Theory of Solid-Propellant Nonsteady Combustion

Boris V. Novozhilov and Vasily B. Novozhilov

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Boris V. Novozhilov
The Semenov Institute of Chemical Physics
Russian Academy of Sciences
117977 Moscow, Russia

Vasily B. Novozhilov
Professor of Mathematics
Victoria University
Melbourne Victoria 8001
Australia

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To
Ludmila Novozhilova
Natalia Golubnichaya
Inga Novozhilov
Natalia Novozhilova

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About the Authors

Professor Boris V. Novozhilov

Professor Boris V. Novozhilov (1930–2017) was born in Alma-Ata (Kazakhstan, which at that time was part of the Soviet Union). He graduated with honors in Applied Physics from the Leningrad (currently Peter the Great St. Petersburg) Polytechnic Institute in 1953. He received his PhD (1959) and DrSc (1968) degrees in Physical and Mathematical Sciences.

From 1954 to 2017, Professor B.V. Novozhilov worked at the Institute of Chemical Physics (currently the Semenov Institute of Chemical Physics) of the USSR (later Russian) Academy of Sciences in various roles, including the Head of Laboratory of Mathematical Methods in Chemical Physics (1976–1992) and Chief Researcher.

Professor B.V. Novozhilov is best known for his outstanding fundamental contribution to the theory of propellant combustion and, together with Ya. B. Zeldovich, is a founder of the Zeldovich–Novozhilov theory of nonsteady solid propellant combustion.

Professor B.V. Novozhilov's other research interests include nuclear physics (propagation of gamma quanta in matter), the theory of spin combustion, and the theory of "cold" flame propagation.

Professor B.V. Novozhilov is a recipient of the Ya. B. Zeldovich Gold Medal from The Combustion Institute "for outstanding contributions to the theory of combustion" (1996). He has also received a number of Russian Federation Government Awards in Science and Technology.

Professor B.V. Novozhilov is the author of over 150 journal papers and 12 books.

Professor Vasily B. Novozhilov

Professor Vasily B. Novozhilov was born in 1963 in Moscow. He graduated with an MSc in Applied Mathematics from the Russian State University of Oil and Gas in 1986. He later received a PhD in Physical and Mathematical Sciences (Mechanics of Fluid, Gas and Plasma) from the Moscow Aviation Institute in 1993.

Professor V.B. Novozhilov held research positions at the Russian Academy of Sciences (Institute for Problems in Mechanics) and the University of Sydney. Furthermore, he held academic appointments at Nanyang Technological University (Singapore), as a Professor in Fire Dynamics at The University of Ulster (UK), and as a Professor of Mathematics at Victoria University (Australia). From 2014 to 2017 he was a Director of the Centre for Environmental Safety and Risk Engineering at Victoria University, Australia.

Major research interests of Professor V.B. Novozhilov include combustion (solid propellants, combustion theory and fire research) and the theory of heat transfer. He is a leading expert in theoretical and computational methods in the areas of combustion and fire research, in particular, computational fluid dynamics modelling of compartment fires. He has also made important contributions to the application of dynamical system methods in fire dynamics, and has also been greatly involved with analytical methods of the heat transfer theory in application to ultra-fast heat transfer processes.

Professor V.B. Novozhilov is the author of over 60 journal papers and four book chapters. He was a Keynote Speaker at the 68th International Astronautical Congress (2017) delivering an overview of the fundamentals of the Zeldovich–Novozhilov propellant combustion theory, as well as of other contributions by Professor B.V. Novozhilov to the physics of combustion.

Preface

Nonsteady operating regimes, where fuel burning rates vary in time, are common for solid rocket motors. Under such conditions, combustion chamber pressure and, consequently, specific impulse are also functions of time. Some examples of such processes are combustion under variable pressure, a transition from one operating regime to another, oscillating combustion, erosion combustion in a dynamical regime, and propellant charge ignition and extinction under rapid depressurization.

