

Advances in Industrial Control

Victor Manuel Hernández-Guzmán  
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# Energy-Based Control of Electromechanical Systems

A Novel Passivity-Based Approach

**AIC**

 Springer

# Advances in Industrial Control

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A Novel Passivity-Based Approach

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## Series Editor's Foreword

A new book on control of electromechanical systems may seem pleonastic in the literature on industrial control systems. Actually, this is an original book both from the point of view of the methodological tools introduced, and of the results that the use of such tools allows to obtain in terms of industrial applications.

Electromechanical systems are of fundamental importance in industrial automation and process control. The actuators of robotic systems, of goods-handling systems, of numerical control machines, of servomotors that open and close the valves and shift the moving surfaces of relevant industrial plants are, in the majority of the cases, electromechanical systems. The dynamic models of electromechanical systems are typically formulated by referring to the balance between the potential energy and the kinetic energy associated with the systems themselves. For this reason, the use of a control approach based on passivity concepts appears to be absolutely natural: the most direct and obvious way of controlling that class of systems.

Passivity is a classical concept in control theory. It is strictly related to the concept of energy conservation. As a matter of fact, given a dynamical system and a certain time interval, the energy stored in the system is equal to the energy transferred to the system minus the energy dissipated. Then, in a physical system, since the dissipated energy is non-negative, the energy transferred to the system is always larger than or equal to the energy stored. In other words, a passive system is a system not able to produce energy. Passivity in linear time-invariant systems implies stability, as was highlighted in the early sixties with the well-known Kalman–Yakubovich–Popov Lemma. The extension of passivity concepts to nonlinear systems and its implications for Lyapunov stability was worked out through a series of fundamental papers published in the nineteen-seventies and nineties.

Passivity can be enforced in a system, under certain conditions, by means of an appropriate control synthesis, and this is often done with the main purpose of conferring the prescribed stability properties to the controlled system. The application of the classical theory of passivity to electromechanical systems can be found in many works that have appeared in the scientific literature in recent decades. Yet,

what distinguishes this book from the previously published works is the use of the classical concept of passivity revisited in a new and original way.

Different types of motors are considered in the book: permanent magnet brushed DC-motors and synchronous motors, induction motors, switched and synchronous reluctance motors, bipolar permanent magnet stepper motors and brushless DC-motors. They are all motors widely used in industrial plants. They often constitute the set of actuators upon which most of the process control and industrial automation systems rely. For every motor the authors describe the dynamic model formulation, starting from the working principle, and also introduce the standard control schemes. Then, they discuss and illustrate multi-loop control schemes based on their novel approach to passivity. The advantage of these schemes is that, by virtue of their simplicity, they are easily implementable and understandable, even to non-experts in control theory.

The book also includes a part dedicated to magnetic levitation systems and micro-electromechanical systems, as well as a final part where robotic systems driven by permanent magnet motors and by synchronous reluctance motors are considered. Given the importance that robotic applications are acquiring on the international industrial scene, this part may be of great interest even to practitioners wishing to improve the performance of robotic automation systems by adopting simple control schemes, which do not require complex computations and are actually compatible with the typical sampling times of the industrial world.

This book is very rich in content. The theoretical part is detailed and treated with precision. It is however, pleasant to read and understandable to an audience even of non-engineers, precisely because it focuses on very simple physical concepts, on which it is possible to have a natural intuition, even if the theoretical bases are lacking. The part in which electromechanical drives are described can also be useful to students who do not deal with control, but only with the modeling and design of electric machines. The robotic part can also be appreciated by readers who are interested in robot control, without necessarily having an expertise on electromechanical drives. For its richness of theoretical details, its methodological rigor and its well-defined structure, I think that this book will also be beneficial to researchers and doctoral students.

For all the reasons mentioned, I am particularly happy to welcome this new monograph in the series on *Advances in Industrial Control*, certain that readers will also be able to appreciate it and get ideas for their application activities in the field of industrial control.

Antonella Ferrara  
University of Pavia  
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*To Judith, my parents and my brothers.*

*Victor Manuel Hernández-Guzmán.*

*To my wonderful children—Rhomy, Robert,  
Josephamón, and Alessa—and to my mother.*

*Ramón Silva-Ortigoza.*

*To God, Virgin Mary, my parents and my  
brother.*

*Jorge Alberto Orrante-Sakanassi.*



# Preface

Electromechanical systems were introduced when electricity was employed for the first time to generate force and torque. At the beginning, electromechanical systems were controlled in open-loop. Once Automatic Control became a mature discipline, it was recognized that closed-loop control of electromechanical systems is instrumental to improve performance. Since then, much research work has been devoted to closed-loop control of electromechanical systems and many control techniques have been applied.

However, since many modern control techniques employ complex mathematical tools, most works on closed-loop control of electromechanical systems have resulted in complex mathematical algorithms which are difficult to understand and, hence, they have not been welcome by practitioners. Moreover, many formally supported controllers result in control laws that require lots of on-line computations which, besides requiring powerful and, hence, expensive hardware, deteriorate performance because they amplify noise, increase numerical errors and produce actuator saturation.

The above situation has motivated the application, by several authors in the past, of passivity-based ideas for closed-loop control of electromechanical systems. This approach takes advantage of the natural structure of the plant to be controlled and, hence, it has been demonstrated that results in simpler control laws. Thus, the designed controllers require a fewer number of on-line computations, improve performance because noise amplification and numerical errors are reduced and actuator saturation is avoided. Moreover, since passivity-based control is supported by energy ideas, a fundamental concept in engineering, this approach can be better understood by practitioners.

However, despite these advancements, passivity-based controllers are not as simple as controllers that are employed in industrial practice for electromechanical systems. This is the case of field oriented control (FOC) of alternating current (AC) motors. It is important to stress at this point that FOC of AC-motors is a control scheme which is not provided, until now, with a formal global asymptotic stability proof although presenting such a result has been the aim of several authors

in the past. The reason for such a search is to explain why FOC of AC-motors works well in practice and to provide tuning guide lines.

