Fernando de Holanda Barbosa
Luiz Antônio de Lima Junior

Workbook for Macroeconomic Theory

Fluctuations, Inflation and Growth in Closed and Open Economies

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Fernando de Holanda Barbosa
Brazilian School of Economics
and Finance
Fundação Getulio Vargas
Rio de Janeiro
Rio de Janeiro, Brazil

Luiz Antônio de Lima Junior
Faculty of Economic Sciences
Federal University of Juiz de Fora,
campus Governador Valdares (Brazil)
Governador Valadares
Minas Gerais, Brazil

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Preface

This book presents the answers to the exercises in *Macroeconomic Theory, Fluctuations, Inflation and Growth in Closed and Open Economies* by Fernando de Holanda Barbosa (Cham, Switzerland: Springer, 2018), hereafter referred as *Macro Theory*.

Altogether, there are 166 exercises in eleven chapters and three appendices. *Macro Theory* points out that many of these exercises are based on, or inspired in, the literature listed in the bibliography, although the sources are not documented. We would like to thank the authors of these papers for shaping our understanding of the issues addressed in *Macro Theory*.

It is a science tenet that models should be falsifiable representation of a phenomenon. The goal of a good number of exercises is to help the student to develop the skills necessary to obtain the models' empirically testable predictions. You should try to solve each exercise by yourself, but do not be upset if you cannot. Some exercises are very hard and take time to work out. However, in order to learn, you should persevere and try again and again. We hope that this workbook will help you in the learning process of macroeconomic theory.

Macro Theory presents almost all models with continuous variables because it is very easy to derive the qualitative results with phase diagrams. There are a few exceptions, like Appendix C, where we present the new Keynesian model using discrete variables.

Macro Theory departs from the praxis of using the representative agent model to analyze the small open economy. Simpler models should be preferred to more complex models, according to Occam's razor. Thus, why should one not use the very simple representative agent model to analyze the small open economy? The answer to this question is based on an empirical stylized fact, namely that some small economies are creditor countries, while others are debtor countries. The representative agent model is unable to provide a long run net foreign asset equilibrium either as a creditor or as a debtor country.

Appendix C of *Macro Theory* takes stock of the new Keynesian model. Its IS curve turns the consumption Euler equation into a level curve. It is common knowledge (so, we do not have to provide references) that the Euler equation is a statement about the slope (smoothing) of the optimal consumption path. It does not say anything about the level of consumption but just states that given the level of current consumption the Euler equation can be used to forecast the expected level of future consumption. The new Keynesian IS curve turns this feature on its head: the level of consumption today depends on the expected level of consumption tomorrow. This IS curve is solved forward and implies that the effect of the rate of interest on output gap is the same today, tomorrow and at any time in the future. Empirical observation rejects this hypothesis. A very clever, but flawed, idea to solve this "puzzle" is the discounted Euler equation, which transforms this equation into a statement about the level of consumption. Exercise 14 of Chap. 7 deals with this issue.

The organization of this workbook is the same as that of *Macro Theory*. The first part deals with flexible price models and has five chapters and 41 exercises. Chapter 1 presents the representative agent model, Chap. 2 analyzes the open economy representative agent model, Chap. 3 addresses the overlapping generations model, Chap. 4 presents the Solow growth model and Chap. 5 introduces endogenous savings and endogenous growth in models of economic growth.

The second part covers sticky price models, both Keynesian and new Keynesian, and has four chapters. Chapter 6 presents the IS, LM and Phillips curves and the Taylor monetary policy rule. Chapter 7 analyzes models of economic fluctuations and stabilization in closed economies as well as deals with the issue of chronic inflation. Chapter 8 introduces the basic concepts of open economy macroeconomics such as arbitrage in markets for goods and services and in asset markets. This chapter also presents the specifications of the IS curve, Phillips curve and monetary policy rules in an open economy. Chapter 9 deals with the models of fluctuations and stabilization in open economies. This part has 55 exercises.

The third part has 2 chapters and 33 exercises. Chapter 10 introduces the government budget constraint and analyzes the following topics: (1) public debt sustainability, (2) hyperinflation, (3) Ricardian equivalence and (4) the fiscal theory of the price level. Chapter 11 addresses several monetary theory issues, such as price level determination, the optimum quantity of money, dynamic inconsistency, smoothing of the interest rate by central banks, inflation targeting, operational procedures of monetary policy and the term structure of interest rates.

Macro Theory as well as this workbook has three appendices with 37 exercises. These appendices make the *Macro Theory* book self-contained. Appendix A deals with differential equations, Appendix B presents the essential of optimal control theory and Appendix C gives the basic tools of difference equation needed to understand the new Keynesian model.

There are two types of exercises in *Macro Theory*. The first type aims to provide the student with material to practice for a full understanding of the subjects presented in the book. The second type of exercises addresses issues that were left out of the book because we chose to limit the size of *Macro Theory* to be less than 500 pages. Those exercises can be solved using the tools presented in the chapter, or appendix, where they belong. The exercises marked with an asterisk were prepared for this workbook. They cover the topics that are not dealt within *Macro Theory*, but we have decided to include them for the sake of completeness.

