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Nicolae Herisanu
Vasile Marinca *Editors*

Acoustics and Vibration of Mechanical Structures—AVMS 2019

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Nicolae Herisanu · Vasile Marinca
Editors

Acoustics and Vibration of Mechanical Structures—AVMS 2019

Proceedings of the 15th AVMS, Timisoara, Romania, May 30–31, 2019

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Preface

The Proceedings of the XV-th International Conference “Acoustics and Vibration of Mechanical Structures”—AVMS-2019 contains selected papers contributed to the Conference held during May 30–31, 2019 in Timişoara (Romania).

This book is organized on five chapters covering a broad range of topics related to acoustics and vibration problems, such as:

- Noise and vibration control;
- Noise and vibration generation and propagation;
- Effects of noise and vibration;
- Condition monitoring and vibration testing;
- Nonlinear acoustics and vibration;
- Analytical, numerical and experimental techniques for noise and vibration;
- Modeling, prediction and simulations of noise and vibration;
- Environmental and occupational noise and vibration;
- Noise and vibration attenuators;
- Biomechanics and bioacoustics.

There are presented some analytical, numerical and experimental techniques applicable to analyze linear and nonlinear noise and vibration problems.

Each paper went through a rigorous review process performed by the members of the International Scientific Committee and specialized external reviewers, and the accepted papers are reported in this volume.

We would like to express our sincere appreciation to keynote speakers and all contributors of the presented papers for sharing their knowledge and experiences with all the participants. We are also expressing our sincere thanks to the members of the International Scientific Committee for spending their valuable time to review the papers and also to the members of the Organizing Committee for ensuring the success of this Conference which would not have been possible without their efforts.

Finally, special thanks are given to Springer for producing this volume.

Timisoara, Romania

Nicolae Herisanu
Vasile Marinca

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Analytical Approaches to Vibration Problems

Modelling of Frictional Contacts in 3D Dynamics of a Rigid Body

Jan Awrejcewicz and Grzegorz Kudra

Abstract There is considered a system of a spatial double pendulum with rigid movable obstacle, consisting of two links connected to each other and suspended on a shaft performing rotational motion about its horizontal axis according to a given function of time (kinematic driving). The links are connected by the use of two universal joints. The second link ends with a ball which can come into contact (impacts and permanent contact) with a planar and rotating obstacle situated below the pendulum. There is presented mathematical model of dynamics based on the Lagrange formulation. In this work, we use and expand our earlier developed models of contact forces (resulting friction force and rolling resistance). The friction models are based on the integral model developed assuming developed sliding on a planar contact area, where at each point, the classical Coulomb's friction law is valid. The integral models are then replaced by special approximations being more suitable for fast numerical simulations. In the present work, we model impacts with non-point frictional contacts assuming Hertzian compliance of the obstacle. The constructed models of 3D dynamics of a rigid body and the planned experimental investigations allow us to perform the tests of importance of the particular individual elements of the models and may lead to general conclusions about modelling and effective computer simulations of mechanical systems with 3D frictional contacts. We report bifurcation dynamics using bifurcation diagrams, Poincaré sections as well as the largest Lyapunov exponent.

1 Introduction

Pendulum-based mechanical systems serve as a paradigmatic model for analysis of many problems in nonlinear dynamics, mechanical engineering, biomechanics, control theory and mechatronics. Among different mechanical systems, one can encounter models based on spherical pendulum [1]. For example, in the work [2], the

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authors analysed the problem of control of the spherical pendulum. In the paper [3], there is presented analysis of 3D frictionless double pendulum rotating with constant angular velocity about its vertical axis. Another example of a multi-pendulum system in 3D space is investigated in the work [4]. Since in robotics, many tasks require making and braking contact with different subjects, some group of works concerns the problems of pendulum dynamics with impact and friction phenomena [5, 6].

