin Uni.), Jason Whyte (Uni. Melbourne), Tania Pencheva (Bulgarian Acad. Sci.), Anuroop Gaddam (Deakin Uni.)

This research retreat was devoted to novel dynamical system methodologies underpinning the modelling of complex physiological systems, and focused on four main topics: Network Physiology, Brain, Diabetes, and Sleep.

The aim was to unite and combine current trends in dynamical systems and time series analysis for solving problems in physiology which are governed by repeating processes. Examples are cardio-dynamics, sleep processes, glucose-insulin regulation and diabetes, and many others. The invited participants were experts in mathematics, physics and computer sciences working in applications of dynamical systems and time series in physiology, biology and medicine. The program explored the state-of the-art research underlying the mathematics of periodic and periodic-like processes in human physiology.

The program was attended by 20 participants funded by the MATRIX Institute, and three participants funded by other institutions. The participants were involved in four discussion and collaborative sessions each afternoon led by one of the plenary speakers. Each session was devoted to one of the main topics of the program. The participants were from five countries: Australia, USA, UK, Mexico and Bulgaria.

Women were well represented, four were plenary speakers, one invited speaker and one a PhD student. Furthermore, two out of the four organisers were women. The participants included world leading researchers in the field, early career researchers, postdocs and PhD students. They were experts in mathematical physiology, mathematical biology, differential equations, functional analysis, time series, fractals, statistical mechanics and phase transitions. A number of participants were also experts in data mining and machine learning, which would facilitate the use of such methods for parameter estimation.

The retreat focused on models based on deterministic and stochastic differential equations and delay differential equations, dynamical system approach to time series, statistical mechanics, phase transitions and mean field approaches. The mathematical models of regulation processes are often informed by data driven models, derived from spectral analysis and signal processing. Furthermore, as the large number of physiological parameters are difficult to measure, machine learning and statistical approaches were exploited to evaluate parameters. The models are based on real data measured from humans (ECG, EEG, actigraphy, eye movements), and complexity for building models from such data was discussed. The program addressed the aims of MATRIX by focusing on new mathematical models governing regulation and control processes in human physiology.

The program had one keynote talk and one invited talk each morning. The afternoons were spent on directed discussions around current trends and coordinated collaborative work. The first and the second day were devoted to Network Physiology and Diabetes. Plamen Ivanov (Boston) gave a fascinating lecture on Network Physiology. During the afternoons there was a session on Open Problems in Network Physiology led by Plamen Ivanov. On Tuesday morning Aneta Stefanovska (Lancaster) continued the theme on Network Physiology, and facilitated a 3 hour workshop on the new time series software tool, MODA, developed in Lancaster. This workshop was very useful for the PhD students attending the program. Another plenary session was focused on diabetes, where Adelle Coster gave a plenary talk and led a discussion session in the afternoon on Open Problems in Diabetes research. The topic on Wednesday was Brain. Krasimira Tsaneva-Atanassova (Exeter) gave the first plenary talk, followed by the talk given by David Liley. David led the discussion session before lunch on fitting complex mathematical models with a large number of parameters. The plenary talks on Thursday were on Sleep, given by Maia Angelova (Deakin) and Andrew Phillips (Monash). The afternoon was focused on collaborative work on Sleep. The closing session on Friday by plenary speaker Christopher Stephens (UNAM), a renowned expert in Data Mining in Healthcare, was a part of Network Physiology topic. The two invited talks given by Tania Pencheva (BAS) and Ruben Fossion (UNAM), and another two short “ignit” talks, were presented by Sutharshan Rajasegarar (Deakin) and Anuroop Gaddam (Deakin). A working group on Sleep was formed to work on models of insomnia; this group met daily during all days of the program. The group is currently active, submitting jointly co-authored papers and preparing an ARC grant proposal. On Wednesday afternoon a walk around Creswick wss organised to facilitate networking and the forming of new research links. This was particularly useful for PhD

students, early- and mid-career researchers, as it allowed them to talk to the leaders in the field in a relaxed atmosphere.

The aim and objectives of the program were completed. The participants expressed their gratitude to The MATRIX Institute for providing excellent conditions that enabled new research collaborations. The program was very useful and contributed to our long term goals to develop Deakin University as a Hub for Mathematical and AI modelling translated to health, physiology, wellbeing and health care. In addition to the papers appearing in this book, a number of papers arising from the program will appear in a special issue of *Frontiers of Physiology*. The organisers and participants gratefully acknowledge funding from the MATRIX Institute and Deakin University, School of IT.

Maia Angelova
Guest editor

Conservation Laws, Interfaces and Mixing

4 – 8 November 2019

Organisers

Snezhana I. Abarzhi
Uni. Western Australia

Neville Fowkes
Uni. Western Australia

Alik Nepomnyashchy
Technion, Israel

Anthony J. Roberts
Uni. Adelaide

Yvonne Stokes
Uni. Adelaide

Participants

Snezhana I. Abarzhi (Uni. Western Australia), Yasuhide Fukumoto (Kyushu Uni.), Ashleigh Hutchinson (Uni. Witwatersrand), Alexander Klimenko (Uni. Queensland), Joseph Klewicki (Uni. Melbourne), Alexander Nepomnyashchy (Technion), Xiaolin Li (State Uni. New York, Stony Brook), Tony Roberts (Uni. Adelaide), Mako Sato (Osaka City Uni.), Helen Wang (Zeta Global Inc.), Kurt Williams (Uni. Western Australia), Paulo de Almeida (Altron Bytes Systems Integration), Cameron Wright (Uni. Western Australia), Tanmay Agrawal (Uni. Melbourne), Saleh Tanveer (Ohio State Uni.), Ash Khan (RMIT Uni.)