It must be recognized that presenting a solution for the above described control problem has not attracted attention of the control community in the recent years. This, however, is neither because the problem has lost relevance or nothing is remaining to solve. We believe that the mere reason for such a lack of interest is that the leading researchers have moved to other subjects. As a matter of fact, one of the leading researchers that tried in the past to present a global asymptotic stability proof for FOC of AC-motors has recently presented in [205] a work on such a subject. He states there the importance of presenting a global asymptotic stability proof for FOC of AC-motors providing tuning guide lines.

The present book is devoted to introduce recent advancements in the design of controllers for electromechanical systems. Most of our proposals consist of multi-loop control schemes possessing an internal proportional-integral (PI) electric current loop and an external PI velocity loop or proportional-integral-derivative (PID) position loop. Aside from these simple controllers, some additional simple terms are included to ensure global asymptotic stability. As we demonstrate along the book, these proposals are simpler control laws than the passivity-based controllers that have been proposed in the past.

The theoretical key to achieve these results is a novel passivity-based approach that we have developed during the last 12 years. This approach exploits the fact that the electrical and the mechanical subsystems exchange energy naturally during their normal operation. From the stability proof point of view, this allows the natural cancellation of several high-order terms. This allows to obtain simpler control laws because these terms must be computed on-line to be exactly cancelled when employing other control approaches, including the passivity-based approaches presented in the past.

It is the authors belief that, because of simpler control laws and a simpler rationale behind their design, our proposals can be welcome by practitioners. At this point, it is important to stress that simple Lyapunov stability analysis is the main mathematical tool that is employed to present the complete stability proofs. In this respect, in order to render attractive the book for both theorists and practitioners, we include the complete general mathematical modeling of all the electromechanical systems that we control. We also include, at the end of each chapter, how to obtain the mathematical model of practical AC-motors: (i) we dismantled the motor to analyze how the phase windings are distributed on the stator and how the permanent magnet (PM) poles are distributed on the rotor, (ii) based on the previous analysis, we compute the magnetic flux at the air gap using Ampère's Law, (iii) these results were employed to derive the motor mathematical model.

Finally, let us say that most of the complete formal stability proofs are presented in appendices, for the interested theorist readers. Simple sketches of the proofs are presented in the corresponding chapters to allow practitioners to understand the main results using energy interpretations.

We apply our approach to PM bushed-DC motors, PM synchronous motors, induction motors, switched reluctance motors, synchronous reluctance motors, PM stepper motors, brushless DC-motors, magnetic levitation systems, microelectromechanical systems, and rigid robot manipulators equipped with PM synchronous motors and switched reluctance motors.

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The ideas that have resulted in this book arose during the early years when the first author began his career as a researcher. Permanent magnet brushed DC-motors were studied first during the Ph.D. studies where he has Dr. Hebertt Sira-Ramírez as advisor. Very special thanks to him. After realizing that energy exchange between the electrical and the mechanical subsystems in these motors allows the natural cancellation of some cross terms, it was also natural to wonder whether these cancellations also exist in other classes of electric motors. After all, energy exchange between the electrical and the mechanical subsystems are the fundamental Physics phenomenon behind any electric motor operation. The present book is the answer for such a question.

Since those early years, a source of motivational support has been Dr. Victor Santibanez, a researcher at Instituto Tecnológico de La Laguna, in Torreón, Coah., México. Thanks and a special acknowledgment to him. Also thanks to my former Ph.D. students, Fortino Mendoza-Mondragón, Moises Martínez-Hernández, Mayra Antonio-Cruz, José Rafael García-Sánchez, and Celso Márquez-Sánchez, for their collaboration in diverse research subjects. Special thanks to my wife Judith for her continuous moral support and understanding when I spend so much time in researching and writing. Also thanks to my parents and my brothers for their fundamental teachings about life.

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# Contents

<b>1</b>	<b>Introduction</b>	1
<b>2</b>	<b>Mathematical Preliminaries</b>	7
2.1	Control of Linear Systems	7
2.2	Mathematical Tools for the Study of Nonlinear Control Systems	12
2.3	Passivity	31
2.4	A Novel Passivity-Based Approach for Control of Electromechanical Systems	39
2.5	The Electromechanical Systems that Are Studied in This Book	45
<b>3</b>	<b>Permanent Magnet Brushed DC-Motor</b>	49
3.1	Motor Modeling	49
3.1.1	A Simple Methodology	49
3.1.2	A General Methodology	53
3.2	Standard Control	54
3.2.1	Case 1	56
3.2.2	Case 2	60
3.2.3	Case 3	61
3.2.4	Case 4	62
3.2.5	An Improved PI Velocity Controller	63
3.2.6	An Improved PID Position Controller	67
3.3	The Standard Control Scheme Revisited	69
3.4	Open-Loop Energy Exchange	71
3.4.1	The Velocity Model	71
3.4.2	The Position Model	72
3.5	Velocity Control	73
3.6	Position Control	77
3.7	Velocity Control Using a DC/DC Buck Power Converter As Power Amplifier	83

3.7.1	Dynamic Model . . . . .	84
3.7.2	Open-Loop Energy Exchange . . . . .	86
3.7.3	Control of the DC to DC Buck Converter DC-Motor System . . . . .	87
3.7.4	Experimental Results . . . . .	92
<b>4</b>	<b>Permanent Magnet Synchronous Motor . . . . .</b>	<b>97</b>
4.1	Motor Modeling . . . . .	98
4.1.1	The Working Principle . . . . .	98
4.1.2	Three-Phase Dynamic Model . . . . .	99
4.1.3	Park's Transformation or dq Transformation . . . . .	105
4.1.4	The dq Dynamic Model . . . . .	108
4.1.5	dq Decomposition of the Stator Magnetic Flux . . . . .	112
	4.1.5.1 The Open-Loop Working Principle . . . . .	112
	4.1.5.2 Closed-Loop Operation . . . . .	113
	4.1.5.3 Field Weakening . . . . .	113
	4.1.5.4 The Home Position . . . . .	114
4.1.6	Standard Field-Oriented Control . . . . .	114
4.2	Open-Loop Energy Exchange . . . . .	117
4.2.1	The Velocity Model . . . . .	117
4.2.2	The Position Model . . . . .	119
4.3	Velocity Control . . . . .	119
4.3.1	Simulation Results . . . . .	130
4.4	Position Control . . . . .	133
4.4.1	Simulation Results . . . . .	147
4.5	Velocity Ripple Minimization . . . . .	150
4.5.1	Mutual Torque . . . . .	150
	4.5.1.1 Nonsinusoidal Distribution of Stator Windings . . . . .	151
	4.5.1.2 Errors in Stator Electric Current Measurements . . . . .	152
4.5.2	Reluctance Torque . . . . .	154
4.5.3	Cogging Torque . . . . .	154
4.5.4	Torque Ripple . . . . .	155
4.5.5	The Problem to Solve . . . . .	155
4.5.6	Stability Analysis . . . . .	156
4.5.7	Experimental Results . . . . .	162
4.6	A Practical PM Synchronous Motor . . . . .	165
4.6.1	Magnetic Field at the Air Gap . . . . .	167
	4.6.1.1 Magnetic Field Produced by the Stator Windings . . . . .	167
	4.6.1.2 Magnetic Field Produced by Rotor . . . . .	174

