

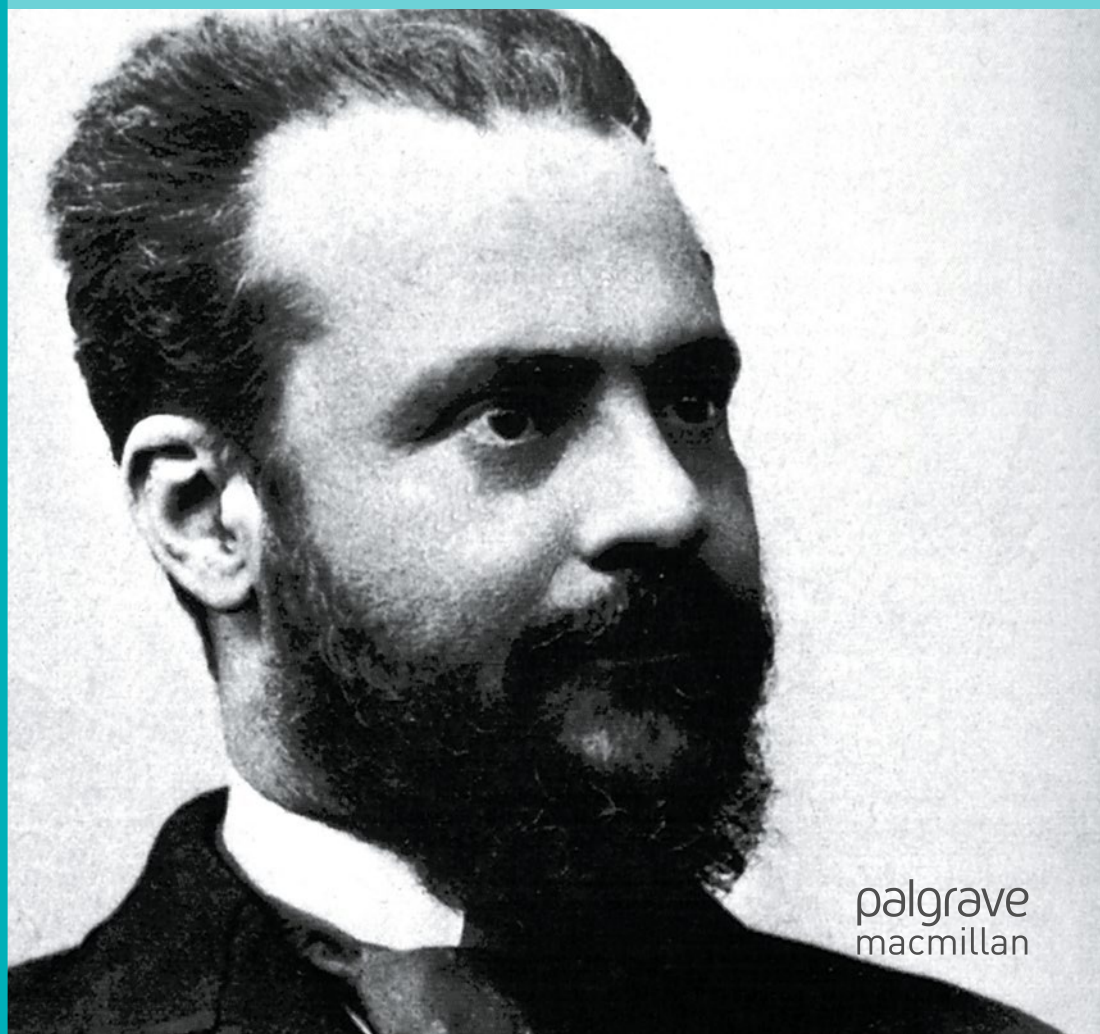


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Vilfredo Pareto:
An Intellectual Biography
Volume III

From Liberty to Science (1898–1923)

Fiorenzo Mornati



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Fiorenzo Mornati
Dipto di Econ e Statistica
University of Turin
Torino, Italy

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PREFACE

In this volume, the third of the trilogy, we shall deal with the final, very intense period in Pareto's intellectual biography when, by now largely free of political distractions, he was able to further investigate and to attempt a synthesis of the two disciplines to which he had devoted most attention, economics and sociology.

