An Introduction to Formal Logic

Second Edition

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www.AdvancedReasoningForum.org/intro_formal_logic

Dedicated to my brother

Robert S. Epstein

whose help and encouragement made this book possible.

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Preface to the Instructor

Logic is a tool we use to investigate how our language connects to the world so that we can reason better and, we hope, understand the world better. This book tells that story. It has a beginning, a middle, and an end.

We begin by setting out what formal logic is: the study of inferences for validity based on their form. Classical propositional logic is then presented as the simplest formal logic. In the development of that logic, the most important tools of formal logic are presented: a formal language, realizations, models, formal semantic consequence, and an axiom system. Examples of formalizing ordinary-language propositions and inferences show how to use classical propositional logic and also show some of its limitations.

The middle of the book relates form to the world. Predicate logic is motivated as a way to widen the scope of classical propositional logic to investigate more kinds of inferences whose validity depends on form. The large assumption that the world is made up of individual things is the basis for both the syntax and semantics of predicate logic. Besides form and the truth or falsity of atomic propositions, only the idea of assigning reference to terms as a kind of naming is needed. Many examples of formalizing show both the scope and limitations of classical predicate logic. Those depend on establishing criteria for what counts as a good formalization. The emphasis in those discussions is how the assumption that the world is made up of individual things is both useful and limiting.

The final chapter reflects on the success of the formal methods we've developed as means to reason to truths. Our formal logics circumscribe what we mean by "individual thing", namely, what can be reasoned about in predicate logic. Our formal logics give us a way to be precise about how we understand possibilities, though only relative to the assumptions we make about form and meaning and what there is in the world.

This story gives the basics, the fundamentals of formal logic. Along the way I point out how the work here can be extended and modified to apply to a wider scope of what we can formalize from ordinary-language reasoning. The story is not finished. Not here, not elsewhere.

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Some Points about the Organization and Content

- Appendices

  The appendices contain material that is either more technical than many students want, or too philosophical for many students, or is supplementary to the main line of the story.

- The form of atomic wffs

  Rather than take "Ralph is a dog" as a wff, we separate the roles of names and predicates. Thus, we write "(— is a dog) (Ralph)". We have a choice between "(— lives in —) (Arf, New Mexico)" or "(— lives in New Mexico) (Arf)" depending on whether we take "New Mexico" to be a name of a thing. This leads the student to see more clearly the roles of names and predicates and is crucially important in extending classical predicate logic to allow for formalizing reasoning that involves relative adjectives and adverbs in *The Internal Structure of Predicates and Names*.

- Superfluous quantification in the formal language of predicate logic

  In most logic texts, the definition of a formal language for predicate logic allows for superfluous quantifications. The rationale for including such formulas is that it simplifies the definition of the formal language, allowing a definition of bound and free variables to be made later. But the disadvantage is that a formula such as "$\forall x$ ((— is a dog) (Ralph))" that would correspond to the nonsensical "For everything, Ralph is a dog" is deemed acceptable. The semantics for superfluous quantifiers treat that formula as equivalent to "Ralph is a dog", which can be true. That is not consonant with our normally treating nonsense as false in our reasoning, as you can see in "Truth and Reasoning" in my *Reasoning and Formal Logic*. The advantages of not allowing superfluous quantification, beyond ridding our semi-formal languages of nonsense, are significant: we need no axiom schemes for superfluous quantification, and many proofs about the language are simplified by no longer having to treat cases of superfluous quantification separately.

- Proof theory

  Hilbert-style axiomatizations of classical propositional logic, classical predicate logic, and classical predicate logic with equality are presented in the text, and their completeness proofs appear in an appendix. Natural deduction and other methods of proof can be left until those skills are needed.

- Functions

  Only in some parts of mathematics and science are functions used, and then partial functions are essential. To extend classical predicate logic to allow for formalizing reasoning with partial functions we would need to analyze how to reason with non-referring names, descriptive names, and descriptive functions. I do that in *The Internal Structure of Predicates and Names*.

- The History of Logic

  It's a big subject. It's complicated. And it's not illuminating at this level. I present a lot in my books *Propositional Logics*, *Predicate Logic*, *Computability*, and *Classical Mathematical Logic*.