- 4.6.2 Magnetic Flux Linkages . . . . . 177
- 4.6.3 The Motor dq Dynamical Model . . . . . 180
- 4.7 Another Practical PM Synchronous Motor . . . . . 181
  - 4.7.1 Magnetic Field at the Air Gap . . . . . 186
    - 4.7.1.1 Magnetic Field Produced by the Stator Windings . . . . . 186
    - 4.7.1.2 Magnetic Field Produced by Rotor . . . . . 193
  - 4.7.2 Magnetic Flux Linkages . . . . . 196
  - 4.7.3 The Motor dq Dynamical Model . . . . . 200
- 5 Induction Motor . . . . . 205**
  - 5.1 Motor Modeling . . . . . 206
    - 5.1.1 The Working Principle . . . . . 206
    - 5.1.2 Three-Phase Dynamical Model . . . . . 207
    - 5.1.3 ab Transformation . . . . . 211
    - 5.1.4 The ab Dynamical Model . . . . . 213
    - 5.1.5 Park’s Transformation or dq Transformation . . . . . 217
    - 5.1.6 The dq Dynamical Model . . . . . 221
    - 5.1.7 dq Decomposition of the Magnetic Flux . . . . . 222
      - 5.1.7.1 The Open-Loop Working Principle . . . . . 222
      - 5.1.7.2 Closed-Loop Operation . . . . . 223
      - 5.1.7.3 Field Weakening . . . . . 224
    - 5.1.8 Standard Indirect Field-Oriented Control . . . . . 224
  - 5.2 Open-Loop Energy Exchange . . . . . 228
    - 5.2.1 The Velocity Model . . . . . 228
    - 5.2.2 The Position Model . . . . . 230
  - 5.3 Velocity Control . . . . . 231
    - 5.3.1 Simulation Results . . . . . 248
  - 5.4 Position Control . . . . . 251
    - 5.4.1 Simulation Results . . . . . 255
  - 5.5 A Practical Induction Motor . . . . . 258
    - 5.5.1 Magnetic Field at the Air Gap . . . . . 259
      - 5.5.1.1 Magnetic Field Produced by the Stator Windings . . . . . 259
      - 5.5.1.2 Magnetic Field Produced by Rotor . . . . . 270
    - 5.5.2 The Magnetic Flux Linkages . . . . . 271
    - 5.5.3 The Motor Dynamic Model . . . . . 276
- 6 Switched Reluctance Motor . . . . . 277**
  - 6.1 Motor Modeling . . . . . 278
    - 6.1.1 The Working Principle . . . . . 278
    - 6.1.2 Magnetic Circuits . . . . . 278
    - 6.1.3 SRM Unsaturated Dynamical Model . . . . . 281
    - 6.1.4 SRM Saturated Dynamical Model . . . . . 284



6.1.5	The Torque Sharing Approach . . . . .	285
6.1.6	Standard Control . . . . .	287
6.2	Open-Loop Energy Exchange . . . . .	289
6.2.1	The Unsaturated Velocity Model . . . . .	289
6.2.2	The Saturated and Unsaturated Position Models . . . . .	291
6.2.2.1	The Unsaturated Model . . . . .	291
6.2.2.2	The Saturated Model . . . . .	292
6.3	Velocity Control . . . . .	293
6.3.1	Simulation Results . . . . .	301
6.4	Position Control . . . . .	305
6.4.1	Position Control Without Velocity Measurements . . . . .	305
6.4.1.1	Closed-Loop Dynamics . . . . .	307
6.4.1.2	A Positive Definite Decrescent Function . . . . .	309
6.4.1.3	Stability Analysis . . . . .	310
6.4.1.4	Proof of Proposition 6.11 . . . . .	313
6.4.2	Position Control Taking into Account Magnetic Saturation . . . . .	316
6.4.2.1	Simulation Results . . . . .	325
6.5	A Practical Switched Reluctance Motor . . . . .	328
6.5.1	Magnetic Field at the Air Gap . . . . .	328
6.5.2	Magnetic Flux Linkages . . . . .	337
6.5.3	SRM Dynamical Model . . . . .	338
<b>7</b>	<b>Synchronous Reluctance Motor</b> . . . . .	<b>343</b>
7.1	Motor Modeling . . . . .	344
7.1.1	The Working Principle . . . . .	344
7.1.2	dq Dynamical Model . . . . .	345
7.1.3	Standard Field-Oriented Control of a SYRM . . . . .	346
7.2	Open-Loop Energy Exchange . . . . .	347
7.2.1	The Velocity Model . . . . .	347
7.2.2	The Position Model . . . . .	349
7.3	Velocity Control . . . . .	349
7.4	Position Control . . . . .	354
<b>8</b>	<b>Bipolar Permanent Magnet Stepper Motor</b> . . . . .	<b>361</b>
8.1	Motor Modeling . . . . .	362
8.1.1	The Working Principle . . . . .	362
8.1.2	Dynamical Model . . . . .	364
8.1.3	Standard Control . . . . .	366
8.2	Open-Loop Energy Exchange . . . . .	368
8.2.1	The Velocity Model . . . . .	368
8.2.2	The Position Model . . . . .	369