The topics covered in the second type of exercises in each chapter are:
Chapter 1: Discount rate and time inconsistency; cash-in-advance (CIA) constraint and money superneutrality; unpleasant monetarist arithmetic; incorrect specification of the Ramsey/Cass/Koopmans model.
Chapter 2: Variable rate of interest; habit formation and small open economy model; intertemporal approach to the balance of payments.
Chapter 3: Diamond growth model; social security system: fully funded versus pay-as-you-go.
Chapter 4: CES production function, Inada conditions and endogenous growth; Solow growth model with money.
Chapter 5: Human capital growth model with leisure in the utility function; human capital model with externalities.
Chapter 6: Calvo Phillips curve in continuous time with discount.
Chapter 7: Discounted Euler equation in the new Keynesian model.
Chapter 8: Monetary approach to the balance of payments with fixed and flexible exchange rates; Harberger–Laursen–Metzler effect; portfolio balance approach to exchange rates.
Chapter 9: Incorrect specification of a small open economy new Keynesian model; tradable and nontradable goods.
Chapter 10: Tax smoothing.
Chapter 11: Consumption asset pricing model.
Appendix A: Present value models: fundamentals and bubbles; housing model; Tobin's q model.
Appendix B: Tobin's q model with installation costs; dynamic inconsistency in monetary models.
Appendix C: Taking stock of the new Keynesian model.

We would like to thank the excellent work of LaTeX expert Cristina Maria Igreja for processing and editing our original files.

Rio de Janeiro, Brazil Fernando de Holanda Barbosa

Governador Valadares, Brazil Luiz Antônio de Lima Junior

Contents

Author Biographies

Dr. Fernando de Holanda Barbosa is Professor of Economics at the FGV EPGE Brazilian School of Economics and Finance. He has a Ph.D. in economics from the University of Chicago (US). He is the author of many academic articles in monetary economics and some are collected in the book *Exploring the Mechanics of Chronic Inflation and Hyperinflation* (Springer, 2017).

Dr. Luiz Antônio de Lima Junior is Adjunct Professor of Economics, Federal University of Juiz de Fora, campus Governador Valdares (Brazil).

Part I
Flexible Price Models

Chapter 1
The Representative Agent Model

(1) The representative agent maximizes the objective function:

$$\int_0^\infty \beta(t)u(c)dt$$

subject to the constraints

$$\dot{a} = ra + y - c$$

$$a(0) = a_0 \text{ given}$$

The Hamiltonian is:

$$H = \beta(t)u(c) + \lambda\,(ra + y - c)$$

The first-order conditions are:

$$\frac{\partial H}{\partial c} = \beta(t)u'(c) - \lambda = 0 \tag{1.1}$$

$$\frac{\partial H}{\partial a} = \lambda r = -\dot{\lambda} \tag{1.2}$$

$$\frac{\partial H}{\partial \lambda} = ra + y - c = \dot{a} \tag{1.3}$$

The derivative of (1.1) with respect to time is:

$$\dot{\beta}u'(c) + \beta u''(c)\dot{c} = \dot{\lambda}$$

F. de Holanda Barbosa, L. Antônio de Lima Junior, *Workbook for Macroeconomic Theory*, https://doi.org/10.1007/978-3-030-61548-2_1

By taking into account (1.2) we get:

$$-\frac{u''(c)}{u'(c)}\dot{c} = r + \frac{\dot{\beta}}{\beta}$$

Now let's think about a second agent that maximizes, for $s > 0$:

$$\int_s^{\infty} \beta\,(t-s)\,u(c)dt$$

subject to the constraints

$$\dot{a} = ra + y - c$$

$$a(s) = a_s \text{ given}$$

The Hamiltonian is:

$$H = \beta\,(t-s)\,u(c) + \lambda\,(ra + y - c)$$

The first-order conditions are:

$$\frac{\partial H}{\partial c} = \beta\,(t-s)\,u'(c) - \lambda = 0 \tag{1.4}$$

$$\frac{\partial H}{\partial a} = \lambda r = -\dot{\lambda} \tag{1.5}$$

$$\frac{\partial H}{\partial \lambda} = ra + y - c = \dot{a} \tag{1.6}$$

The derivative of (1.4) with respect to time is:

$$\dot{\beta}\,(t-s)\,u'(c) + \beta\,(t-s)\,u''(c)\dot{c} = \dot{\lambda}$$

By taking into account (1.2) and rearranging the terms of the equation we get:

$$-\frac{u''(c)}{u'(c)}\dot{c} = r + \frac{\dot{\beta}\,(t-s)}{\beta\,(t-s)}$$

The solution for the two agents is the same when:

$$\frac{\dot{\beta}(t)}{\beta(t)} = \frac{\dot{\beta}\,(t-s)}{\beta\,(t-s)}$$

Thus:

$$\beta(t) = e^{-\rho t}$$

$$\beta\,(t - s) = e^{-\rho(t-s)}$$

$$\rho = \text{constant}$$

(2) The representative agent maximizes the objective function:

$$\int_0^\infty e^{-\rho t}\,[u(c) + \upsilon(m)]\,dt$$

subject to the constraints

$$y + \tau = c + \frac{\dot{M}}{P}$$

$$M(0) \text{ given}$$

Since $m = \frac{M}{P}$, the derivative with respect of time of real per capita money stock is:

$$\dot{m} = \frac{\dot{M}}{P} - \pi m$$

$\pi = \frac{\dot{P}}{P}$(inflation rate). The constraint can be rewritten:

$$\dot{m} = y - c + \tau - m\pi$$

(a) The current value Hamiltonian is:

$$H = u(c) + \upsilon(m) + \lambda\,(y - c + \tau - \pi m)$$

The first-order conditions are:

$$\frac{\partial H}{\partial c} = u'(c) - \lambda = 0$$

$$\rho\lambda - \frac{\partial H}{\partial m} = \rho\lambda - \upsilon'(m) + \rho\pi = \dot{\lambda}$$

$$\frac{\partial H}{\partial \lambda} = y - c + \tau - \pi m = \dot{m}$$

(b) From $u'(c) = \lambda$ we may write c as a function of $\lambda : c = c(\lambda)$, $\frac{\partial c}{\partial \lambda} < 0$, since $u''(c) < 0$. The dynamical system is:

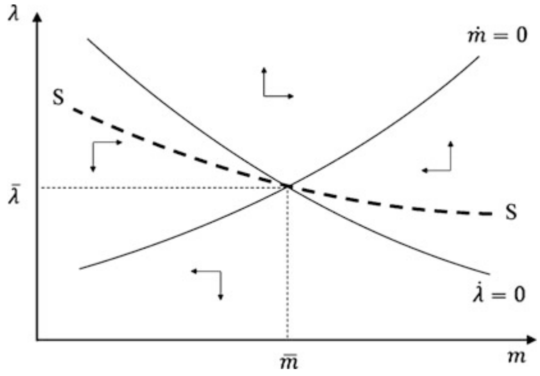


Fig. 1.1 Phase diagram for the λ and m system

$$\begin{cases} \dot{\lambda} = \lambda\,(\rho + \pi) - \upsilon'(m) \\ \dot{m} = y - c(\lambda) + \tau - \pi m \end{cases}$$

The Jacobian of the system is:

$$J = \begin{bmatrix} \frac{\partial \dot{\lambda}}{\partial \lambda} & \frac{\partial \dot{\lambda}}{\partial m} \\ \frac{\partial \dot{m}}{\partial \lambda} & \frac{\partial \dot{m}}{\partial m} \end{bmatrix} = \begin{bmatrix} \rho + \pi & -\upsilon''(m) \\ -\frac{\partial c}{\partial \lambda} & -\pi \end{bmatrix}$$

The determinant of this Jacobian is:

$$|J| = -\,(\rho + \pi)\,\pi - \frac{\partial c}{\partial \lambda}\upsilon''(m) < 0$$

because $\frac{\partial c}{\partial \lambda} < 0$ and $\upsilon''(m) < 0$. Thus, the steady-state equilibrium is a saddle point. The $\dot{m} = 0$ curve slopes upward and $\dot{\lambda} = 0$ slopes downward, as shown in Fig. 1.1. The arrows show the dynamics of the system and SS is a saddle path.

(c) The marginal utility of consumption is equal to the costate $u'(c) = \lambda$. By taking derivatives with respect to time we get:

$$u''(c)\dot{c} = \dot{\lambda}$$

Therefore, the equation for $\dot{\lambda}$ can be written as:

$$u''(c)\dot{c} = (\rho + \pi)\,u'(c) - \upsilon'(m)$$

The dynamical system is given by:

$$\begin{cases} \dot{c} = \frac{u'(c)}{u''(c)}\,(\rho + \pi) - \frac{\upsilon'(m)}{u''(c)} \\ \dot{m} = y - c + \tau - \pi m \end{cases}$$

The Jacobian of the system is:

$$J = \begin{bmatrix} \frac{\partial \dot{c}}{\partial c} & \frac{\partial \dot{c}}{\partial m} \\ \frac{\partial \dot{m}}{\partial c} & \frac{\partial \dot{m}}{\partial m} \end{bmatrix} = \begin{bmatrix} \frac{\partial \dot{c}}{\partial \dot{c}} & -\frac{\upsilon''(m)}{u''(c)} \\ -1 & -\pi \end{bmatrix}$$

The determinant of this Jacobian is:

$$|J| = -\pi \frac{\partial \dot{c}}{\partial c} - \frac{\upsilon''(m)}{u''(c)}$$

The derivative of $\dot{c}$ with respect to c is:

$$\frac{\partial \dot{c}}{\partial c} = (\rho + \pi) \left[\frac{\left(u''(c)\right)^2 - \left(u'(c)\left(u'''(c)\right)\right)}{(u''(c))^2} \right] + \frac{\upsilon'(m)u'''(c)}{(u''(c))^2}$$

which can be written as:

$$\frac{\partial \dot{c}}{\partial c} = \frac{(\rho + \pi)\left(u''(c)\right)^2 - u'''(c)\left[(\rho + \pi)\, u'(c) - \upsilon'(m)\right]}{(u''(c))^2}$$

In steady-state:

$$(\rho + \pi)\, u'(c) = \upsilon'(m)$$

Thus

$$\frac{\partial \dot{c}}{\partial c} = (\rho + \pi) \geq 0 \text{ if } \pi \geq -\rho$$

It follows that the determinant of the Jacobian, evaluated at the steady-state point is negative. Thus, the system has a saddle point.

The $\dot{c} = 0$ curve slopes upward and $\dot{m} = 0$ slopes downward, as depicted in Fig. 1.2. The arrows show the dynamics of the system and SS is the saddle path.

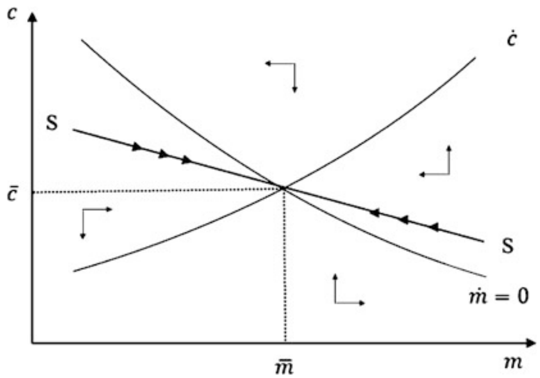


Fig. 1.2 Phase diagram for the λ and m system

(3) Since $c = y$, $u'(c) = \lambda =$ constant. Thus $\dot{\lambda} = 0$ and $\frac{\upsilon'(m)}{u'(c)} = (\rho + \pi)$.