In contrast to a steady-state regime, propellant burning rate in such situations depends not only on instantaneous parameters (initial pressure, temperature, and the velocity of a tangential gas stream), but is also determined by the full history of the process. This is due to the inertia of the combustion wave, which includes a heated layer of the condensed phase, a chemical reaction zone, and a certain region in space that is occupied by combustion products.

The natural way of describing nonsteady propellant combustion would be to use the theory of steady-state burning regimes. The transition to unsteady theory would require a simple addition of time derivatives into the relevant set of differential equations. At the current level of available computational resources, this additional mathematical complexity does not present a problem. However, the described hypothetical approach is not possible for a very simple reason: the consistent and universal theory of steady-state propellant combustion describing experimental observations does not exist.

Each physical and chemical process which occurs during the combustion of propellants is immensely complex. For the overwhelming majority of substances, the burning rate is determined by chemical kinetics. Therefore, the kinetic parameters of reactions are a substantial part of practically any combustion theory. However, with some exception, this knowledge of the kinetics of combustion reactions is as yet incomplete. In particular, information on chemical transformations which occur during the combustion of condensed substances is very scarce so it is necessary to involve model kinetic schemes, which only remotely resemble real chemical processes (typically, Arrhenius dependence on temperature and power dependence on reactant concentration are adopted).

There is a large body of steady-state homogeneous and composite propellant combustion models presented within this work. Naturally, such models contain a large number of parameters (reaction rate constants, activation energies, heat of combustion, transfer coefficients, thermophysical properties of gas, condensed phases, etc.) which are unknown in most cases. Evidently, an adjustment of numerous parameters allows the experimental data to be approximated, which of course does not imply a proper description of real fuel. Such studies are therefore of qualitative nature only, and are hardly suitable for comparison with experiments. Moreover, it would probably be impossible to develop a quantitative steady-state combustion theory applicable to a wide range of substances due to the large variation in their properties.

A drastically different approach to the development of nonsteady theory (avoiding the necessity to create a detailed description of the steady-state regime) was proposed by Ya.B. Zeldovich in 1942. It invokes an elegant and powerful idea of using the experimentally determined steady-state dependence of a propellant burning rate on pressure and an initial temperature for studying nonsteady combustion regimes. It was demonstrated that this is only possible taking into account the thermal inertia of the condensed phase. The idea was formulated in the original paper by Zeldovich (1942) in the following way: 'Since the relaxation time of combustion in gas is very small, we have the right to consider gaseous combustion as determined by the thermal condition of the thin condensed phase layer adjacent to the interface; the temperature distribution within deeper layers does not have a direct effect on the processes near the surface. The conditions of gas must be fully determined by instantaneous values of the surface temperature, and a temperature gradient in the condensed phase at the surface. Consider the surface temperature as being constant.'

Thus, the nonsteady propellant combustion theory was reduced to a consideration of a relatively slow variation of temperature distribution in the condensed phase. This is achieved by the solution of the heat transfer equation, combined with the known dependency of the burning rate on instantaneous values of pressure and the temperature gradient in a condensed phase at the surface. The latter may be obtained from the (theoretical or experimental) steady-state dependency of the burning rate on pressure and initial temperature.

This theory explained some nonsteady combustion phenomena qualitatively, but its quantitative comparison with experimentation leads to contradiction. Most remarkably, this contradiction manifests itself in the conclusion that, according to this theory, a steady-state burning regime of real systems is actually unstable. The reason for this discrepancy is an oversimplification of the theory, which considered propellant surface temperature as constant.

For all practically used compositions, the temperature at the interface between the condensed and gas phases depends on external conditions: the pressure, initial temperature, and velocity of the tangential gas stream. The theory proposed by Zeldovich (1942) was generalized and transformed to its contemporary state by B.V. Novozhilov in 1965 (Novozhilov 1965a,b). An additional function characterizing the steady-state regime was introduced: the dependence of the propellant surface temperature on external parameters. Such a generalization of the Zeldovich theory allowed the majority of phenomena related to nonsteady combustion regimes to be explained.