Real natural and engineering objects with frictional 3D contacts very often cannot be modelled using classical one-dimensional friction model. As examples, one can indicate rolling bearings, billiard ball, Celtic stone or other contact phenomena encountered for example in robotics. Exact numerical modelling and simulation require in general space discretization methods and lead to high computational costs. Looking for simplified but realistic models, Contensou proposed an integral model of resultant friction force assuming fully developed sliding and Coulomb friction law at each element of the contact area [7]. The integral model can be then approximated by the use of special algebraic functions in order to make the simulation faster [8]. Some authors developed approximations of the friction force and moment assuming special contact pressure distributions allowing to model rolling resistance [9, 10]. The approximated models were tested during modelling of the selected mechanical systems: wobblestone, billiard ball and full solid ellipsoid of the revolution [11–13].

In multibody system dynamics, the impact phenomena can be modelled as the so-called hard or soft impacts [14, 15]. However, the impacts are usually modelled as phenomena occurring at a certain point. Even if friction torque is taken into account, the coupling between friction force and moment is neglected.

In this work, we join the friction models developed and presented in the works [10–13] with impact model based on Hertzian stiffness and special model of nonlinear damping. The present work is continuation and extension of the conference papers [16, 17].

2 Mathematical Model

In Fig. 1a, there is presented a physical concept of the investigated double spatial pendulum, where one can observe the fixed frame F, three connected solids (the body 0, the pendulums 1 and 2) and the obstacle 3 in the form of the disc performing rotational motion. The solid 0 performs rotational motion with respect to the fixed frame. The solid 0, pendulum 1 and 2 are connected by the use of two Cardan–Hook joints. There are introduced the following coordinate systems: the fixed (global) reference frame O_1xyz, pendulum 1 fixed reference frame $O_1x_1y_1z_1$ and pendulum 2 fixed coordinate system $O_2x_2y_2z_2$. It is assumed that the origins O_1 and O_2 of the introduced coordinate systems lie in the centres of the corresponding Cardan–Hook joints (intersections of their axes) and that the body 0 rotates about the axis z.

It is assumed that the initial position of the system corresponds to the reference frames $O_1x_1y_1z_1$ and O_1xyz overlapping each other and the axes of the coordinate system $O_2x_2y_2z_2$ being parallel to the corresponding axes of the reference frame

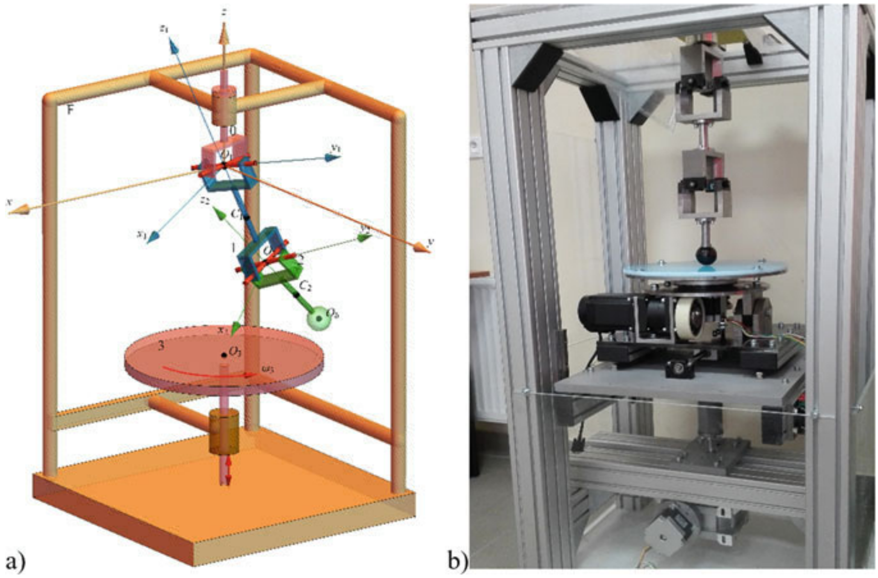

Fig. 1 Double pendulum with obstacle—model and experimental rig

$O_1x_1y_1z_1$. Then, the position of the system is described by the use of the following sequence of rotations: by angle ψ_1 about axis z_1 (rotation of the body 0), by angle θ_1 about axis x_1, by angle φ_1 about axis y_1, by angle θ_2 about axis x_2 and by angle φ_2 about axis y_2. We assume also that the centre O_2 of the second Cardan–Hook joint lies on the axis O_1z_1.