Considering pure economics to be now capable of autonomous development, that is free of the involvement of specific external factors, we shall initially turn our attention to the definitive conclusions reached by Pareto with particular regard to the architecture of general equilibrium, based exclusively on the empirical factor represented by the curve of indifference.

The final form of Pareto's sociology appears, instead, to spring from the disenchantment he now felt in observing the irreversible decline of liberalism together with the apparently irresistible rise of socialism. These two phenomena, with their apparent negation of the logic whereby societies will constantly seek to attain maximum well-being, appear to have stimulated Pareto to attempt the formulation of the comprehensive account appearing in the *Treatise* on general sociology, of which we will examine the salient features. The war and the post-war period provided Pareto with an opportunity to perform a—somewhat self-congratulatory—verification of the plausibility of the sociological model he had developed.

Again, in this volume we will draw on the resources found in the Paretoology in a highly selective manner, presenting the formal arguments in as complete but accessible a manner as possible.

We extend our thanks to Roberto Marchionatti for our exchanges on a variety of topics relating also to this volume and to the following for their valuable and patient collaboration in archive research (in alphabetical order): the Banca Popolare di Sondrio (owners of the Vilfredo Pareto letter archive held at the Luigi Credaro library in Sondrio), Piercarlo Della Ferrera (custodian of the archive), the Interlibrary loans service of the Norberto Bobbio Library at the University of Turin and the Vaud Cantonal Archives in Lausanne.

Torino, Italy

Fiorenzo Mornati

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A New Pure Economics

In the decade following his address at the “Stella” student association in Lausanne and culminating in the publication of the French version of his *Manual of Political Economy*, Pareto’s conception of theoretical (or “pure”) economics, hitherto constituting an introduction to the field of applied economics which followed broadly in the footsteps of Walras,¹ appears to have undergone a definitive and largely self-sustained development which was distinctly original, even if never disconnected from Pareto’s other interests in the social sciences.² Hence, in this chapter, we will describe his ground-breaking theory of choice (Sects. 1.1 and 1.2) as well as the aspects of the *Manual* which display innovations in relation to Pareto’s economic thinking of the immediately preceding period (Sect. 1.3). We will then characterise the definitive description of pure economics offered by Pareto (Sect. 1.4) together with a selection of his critical—and self-critical—remarks on the discipline (Sect. 1.5).

1.1 THE BEGINNINGS OF PARETIAN PURE ECONOMICS: EARLY REFERENCES TO THE THEORY OF CHOICE

On the 14th of December 1898³ Pareto informed Pantaleoni that for the meeting of the Lausanne student association “Stella” on the following 17th of December, when he was due to be nominated an honorary member,⁴ he had prepared the paper *Comment se pose le problème de l’économie*

pure (Expounding the pure economics). In this paper he showed “how to circumvent the difficulty arising from the impossibility of measuring ophelimity”. This is recognised as constituting a decisive step forward in the development of Pareto’s theory of utility which was formalised with the abandoning of the always problematical cardinal conception of utility in favour of an ordinal alternative, at least for theoretical purposes. In his paper Pareto affirmed that in order to construct a pure economics, it was necessary, firstly, to conceive the economic characteristic of mankind to be “the pursuit of pleasure and the avoidance of pain”; secondly, to conceive of both pleasure and pain as quantities and, thirdly, to bear in mind that recognising the existence of quantities and actually measuring them are two different things.⁵

Consequently, Pareto defined pure economics as “a type of rational mechanics” dealing not with points but with *homines oeconomici* (“economic agents”).⁶ If one of these agents possesses variable quantities q_a and q_b of commodities A e B , each possible combination of these quantities “will generate different degrees of utility”.⁷