- English and Formal Logic

  Some might object that the development of formal logic here is too closely tied to motivations and examples from English. But in slightly modified form the examples and motivation here will apply to reasoning in many other languages. If they do not serve, then that would be evidence that the notion of thing is not as deeply embedded in the other language, as I explain in *Predicate Logic* and "Nouns and Verbs" in *Language and the World: Essays New and Old*.

1 The Basics of Logic

We want to know what to believe. What counts as good reason to believe? How can we codify what we know? What are the consequences of assuming this rather than that? Logic helps us answer these questions. It's a tool to help us reason well.

A. Propositions

We want rules to help us find what's true. But what is it that's true or false? Not apples or cars or people, except in a metaphorical sense. What we say is what is true or false.

Example 1: Suzy: *Spot is out of the yard.*
Analysis This is true or false.

Example 2: Dick: *No cat can swim.*
Analysis This is true or false.

Example 3: Zoe (to Dick): *Spot is chewing your shoe.*
Analysis This is true or false.

Example 4: Dick: *Bad dog, Spot, bad dog!*
Analysis This isn't true or false; it's just Dick trying to influence Spot, like a command.

Example 5: Zoe: *What time does the movie start?*
Analysis This isn't true or false.

Example 6: Dick: *Get me a beer, Zoe.*
Analysis This isn't true or false.

Questions, commands, and a lot more that we say isn't true or false. But it's not just the words we say. It's the way we use them.

Example 7: Maria: *I wish I could get a job.*
Analysis Maria has been trying to get a job for three weeks and said this to herself late at night. It isn't true or false. It's more like a prayer or an extended sigh.

Example 8: Dick: *I wish I could get a job.*
Analysis Dick's parents have been berating him for not getting a job and he told them that it's not that he's not trying. Then he said this, which is true or false.

Proposition A *proposition* is a written or uttered sentence used in such a way that it is true or false, but not both.

We call truth and falsity the *truth-values* of propositions. In what follows, I'll just say "uttered" when I mean written or uttered.

We don't have to make a judgment about whether a sentence is true or whether it's false in order to classify it as a proposition. We need only judge that in the context in which the sentence is uttered, it's one or the other.

Example 9: Zoe: *Dick is upset*.
Analysis Zoe said this in a whisper to Tom when she and Dick were visiting him. Tom might not know whether it's true or false, but he knows it's one or the other. It's a proposition.

Example 10 : *2 + 2 = 5*
Analysis Mathematical formulas are meant to be true or false, so this is a proposition, a false one. A proposition need not be a sentence put forward as true.

B. Propositions as Types

We want to reason together. When I write "Spot is a dog", you understand the this as a proposition. If you then write "Spot is a DOG", or someone else writes "Spot is a dog", or a friend shouts "Spot is a dog", we understand those as the same proposition. They are the same words in the same order.

Words are Types In the course of any reasoning, any word we use will continue to have the same properties when used again. We agree to identify the two uses as the same word. Briefly, *a word is a type*.

Propositions are Types In the course of any reasoning, we will consider a sentence to be a proposition only if any other sentence or phrase that is composed of the same words and punctuation in the same order can be assumed to have the same properties of concern to us during that discussion. We agree to identify such sentences or phrases and treat them as the same. Briefly, *a proposition is a type*.

Example 11: Flo: *Spot is a dog*.
Berta: *Spot is not a dog. He's a cat*.
Analysis It seems that Flo and Berta disagree. But Flo is talking about Dick and Zoe's dog Spot, and Berta is talking about Mrs. Zerba's cat Spot. They are not using the word "Spot" the same.

Example 12: *Spot es un perro*.
Analysis Dick and Pancho are going back and forth between English and Spanish in reasoning about dogs and cats. They agree that they'll treat this sentence as being the same as "Spot is a dog."

Example 13: *Rose rose and picked a rose.*
Analysis We can't use this in our reasoning unless we distinguish the three inscriptions, using perhaps "Rose_1 rose_2 and picked a rose_3" or "$\text{Rose}_{\text{name}}$ $\text{rose}_{\text{verb}}$ and picked a $\text{rose}_{\text{noun}}$".

Example 14: *I am 1.80 m tall.*
Analysis This is true if Dick says it. It's false if Zoe says it.