The second pendulum ends with a spherical solid of radius R_b and centred at the point O_3 which lies on the axis O_2z_2. It is also assumed that the mass centres C_1 and C_2 of the both links lie on the axes O_1z_1 or O_2z_2, respectively. Moreover, the axes of the reference frames $O_1x_1y_1z_1$ and $O_2x_2y_2z_2$ are the principal axes of inertia of the corresponding pendulums. The geometric and mass properties of the pendulums are defined by the following parameters: $L_1 = O_1O_2$, $L_2 = O_2O_3$, R_b, $e_1 = O_1C_1$, $e_2 = O_2C_2$, m_1 and m_2 (masses of the corresponding links), I_{xi}, I_{yi} and I_{zi} ($i = 1, 2$; the corresponding principal central moments of inertia of the link number i with respect to the axis parallel to the corresponding axis O_ix_i, O_ix_i or O_ix_i). It is assumed that the Cardan–Hook joints are massless. Moreover, the rotational motion of the body 0 is known in advanced as a kinematic driving of the system.

The spherical solid at the end of the second link can come into a contact with the obstacle 3, which has the form of a disc rotating around the axis z with angular velocity ω_3. Vertical position of the obstacle is defined by the parameter z_{O_3}—describing the z coordinate of the disc's centre O_3. All the bodies are assumed to be rigid during modelling of their global dynamics. Introduced further local compliance of the bodies in the vicinity of the contact is assumed to not influence global geometry of the system in a significant way.

Angular velocity $\omega = \mathrm{d}\psi_1/\mathrm{d}t$ of the body 0 is assumed to be the following function of time

$$\omega(t) = \omega_0 + q \cos \Omega t, \tag{1}$$

where ω_0, q and Ω are parameters representing its constant component, amplitude and frequency.

The governing equations of motion are expressed using the Lagrange's formalism:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = \tau_i, \quad i = 1, 2, 3, 4 \tag{2}$$

where q_i is the ith generalized coordinate and element of the following vector

$$\mathbf{q} = \begin{bmatrix} q_1 \; q_2 \;\; q_3 \; q_4 \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} \theta_1 \; \varphi_1 \;\; \theta_2 \; \varphi_2 \end{bmatrix}^{\mathrm{T}} \tag{3}$$

and T—is kinetic energy, V—potential energy of gravity forces, τ_i—is the ith generalized force and the element of the following vector

$$\boldsymbol{\tau} = \begin{bmatrix} \tau_1 \; \tau_2 \;\; \tau_3 \; \tau_4 \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} \tau_{\theta_1} \; \tau_{\varphi_1} \;\; \tau_{\theta_2} \; \tau_{\varphi_2} \end{bmatrix}^{\mathrm{T}}. \tag{4}$$

The kinetic energy T of the system reads

$$\begin{aligned} T = {} & \frac{1}{2}m_1\left(v^2_{C_1x_1} + v^2_{C_1y_1} + v^2_{C_1z_1}\right) + \frac{1}{2}m_2\left(v^2_{C_2x_2} + v^2_{C_2y_2} + v^2_{C_2z_2}\right) \\ & + \frac{1}{2}\left(I_{x_1}\omega^2_{1x_1} + I_{y_1}\omega^2_{1y_1} + I_{z_1}\omega^2_{1z_1}\right) + \frac{1}{2}\left(I_{x_2}\omega^2_{2x_2} + I_{y_2}\omega^2_{2y_2} + I_{z_2}\omega^2_{2z_2}\right), \end{aligned} \tag{5}$$