Thus, if we are able to measure the separate increases in utility

$$\varphi_a d_a \text{ and } \varphi_b d_b$$

which the economic agent will obtain in passing from q_a to dq_a (or similarly from q_b to dq_b), then the agent will enjoy an overall increase in utility equal to

$$\varphi_a dq_a + \varphi_b dq_b$$

which will be maximised with the occurrence of the quantities of A and B such that⁸

$$\varphi_a dq_a + \varphi_b dq_b = 0$$

If we further posit that “the utility is independent of the order of consumption”,⁹ then φ_a and φ_b constitute the first-order partial derivatives of a function Φ ¹⁰ and so the previous equation can be substituted by¹¹

$$d\Phi = 0$$

In reality, φ_a and φ_b cannot be measured, and therefore neither can Φ . But every human being knows, “with certainty, since this is a matter of logical actions”, whether, in passing from one combination to the next, his utility increases or diminishes, that is, whether $d\Phi$ increases or diminishes.¹²

A further Ψ function can also be constructed “whose values, while partially arbitrary, are such that $d\Psi$ will always have the same sign as $d\Phi$ ”.¹³

Hence,

$$d\Phi = 0$$

can be replaced with

$$d\psi = 0$$

Pareto emphasises¹⁴ that the economic agent’s passage from one combination to another of quantities of goods is not unrestricted but is constrained by the conditions of production which, if the goods can be transformed in a fixed proportion, can be represented graphically by a negatively inclined curve and in algebraic terms by the function

$$F(q_a, q_b) = 0$$

which can be specified as

$$a - q_a + (b - q_b) = 0$$

where a and b are the quantities initially possessed of A and B ; q_a and q_b are the quantities possessed after the transformation; $a - q_a$ and $b - q_b$ are the quantities in which the transformation of A to B is manifested and α is the constant which represents the fixed ratio at which a quantity of A can be transformed into a quantity of B .

Shortly afterwards Pareto took the opportunity of a brief but seminal exchange of correspondence with the French mathematician Hermann Laurent¹⁵ to clarify his new theory. He agreed with his correspondent that only “elements which are capable of equivalence and addition” can be measured, but added that, in his view, utility possesses these qualities on an empirical level, since “each day, we weigh one type of pleasure against another, we judge them to be equal, greater or lesser”. Nevertheless,

Pareto was also prepared to admit that the possession of such qualities represented simply a postulate, a procedure to which, for that matter, every science resorts in its infancy.¹⁶

In the light of this, Pareto specified that this general problem of economics implies the existence of the equation¹⁷

$$d\varphi = \varphi_a dx_a + \varphi_b dx_{ab}$$

where the value of φ “depends on the order of consumption”¹⁸ while x_a, x_b, \dots represent “the successive values of the quantities exchanged”.¹⁹

Further, Pareto observed that in the differential of what we now call the budget constraint

$$p_a dx_a + p_b dx_{ab} + \dots = 0$$

the prices do not depend on $x_a, x_b \dots$ but on the values which the quantities of goods have at the conclusion of the exchange (designated $r_a, r_b \dots$ by Pareto) so that the integral of the equation at the end of the exchange is

$$p_a r_a + p_b r_{ab} + \dots = 0$$

representing “simply the statement of condition of the individual concerned: income = expenditure”, which does not thus permit the determination of the quantities of goods exchanged.²⁰

These quantities are in fact the product of the system consisting of

$$d\varphi = \varphi_a dx_a + \varphi_b dx_b + \dots$$

and of the $n - 1$ equations

$$p_a \left(\frac{dx_a}{dx_b} \right) + p_b = 0; p_a \left(\frac{dx_a}{dx_c} \right) + p_c = 0; \dots$$

which are obtained from

$$p_a dx_a + p_b dx_b + \dots = 0$$

bearing in mind that the latter establishes “a relation between the n quantities” $x_a, x_b \dots$, with the implication that one of these, for example x_a itself, can be considered a function of the remaining $n - 1$.²¹