Interfacial transport and mixing are non-equilibrium processes coupling kinetic and macroscopic scales. They occur in molecules, fluids, plasmas and materials over celestial events. Examples include supernovae and fusion, planetary convection and reactive fluids, wetting and adhesion, turbulence and mixing, nano-fabrication and bio-technology. Addressing the societal challenges posed by alternative energy sources, efficient use of non-renewable resources, and purification of water requires a better understanding of non-equilibrium interfacial transport and mixing.

The dynamics of interfacial transport and mixing often involve sharp changes of vector and scalar fields, and may also include strong accelerations and shocks, radiation transport and chemical reactions, diffusion of species and electric charges, among other effects. Interfacial transport and mixing are inhomogeneous, anisotropic, non-local, and statistically unsteady. At macroscopic scales, their spectral and invariant properties differ substantially from those of canonical turbulence. At atomistic and meso-scales, the non-equilibrium dynamics depart dramatically from the standard scenario given by Gibbs ensemble averages and the quasi-static Boltzmann equation. At the same time, non-equilibrium transport may lead to self-organization and order, thus offering new opportunities for diagnostics and control. Capturing

the properties of interfaces and mixing enables: the accurate description of conservation properties, the solution of boundary value problems, better understanding of Eulerian and Lagrangian dynamics, and the development of methods for control of non-equilibrium transport in nature and technology.

Significant success was recently achieved in the understanding of interfacial transport and mixing in terms of of theoretical analysis, large-scale numerical simulations, and data analysis. This success opened new opportunities for the study of the fundamentals of non-equilibrium dynamics across the scales, for developing a unified description of particles and fields on the basis of the synergy of theory and numerical data, and for applying the fundamentals of non-equilibrium transport to address the contemporary challenges of modern science, technology and society.

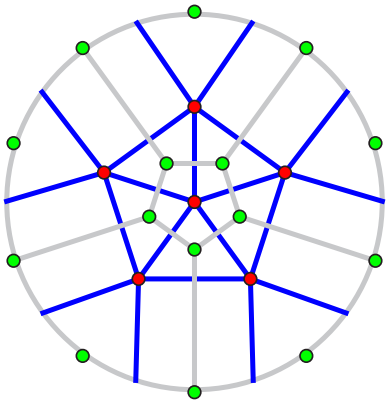
This program built upon recent achievements in understanding interfacial transport and mixing using theoretical analysis, large-scale numerical simulations, and data analysis. The focus was on conservation laws and boundary value problems. The program brought together researchers from applied mathematics, applied analysis, dynamical and complex systems, stochastic processes and data analysis, dynamics of fluid and plasmas, industrial mathematics and materials science. The program motivated discussions of rigorous mathematical problems, theoretical approaches and state-of-the-art numerical simulations along with advanced data analysis techniques. The program explored the state-of-the-art in the areas of interfaces and non-equilibrium transport, and charted new research directions in this field.

The participants included leading experts and researchers at all career stages from Australia and from abroad.

Snezhana Abarzhi, Alexander Nepomnyashchy, Anthony Roberts, Joseph Klewicki
Guest editors

Structural Graph Theory Downunder

25 November – 1 December 2019



Organisers

David Wood
Monash Uni.

Anita Liebenau
UNSW Sydney

Alex Scott
Uni. Oxford

Participants

Maria Chudnovsky (Princeton Uni.), Zdeněk Dvořák (Charles Uni.), Kevin Hendrey (IBS Korea), Tony Huynh (Monash Uni.), Gwenaël Joret (ULB Belgium), Nina Kamčev (Monash Uni.), Ringi Kim (KAIST Korea), Tereza Klimošová (Charles Uni.), Anita Liebenau (UNSW Sydney), Chun-Hung Liu (Texas A&M Uni.), Natasha Morrison (Uni. Cambridge), Marcin Pilipczuk (Uni. Warsaw), Bruce Reed (McGill Uni., Montréal), Alex Scott (Uni. Oxford), Paul Seymour (Princeton Uni.), Maya Stein (Uni. Chile), Jane Tan (Uni. Oxford), David Wood (Monash Uni.), Liana Yepremyan (London School Econs.), Yelena Yuditsky (Ben-Gurion Uni.), Xuding Zhu (Zhejiang Uni.)

This program consisted of a 1-week intensive research workshop, where mathematicians from across the globe came together to work on open problems in structural graph theory. The program featured a mix of early-career, mid-career and senior researchers; a mix of women and men; and a mix of people from Australia, Europe, North America, South America, Israel, China, and Korea. The goal was to create an environment where mathematicians at all career stages worked side-by-side. This goal was certainly achieved. Many participants commented on how conducive the MATRIX House was for doing collaborative research.

The majority of the time was allocated to collaborative research. In addition, there were six research talks about recent significant results:

- Xuding Zhu (Zhejiang Uni.) surveyed recent developments on Hedetniemi's Conjecture and the Poljak-Rödl function, including Shitov's recent breakthrough;
- Liana Yepremyan (London School Econs.) presented a proof of the size-Ramsey number of graphs of bounded degree and bounded treewidth;
- Tereza Klimošová (Charles Uni.) talked about edge-partitioning 3-edge-connected graphs;
- Chun-Hung Liu (Texas A&M Uni.) talked about clustered graph colouring, in particular, clustered variants of Hajós' Conjecture;