and one can be presented using the following matrix notation

$$T(\mathbf{q}, \dot{\mathbf{q}}, \omega) = \frac{1}{2}\dot{\mathbf{q}}^{\mathrm{T}}\mathbf{M}(\mathbf{q})\dot{\mathbf{q}} + \dot{\mathbf{q}}^{\mathrm{T}}\mathbf{b}_1(\mathbf{q})\omega + \frac{1}{2}b_0(\mathbf{q})\omega^2. \tag{6}$$

where $\mathbf{v}_{C_1}$ and $\mathbf{v}_{C_2}$ denote velocities of mass centres C_1 and C_2 of the first and the second link, $\boldsymbol{\omega}_1$ and $\boldsymbol{\omega}_2$—stand for angular velocities of the bodies, while mass matrix $\mathbf{M}(\mathbf{q})$, $\mathbf{b}_1(\mathbf{q})$ and $b_0(\mathbf{q})$ are certain functions.

Potential energy $V(\mathbf{q})$ of gravitational forces reads as follows

$$V = m_1 g z_{C_1} + m_2 g z_{C_2}, \tag{7}$$

where z_{C_1} and z_{C_2} denote the coordinates along the axis z of the mass centres C_1 and C_2.

The differential equations of motion can be finally presented in the following way

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{b}_1(\mathbf{q})\dot{\omega} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{c}_1(\mathbf{q}, \dot{\mathbf{q}})\omega + \mathbf{c}_2(\mathbf{q})\omega^2 + \mathbf{w}(\mathbf{q}) = \boldsymbol{\tau}, \tag{8}$$

where

$$\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) = \frac{\mathrm{d}\mathbf{M}(\mathbf{q})}{\mathrm{d}t} - \frac{1}{2}\frac{\partial \dot{\mathbf{q}}^{\mathrm{T}}\mathbf{M}(\mathbf{q})}{\partial \mathbf{q}},$$

$$\mathbf{c}_1(\mathbf{q}, \dot{\mathbf{q}}) = \frac{\mathrm{d}\mathbf{b}_1(\mathbf{q})}{\mathrm{d}t} - \frac{\partial \mathbf{b}_1^{\mathrm{T}}(\mathbf{q})}{\partial \mathbf{q}}\dot{\mathbf{q}},$$

$$\mathbf{c}_2(\mathbf{q}) = -\frac{1}{2}\frac{\partial b_0(\mathbf{q})}{\partial \mathbf{q}},$$

$$\mathbf{w}(\mathbf{q}) = \frac{\partial V(\mathbf{q})}{\partial \mathbf{q}}.$$

The generalized forces are organized in the following way:

$$\boldsymbol{\tau} = \boldsymbol{\tau}_c + \boldsymbol{\tau}_b, \tag{9}$$

where $\boldsymbol{\tau}_c$ represents generalized contact forces and $\boldsymbol{\tau}_b = [\ \tau_{\theta_1 b}\ \tau_{\varphi_1 b}\ \ \tau_{\theta_2 b}\ \tau_{\varphi_2 b}\ ]^{\mathrm{T}}$ is a vector of damping torques in the corresponding joints.

Let us denote by A_2 and A_3 two points belonging to the bodies 2 and 3, respectively, where they can potentially come into a contact with each other. The elements of vector of generalized contact forces are computed in the following way

$$\tau_{ci} = \mathbf{F}_c \cdot \frac{\partial \mathbf{v}_{A_2}}{\partial \dot{q}_i}, \quad i = 1, 2, 3, 4 \tag{10}$$

where $\mathbf{F}_c$ is resultant contact force acting on the ball of the second link at the point A_2 and $\mathbf{v}_{A_2}$ is velocity of the point A_2.