- 8.3 Velocity Control . . . . . 370
  - 8.3.1 Simulation Results . . . . . 376
- 8.4 Position Control . . . . . 378
  - 8.4.1 Simulation Results . . . . . 382
- 8.5 A Practical PM Stepper Motor . . . . . 385
- 9 Brushless DC-Motor . . . . . 393**
  - 9.1 Motor Modeling . . . . . 394
    - 9.1.1 Dynamic Model . . . . . 395
    - 9.1.2 Standard Control . . . . . 398
  - 9.2 Open-Loop Energy Exchange . . . . . 400
    - 9.2.1 The Velocity Model . . . . . 400
    - 9.2.2 The Position Model . . . . . 401
  - 9.3 Velocity Control . . . . . 402
    - 9.3.1 Simulation Results . . . . . 409
  - 9.4 Position Control . . . . . 410
    - 9.4.1 Simulation Results . . . . . 416
  - 9.5 A Practical BLDC Motor . . . . . 417
    - 9.5.1 Magnetic Field at the Air Gap . . . . . 419
      - 9.5.1.1 Magnetic Field Produced by the Stator Windings . . . . . 419
      - 9.5.1.2 Magnetic Field Produced by Rotor . . . . . 427
    - 9.5.2 Magnetic Flux Linkages . . . . . 428
    - 9.5.3 Mathematical Model . . . . . 429
- 10 Magnetic Levitation Systems and Microelectromechanical Systems . . . . . 435**
  - 10.1 Magnetic Levitation Systems . . . . . 436
    - 10.1.1 Mathematical Model . . . . . 436
    - 10.1.2 Open-Loop Energy Exchange . . . . . 438
    - 10.1.3 Position Control . . . . . 439
    - 10.1.4 Simulation Results . . . . . 448
  - 10.2 Microelectromechanical Systems . . . . . 451
    - 10.2.1 Mathematical Model . . . . . 451
    - 10.2.2 Open-Loop Energy Exchange . . . . . 454
    - 10.2.3 Position Control . . . . . 456
    - 10.2.4 Simulation Study . . . . . 463
- 11 Trajectory Tracking for Robot Manipulators Equipped with PM Synchronous Motors . . . . . 469**
  - 11.1 Dynamical Model of Robot Manipulators Equipped with PM Synchronous Motors . . . . . 471
  - 11.2 Main Result . . . . . 473

11.3	Simulation and Experimental Results . . . . .	480
11.3.1	Simulation Study . . . . .	480
11.3.2	Experimental Results . . . . .	485
<b>12</b>	<b>PID Control of Robot Manipulators Equipped with SRMs</b> . . . . .	<b>489</b>
12.1	Dynamical Model . . . . .	490
12.2	The Torque Sharing Approach. . . . .	492
12.3	Main Result . . . . .	493
12.3.1	The Rationale Behind Controller in Proposition 12.1 . . . . .	495
12.3.2	Sketch of Proof of Proposition 12.1. . . . .	496
12.3.3	Closed-Loop Dynamics. . . . .	496
12.3.4	A Positive Definite and Decrescent Function . . . . .	498
12.3.5	Time Derivative of $V(y)$ . . . . .	500
12.3.6	Proof of Proposition 12.1 . . . . .	505
12.4	Simulation Study . . . . .	506
	<b>Appendix A: Energy Functions</b> . . . . .	<b>511</b>
	<b>Appendix B: Proofs of Propositions for Brushed DC-Motors</b> . . . . .	<b>533</b>
	<b>Appendix C: Proofs of Propositions for PM Synchronous Motors</b> . . . . .	<b>545</b>
	<b>Appendix D: Proofs of Propositions for Induction Motors</b> . . . . .	<b>561</b>
	<b>Appendix E: Proofs of Propositions for Switched Reluctance Motors</b> . . . . .	<b>577</b>
	<b>Appendix F: Proofs for BLDC Motors</b> . . . . .	<b>589</b>
	<b>Appendix G: Derivation of Some Expressions for the Proofs in Chap. 10</b> . . . . .	<b>599</b>
	<b>References</b> . . . . .	<b>603</b>
	<b>Index</b> . . . . .	<b>617</b>

# Chapter 1

## Introduction



Electric motors are fundamental devices for force and torque generation in industrial facilities. In fact, electric motors were introduced in the nineteenth century, by the incipient companies that tried to expand the use of electric energy, to provide industry with a device capable to generate force and torque. These companies distributed the electric energy in the form of what we know now as direct current (DC). This situation motivated T.A. Edison to introduce the brushed DC-motor. The main advantage of brushed DC-motor is that it can be easily controlled. In fact, in order to put to work a brushed DC-motor it suffices to connect its terminals.

Alternating current (AC) was introduced later by N. Tesla as a means to improve the distribution of electric energy. Tesla recognized the need for a new motor working on the basis of AC, and thus, he introduced the induction motor. One important advantage of induction motor is that it does not employ brushes nor commutators to operate. However, it was soon realized that induction motor was difficult to control compared to brushed DC-motor. For instance, in order to vary the motor velocity it is required to vary the frequency of the AC flowing through the motor phase windings. Hence, complex hardware was required to operate induction motors compared to hardware required to operate brushed DC-motors. Moreover, performance was not as good as that achieved by brushed DC-motors: this control strategy, also known as open-loop, was found to be stable in practice only if the motor is lightly loaded.<sup>1</sup>

At that point of time, electric motors were controlled empirically, i.e., without taking into account their mathematical models. When Automatic Control became a mature discipline, it was recognized the necessity to take into account the motor model to improve performance. It was then found that brushed DC-motor has a linear and single-input single-output model which facilitates the control design task. On the other hand, the induction motor model is nonlinear and it has several inputs. Thus, controller design for induction motors was soon recognized to be a difficult task. Several classes AC-motors were introduced later in the twentieth century which,

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<sup>1</sup>It is interesting to remark that this was mathematically demonstrated recently in [212]. Moreover, global exponential stability was demonstrated later in [270].

contrary to brushed DC-motors, do not require brushes nor commutators to work, an attractive feature because of the reductions in maintenance periods and costs. However, all of these AC-motors have a similar disadvantage than induction motors: they are difficult to control because they have nonlinear models and multiple inputs.

The ease to operate brushed DC-motors motivated their employment in most of the applications of electric motors in the twentieth century. In 1972, a new control methodology for induction motors was empirically introduced [22]. It is known now as vectorial control of AC-motors or field-oriented control (FOC) of AC-motors. However, because of the hardware limitations at that moment, the practical applications of FOC appeared after several years when microprocessor and digital-signal-processor (DSP) technologies became popular [20, 264]. The use of what we know now as standard FOC (SFOC) importantly improved performance of induction motors in practical applications and SFOC has become now the standard in industrial applications. This success motivated the development of SFOC ideas to control other AC-motors.