It is important to note that this generalization of the theory by Zeldovich (1942) preserved its main idea, that is, the possibility of investigating nonsteady phenomena using steady-state dependencies. The theory contains experimental dependencies related to a steady-state combustion regime. These dependencies carry all the information on kinetics of chemical reactions as well as on various physical processes (thermal conduction and diffusion in the gas phase, devolatilization of the fuel, etc.).

Even in the absence of such dependencies, the theory still turns out to be helpful when considering a comparison of its conclusions with experimental data on nonsteady combustion. Thus, the comparison of theoretical and experimental data on acoustic combustion instability enables prediction of the behaviour of the same fuel under different nonsteady conditions, such as during combustion in a semi-enclosed volume.

Moreover, the theory allows some fuel parameters, related to the kinetics of reactions at its surface, to be extracted from experimental data. For example, it is possible to obtain the effective activation energy of the chemical reaction at the fuel surface from the data on acoustic admittance.

The theory developed on the basis of results by Zeldovich (1942) and Novozhilov (1965a,b) is usually called the Zeldovich–Novozhilov theory (the ZN theory). The other titles that are used include the phenomenological theory of nonsteady combustion or the t_c approximation. The latter emphasizes that the time of thermal relaxation of the condensed phase is the only fuel characteristic time. In the following text terms such as energetic material, solid rocket fuel, volatile condensed combustion system, and propellant are used as synonyms.

Within the framework of the theory presented in this monograph, the nonsteady process of propellant combustion is investigated by means of solving the heat transfer equation in the condensed phase with relevant initial and boundary conditions. Other necessary elements of the theory are the steady-state dependencies of the burning rate and surface temperature on the pressure and initial temperature. These may be obtained experimentally or by considering a specific theoretical propellant combustion model. It is clear that all conclusions of the theory are applicable to real systems as the aforementioned dependencies are obtained from experiments with the exactly same systems.

Let us discuss briefly the assumptions which form the foundation of the theory. It should be noted that, with a few exceptions, these assumptions are adopted in all studies of the nonsteady combustion of solid rocket fuels. First of all, fuel is assumed to be homogeneous and isotropic. The scale of nonhomogeneity must be much smaller than the characteristic scale following from steady-state theory, that is, the Michelson length. This requirement is undoubtedly fulfilled for ballistites. In the case of composite fuels, this assumption is valid for sizes of fuel and oxidizer particles much smaller than the thickness of the heated layer of the condensed phase. In the following discussion, one-dimensional problem formulation assuming flat flame front and interface between the phases is considered nearly everywhere. Second, the basic assumption of the discussed theory is that thermal decomposition of the condensed phase and combustion in the gaseous phase occur much faster than the heating up of the condensed phase. This proposition may be justified by simple estimations, which are presented in the main body of the monograph.

The first review of the proposed theory was presented by Novozhilov (1968). It considered the major results obtained by that time: combustion stability at constant pressure, linear oscillating combustion regimes, acoustic admittance of the surface of burning propellant, combustion stability in a semi-enclosed volume, nonlinear oscillations of burning rate, transitional combustion regimes, and propellant extinction. Later, the monographs (in Russian) by Novozhilov (1973a) and Zeldovich et al. (1975), as well as the more recent review by Novozhilov (1992a) appeared.

It seemed initially (to many, including the first author) that the phenomenological theory would be replaced soon by a more sophisticated approach to nonsteady propellant combustion. Such an approach could be built on the consideration (by numerical methods) of a set of differential equations, complete and consistent from the point of view of macroscopic chemical physics, describing the specific problem. This may eventually happen. However, progress beyond the framework of the ZN theory has been very slow owing to the significant complexity (even for homogeneous systems) of propellant combustion. This complexity, in the first place, is due to phase transition from condensed to gaseous, complicated even further by chemical reactions.