We have to avoid words such as "I", "my", "now", or "this", whose meaning depends on the circumstances of their use. Such words are called *indexicals*, and they play an important role in reasoning. Yet our demand that words be types requires that they be replaced by words we can treat as uniform in meaning throughout a discussion, such as "Dick" for "I", or "March 9, 1991" for "now".

Example 15: *It's raining.*
Analysis Dick said this to Zoe today. It's true. Last Thursday, Zoe was outside when it was raining, then she went inside to answer the phone and told her mother, "It's raining" though it had actually stopped a moment before. That's false. So it seems a proposition can be true at one time and false at another. But it's incorrect to identify the two utterances of "It's raining" as being the same proposition. They're the same words in the same order, but they're different for our reasoning because one is true and one is false.

The device I've been using of putting quotation marks around a word or phrase is a way of naming that word or phrase, or any piece of language. We need some convention because sometimes it's not clear whether we're talking about a word or phrase. For example, if a professional tennis player tells Dick, "Love means nothing in tennis", it's not clear whether she's telling Dick what the word "love" means in tennis or asserting that there's no room for sentiment in tennis. When we talk about and mark off a piece of language with quotation marks, we've *mentioned* it. Otherwise, we *use* the word or phrase, as we normally do. Sometimes we use italics rather than quotation marks.

Sometimes people use quotation marks as the equivalent of a wink or a nod in conversation, a nudge in the ribs indicating that they're not to be taken literally or that they don't really subscribe to what they're saying. Used that way they're called *scare quotes*, and they allow us to get away with "murder".

C. Inferences

We're concerned not only with which propositions are true, but which propositions follow from other propositions.

Example 16: *All dogs bark, and Humberto barks. So Humberto is a dog.*
Analysis Does "Humberto is a dog" follow from "All dogs bark" and "Humberto barks"? What does it mean for one proposition to follow from one or several other ones?

Inference An *inference* is a collection of two or more propositions, one of which is the *conclusion* and the others the *premises*, that is intended by the person who sets it out as either showing that the conclusion follows from the premises or investigating whether that is the case.

When does an inference show that the conclusion follows from the premises? That depends in part on what kind of reasoning we are analyzing. Different conditions apply depending on whether we are concerned with arguments, explanations, mathematical reasoning, reasoning about cause and effect, or reasoning with prescriptive propositions, as you can read in my series of books *Essays on Logic as the Art of Reasoning Well*. However, for all kinds of reasoning, a fundamental criterion for whether the conclusion follows is that the inference is valid or strong.

Valid and strong inferences An inference is *valid* means that there is no way the premises could be true and the conclusion false at the same time.

An inference is *strong* means that there is a way for the premises to be true and the conclusion false, but all such ways are unlikely.

An invalid inference that is not strong is *weak*.

If an inference is valid or strong, then the conclusion *follows from* the premises; the conclusion is a *consequence* of the premises.

So Example 16 is a weak inference: there are lots of ways the premises could be true and conclusion false: Humberto could be a seal, or a parrot that's learned how to bark, or a philosophy professor who thinks he is a dog.

Example 17: *All dogs bark, and Humberto is a dog. So Humberto barks.*
Analysis This is valid. There's no way the premises could be true and conclusion false. I can't explain to you why that's so—either you recognize it or you don't.

But just because a proposition follows from some others doesn't mean that the reasoning is good. Example 17 shouldn't give you reason to believe that Humberto barks, because the first premise is false: some dogs have had their vocal cords cut, and others are ridgebacks that can't bark. An inference that is meant to convince someone, possibly yourself, that its conclusion is true is called an *argument*.

Example 18: *Dogs bark. So Ralph is a dog or Ralph is not a dog.*
Analysis This inference is valid because there is no way that the conclusion could be false. It's a tautology.

Tautology A *tautology* is a proposition if there is no way it could be false.