In geometric terms, if the quantities r_a and r_b are connected by the function

$$\varphi(r_a, r_b) = \text{constant}$$

obtainable only “through experience”,²² this latter can be represented on the plane “of the contour lines of any other surface”²³

$$\varphi_a dr_a + \varphi_b dr_b = 0.$$

The equation

$$p_a dx_a + p_b dx_b = 0$$

(which is a straight line, if the prices do not depend on the quantities x_a, x_b) itself represents “the differential equation of a path followed during the exchange ... [which] stops when the path is tangential to a contour line”,²⁴ where it assumes the form

$$p_a dr_a + p_b dr_b = 0$$

Pareto then reiterates that in order to achieve the condition of tangency of the straight line to the contour lines, both the equation of the straight line and the equation of a contour line are necessary and hence the system²⁵

$$\varphi_a dr_a + \varphi_b dr_b = 0$$

$$p_a dr_a + p_b dr_b = 0$$

whose solution is yielded by the combination of r_a and r_b such that

$$\frac{\varphi_a}{\varphi_b} = - \left(\frac{dr_a}{dr_b} \right) = \frac{p_a}{p_b}$$

or such that

$$\frac{\varphi_a}{p_a} = \frac{\varphi_b}{p_b}$$

Pareto deduces that, as also “generally in practice”,²⁶

$$p_a = f_a(r_a, r_b, \dots); p_b = f_{ab}(r_a, r_b, \dots); \dots$$

Pareto explained to Pantaleoni that while “Edgeworth and the others start from the notion of final degree of utility to arrive at the determination of the indifference curve”, he, and “this is the only novelty”, “leave[s] completely aside the final degree of utility and start[s] from the curves of indifference which emerge directly from experience”.²⁷ Thus, in the end, in response to “the objections made because the final degree of utility [can] not be measured”, we can retort that “there is no need to measure [it]”.²⁸

Pareto then underlined that for the construction of a curve of indifference, an operation consisting simply in having someone state the different combinations of quantities of two goods which for him are “exactly the same” without asking him “why”, there is no “talk of transformations or reasons for transformations or prices” and neither “is it necessary that the elements should be measurable”.²⁹ Moreover, reflecting on a curve of indifference which today we would call standard, that is, continuous, differentiable, strictly convex and negatively inclined, Pareto remarked that “the reason for the exchange or the transformation” between the two commodities “varies constantly”.³⁰ and indeed it is also possible to have curves inclined negatively at right angles (today defined as curves describing lexicographical orders of preference) where “the individual, provided he has b of B , is indifferent to having any quantity of A greater than a ”.³¹ However, positively inclined indifference curves cannot exist because a combination containing greater quantities of both commodities cannot be indifferent in relation to one containing lesser quantities of both.³²

1.2 THE DEFINITIVE FORMULATION OF THE THEORY OF CHOICE

In March 1900, Pareto, proceeding with this reflection, pointed out that “the most basic observation suffices to demonstrate that animals and humans make choices [that] very often [are] constantly repeated”, which allows these choices to constitute “the object of a science”.³³ Pareto underlined that he was interested exclusively in the fact of the choice, without “in any way” seeking “the reason” for it,³⁴ which “is only a matter of taste”.³⁵ Pareto also specified that choices “fall on items whose quantities are variable and subject to measurement”,³⁶ with a variability which in practice is discontinuous and which is replaced by a variability which is considered as continuous³⁷ simply in order to “facilitate the application of the mathematical method”. Pareto also thought it advisable to distinguish choices an individual makes on the basis of his own personal preferences³⁸ from those made “considering the effects they will have on other individuals”.³⁹ Finally, Pareto noted that “in his choices, the individual encounters obstacles”⁴⁰ and that he aims to make a choice, referred to as an equilibrium choice, “which he favours over all others, taking into account all the obstacles”.⁴¹