All the considerations below are applicable, strictly speaking, to homogeneous propellants only. The theory of composite systems is at a rudimentary stage since the processes in the combustion wave of such substances are much more complicated compared to homogeneous propellants. Apart from a nearly complete absence of data on the kinetics of chemical reactions, there are additional obstacles to a quantitative consideration of the combustion process of composite systems. Although during steady-state conditions the mean burning rate is constant in time, the processes occurring in the vicinity of the surface are nonsteady. The geometry of the surface continuously changes in time as burnt particles are replaced by virgin ones at other locations at the surface. Temperature distribution in the vicinity of the interface between the phases, and on the interface itself, is a random function of time. It should be noted that attempts were made (Romanov 1976) to expand the theory to heterogeneous systems. It was proposed, in addition to steady-state dependencies of the burning rate and surface temperature on pressure and initial temperature, to use dependencies of average values of these quantities on fuel (or oxidizer) mass fraction at the interface. Unfortunately, significant difficulties in obtaining such dependencies experimentally did not allow this approach to proceed.

Nevertheless, one may hope that some of the results obtained for homogeneous propellants would also be qualitatively applicable to composite systems. For example, the resonance response of the burning rate to periodically varying pressure may be expressed in the same terms as for homogenous systems. Naturally, the parameters characterizing such composite systems would have to be considered as adjustable values.

Let us discuss briefly the comparison of conclusions (which are discussed within the book) of the theory, with experimentation. As with any other theory, ZN theory requires, first of all, some experimental input data. These are steady-state burning laws, that is, steady-state dependencies of the burning rate and surface temperature (and, in some cases, of other properties of the combustion wave, e.g. combustion temperature) on external parameters, that is, on pressure and initial temperature. The theory demands a rather high accuracy of input experimental data as its conclusions follow from peculiarities of steady-state burning laws. For example, the study of linear nonsteady phenomena is only possible if first derivatives of burning laws, with respect to external parameters, are known. Calculation of these derivatives obviously involves large errors as such a mathematical operation is ill-posed.

The same applies to experimental data which is used for comparison with the theory outcomes. Observation of various unsteady combustion phenomena are associated with significant difficulties. At best, relative errors are of the order of tens of percent. The theory, however, predicts a number of quite distinctive qualitative effects, for example the existence of the natural frequency of propellant combustion and, as a consequence, a resonance response of the nonsteady burning rate to harmonically oscillating pressure. Such effects are actually being observed, and it is usually possible to reconcile the theory and experimental results quantitatively for the values of parameters within the experimental uncertainty region.

Before briefly outlining the monograph content, let us notice that the overwhelming majority of presented results are obtained using a universal approach. The following is its mathematical formulation.

A one-dimensional unsteady heat transfer equation

$$\frac{\partial \theta}{\partial \tau} = \frac{\partial}{\partial \xi}\left(\frac{\partial \theta}{\partial \xi} - v\theta\right), \quad -\infty < \xi \le 0, \quad \tau \ge 0$$

is considered, along with the relevant boundary and initial conditions

$$\xi \to -\infty, \quad \theta = 0$$

$$\xi = 0, \quad \theta = \vartheta(\tau)$$

$$\theta(\xi, 0) = \theta_i(\xi)$$

Nonsteady relations between the burning rate $v(\tau)$ and the surface temperature $\vartheta(\tau)$, on the one hand, and some external parameter $\eta(\tau)$ (most often pressure) and the temperature gradient $\varphi(\tau) = (\partial\theta/\partial\xi)_{\xi=0}$, on the other

$$v = \Phi_u(\varphi, \eta), \quad \vartheta = \Phi_s(\varphi, \eta)$$

are prescribed.

There must also be prescribed the function

$$\eta = \Pi(\tau)$$

or some auxiliary equation which determines this function.

A specification of the functions $\Phi_u(\varphi, \eta)$ and $\Phi_s(\varphi, \eta)$, and the external parameter dependence on time $\eta(\tau)$ lead to a class of problems that are related to rapid development in recent decades in the multidisciplinary area of synergetics (Mikhailov 2011).

The majority of results in this area are obtained by a numerical analysis of various model sets of differential equations. Most often, the systems with a finite number of degrees of freedom are considered.