Exercises

1. What is a proposition?
2. Which of the following are propositions? If necessary, supply a context.
   a. Ralph is a dog.
   b. I am 2 meters tall.
   c. "The Queen, my lord, is dead", said by Seton to Macbeth in *Macbeth*.
   d. Feed Ralph.
   e. Did you feed Ralph?
   f. Strike three!
   g. Ralph believes that George is a goose.
   h. Ralph didn't see George.
   i. Whenever Juney barks, Ralph gets mad.
   j. If anyone should say that cats are nice, then he is confused.
   k. If Ralph should say that cats are nice, then he is confused.
   l. If Ralph should say that cats are nice, then Ralph is confused.
   m. There are an odd number of stars in the universe.
3. a. What does it mean to say that a word is a type?
   b. What does it mean to say that a proposition is a type?
   c. Should we identify the following as being the same proposition?
      "Ralph is a dog", said about my Ralph.
      "Ralph is a dog", said about Ralph Abernathy, whom I never met.
   d. Should we identify the following as being the same proposition?
      "All the dogs in the yard are barking", said by Zoe at 3 p.m.
      "All the dogs in the yard are barking", said by Dick at 8 p.m.
4. a. What is an indexical?
   b. Give two examples of an indexical used in a sentence.
   c. Explain why we choose not to allow indexicals to appear in the propositions we're considering here.
5. a. What is an inference?
   b. What is a valid inference? Give an example.
   c. What is a strong inference? Give an example.
   d. What does it mean to say that one proposition follows from one or more other propositions?
   e. What does it mean to say that one proposition is a consequence of one or more other propositions?
6. What is a tautology. Give two examples.

Aside: *Other conceptions of propositions*
Some say that what is true or false is not the sentence but the "meaning" or "thought" expressed by the sentence. Thus "Spot is a dog" is not a proposition; it expresses a proposition, the very same one expressed by "Spot is a domestic canine" and by "Spot es un perro".

Platonists take this one step further. A *platonist* is someone who believes that there are abstract objects not perceptible to our senses that exist independently of us. Such objects can be perceived by us only through our intellect. The independence and timeless existence of such objects account for objectivity in logic and mathematics. In particular, propositions are abstract objects, and a proposition is true or is false, though not both, independently of our even knowing of its existence. Thus each of the following, if uttered at the same time and place, expresses or stands for the same abstract proposition: "It is raining", "Pada deszcz", "Il pleut". Platonists say that the word "true" can be properly used only for things that cannot be seen, heard, or touched. Sentences are understood to "express" or "represent" or "participate in" such propositions. The assumption that propositions are types is said by them to be about which inscriptions and utterances represent or express or point to the same abstract proposition.

Those who take abstract propositions as the basis of logic argue that we cannot answer precisely the questions: What is a sentence? What constitutes a use of a sentence? When has one been been put forward for discussion? These questions, they say, can and should be avoided by taking things inflexible, rigid, and timeless as propositions. But then we have the no less difficult questions: How do we use logic? What is the relation of these theories of symbols to our arguments, discussions, and search for truth? How can we tell if this utterance is an instance of that abstract proposition?

In the end, though, the platonist as well as a person who thinks a proposition is the meaning of a sentence or a thought reasons in language, using sentences that they call "representatives" or "expressions" of propositions. We can and do reason together using those, and to that extent our definition of "proposition" can serve those folks, too.

Key Words		
	proposition	inference
	truth-value	conclusion
	types	premise
	indexical	valid inference
	quotation marks	strong inference
	use of a word or phrase	a proposition follows
	mention of a word or phrase	tautology
	scare quotes	

2 Compound Propositions

A. Compound Propositions

Compound proposition A *compound proposition* is one that has another proposition as part but has to be viewed as just one proposition.

Example 1: Dick: Who won the election for mayor?
Zoe: *Either a Democrat won the election or a Republican won the election.*
Analysis This is a single proposition made up of two propositions "A Democrat won the election" and "A Republican won the election." They're joined by the word "or." Whether it's true depends on whether one or both of its parts are true. But the entire sentence is just one proposition.

Example 2: *If Suzy studies hard, then Suzy will pass the exam.*
Analysis This is just one proposition, made up of the two propositions "Suzy studies hard" and "Suzy will pass the exam."

Example 3: *Lee will pass his exam because he studied so hard.*
Analysis This is not a compound proposition: "because" tells us that this meant as an inference.

B. "Or" Propositions

Example 4: *Dick or Zoe will go to the grocery to get eggs.*
Analysis We can view this as an "or" proposition compounded from "Dick will go to the grocery to get eggs" and "Zoe will go to the grocery to get eggs." The propositions that make up an "or" proposition are called the *alternatives*.