However, despite the success of SFOC in practice, any formal study does not exist ensuring its global asymptotic stability. This has motivated several formal works trying to (a) propose new control design methodologies which be provided with a formal stability proof and improving the performance achievable with SFOC and (b) find a global asymptotic stability proof for SFOC which formally explains its practical success providing control tuning guidelines.

Following the line of (a), the book [60] has summarized the work of a group of researchers who proposed a series of controllers for different AC-motors which exploit the *backstepping* ideas first introduced in [152]. The main drawback of such proposals is the large amount of online computations that are required by the corresponding control laws. It is important to stress that this is not just a matter of finding fast enough hardware to perform the required computations, but it is a matter that pertains to performance deterioration due to numerical errors and noise amplification introduced by complex computations [204]. Furthermore, as it is stated in [222], the drives community does not like complex controllers.

On the other hand, following the line of (b), the book [204] has summarized the work of a group of researchers who proposed a series of controllers for different AC-motors which rely on *standard passivity-based control*, an approach that has its roots in the work published in [209]. It is shown that using this approach, several basic concepts of FOC for AC-motors can be explained. In particular, in [210] is formally proven global asymptotic stability of standard indirect field-oriented control (SIFOC) for current-fed induction motors. This means that the stator electrical dynamics is neglected, i.e., the stator electric current is assumed to be the control signal for motor instead of the applied voltages. It is important to stress that SIFOC is the workhorse for induction motor control in industrial applications at present. However, when taking into account the stator electrical dynamics, i.e., when the motor control signals are voltages applied at the stator terminals, a number of important

differences appear between the solutions in [204] and SIFOC of induction motors.<sup>2</sup> In particular, a number of additional terms which require the exact knowledge of several motor parameters have to be computed on line. The proposed solution relies on torque observers instead of PI velocity controllers. Moreover, the PID position control problem is not studied. This is important to stress since PID control defines the application of SIFOC in position control problems. It is interesting to remark that instead of PID position regulation, position trajectory tracking control of robot manipulators equipped with induction motors is solved in [204]. Notice that trajectory tracking requires more complex controllers than simple PID regulators. Furthermore, recall that the motor controllers proposed in [204] are more complex than SIFOC. Thus, it is the authors belief that solving the trajectory tracking control problem instead of the simpler PID position regulation problem can be explained by recalling that it is easier to justify a complex controller if the task is also complex.

After [60, 204] were published, the interest in these problems diminished and the authors that led those works moved to other research subjects. Moreover, current research on AC electric machines is focused on sensorless control. See [18, 54, 86, 117, 118, 156, 183, 193, 207, 208, 245, 268, 281] and references therein for instance. Despite this, as we have shown in the above discussion, it is not because the problem has been solved or it has no relevance. As a matter of fact, in the recent paper [205], the leading author of [204] recognizes the importance of presenting a global asymptotic stability proof for field-oriented control. Moreover, efforts in that paper focus in proving global asymptotic stability when internal PI electric current controllers are employed. However, this is performed when controlling the mechanical subsystem in open-loop and the use of either a PI velocity controller or a PID position controller still remains without a formal solution.

Motivated by the control problems that remain open, which we describe below, in the present book, we introduce a novel passivity-based control approach for electromechanical systems. The main advantages of this new approach with respect to previous passivity-based methodologies in the literature are the following:

1. The energy exchange that naturally appears between the electrical and the mechanical subsystems is exploited. This feature is important to stress because it allows the derivation of simpler control laws and it has not been exploited in previous works in the literature. Moreover, previous passivity-based controllers rely on exact cross term cancellations that require the exact knowledge of several motor parameters.
2. The previous passivity-based approaches in the literature rely on deriving isolated closed-loop error equations for the electrical dynamics in order to take advantage of the system passivity properties. Instead of this, we dominate the cross terms existing between the electrical and the mechanical dynamics.

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<sup>2</sup>We consider important to stress that it is stated in [204] that their control approach is also valid for many AC-motors; however, an explicit controller is only presented for induction motors. As we show in the present book much simpler controllers are designed for other AC-motors when using our approach than following the steps suggested by the approach in [204].

3. In order to achieve the previous item, the previous passivity-based approaches in the literature require to compute online and to cancel the complete expression for the time derivative of the desired electric current. This commonly requires an important number of additional online computations rendering the controller more complex. The novel passivity-based approach that we propose does not require to perform these computations nor cancellations because we dominate these terms.
4. A *nested-loop passivity-based control* approach is exploited in [204]. This means that the electric current error is first proven to converge exponentially to zero and this allows to use this variable as a vanishing perturbation for the mechanical subsystem. This, however, requires the online computations referred in the previous item. Instead of that we use an approach which is similar to what was called in [204], pp. 243, *passivity-based control with total energy shaping*. Although the latter approach has been disregarded in [204] arguing that it results in more complex controllers; we prove the opposite in the present book. This is another important contribution of our proposal.
5. The previous passivity-based approaches in the literature replace velocity measurements with high-pass position filtering explaining that this reduces the effects of noise that is present in velocity measurements, which is true. However, it is also true that the time derivative of the desired electric current would become even more complex if velocity measurements were allowed. Hence, an even more complex controller would result. On the other hand, it is important to stress that allowing velocity measurements is instrumental to successfully design PID position controllers in nonlinear systems.<sup>3</sup> This is concluded from results in PID robot control where the few control schemes that have been proposed without requiring velocity measurements impose severe constraints to the controller gains that can be employed and, hence, the achieved performances are far from satisfactory (try to perform simulations with the controllers in [98, 172, 203, 252, 253], for instance). Furthermore, some of these works rely on the presence of significant viscous friction that the mechanical subsystem must naturally possess. This might be the reason why PID position regulation is not reported together with SIFOC in AC-motors as pointed out above. It is shown in the subsequent chapters that the novel passivity-based approach that we propose allows velocity measurements without increasing the complexity of the resulting controller. As a matter of fact, simple PID position controllers are presented for several AC-motors.
6. Aside from three simple nonlinear terms, the main controllers that we propose are identical to SFOC for voltage-fed AC-motors, i.e., when the complete motor dynamics is taken into account. Hence, we propose the most similar control scheme to SFOC but provided with a global asymptotic stability proof.