(a) The derivative with respect to time of the real quantity of money $m = \frac{M}{P}$ is:

$$\frac{\dot{m}}{m} = \frac{\dot{M}}{P} = \mu - \pi$$

It follows that:

$$\dot{m} = (\mu - \pi)\, m = [(\mu + \rho) - (\rho - \pi)]\, m$$

By taking into account the previous equilibrium conditions we get:

$$\dot{m} = (\mu + \rho)\, m - \frac{m\upsilon'(m)}{u'(c)}$$

If $\lim m\upsilon'(m) = 0$ we get two equilibria, $m = 0$ and $m = \bar{m}$, as shown in Fig. 1.3. The path EA is a hyperdeflation bubble. The path E0 is a hyperinflation path.

If $\lim_{m\to 0} m\upsilon'(m) > 0$ there is one equilibrium, as shown in Fig. 1.4. The path EA is a hyperdeflation bubble and the path EB is a hyperinflation bubble.

(b) When $\frac{\dot{M}}{P} =$ constant $= f$, the previous real quantity of money equation can be written:

$$\dot{m} = f - m\pi - \rho m + \rho m$$

which is equivalent to:

$$\dot{m} = \rho m + f - m\,(\rho + \pi)$$

Fig. 1.3
$\lim_{m\to 0} m\upsilon'(m) = 0$

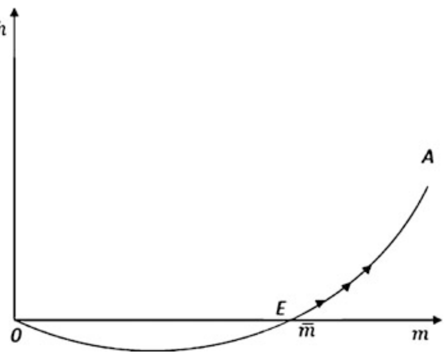


Fig. 1.4
$\lim_{m\to 0} m\upsilon'(m) > 0$

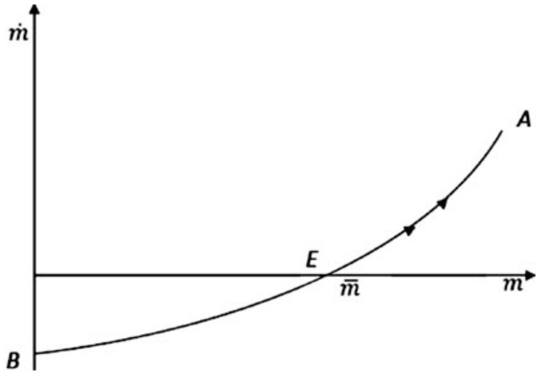


Fig. 1.5
$\lim_{m\to 0} m\upsilon'(m) = 0$

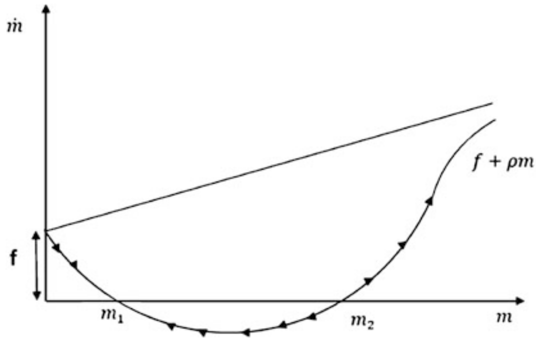
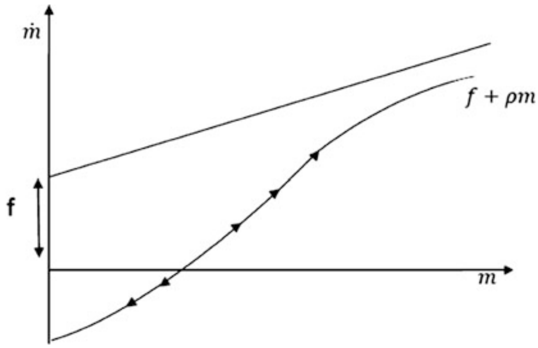


Fig. 1.6
$\lim_{m\to 0} m\upsilon'(m) > 0$



By taking into account the equilibrium condition we get:

$$\dot{m} = \rho m + f - \frac{m\upsilon'(m)}{u'(c)}$$

If $\lim_{m\to 0} m\upsilon'(m) = 0$ we may get two equilibrium points as shown in Fig. 1.5. There is no hyperinflation (bubble) but there is a hyperdeflation bubble path, as shown by the arrows. There is a stable low equilibrium (point m_1) and an unstable high equilibrium (point m_2).

If $\lim_{m\to 0} m\upsilon'(m) > 0$ there is just one equilibrium, which is unstable (Fig. 1.6). There are a hyperinflation and a hyperdeflation, both are bubbles.