Example 5: *Either Dick picked up Zoe at the market, or Zoe went to see Suzy. Zoe didn't go to see Suzy. So Dick picked up Zoe at the market.*
Analysis This is a valid inference. We don't need to know anything about Dick or Zoe or Suzy or the market to see that. It's valid just because of its form.

Excluding Possibilities

A or B + not A
↓
B

Valid

I've used the letters A, B, and C to stand for any propositions, the symbol "+" to indicate that we have an additional premise, and ↓ to stand for "therefore".

C. Contradictories

Example 6: *Humberto got out of jail or he didn't get parole. Humberto did get parole. So Humberto got out of jail.*
Analysis This is an example of excluding possibilities. Here B is "Humberto didn't get parole" and the not-version of that is "Humberto did get parole." We can read "not-A" in excluding possibilities to mean the contradictory of A.

Contradictory of a proposition A *contradictory* of a proposition is one that must have the opposite truth-value.

Example 7: *Spot is barking.*
Analysis A contradictory of this is "Spot is not barking."

Example 8 : *Inflation will be at least 3% this year.*
Analysis A contradictory of this is "Inflation will be less than 3% this year", which doesn't contain "not".

Contradictory of an* "or" *proposition *A or B* has contradictory *not A and not B*.
Contradictory of an* "and" *proposition *A and B* has contradictory *not A or not B*.

Example 9: *Maria got the van or Manuel won't go to school.*
Analysis A contradictory is "Maria didn't get the van, and Manuel will go to school."

Example 10: *Tom or Suzy will pick up Manuel for class today.*
Analysis A contradictory is "Neither Tom nor Suzy will pick up Manuel for class today."

D. Conditional Propositions

Conditional propositions A *conditional proposition* is a proposition that is or can be rewritten as one in the form *If A then B* that must have the same truth-value. The proposition *A* is the *antecedent* and *B* is the *consequent*.

Example 11: *If Spot ran away, then the gate was left open.*
Analysis This is a conditional with antecedent "Spot ran away" and consequent "The gate was left open." The consequent need not happen later.

Example 12: *I'll never talk to you again if you don't apologize.*
Analysis This is a conditional with antecedent "You don't apologize" and consequent "I'll never talk to you again."

Example 13: *Loving someone means you never throw dishes at him.*

Analysis This is a conditional with antecedent "You love someone" and consequent "You never throw dishes at him". It's not a definition.

Example 14: *A mammal is an ungulate if it has hoofs.*
Analysis This is not a conditional or a compound. It's a definition that uses "if" instead of "means that". We have to use our judgment to decide whether a proposition is a conditional.

Example 15: *If Dick goes to the basketball game, then either he got a free ticket or he borrowed money for one.*
Analysis This is a conditional whose consequent is a compound proposition.

Example 16: *Dick will go into the army only if there is a draft.*
Analysis What does "only if" mean? It means that if there is no draft, then Dick won't go into the army. And that is true just in case "If Dick goes into the army, then there is a draft". Generally, "A only if B" is true just in case "If A then B" is true. "Only if" does not mean the same as "if".

Example 17: *Dick will go into the army if and only if there is a draft.*
Analysis This is how we say "Both if Dick goes into the army then there is a draft, and if there is a draft, then Dick will go into the army".

"only if" propositions *A only if B* is equivalent to *If A*, *then B*.

Equivalent propositions Two propositions are *equivalent* means that the one is true exactly when the other is true.

"if and only if" propositions *A if and only if B* means *If A*, *then B*, *and if B then A*. We abbreviate "if and only if" as as "iff".

A contradictory of a conditional is not another conditional.

Contradictory of a conditional *If A, then B* has contradictory *A but not B*.

Example 18: *If Spot barks, then Suzy's cat will run away.*
Analysis Contradictory: Spot barked, but Suzy's cat did not run away.

Example 19: *If Spot got out of the yard, he was chasing a squirrel.*
Analysis Contradictory: Spot got out of the yard, but he wasn't chasing a squirrel.