In the light of all this, Pareto claimed that, conceptually, in order to deal with the problem of choice, “the first operation is the creation of a table of the possible choices⁴² faced by the individuals under consideration ... indicating their order of preference”.⁴³ In order to proceed, it is necessary to observe the following limitations: “to consider only the choice of economic goods whose quantities are variable and can be measured”;⁴⁴ to consider “only the state of equilibrium”,⁴⁵ in the context of a given combination of quantities of goods, to consider “the choice to be indifferent with regard to the order of consumption”.⁴⁶ Thus, were it not for the obstacles, “the solution to the problem of equilibrium would be very simple. The individual would stop at the point where he was sated with everything”.⁴⁷ However, the obstacles exist, in the sense that “to obtain certain things he is obliged to forgo others” and therefore it is necessary also to construct “a table of obstacles”.⁴⁸

Consequently, the economic problem, described verbally, consists in representing “experience” in the form of a volume on each of whose pages are recorded the combinations which are indifferent among themselves, taking care to “arrange the combinations in order of preference according

to the number of pages” and remembering that the obstacles will identify all the pages among which a choice can in fact be made.⁴⁹

In geometric terms, after placing the quantities available of commodities X and Y on a Cartesian axis, the set of combinations of the quantities X and Y to which the individual’s choice is limited by the obstacles can be represented on a continuous negatively inclined curve.⁵⁰ Once this wave curve is drawn,⁵¹ Pareto underlines that the individual, arriving at the curve along “his favoured path”, is not obliged to follow it but neither can he pass beyond it.⁵² Although for Pareto it is an “exception”,⁵³ the individual may halt at that point neighbouring which there are only points which are dispreferred by him; the “general case”, however, is that where the individual stops at a point neighbouring which there are other points which he considers indifferent to the stopping point.⁵⁴

Pareto then introduced the concept of the line of indifference as being the line which “represents types of consumption among which the *economic agent* does not distinguish”,⁵⁵ while specifying that “only one line of indifference can pass through any given point”⁵⁶ and that the set of infinite lines of indifference which can be constructed,⁵⁷ each starting from any given combination of quantities of goods is sufficient to “characterise the *economic agent* in regard to all problems of equilibrium”.⁵⁸

On this basis, Pareto declared that “the equilibrium position occurs when the curve indicating the effects of the obstacles [and] one of the curves of indifference have a common tangent”,⁵⁹ which in general is made up of a number of points. The resulting situation of equilibrium “corresponds to the best choice possible, and also to the worst”; which of these is the case is indicated by “the nature of the problem itself”.⁶⁰

The graphical representation can be completed on the basis of the following considerations:

to any given combination we can assign “an arbitrary numerical index” and this same index will also be attributed to all the other combinations lying along the line of indifference of which this combination forms part; let any given combination (such as A) on a curve of indifference be compared with another combination B which does not form part of the same curve: if B is preferred (dispreferred) with regard to A , an arbitrary numerical index is assigned to B (and to all the combinations lying along the curve of indifference to which it belongs) which, however, is greater (or smaller) than that assigned to A ;⁶¹

in this manner we will cover “the plane with an infinite number of lines of indifference which are infinitely close to each other ... each [of which] will have an index showing the individual’s order of preference”,⁶² thus generating a “complete representation of the individual’s preferences”.⁶³

Lastly, from the analytical point of view, Pareto stated that “by interpolation”, the whole set of lines of indifference can yield the equation

$$G(x,y,\beta) = 0 \text{ or } \beta = I(x,y)$$

where the equation of a line of indifference corresponds to each value of the parameter β .⁶⁴

By differentiating

$$\beta = I(x,y)$$

we obtain⁶⁵

$$\left(\frac{\partial I}{\partial x}\right)dx + \left(\frac{\partial I}{\partial y}\right)dy = 0$$

By hypothesising the equation of the obstacles

$$f(x,y) = a \text{ (where } a \text{ is a constant)}$$

and differentiating it, we obtain

$$\left(\frac{\partial f}{\partial x}\right)dx + \left(\frac{\partial f}{\partial y}\right)dy = 0$$