(4) The Leibnitz rule (*Macro Theory*, p. 337) for the particular case where

$$V(r) = \int_{\alpha}^{\beta(r)} f(x)dx$$

is given by:

$$\frac{dV(r)}{dr} = f\left(\beta(r), r\right)\frac{d\beta(r)}{dr}$$

Applying this rule to:

$$F(\theta) = \int_{t}^{t+\theta} c(s)ds$$

we obtain:

$$\frac{dF(\theta)}{d\theta} = c\left(t+\theta\right)\frac{d\left(t+\theta\right)}{d\theta} = c\left(t+\theta\right)$$

It follows that

$$\frac{d^2F(\theta)}{d\theta} = \frac{dc\left(t+\theta\right)}{d\theta} = \dot{c}\left(t+\theta\right)$$

The Taylor Expansion of $F(\theta)$ around the point $\theta = 0$ is:

$$F(\theta) = F(0) + F'(0)\theta + \frac{F''(0)}{2}\theta^2 + \cdots$$

Thus

(a) $F(\theta) = c(t)\theta + \frac{\dot{c}(t)}{2}\theta^2 + \ldots$

(b) $M(t) \geq F(\theta)$

From (a) we can write the second-order approximation:

$$M(t) \geq c(t)\theta + \frac{c(t)\theta}{2}\theta^2 + \cdots$$

It follows that:

$$M(t) \geq c(t)\theta$$

(5) Total assets are given by:

$$a = k + m$$

Thus the dynamic equation can be written as:

$$\dot{a} = f(a - m) + \tau - c - \delta\,(a - m) - \pi m$$

The cash-in-advance constraint (CIA) is $m = c$. The $\dot{a}$ equation becomes:

$$\dot{a} = f\,(a - c) + \tau - c - \delta\,(a - c) - \pi c$$

The Hamiltonian of this problem is:

$$H = u(c) + \lambda\,[f\,(a - c) + \tau - (1 - \delta + \pi)\,c - \delta a]$$

(a) The first-order conditions are:

$$\frac{\partial H}{\partial c} = u'(c) + \lambda\left[f'\,(a - c)\,(-1) - (1 - \delta + \pi)\right] = 0$$

$$\rho\lambda - \frac{\partial H}{\partial a} = \rho\lambda - \lambda\left[f'\,(a - c) - \delta\right] = \dot{\lambda}$$

$$\frac{\partial H}{\partial \lambda} = f\,(a - c) + \tau - (1 - \delta + \pi)\,c - \delta a = \dot{a}$$

(b) In the steady-state equilibrium:

$$\dot{\lambda} = \left[\rho - f'(\bar{k}) + \delta\right]\lambda = 0$$

Thus:

$$\rho = f'(\bar{k}) - \delta$$

Therefore, money is superneutral in this model because $\bar{k}$ does not depend on the rate of growth of money.

(6) By taking into account the CIA and the fact that:

$$\frac{\dot{M}}{P} = \dot{m} + m\pi$$

we can write:

$$f(k) + \tau = c + \dot{k} + \delta k + \frac{\dot{M}}{P} = m + \dot{m} + m\pi$$

and

$$\dot{m} = f(k) + \tau - (1 + \pi)\, m$$

The CIA can be written as:

$$\dot{k} = m - c - \delta k$$

The representative agent maximizes the objective function:

$$\int_0^{\infty} e^{-\rho t} u(c) dt$$

subject to the constraints

$$\dot{m} = f(k) + \tau - (1 + \pi)\, m$$

$$\dot{k} = m - c - \delta k$$

$$k(0) \text{ and } M(0) \text{ given}$$

(a) The Hamiltonian of this problem is:

$$H = u(c) + \lambda\,(m - c - \delta k) + \mu\,[f(k) + \tau - (1 + \pi)\, m]$$

The first-order conditions are:

$$\frac{\partial H}{\partial c} = u'(c) - \lambda = 0$$

$$\dot{\lambda} = \rho\lambda - \frac{\partial H}{\partial k} = \rho\lambda - \left[-\lambda\delta + \mu f'(k)\right]$$

$$\dot{\mu} = \rho\mu - \frac{\partial H}{\partial m} = \rho\mu - [\lambda - \mu\,(1 + \pi)]$$

In steady-state equilibrium

$$\dot{\lambda} = 0$$

$$\dot{\mu} = 0$$

Thus:

$$\dot{\lambda} = \left[(\rho + \delta)\, \lambda - \mu f'(k) \right] = 0$$

$$\dot{\mu} = \mu\, (\rho + 1 + \pi) - \lambda = 0$$

From the first expression we get:

$$\rho + \delta = \frac{\mu}{\lambda} f'(k)$$

and from the second expression, we obtain:

$$(1 + \rho + \pi) = \frac{\lambda}{\mu}$$

The marginal product of capital is:

$$f'(k) = (\rho + \delta)\,(1 + i)$$

where $i = \rho + \pi$ is the nominal interest rate. The real rate of interest is:

$$f'(k) - \delta = \rho + i\,(\rho + \delta)$$

(b) Money is neutral since a change of its level does not change the nominal interest rate.
(c) Money is not superneutral because the rate of growth of the stock of money changes the nominal interest rate, and a change of the nominal interest rate affects the quantity of capital in steady-state equilibrium.