Exercises

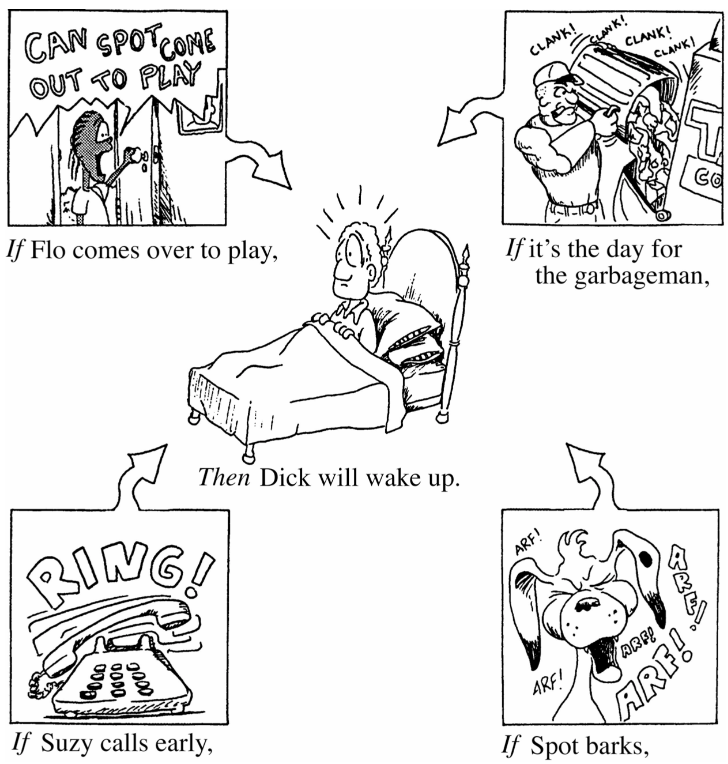
For each of the following: a. Say if it's a compound proposition. b. If it's a conditional, state the antecedent and consequent. c. Write a contradictory of it.

1. Maria or Lee will pick up Manuel after classes.
2. Neither Maria nor Lee has a bicycle.
3. AIDS cannot be contracted by touching or by breathing air in the same room as a person infected with AIDS.

4. Zoe (to Dick): Will you take the trash out, or do I have to?
5. If Spot barks, then Puff will run away.
6. Lee will take care of Spot next weekend if Dick will help him with his English exam.
7. Since 2 times 2 is 4, and 2 times 4 is 8, I should be ahead $8, not $7.
8. If Manuel went to the basketball game, then he either got a ride with Maria or he left early in his wheelchair to get there.
9. Drop the gun and no one will get hurt.
10. Maria will get a raise at work if and only if she is on time for work for a month.

E. Valid Forms of Inferences using Conditionals

Let's look at some valid forms of reasoning using conditionals and some forms that look a like those but are usually weak. Here is an illustration for some of the examples.



The Direct Way of Reasoning with Conditionals

If A, then B + A
↓
B

Valid

Affirming the Consequent

If A, then B + B
↓
A

Weak

The direct way of reasoning with conditionals is often called *modus ponens*.

Example 20: *If Spot barks, then Dick will wake up. Spot barked. So Dick woke up.*
Analysis This is a valid inference. It is impossible for the premises to be true and conclusion false at the same time. It's an example of the direct way of reasoning with conditionals.

Example 21: *If Spot barks, then Dick will wake up. Dick woke up. So Spot barked.*
Analysis This is weak. Maybe Suzy called, or Flo came over to play. It's affirming the consequent, reasoning backwards.

The Indirect Way of Reasoning with Conditionals		***Denying the Antecedent***	
If A, then B + not B ↓ *not A*	*Valid*	*If A, then B + not A* ↓ *not B*	*Weak*

The indirect way of reasoning with conditionals is often called *modus tollens*.

Example 22: *If Spot barks, then Dick will wake up. Dick didn't wake up. So Spot didn't bark.*
Analysis This is valid, an example of the indirect way of reasoning with conditionals.

Example 23: *If it's the day for the garbageman, then Dick will wake up. It's not the day for the garbageman. So Dick didn't wake up.*
Analysis This is weak. Even though the garbageman didn't come, maybe Spot barked or Suzy called early. It overlooks other possible ways the premise could be true and conclusion false.

Example 24: *If Maria doesn't call Manuel, then Manuel will miss his class. Maria did call Manuel. So Manuel didn't miss his class.*
Analysis This is weak, denying the antecedent. Remember that "not" in the form indicates a contradictory.