The formulation discussed above is probably the simplest for distributed dynamical systems. Despite this simple form, however, the set of system behaviour scenarios is quite rich. For example (and this is demonstrated in the relevant section of the book), studying the system behaviour under constant external conditions is directly related to the problem of turbulence. Variation of one of the control parameters leads to successive bifurcations of combustion regimes, ending up with chaotic behaviour.

In contrast to the majority of studied examples of dynamical systems, the model described above expresses real physical and chemical processes. Within the framework of the presented formulation, such practically important phenomena as propellant burning interaction with the acoustics of the combustion chamber, combustion extinction under depressurization, burning stability under constant pressure, etc. may be investigated.

Since experimental data on steady-state combustion are an essential input into the theory, this book begins with an establishing chapter describing the steady-state burning regime and presenting various fuel property dependencies on external conditions. The theoretical estimations and experimental results provided in the first chapter are necessary for a justification of nonsteady combustion theory. Analytical dependencies of the burning rate and surface temperature on external parameters (i.e. steady-state burning laws) are useful for quantitative analysis. Their physically sensible form may only be obtained from the current understanding of steady-state combustion regimes of the simplest systems. These issues are also discussed in the first chapter.

The second chapter, which is fundamental for the monograph, formulates major assumptions of the theory of nonsteady combustion of solid rocket fuels. A simplified case of constant propellant surface temperature (Zeldovich theory) is considered in detail. The two formulations of the ZN theory, differential and integral, are then presented.

In the first of these formulations, temperature distribution in the condensed phase is a necessary element of consideration. This profile, however, is rarely used in practice. It turns out that an alternative, integral formulation, involving only the most relevant quantities – pressure and burning rate, as an example – may be developed. Readers interested in a formal mathematical justification of the theory should pay attention to the last section of this chapter.

The third chapter considers propellant combustion at a constant pressure. In one-dimensional problem formulation, the stability criterion for the steady-state combustion regime, relating burning rate and surface temperature derivatives with respect to initial temperature is obtained. The possibility of two-dimensional instability is discussed using the example of the simplest combustion system. Numerical modelling is used to investigate nonsteady modes of propellant combustion under constant pressure beyond the stability boundary of the steady-state regime. The simplest propellant combustion model containing just two control parameters is considered. With a fixed value of one of these, the other plays the role of a bifurcation parameter. It is demonstrated that on variation of the bifurcation parameter, the system may transit from the steady-state to the chaotic combustion regime following the Feigenbaum scenario. A sequence of period doubling bifurcations of the burning rate oscillations, leading eventually to a chaotic combustion regime, is studied. A comparison with experimental data is discussed in the last section of the chapter.

The following three chapters are devoted to the influence of external parameters, that is, pressure, erosive gas stream, and thermal radiation, on propellant combustion under periodically varying pressure. Practical interest in these processes is due to the need to understand the causes of the development of various nonsteady effects, for example soft or hard excitation of burning rate and pressure oscillations, superimposed on the designed steady-state solid rocket motor regime.

First, the problem of combustion and acoustics interaction (Chapter 4) is considered in the linear approximation with the burning rate amplitude being proportional to pressure amplitude. An analytical expression for the response function of the burning rate to oscillating pressure is obtained. Furthermore, its properties and relation with the most important property of burning propellant surface (acoustic admittance) are discussed. Then, the notion of response functions of higher orders, with respect to pressure amplitude, is discussed. These would find an application in investigations of sustained and transitional regimes with finite burning rate and pressure amplitudes. A conducted nonlinear analysis reveals a fundamentally new phenomenon: period doubling bifurcations of burning rate oscillations that occur on an increase of amplitude or change of frequency of pressure oscillations. A sequence of bifurcations leading eventually to the chaotic combustion regime is investigated for a propellant combustion model containing a minimal number of parameters in nonsteady burning laws.