(7) First, let us change notation. We denote the nominal interest rate by i instead of r. Thus, the representative agent budget is:

$$(1 - \tau)\,(ib + y) = c + \frac{\dot{B}}{P} + \frac{\dot{M}}{P}$$

and the government budget constraint is:

$$g + ib - \tau\,(ib + y) = \frac{\dot{M}}{P} + \frac{\dot{B}}{P}$$

Total assets are defined by:

$$a = m + b$$

By using the a equation we can rewrite the agent budget constraint as:

$$(1-\tau)(ib+y) = c + \dot{b} + \pi b + \dot{m} + \pi m = c + \dot{a} + \pi a$$

Taking into account that $b = a - m$, we may write:

$$\dot{a} = (1-\tau)[i(a-m)+y] - \pi a - c$$

or:

$$\dot{a} = [(1-\tau)i - \pi]a - (1-\tau)im + (1-\tau)y - c$$

The representative agent maximizes the objective function:

$$\int_0^\infty e^{-\rho t}[u(c) + \upsilon(m)]\,dt$$

subject to the previous transition equation. The Hamiltonian of this problem is:

$$H = u(c) + \upsilon(m) + \lambda[((1-\tau)i - \pi)a - (1-\tau)im + (1-\tau)y - c]$$

The first-order conditions are:

$$\frac{\partial H}{\partial c} = u'(c) - \lambda = 0$$

$$\frac{\partial H}{\partial m} = \upsilon'(m) - \lambda(1-\tau)i = 0$$

$$\rho\lambda - \frac{\partial H}{\partial a} = \rho\lambda - \lambda[(1-\tau)i - \pi] = \dot{\lambda}$$

The goods and services market is in equilibrium. Thus, c is constant and the costate variable is constant, so $\dot{\lambda} = 0$. Therefore:

$$(\rho + \pi) = (1-\tau)i$$

From the second first-order condition equation we get:

$$\upsilon'(m) = \lambda(1-\tau)i$$

Since $u'(c) = \lambda$, we obtain:

$$\frac{\upsilon'(m)}{u'(c)} = (1-\tau)\, i$$

From the differential equation of the real stock of money, we get:

$$\dot{m} = \bar{\mu} m - m\pi - \rho m + \rho m$$

or:

$$\dot{m} = \bar{\mu} m - (\pi + \rho)\, m + \rho m = (\bar{\mu} + \rho)\, m - \frac{\upsilon'(m) m}{u'(c)}$$

The government budget constraint can be written as:

$$g + ib - \tau\,(ib + y) = \dot{m} + m\pi + \dot{b} + b\pi$$

From the monetary policy rule:

$$\dot{m} = m\,(\mu - \pi)$$

Thus, we get the following equation for the government budget constraint:

$$\dot{b} = g - \tau y + [(1-\tau)\, i - \pi]\, b - \bar{\mu} m$$

or

$$\dot{b} = \rho b - \bar{\mu} m + g - \tau y$$

The dynamical system has two equations, one for m and another for b, that is:

$$\begin{cases} \dot{m} = (\bar{\mu} + \rho)\, m - m \frac{\upsilon'(m)}{u'(c)} \\ \dot{b} = \rho b - \bar{\mu} m + g - \tau y \end{cases}$$

The Jacobian of the system is:

$$J = \begin{bmatrix} \frac{\partial \dot{m}}{\partial m} & \frac{\partial \dot{m}}{\partial b} \\ \frac{\partial \dot{b}}{\partial m} & \frac{\partial \dot{b}}{\partial b} \end{bmatrix} = \begin{bmatrix} \frac{\partial \dot{m}}{\partial m} & 0 \\ -\bar{\mu} & \rho \end{bmatrix}$$

where:

$$\frac{\partial \dot{m}}{\partial m} = (\bar{\mu} + \rho) - \left[\frac{\upsilon'(m) + m\upsilon''(m)}{u'(c)}\right] = \bar{\mu} + \rho - \frac{\upsilon'(m)}{u'(c)} - \frac{m\upsilon''(m)}{u'(c)}$$

Since:

$$\frac{\upsilon'(m)}{u'(c)} = (1-\tau)\, i = \rho + \pi$$

we get

$$\frac{\partial \dot{m}}{\partial m} = \bar{\mu} + \rho - (\rho + \pi) - \frac{m\upsilon''(m)}{u'(c)}$$

In equilibrium $\bar{\mu} = \pi$, therefore

$$\frac{\partial \dot{m}}{\partial m} = -\frac{m\upsilon''(m)}{u'(c)}$$

(a) The determinant of this Jacobian is positive:

$$|J| = \frac{\partial \dot{m}}{\partial m}\rho > 0$$

The trace of J is positive:

$$trJ = \frac{\partial \dot{m}}{\partial m} + \rho > 0$$

We may conclude that the steady-state equilibrium is unstable. Figure 1.7 shows the phase diagram for the dynamical system of two differential equations, one for the real quantity of public debt (b) and the other for the real quantity of money (m). The arrows in this figure show the dynamics of the system.

(b) Figure 1.8 shows the monetary experiment whereby the central bank reduces the monetary expansion rate from μ_0 to μ_1 at instant zero. The adjustment of the economy is shown in Fig. 1.9. The path E_0E_T shows the system adjustment to the tight monetary policy with a ceiling of the