From the equations

$$\left(\frac{\partial I}{\partial x}\right)dx + \left(\frac{\partial I}{\partial y}\right)dy = 0$$

$$\left(\frac{\partial f}{\partial x}\right)dx + \left(\frac{\partial f}{\partial y}\right)dy = 0$$

we obtain

$$\left(\frac{\partial I}{\partial y}\right)\left(\frac{\partial f}{\partial x}\right) - \left(\frac{\partial I}{\partial x}\right)\left(\frac{\partial f}{\partial y}\right) = 0$$

which, in combination with the equation of the obstacles, identifies the coordinates of the point of tangency.⁶⁶

At this juncture Pareto pointed out that in the new formulation, the problem of economic equilibrium emerges only from “real experience, viz.: firstly, the individual’s order of preference; secondly, the obstacles he encounters in these choices”.⁶⁷ Hence, economic theory can finally “study the (economic) facts directly” and not through “the notions that men are possessed of”.⁶⁸

Pareto then specified that “if p_1, p_2, \dots are the prices of the goods, q the cost of labour, i the interest rate, r, \dots the price for lease of the land, etc. and if a_1, a_2, a_3 represent certain parameters, mathematical economics shows that the variables p_1, p_2, \dots, q, i, r , are determined by all the parameters”. In other words, given the system of equations

$$\begin{aligned} ax + by &= c \\ a'x + b'y &= c' \end{aligned}$$

it makes no sense to ask “which of the parameters a, b, c, a', b', c' determines the value of x and y ”.⁶⁹

Furthermore, in general equilibrium expressed as a system of equations, “the prices disappear by elimination” too, so that “for the determination of the quantities received by each person only the parameters of tastes, obstacles and wealth distribution remain”.⁷⁰

Pareto expounded his new conception of pure economics publicly for the first time in a short course he gave at the *École des Hautes Études Sociales* in Paris from the 10th to the 18th of November 1901. Here he began by specifying that economic equilibrium requires only the equations representing

the preferences and the balance of income and expenditure for each individual;⁷¹

the hypothetical social system (where the combination between private property and free competition is described by the equivalence between “the selling price of the products and their cost of production”; that between private property and monopoly is described by the condition that “the difference between the selling price [and] the cost of production is at a maximum”; collectivism is represented by production being performed in such a way as to “procure the greatest possible well-being for the citizens of the socialist State”);⁷²

“the relationships (not only technical [but] also economic) between the quantities of goods being transformed and the products derived from them”.⁷³

Pareto added that

after Irving Fisher, the conditions of capitalisation (relating to the production of those elements “which do not constitute the direct goal of production but a means of production”) can “be based exclusively on the notion of the transformation of economic goods”, thus allowing the notion of capital “[which is] arbitrary and not very rigorous, scientifically speaking”⁷⁴ to be dispensed with;

it is possible to limit ourselves to addressing the equilibrium of exchange by substituting the equations for production with those for exchange, since the latter encapsulate the fundamental idea that “what one individual finds himself short of, another acquires a surplus of”.⁷⁵

In the same period Pareto formalised these ideas in describing “any given state of equilibrium” characterise, for θ individuals, by the equations for maximum ophelimity (for commodities m , these are $\theta(m - 1)$), by the θ equations of equivalence between inflows and outflows and by m equations, one for each commodity, which “serve to indicate the obstacles which individuals will encounter in order to procure the economic goods for themselves”, particularly the equivalence between quantities sold and purchased.⁷⁶ Thus, we have a system of $\theta m + m - 1$ equations (given that one of these depends on all the others) which allow us to identify the θm quantities exchanged and the $m - 1$ prices (one of the commodities is considered as the *numéraire* and hence a unit of this by definition constitutes the price).⁷⁷

1.3 THE *MANUAL OF POLITICAL ECONOMY*: THE INNOVATIONS IN RELATION TO PARETO'S PREVIOUS ECONOMIC IDEAS

On the 19th of November 1899, Pareto informed Pantaleoni that following up “the idea already touched on” in *Expounding the pure economics*, he was writing “a treatise on mathematical economics” in which he formulated “the fundamental equations without making use of either the final degree of utility, or ophelimity or even prices”.⁷⁸