The fifth chapter considers nonsteady propellant burning in the tangential stream of combustion products. Within the framework of the phenomenological theory, the propellant burning rate response to periodically varying pressure and tangential mass flux of combustion products is investigated. An elementary acoustic perturbation in the form of a monochromatic travelling sound wave is considered. Analytical and numerical results are obtained for the simplest propellant model described by a minimal number of parameters. The role of steady-state and nonsteady erosion contributions at small and large values of the erosion ratio is also revealed.

The following chapter deals with nonsteady combustion under external radiation. In this case, the heat transfer equation includes an additional source term, while both steady-state and nonsteady burning laws also change. The stability of the steady-state combustion regime is investigated in the linear approximation. The response functions of the burning rate to harmonically oscillating pressure in the presence of constant radiative flux, as well as to harmonically oscillating radiative flux, are obtained. In the linear approximation of the ZN theory, an analytical relationship between the response function to oscillating pressure, obtained at a certain initial temperature, and response function to oscillating radiative flux, obtained at the same pressure but different (lower) initial temperature, is established. The difference of initial temperatures satisfies the requirement that steady-state burning rates with and without radiative flux are equal, and is directly proportional to radiative flux. It is likely that this relationship will be useful for obtaining experimental data on the response function to oscillating pressure.

Chapter 7 presents theoretical considerations and experimental data related to combustion regimes where pressure varies according to the law, which is different from harmonic oscillations. Such processes include, for example, propellant combustion during transition from one operational regime to another (at higher or lower pressure), extinction on rapid and deep depressurization, and others.

The eighth chapter describes nonsteady propellant combustion regimes in the combustion chamber of a rocket engine. There are three time scales that are relevant for this problem: the thermal relaxation time of the heated layer of the condensed phase t_c, the acoustic time t_a, and the time of combustion products efflux from the chamber t_{ch}.

If the relaxation time of the condensed phase is close to the efflux time, $t_c \sim t_{ch}$ (which occurs in small engines at low pressures), then such regimes may be called nonacoustic. Time scales in such problems are much larger than the acoustic time. This area of research may also be referred to as propellant combustion in semi-enclosed volume.

On the other hand, over the last few decades a specific and dedicated area of research which may be termed ‘acoustics and combustion’ has taken shape. It deals with the case where acoustic time is close to the condensed phase thermal relaxation time $t_a \sim t_c$, which leads to a possibility of sonic (in the general case nonlinear) oscillations development in the engine. The latter relation between the time scales applies to engines of large size with high pressure values in combustion chambers. The theory of such processes is still at a rudimentary stage. As an example, possible combustion regimes in a solid rocket engine with end burner grain geometry are investigated. A set of equations which allows the interaction between combustion and acoustic processes in a combustion chamber to be modelled is presented. The specific feature of the problem is the existence of the two distinctive time scales, namely the acoustic time and the time of pressure oscillation amplitude variation. These time scales differ by approximately three orders of magnitude, which demands high computational accuracy. A simpler solution method is developed in the quadratic approximation with respect to the amplitude of oscillations. This method accounts only for the effects related to the time scale of oscillation amplitude variation. Numerical results are obtained for the simplest propellant combustion model in the absence of entropic waves in combustion products. Stable and unstable combustion regimes are identified. In the latter regime, nonlinear effects may trigger shock waves in the combustion chamber.

The possibility of expanding the theory beyond the phenomenological framework is discussed in the final chapter. This development requires a more detailed combustion model that would adequately describe processes occurring in low-inertia zones of a combustion wave. The influence of low-inertia zones (the reacting layer of the condensed phase, preheat and reaction zones in the gas phase, the half-space occupied by gaseous combustion products) on various nonsteady phenomena are investigated both analytically and numerically. The consideration is presented within the framework of the Belyaev model. It is demonstrated that under a weak dependence of surface temperature on initial temperature accounting for the above low-inertia zones (even if their thermal inertia is small compared to the inertia of the preheat layer of the condensed phase) leads to significant corrections to the t_c approximation.