In this book, we are interested in proposing control strategies that are simple to implement. In particular, those control strategies that are very similar to SFOC

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<sup>3</sup>In [204], Ch. 8, is designed a PID position controller using velocity low-pass filtering for a magnetic levitation system. However, this is possible because the mechanical subsystem is linear and, hence, linear concepts such as Hurwitz matrices can be used to ensure stability.

or other control schemes that are recognized to be the standard control techniques for each one of the electromechanical systems that we consider in the book. Our contribution consists in providing formal stability proofs for the control schemes that we propose such that, under mild assumptions, our proposals explain to some extent why SFOC and other standard controllers work well in practice.

We are also interested in presenting the details behind the mathematical models that are employed to perform the control design task. It is the authors experience that designing controllers for electromechanical systems without a clear understanding of the principles that determine how the plant works, and in particular how the electromagnetic subsystem works, is something like being blind. Thus, we begin each chapter by explaining how the electromechanical system is modeled and the rationale behind the coordinate changes that are employed. After that, we describe the standard control scheme that is traditionally employed for the electromechanical system under study, and finally, we present the controller that we propose with the corresponding stability proof.

Aside from theorists, this book also tries to attract attention of practitioners. Hence, the complete formal stability proofs are sent to appendices for the interested readers, and simple sketches of the proofs are presented in the body of the chapters. Reason for this is to explain, using simple energy-based arguments, how the main ideas of the novel passivity-based approach that we introduce are exploited. On the other hand, it is usual in the control community concerned with the study of electromechanical systems to use the dynamical models that are at disposal but they have never seen the inside of an AC-motor, for instance. Although this is not necessary to design controllers, performing this gives a lot of insight on the main assumptions employed when modeling an AC-motor and the main ideas behind the coordinate changes that are usual when using SFOC.

In order to reduce this gap between theory and practice, we present the study of the stator windings of several classes of practical AC-motors at the end of most chapters. First, we have dismantled the motor and we describe the physical distribution of the stator phase windings and the rotor permanent magnets if any. Then, we employ a procedure described in [55], based on *Ampère's Law*, to mathematically model the magnetic field distribution produced by the stator phase windings. We also describe mathematically the magnetic field distribution produced by the rotor permanent magnets, if any. This allows to compute the stator three-phase windings flux linkages, and thus, (i) the three-phase electrical subsystem dynamical model is derived and (ii) the electromagnetic torque generated by the three phases of the motor is also computed. After this, the application of *Newton's Second Law* results in the mechanical subsystem dynamical model. Then, as an application example, the formulas derived at the beginning of each chapter are employed to obtain the dq dynamical model of the motor. Finally, we explain how these derived models are correctly represented by the general dq dynamical model derived in the first part of the chapter for a general motor of the class under study.



In the following chapter, i.e., Chap. 2, we review the mathematical tools required by the formal part of the studies presented in this book. The main ideas of our novel control approach are illustrated using the permanent magnet brushed DC-motor as an example. Finally, we also present there a description of the book content and how the subsequent chapters relate among them.

# Chapter 2

## Mathematical Preliminaries



### 2.1 Control of Linear Systems

From the *classical control* point of view physical systems are modeled using the following ordinary linear  $n$ -order differential equation:

$$\begin{aligned}
 y^{(n)} + a_{n-1}y^{(n-1)} + \dots + a_2\ddot{y} + a_1\dot{y} + a_0y \\
 = b_mu^{(m)} + b_{m-1}u^{(m-1)} + \dots + b_2\ddot{u} + b_1\dot{u} + b_0u,
 \end{aligned}
 \tag{2.1}$$

where  $n \geq m$ ,  $a_i, b_j, i = 0, \dots, n - 1, j = 0, \dots, m$ , are real constant scalars,  $y(t)$  is the variable representing the system response or *output* and  $u(t)$  is the system excitation or *input*. Applying the Laplace transform to the previous expression and assuming that all of the initial conditions are zero, we obtain the following *transfer function*:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_ms^m + b_{m-1}s^{m-1} + \dots + b_2s^2 + b_1s + b_0}{s^n + a_{n-1}s^{n-1} + \dots + a_2s^2 + a_1s + a_0},
 \tag{2.2}$$

where  $Y(s) = \mathcal{L}\{y(t)\}$  and  $U(s) = \mathcal{L}\{u(t)\}$  are the Laplace transforms of  $y(t)$  and  $u(t)$ , respectively. The  $n$  roots of polynomial at the denominator are known as the system *poles* and the  $m$  roots of polynomial at the numerator are known as the system *zeros*. Given a known function of time  $u(t)$  such that  $U(s) = \frac{L(s)}{M(s)}$ , where  $L(s)$  and  $M(s)$  are polynomials, the solution  $Y(s)$  is found using fraction expansion, i.e.,

$$\begin{aligned}
 Y(s) &= \frac{b_ms^m + b_{m-1}s^{m-1} + \dots + b_2s^2 + b_1s + b_0}{s^n + a_{n-1}s^{n-1} + \dots + a_2s^2 + a_1s + a_0} U(s), \\
 &= \frac{b_ms^m + b_{m-1}s^{m-1} + \dots + b_2s^2 + b_1s + b_0}{s^n + a_{n-1}s^{n-1} + \dots + a_2s^2 + a_1s + a_0} \frac{L(s)}{M(s)},
 \end{aligned}$$