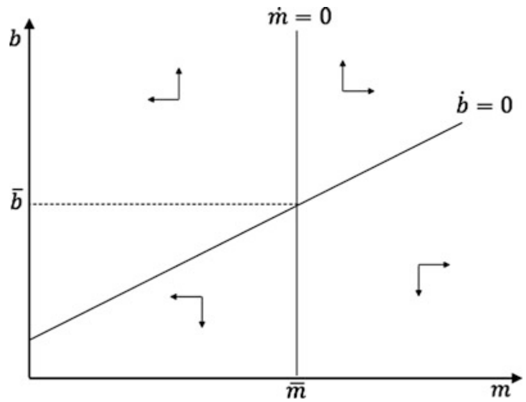


Fig. 1.7 The phase diagram for the b and m system

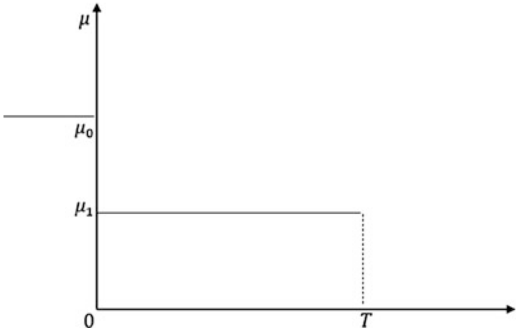


Fig. 1.8 Experiment of a tight monetary policy

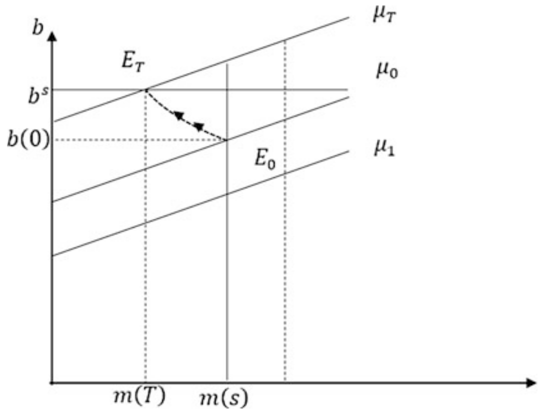


Fig. 1.9 Dynamic adjustment of the system

real stock of debt b^s. The real quantity of money decreases and inflation increases since the announcement of the tight monetary policy.

(8) When $\dot{b} = f + \rho b$ we may write:

$$\frac{d}{ds} b e^{-\rho s} = f e^{-\rho s}$$

or:

$$dbe^{-\rho s} = f e^{-\rho s} ds$$

(a) By integrating this equation, we obtain:

$$\int_t^T dbe^{-\rho s} = \int_t^T f e^{-\rho s} ds$$

or:

$$b(T)e^{-\rho T} - b(t)e^{-\rho t} = \int_t^T f e^{-\rho s} ds$$

It follows that:

$$b(T) = b(t)e^{\rho T} e^{-\rho T} + \left[\int_t^T f e^{-\rho s} ds\right] e^{\rho T}$$

(b) From this expression we get:

$$\lim_{T\to\infty} \lambda b(T)e^{-\rho T} = \lim_{T\to\infty} \lambda b(t)e^{-\rho t} + \lim_{T\to\infty} \lambda \left[\int_t^T f e^{-\rho s} ds\right]$$

Since

$$\int_t^\infty e^{-\rho s} ds = \frac{e^{-\rho t}}{\rho}$$

the former becomes:

$$\lim_{T\to\infty} \lambda b(T)e^{-\rho T} = \lim_{T\to\infty} \lambda b(t)e^{-\rho t} + \lim_{T\to\infty} \lambda \frac{e^{-\rho t}}{\rho}$$

which is equal to:

$$\lim_{T\to\infty} \lambda b(T)e^{-\rho T} = \lambda e^{-\rho t} \left(b(t) + \frac{e^{-\rho t}}{\rho}\right) \neq 0$$

(c) The real deficit is constant:

$$\dot{b} = f$$

Thus,

$$\int_t^T db = \int_t^T f ds$$

and

$$b(T) - b(t) = f\,(T - t)$$

or

$$b(T) = b(t) + f\,(T - t)$$

The transversality condition is:

$$\lim_{T\to\infty} \lambda b(T)e^{-\rho T} = \lim_{T\to\infty} \lambda b(t)e^{-\rho T} + \lim_{T\to\infty} \lambda f\,(T - t)\,e^{-\rho T}$$

It is easy to verify that:

$$\lim_{T\to\infty} \lambda b(t) e^{-\rho T} = 0$$

and

$$\lim_{T\to\infty} \lambda \frac{f(T-t)}{\left(e^{\rho T}\right)} = 0$$

We conclude that

$$\lim_{T\to\infty} \lambda b(T) e^{-\rho T} = 0$$

(d) The conclusions drawn from items (b) and (c) are that a constant primary deficit does not obey the transversality condition, but a constant real deficit satisfies the transversality condition.

(9) The real business cycle model with a government is given by the dynamical system:

$$\begin{cases} \dot{k} = Ak^{\alpha} - \frac{\rho+\delta}{\alpha} k \\ \dot{K} = AKk^{\alpha-1} - \frac{A(1-\alpha)}{\beta} k^{\alpha} - g - \delta k \end{cases}$$

(a) The first experiment is an unanticipated permanent increase in government spending as described in Fig. 1.10. The $\dot{k} = 0$ curve does not shift but the $\dot{K} = 0$ curve shifts downward as Fig. 1.11 shows.
The variable K is predetermined but the variable k can jump. At the instant of the increase in the government expenditure k jumps to the point I, with $k(0_+)$, in saddle path SS, and then converges on the new equilibrium (point E_f).

(b) The second experiment is an anticipated permanent increase in government spending as described in Fig. 1.12. The $\dot{k} = 0$ will shift at instant t.