This project, which he had already conceived a decade earlier,⁷⁹ took shape in the *Manual of Political Economy*⁸⁰ where he pointed out, firstly, that if the quantities of all the goods available to the individual increase (or decrease), “there is no problem to be solved” since evidently “the new position will be more (or less) advantageous for the individual involved”.⁸¹ Instead, economic problems consist precisely in ascertaining whether, following an increase in certain quantities and a decrease in others, “the new combination is or is not advantageous for the individual”.⁸²

In general, among the combinations of quantities of goods available, the individual's choice can be established by reference to “the theory of economic equilibrium”,⁸³ hinting, in conceptual terms, at the result of the “contrast between people's preferences and the obstacles to satisfying them”⁸⁴ and in formal terms, to “the state ... [in which] no further exchanges will occur”⁸⁵ since “the exchanges permitted by the obstacles are prevented by the preferences [and] vice versa”.⁸⁶

Economic problems are particularly manifested in exchanges (involving “giving one thing to receive another”) and in production (where “certain things are transformed into certain other things”).⁸⁷

Further, an individual may engage in the exchange “at [prevailing] market conditions” or may modify these.⁸⁸ Hypothetically, in the first case (corresponding to “free competition”),⁸⁹ the individual aims simply to “satisfy his own desires”⁹⁰ whereas in the second case (corresponding to “monopoly”),⁹¹ he seeks the attainment of the market conditions which will allow him to achieve the end he was aiming for,⁹² with the implication that under equilibrium, the quantities of goods corresponding to each case are different.⁹³ Similarly, an enterprise may accept the prevailing prices or may modify them,⁹⁴ but in either case with the objective of obtaining “the maximum cash monetary profit ... [by] pay[ing] the lowest amount possible for its purchases and obtain[ing] the highest amount possible for what it sells”.⁹⁵

Tastes (Preferences)

Having established this, Pareto observed that if “everyone used the goods he possessed for only as long as he liked”,⁹⁶ then pure economics should consider “not the quantities consumed, but the quantities available to the individual”⁹⁷ as well as “the present anticipation of the future consumption of the goods available, as constituting the motive for the individual’s actions”.⁹⁸

Moreover, Pareto noted that “in general, consumption is dependent” in two ways.⁹⁹ The first of these (relating to “complementary goods”) refers to “the pleasure of consumption being in relation to the pleasures of alternative consumption ... over a broad range of variation [in the quantities of goods]”.¹⁰⁰ The second (relating to substitute goods) refers to “being able to substitute one thing for another so as to provide an individual with sensations which, if not identical, are at least approximately the same”.¹⁰¹ Lastly there also remains the case of independent goods, that is, that whereby, through “restricted variations in the quantities of the goods, the ophelimity deriving from the consumption of an item [is] independent [of] the consumption of the others”.¹⁰²

In the light of all this, an individual’s preferences can be expressed through a series of infinite combinations of quantities of the same goods between which the individual “would be unable to choose”.¹⁰³ For all three of the typologies of goods mentioned,¹⁰⁴ the series in question can be represented graphically by means of a curve of indifference¹⁰⁵ which can be imagined as continuous and which has the properties, recognised thanks to “everyday experience”, $(dy/dx) < 0$,¹⁰⁶ $(d^2y/dx^2) > 0$,¹⁰⁷ that is, in graphic terms a negatively inclined curve which is strictly convex.

In algebraic terms, having defined x, y, z the quantities of commodities X, Y, Z ... available to the individual and b_y, c_z, \dots the increases which X must undergo in order to compensate, in the eyes of the individual, for the decreases in the quantities of Y, Z, \dots ,¹⁰⁸ the curve of indifference can be constructed as follows:¹⁰⁹

if dx represents the increase in X which compensates the reduction in overall ophelimity caused by the reduction dy in the quantity of Y available, then the equation

$$dx + b_y dy = 0$$

or¹¹⁰