$$\begin{aligned}
&= \sum_{i=1}^l \frac{f_i}{s + d_i} \\
&+ \sum_{i=1}^q \left( \frac{g_{1i}}{(s + h_i)^k} + \frac{g_{2i}}{(s + h_i)^{k-1}} + \cdots + \frac{g_{(k-1)i}}{(s + h_i)^2} + \frac{g_{ki}}{s + h_i} \right), \\
&+ \sum_{i=1}^r \frac{\beta_i s + \gamma_i}{s^2 + 2\zeta_{ip}\omega_{nip}s + \omega_{nip}^2} \\
&+ \sum_{i=1}^v \left( \frac{\delta_{1i}s + \alpha_{1i}}{(s^2 + 2\zeta_i\omega_{ni}s + \omega_{ni}^2)^k} + \frac{\delta_{2i}s + \alpha_{2i}}{(s^2 + 2\zeta_i\omega_{ni}s + \omega_{ni}^2)^{k-1}} + \right. \\
&\quad \left. \cdots + \frac{\delta_{(k-1)i}s + \alpha_{(k-1)i}}{(s^2 + 2\zeta_i\omega_{ni}s + \omega_{ni}^2)^2} + \frac{\delta_{ki}s + \alpha_{ki}}{s^2 + 2\zeta_i\omega_{ni}s + \omega_{ni}^2} \right) \\
&+ \frac{Q(s)}{M(s)},
\end{aligned}$$

where it is taken into account that the plant poles may be real, single or repeated  $k$  times, or complex conjugate, single or repeated  $k$  times. The symbols  $f_i$ ,  $g_{ji}$ ,  $\beta_i$ ,  $\gamma_i$ ,  $\delta_{ji}$ ,  $\alpha_{ji}$ ,  $j = 1, \dots, k$ , represent real constants to be computed. It is assumed that  $G(s)$  has (i)  $l$  different real single poles located at  $s = -d_i$ , (ii)  $q$  different real repeated poles located at  $s = -h_i$ , (iii)  $r$  different single pairs of complex conjugate poles located at  $s = -\zeta_{ip}\omega_{nip} \pm j\omega_{dip}$ , where  $\omega_{dip} = \omega_{nip}\sqrt{1 - \zeta_{ip}^2}$ , and (iv)  $v$  different repeated pairs of complex conjugate poles located at  $s = -\zeta_i\omega_{ni} \pm j\omega_{di}$ , where  $\omega_{di} = \omega_{ni}\sqrt{1 - \zeta_i^2}$ . Moreover, the latter fraction  $\frac{Q(s)}{M(s)}$ , where  $Q(s)$  is another polynomial to be computed, represents the effect of all of the input poles. Thus, we can apply the inverse Laplace transform to find the *general response of a linear system*:

$$\begin{aligned}
y(t) &= \mathcal{L}^{-1}\{Y(s)\} = \sum_{i=1}^l f_i e^{-d_i t} \\
&+ \sum_{i=1}^q (g_{1i} t^{k-1} e^{-h_i t} + g_{2i} t^{k-2} e^{-h_i t} + \cdots + g_{(k-1)i} t e^{-h_i t} + g_{ki} e^{-h_i t}), \\
&+ \sum_{i=1}^r \frac{\lambda_i}{\sqrt{1 - \zeta_{ip}^2}} e^{-\zeta_{ip}\omega_{nip} t} \sin(\omega_{dip} t + \phi_{ip}) \\
&+ \sum_{i=1}^v \left( \frac{\rho_{1i}}{\sqrt{1 - \zeta_i^2}} t^{k-1} e^{-\zeta_i\omega_{ni} t} \sin(\omega_{di} t + \phi_{ip}) \right. \\
&\quad \left. + \frac{\rho_{2i}}{\sqrt{1 - \zeta_i^2}} t^{k-2} e^{-\zeta_i\omega_{ni} t} \sin(\omega_{di} t + \phi_i) + \right.
\end{aligned}$$

$$\begin{aligned}
& \dots + \frac{\rho^{(k-1)i}}{\sqrt{1-\zeta_i^2}} t e^{-\zeta_i \omega_{ni} t} \sin(\omega_{di} t + \phi_i) \\
& + \frac{\rho_{ki}}{\sqrt{1-\zeta_i^2}} e^{-\zeta_i \omega_{ni} t} \sin(\omega_{di} t + \phi_i) \Big) \\
& + p(t), \quad \text{where } p(t) = \mathcal{L}^{-1} \left\{ \frac{Q(s)}{M(s)} \right\}. \tag{2.3}
\end{aligned}$$

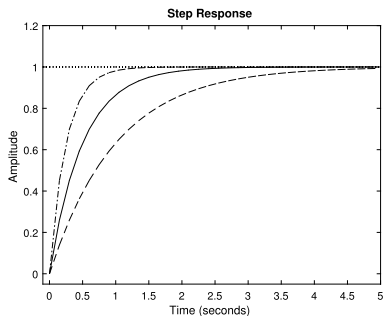
The symbols  $\lambda_i, \rho_{ji}, j = 1, \dots, k$ , represent real constants and  $\phi_{ip} = \arctan\left(\frac{\sqrt{1-\zeta_p^2}}{\zeta_{ip}}\right)$ ,  $\phi_i = \arctan\left(\frac{\sqrt{1-\zeta_i^2}}{\zeta_i}\right)$ .

In the previous expression,  $p(t)$  is known as the *forced response*,  $y_f(t)$ , and all of the remaining terms are known as the *natural response*,  $y_n(t)$ . Since  $p(t) = \mathcal{L}^{-1} \left\{ \frac{Q(s)}{M(s)} \right\}$ , the forced response is “similar” to the input  $u(t)$ . Because of this, in a control system the input  $u(t)$  (through  $p(t)$ ) is employed to specify the behavior that is expected for  $y(t)$ , i.e., a closed-loop control system is designed such that  $\lim_{t \rightarrow \infty} y(t) = y_f(t) = p(t)$ . Hence, it is desirable that  $p(t) = u(t)$  and  $\lim_{t \rightarrow \infty} y_n(t) = 0$ . A transfer function is said to be stable if and only if this limit is accomplished for all initial conditions.

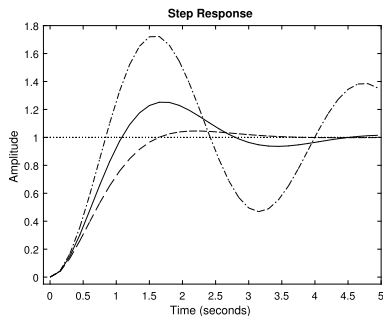
According to the general solution given in (2.3), we have the following conclusions on the *stability of an arbitrary linear system*.