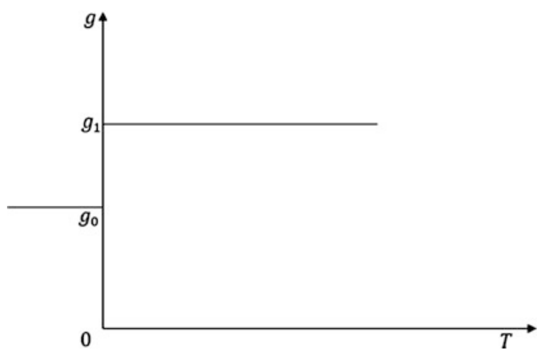


Fig. 1.10 An unanticipated permanent increase in government spending

Fig. 1.11 Dynamic adjustment of the economy to unanticipated change in government spending

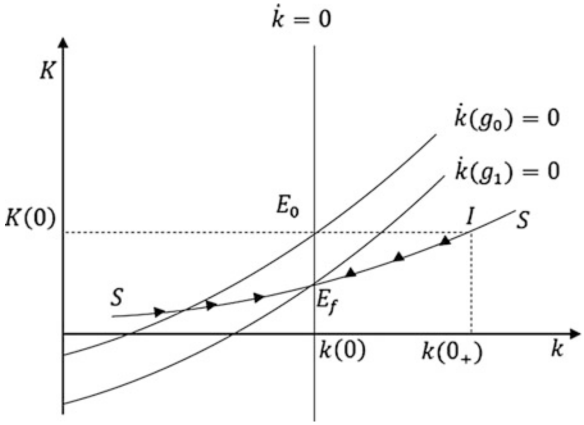
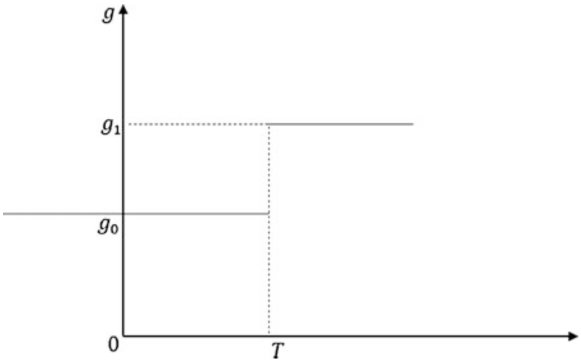


Fig. 1.12 Anticipated permanent increase in government spending



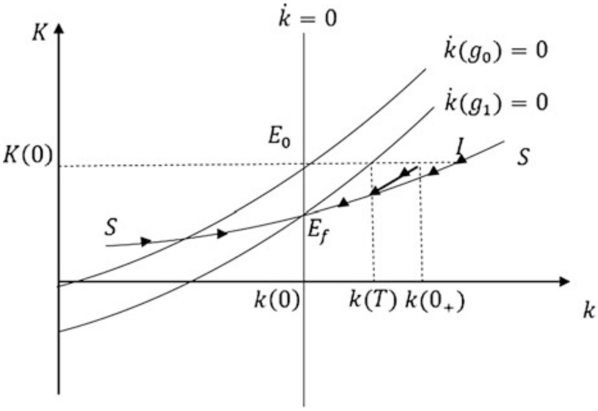


Fig. 1.13 Dynamic adjustment of the economy to anticipated change in government spending

However the economy jumps at instant zero in such way that at instant T will be at the saddle path SS, as shown in Fig. 1.13. The economy converges on the point E_f, the new steady-state equilibrium.

(c) The third experiment is an unanticipated transitory increase in government spending as described in Fig. 1.14.
This transitory increase last until time T, when government goes back to the previous level. Figure 1.15 shows the dynamic adjustment of the economy. The variable k jumps to point I in such a way that by time T it will reach the point I_T, and then converges back on the previous equilibrium E_0.

(d) The fourth experiment is an anticipated transitory increase in government spending as described in Fig. 1.16.
The $\dot{k} = 0$ will change at time T_1 and it will be back to the original position at T_2. Figure 1.17 shows the dynamic adjustment of the economy. The variable k jumps to point I when the fiscal policy is announced. The capital

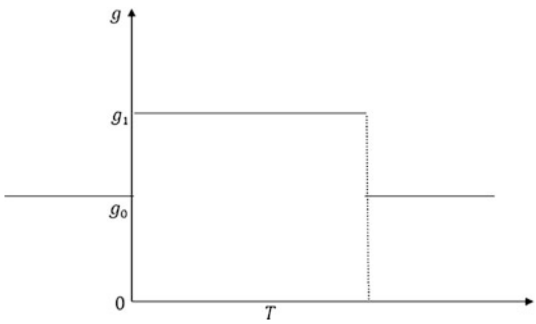


Fig. 1.14 An unanticipated transitory increase in government spending

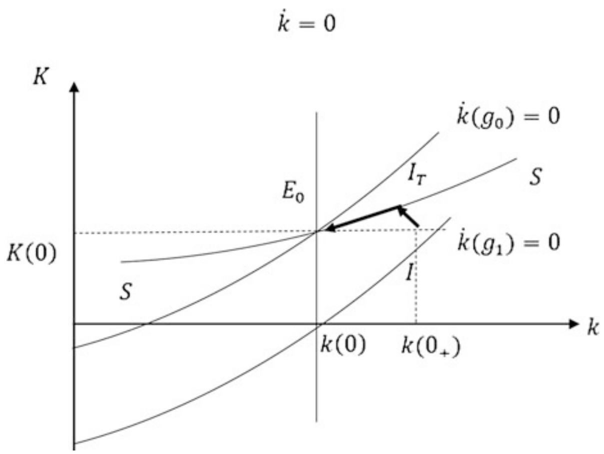


Fig. 1.15 Dynamic adjustment of the economy to an unanticipated transitory change in government spending