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Boris V. Novozhilov
Vasily B. Novozhilov

Important Notation and Abbreviations

Abbreviations

ADN	Ammonium dinitramide
BVP	Boundary value problem
c.c.	Complex conjugate
ZN	Zeldovich–Novozhilov
FM	Flame model
HMX	Cyclotetramethylene tetranitramine
ODE	Ordinary differential equation
PDE	Partial differential equation
PETN	Pentaerythritol tetranitrate
QSHOD	Quasi-steady, homogeneous, one-dimensional
RDX	Cyclotrimethylene trinitramine
SHS	Self-propagating high–temperature synthesis
SRM	Solid rocket motor

Mathematical Functions

L_n	Laguerre polynomials
lg	$\log_{10}$
erfc	Complimentary error function
He_n	Hermite polynomials
W	Whittaker function

Notation

Over-bar	complex conjugate; Laplace–Carson transform
prime	time derivative, case-specific dimension; perturbed value, case-specific dimension

Basic Physical Dimensions	M (mass), L (length), T (time), θ (temperature), N (amount of substance, e.g. mole)
a	speed of sound, LT^{-1}; amplitude, case-specific dimension
a_f	amplitude of forced oscillations, case-specific dimension
A	nozzle discharge coefficient, $L^{-1}T$
b	combustion temperature, nondimensional; correction (Chapter 9)
c	specific heat at constant volume, $L^2T^{-2}\theta^{-1}$
c_p	specific heat at constant pressure, $L^2T^{-2}\theta^{-1}$
D, D_g	gas diffusion coefficient, L^2T^{-1}; amplitude of perturbation (Chapter 3), L; integration constant, nondimensional (Chapter 9)
E	activation energy, $ML^2T^{-2}N^{-1}$
f	temperature gradient at the surface, condensed phase side, θL^{-1}
$\bar{f}(p)$	Laplace–Carson transform of $f(t)$, case-specific dimension
J	Jacobian, $L^2M^{-1}T$
g	mass velocity of gas flow, $ML^{-2}T^{-1}$
G	response function of gas velocity to oscillating pressure, non-dimensional; integration constant, nondimensional (Chapter 9)
h	amplitude of perturbation of relative pressure, nondimensional
I	radiative heat flux, MT^{-3}
k	sensitivity coefficient, nondimensional
K	distortion factor, nondimensional; burning rate amplification coefficient, nondimensional; wave vector, L^{-1}
l	thickness of thermal layer of the condensed phase, L; mean free path for radiation absorption in the condensed phase, nondimensional
l_b	distance away from surface at which heat generation becomes negligible
L	length of cylindrical combustion chamber, L; latent heat of evaporation, L^2T^{-2}
m	mass burning rate, $ML^{-2}T^{-1}$
m_g	mass velocity of the gas stream, $ML^{-2}T^{-1}$
M	Mach number, nondimensional; mass of gas in the chamber, M
p	pressure, $ML^{-1}T^{-2}$
q	heat flux into the condensed phase, MT^{-3}
$Q = Q_s + Q_g$	total heat of combustion/reaction, L^2T^{-2}
Q_g	heat of combustion/reaction in the gas phase, L^2T^{-2}
Q_s	heat of reaction in the condensed phase, L^2T^{-2}
r	sensitivity parameter, nondimensional
r_b	sensitivity parameter, nondimensional

R	Universal gas constant, $ML^2T^{-2}\theta^{-1}N^{-1}$
s	relative change of the nozzle cross-sectional area, nondimensional; correction (Chapter 9), nondimensional; surface temperature in the steady-state regime at initial pressure (Chapter 9), nondimensional
S	cross-sectional area of cylindrical combustion chamber, L^2
t	time, T
T	temperature, θ; period of oscillations, nondimensional
T_k	reference temperature (Chapter 7), θ
u	propellant linear burning rate, LT^{-1}
u_g	dimensional gas velocity normal to the surface, LT^{-1}
u_p	gas velocity normal to the surface in the combustion products zone (Belyaev model), LT^{-1}
U	response function of burning rate to oscillating pressure, non-dimensional; mass burning rate dependence on temperature and pressure (Chapter 9), $ML^{-2}T^{-1}$
v	propellant linear burning rate, nondimensional
v_g	gas velocity normal to the surface, nondimensional
v_p	gas velocity normal to the surface in the combustion products zone (Belyaev model), nondimensional
V	volume of combustion chamber, L^3
w	tangential gas velocity, LT^{-1}
w_t	tangential gas velocity, nondimensional
W	chemical reaction rate, $ML^{-3}T^{-1}$
x	cartesian coordinate, L
y	cartesian coordinate, L, or nondimensional; correction (Chapter 9), nondimensional
Y	reactant mass fraction, nondimensional

