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Theory of Algebraic Surfaces

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Kunihiko Kodaira at his home in Tokyo, 1990 © Springer Japan

Foreword

In the academic year 1967, Kunihiko Kodaira gave a course of lectures at the University of Tokyo on the theory of complex algebraic surfaces. The lecture notes were published in 1968 as Volume 20 in the series of “Seminary Notes” by the University of Tokyo. That was the copy of the handwritten manuscript in Japanese by Shigeho Yamashima, based on his beautiful notes reflecting faithfully the atmosphere of Kodaira’s lectures. The present book is an English translation of that volume with slight modifications, correcting typos, etc. in the Japanese version. The readers are expected to have only the elementary prerequisites on complex manifolds as the background.

The book consists of two parts: Chaps. 1 and 2.

After stating the goal of the lecture in the Introduction, in Chap. 1, basic facts on algebraic surfaces are reviewed, touching upon divisors, linear systems, intersection theory, and the Riemann–Roch theorem. It provides an elegant introduction to the theory of algebraic surfaces covering some classical materials whose modern proofs first appeared in Kodaira’s papers. Among others, one can find a concise analytic proof of Gorenstein’s theorem for curves on a non-singular surface, which is a detailed explanation of the one given in Appendix I to “On Compact Complex Analytic Surfaces, I,” Ann. of Math. 71 (1960). Another highlight is the elementary proof of Noether’s formula for the arithmetic genus of an algebraic surface. Nowadays, the formula is known and treated as a special case of Hirzebruch’s Riemann–Roch theorem. Kodaira’s approach is based on the standard fact that, via generic projections, every algebraic surface can be obtained as the normalization of a surface with only ordinary singularities in the projective 3-space. However, unlike the other modern proofs, the argument does not rely on general facts, such as Porteus’ formula, which require a separate treatment. It is self-contained and follows a classical line, using Lefschetz pencils, much more in the style of Noether’s original proof.

The second part, Chap. 2, discusses the behaviors of the Pluri-canonical maps of algebraic surfaces of general type, as an application of the general theory provided in Chap. 1. It gives a detailed account of “Pluri-canonical Systems on Algebraic Surfaces of General Type,” J. Math. Soc. Japan 20 (1968). The main tool is a vanishing criterion for the first cohomology groups, which is known as Mumford's vanishing theorem. It is shown by the analytic method in essentially the same way as in the proof of Kodaira's vanishing theorem. It gives us, for example, the pluri-genus formula for algebraic surfaces of general type. Then by means of various composition series for pluri-canonical divisors, one can discuss when the pluri-canonical system is free from base points and when the pluri-canonical map is birational onto its image. Such excellent results have been extended by Kodaira himself, Enrico Bombieri, and others. So, we can now answer affirmatively to a question stated in Remark 9 in the Introduction.

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April 2020

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