Reasoning in a Chain with Conditionals

If A, then B + If B, then C
↓
If A, then C *Valid*

Example 25: *If Dick takes Spot for a walk, then Zoe will cook dinner. And if Zoe cooks dinner, then Dick will do the dishes. So if Dick takes Spot for a walk, then he'll do the dishes. But Dick did take Spot for a walk. So he must have done the dishes.*
Analysis This is a valid inference: reasoning in a chain with conditionals followed by the direct way of reasoning with conditionals. We conclude the last consequent because we have the first antecedent.

Reasoning from Hypotheses
If you start with an hypothesis *A* and make a valid inference with conclusion *B*, then you've shown that *If A*, *then B* must be true

Example 26: *Lee: I'm thinking of majoring in biology.*
Maria: That means you'll take summer school. Here's why: You're in your second year now. To finish in four years like you told me you need to, you'll have to take all the upper-division biology courses your last two years. And you can't take any of those until you've finished the three-semester calculus course. So you'll have to take calculus over the summer to finish in four years.

Analysis Maria has not proved that Lee has to go to summer school. Rather, on the assumption (hypothesis) that Lee will major in biology, Lee will have to go to summer school. That is, Maria has proved "If Lee majors in biology, then he'll have to go to summer school."

Exercises

Evaluate the following as valid or weak.
Identify the form of the inference if it's one from this chapter.

1. Tom: Either you'll vote for the Republican or the Democratic candidate for president.
   Lee: No way I'll vote for the Democrat.
   Tom: So you'll vote for the Republican.
2. Dick: Somebody knocked over our neighbor's trash can last night. Either our neighbor hit it with her car again when she backed out, or a raccoon got into it, or Spot knocked it over.
   Zoe: Our neighbor didn't hit it with her car because she hasn't been out of her house since last Tuesday.
   Dick: It wasn't a raccoon because Spot didn't bark last night.
   Zoe: Spot! Bad dog! Stay out of the trash!
3. If Suzy breaks up with Tom, then she'll have to return his letter jacket. But there is no way she'll give up that jacket. So she won't break up with Tom.
4. Steve Pearce is a congressman who meets with his constituents regularly.
   If someone is a good congressman, he meets with his constituents regularly.
   So Rep. Pearce is a good congressman.
5. Dr. E (on an exam day): If students don't like me, they won't show up. But all of them showed up today. So they must really like me.
6. Maria: Lee will take care of Spot Tuesday if Dick will help him with his English paper.
   Manuel: (*later*) Dick didn't help Lee with his English paper, so I guess Lee didn't take care of Spot on Tuesday.

3 Classical Propositional Logic: Form

A. Propositional Logic and the Basic Connectives

We've looked at some inferences that are valid due to their form, and we've seen some whose forms look a lot like the valid ones but are weak. As well, some proposition are true due to their form. For example, we don't need to know anything about Dick and what he likes in order to know that the following is true:

Dick liked the movie or Dick didn't like the movie.

Should we go on, looking at one form after another to try to decide whether it guarantees validity or truth? Do we have any method beyond our intuition?

We can do better. We can try to make precise what forms we're looking at. Then we can say how we'll understand the forms so that we can have a general method for evaluating propositions and inferences.

Formal logic *Formal logic* is (i) the analysis of inferences for validity in terms of the structure of the propositions appearing in an inference, and (ii) the analysis of propositions for truth in terms of their structure.

Formal logic will be a tool we can use in the analysis of inferences and propositions. By itself it can't determine whether any particular reasoning is good. Nor can it help us evaluate strong inferences.

To begin, we'll follow up on what we did in the last chapter, looking at simple forms based on how we can combine propositions.

Propositional logic *Propositional logic* is formal logic where we ignore the internal structure of propositions except as they are built from other propositions in specified ways.

Let's start with four phrases for building up propositions from other propositions we saw in the last chapter: the *connectives* "and", "or", "not", and "if . . . then . . .". These will give us a basis to see the issues in classifying forms and understanding meanings to serve us when we later extend what we pay attention to in the *syntax* (forms) and *semantics* (meaning) of propositions.

We use these English connectives in many ways, some of which may be of no concern to us in our reasoning. We'd like to agree on how we'll understand