Greek

α	linear absorption coefficient, L^{-1}; numerical parameter (Chapter 7), nondimensional
β	burning rate temperature sensitivity coefficient, θ^{-1}
γ	specific heat ratio, nondimensional; frequency (Chapter 9), nondimensional
$\widehat{\gamma}$	frequency, T^{-1}
δ	Jacobian, nondimensional; Feigenbaum constant, nondimensional
$\delta_{m,n}$	Kronecker delta
ε	erosion ratio (coefficient), nondimensional
ζ	channel coefficient of resistance, nondimensional; acoustic admittance, nondimensional
$\widehat{\zeta}, \widehat{\zeta}_1$	nozzle gain coefficient, nondimensional

η	pressure, nondimensional; relative concentration of the product, nondimensional; progress variable of chemical transformation, nondimensional
θ	temperature, nondimensional
Θ	response function of temperature of combustion products to oscillating pressure, nondimensional
ϑ	surface temperature, nondimensional
ι	sensitivity coefficient, nondimensional
κ	propellant thermal diffusivity, L^2T^{-1}
λ	thermal conductivity, $MLT^{-3}\theta^{-1}$; oscillation damping decrement, nondimensional
μ	sensitivity coefficient, nondimensional
μ_b	sensitivity coefficient, nondimensional
$\widetilde{\mu}$, μ_g	molecular weight, MN^{-1}
ξ	cartesian coordinate, nondimensional
ρ	density, ML^{-3}
σ	ratio of instantaneous nozzle cross-sectional area to its initial value, nondimensional; ratio of relaxation times of the gas and condensed phases, nondimensional
τ	time, nondimensional
$\tau_{U,V,r}$	lag times, nondimensional
φ	temperature gradient at the surface, at the condensed phase side, nondimensional
χ	apparatus constant, nondimensional
ψ	phase shift, nondimensional; velocity potential, L^2T^{-1}; auxiliary function of time (Chapter 7)
ω, $\widetilde{\omega}$	complex frequency, nondimensional
Ω	complex frequency, T^{-1}, or no-dimensional (Chapter 9)

Gothic

$\aleph$, $\aleph_{g,p}$	corrections (Chapter 9); nondimensional
$\mathfrak{p}_{g,p}$	complex amplitude (Chapter 9); nondimensional

Superscripts

a	analytical
c	t_c approximation
cl	classical
f	final
i	initial; interval

I	radiation conditions
qs	quasi-steady-state
r	t_r approximation
0	steady-state regime

Subscripts

a	ambient; acoustic
b	burning; combustion temperature
bl	boiling
c	condensed phase
ch	combustion products efflux from the chamber
cr	chemical reaction
cs	condensed phase side, at the surface
d	delay
e	correction to steady-state value (Chapter 4); entropy; extremum
f	final; position of flame front; flame
g	gas; mass velocity
gs	gas phase side, at the surface
i	initial; infinite
I	radiation conditions
na	nonacoustic
p	pressure; products; time scale of unsteady process (e.g. a period of pressure variation); response to variable pressure
r	radiation; reference; relaxation
s	surface; solid; position of liquid–gas interface
ε	erosion; erosive combustion regime
φ	nondimensional temperature gradient at the surface (condensed phase side)

About the Companion Website

This book is accompanied by a companion website:

www.wiley.com/go/Novozhilov/solidpropellantnonsteadycombustion

The Website includes:

- Solution manual
- Chapter Abstracts and keywords

Scan this QR code to visit the companion website