1. The differential equation in (2.1) or, equivalently, the transfer function in (2.2) is stable if and only if all of the poles of  $G(s)$  have negative real parts, i.e., if and only if  $-d_i < 0$ ,  $-h_i < 0$ ,  $-\zeta_{ip}\omega_{nip} < 0$ , and  $-\zeta_i\omega_{ni} < 0$ .
2. The differential equation in (2.1) or, equivalently, the transfer function in (2.2) is unstable if at least one of the poles of  $G(s)$  has a positive real part, i.e., if  $-d_i > 0$  or  $-h_i > 0$  or  $-\zeta_{ip}\omega_{nip} > 0$  or  $-\zeta_i\omega_{ni} > 0$ .
3. The differential equation in (2.1) or, equivalently, the transfer function in (2.2) is unstable if  $G(s)$  has at least one pole with zero real part which is repeated at least two times, i.e., if  $-h_i = 0$  or  $-\zeta_i\omega_{ni} = 0$  and  $k \geq 2$ .
4. The differential equation in (2.1) or, equivalently, the transfer function in (2.2) is marginally stable if all of the poles of  $G(s)$  have negative real parts excepting some single poles which have zero real parts, i.e., if  $-d_i < 0$ ,  $-h_i < 0$ ,  $-\zeta_{ip}\omega_{nip} < 0$ ,  $-\zeta_i\omega_{ni} < 0$  and  $k = 1$  for the poles such that  $-h_i = 0$  or  $-\zeta_i\omega_{ni} = 0$ .

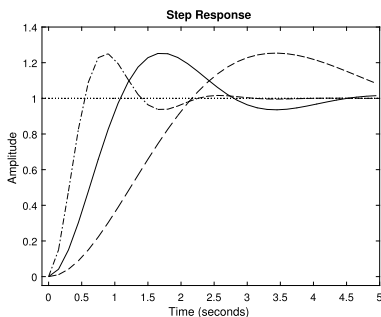
From the expression in (2.3) it is clear that the poles of a transfer function not only determine the system stability but also the system transient response. As a matter of fact, in Fig. 2.1 we present several examples of *typical transient responses* due to real poles and complex conjugate poles when  $u(t) = 1$ . The relative location of these poles on the plane  $s$  is depicted in Fig. 2.2. We conclude that a faster response is accomplished as these poles are located farther from the origin on the left-hand half-complex plane. A more oscillatory response is obtained as the complex conjugate



(a) A real pole. Continuous  $d = 2$ , dashed  $d = 1$ , dash-dot  $d = 4$ .



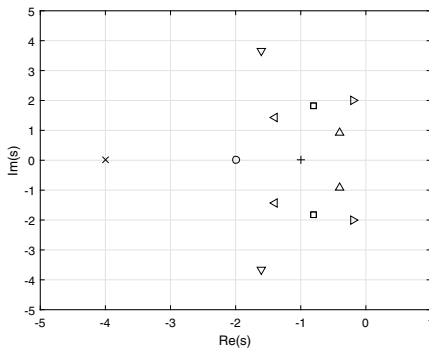
(b) Two complex conjugate poles.  $\omega_n = 2$ . Continuous  $\zeta = 0.4$ , dashed  $\zeta = 0.7$ , dash-dot  $\zeta = 0.1$ .



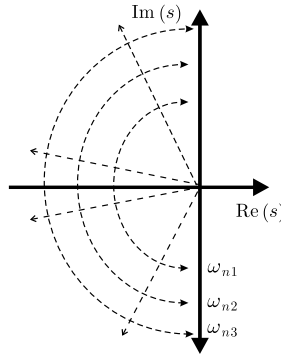
(c) Two complex conjugate poles.  $\zeta = 0.4$ . Continuous  $\omega_n = 2$ , dashed  $\omega_n = 1$ , dash-dot  $\omega_n = 4$ .

**Fig. 2.1** Transient responses when an unit step input is applied

**Fig. 2.2** Location on the plane  $s$  of poles corresponding to the transient responses in Fig. 2.1. Figure 2.1a: dash-dot  $\times$ , continuous  $\circ$ , dashed  $+$ . Figure 2.1b: dash-dot triangle-right, continuous square, dashed triangle-left. Figure 2.1c: dash-dot triangle-down, dashed triangle-up



poles approach the imaginary axis. Real poles do not produce any oscillation. This is depicted in Fig. 2.3.



**Fig. 2.3** Regions of the plane  $s$ . **a** Arrows of the straight lines indicate where to locate poles to achieve a faster response. A slower response is obtained assigning the poles in the opposite direction of the arrows. **b** Arrows of the circular lines indicate where to locate the poles to obtain a more oscillatory response. A less oscillatory response is obtained assigning the poles in the opposite direction of the arrows

Also notice that in the case where  $u(t) = A = \text{constant}$ , that is  $U(s) = \frac{A}{s}$ , the steady-state response can be computed using the final value theorem, i.e.,

$$\lim_{t \rightarrow \infty} y(t) = \lim_{s \rightarrow 0} sY(s) = \lim_{s \rightarrow 0} sG(s)U(s) = \lim_{s \rightarrow 0} sG(s) \frac{A}{s} = G(0)A.$$

This means that the system steady-state response depends on the locations of both poles and zeros of  $G(s)$ .

Thus, the problem of designing a controller in classical control is to choose a new differential equation with transfer function  $G_c(s)$ , known as the controller, such that when it is feedback connected to the plant transfer function  $G_p(s)$ , i.e., as shown in Fig. 2.4, the closed-loop transfer function:

$$\frac{Y(s)}{Y_d(s)} = G_{cl}(s) = \frac{G_c(s)G_p(s)}{1 + G_c(s)G_p(s)},$$

where  $Y_d(s) = \frac{A}{s}$  represents the reference or the desired output, satisfies the following requirements.

- $G_{cl}(s)$  is stable.
- All poles and zeros of  $G_{cl}(s)$  are suitably assigned on the left-hand half-plane such that the transient response satisfies the desired specifications.
- All poles and zeros of  $G_{cl}(s)$  are suitably assigned on the left-hand half-plane such that the steady-state response satisfies  $\lim_{t \rightarrow \infty} y(t) = G_{cl}(0)A = A = y_d(t) = p(t) = y_f(t)$ , where  $y_d(t) = \mathcal{L}^{-1}\{Y_d(s)\}$ .

Two powerful tools are employed in classical control to achieve simultaneously these requirements: the *root locus method* (see Sect. 3.2) and the *frequency response*