The set of admissible positions of the pendulum is limited by the obstacle, which undeformed surface is described by the equation $z = z_{O_3}$, where z is z-coordinate of a point lying on the surface and z_{O_3} is a constant parameter. It results in the following formula for distance h between the undeformed ball and the disc's surface

$$h = z_{O_b} - z_{O_3} - R_b, \tag{11}$$

where z_{O_b} denotes the corresponding global z-coordinate of the ball centre O_b.

The derivative of distance h with respect to time is expressed in the following way

$$\dot{h} = \mathbf{v}_{A_2} \cdot \mathbf{n}, \tag{12}$$

where n is a unit vector normal to the obstacle.

One can write the following relation allowing to calculate velocity of the point A_3

$$\mathbf{v}_{A_3} = \boldsymbol{\omega}_3 \times \mathbf{r}_{A_3}, \tag{13}$$

where $\boldsymbol{\omega}_3$ is angular velocity of the obstacle rotating about the axis z and $\mathbf{r}_{A_3}$ stands for the vector of the beginning at the point O_1 (lying on the rotation axis of the obstacle) and the end at the point A_3.

Translational and angular sliding relative velocities at the centre of the contact area read

$$\begin{aligned} \mathbf{v}_s &= \mathbf{v}_{A_2} - \dot{h}\mathbf{n} - \mathbf{v}_{A_3}, \\ \boldsymbol{\omega}_s &= (\boldsymbol{\omega}_2 \cdot \mathbf{n})\mathbf{n} - \boldsymbol{\omega}_3. \end{aligned} \tag{14}$$

The damping torques in the joints $\boldsymbol{\tau}_b = \begin{bmatrix} \tau_{\theta_1 b} & \tau_{\varphi_1 b} & \tau_{\theta_2 b} & \tau_{\varphi_2 b} \end{bmatrix}^{\mathrm{T}}$ are modelled in the following way

$$\tau_{\xi_i b} = -M_b \frac{\dot{\xi}_i}{\sqrt{\dot{\xi}_i^2 + \varepsilon_b^2}}, \quad \xi = \theta, \varphi; i = 1, 2, \tag{15}$$

where M_b and ε_b are constant parameters common for all the joints.

The contact force acting on the pendulum at the point A_2 consists of two components

$$\mathbf{F}_c = \mathbf{N} + \mathbf{T}, \tag{16}$$

where $\mathbf{N} = N\mathbf{n}$ is normal component of reaction, $\mathbf{T}$—resultant friction force reduced to the centre of the contact and $\mathbf{n}$—is unit vector normal to the disc's 3 surface.

The normal component is expressed in the following way

$$N = k|h|^{3/2}\left(1 - b\dot{h}\right)1(-h)1\left(1 - b\dot{h}\right), \tag{17}$$

where k denotes the coefficient of the nonlinear stiffness of the Hertzian contact, b is coefficient of damping and **1** is the unit step function. In the case of the contact between a ball of radius R_b and an elastic semi-space, the coefficient of stiffness can be computed in the following way

$$k = \frac{4\sqrt{R_b}}{3\left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}\right)}, \tag{18}$$

where ν_1 and ν_2 are Poisson's coefficients of materials of the contacting bodies, while E_1 and E_2 are their Young's modulus.

The approximation of the resulting friction force for circular contact area, based on extensions of Padé approximants, assuming fully developed sliding and Coulomb friction model at each point of the contact, has the form [10–13]

$$\mathbf{T} = -\mu N \frac{\mathbf{v}_s}{\sqrt{\mathbf{v}_s^2 + b_T^2 a_r^2 \boldsymbol{\omega}_s^2 + \varepsilon^2}}, \tag{19}$$

where μ is coefficient of Coulomb friction, b_T—the parameter depending on the contact stress distribution, ε—the parameter introduced in order to regularize functions (19), a_r—radius of the corresponding Hertzian contact, while $\mathbf{v}_s$ and $\boldsymbol{\omega}_s$ are relative linear and angular sliding velocities at the centre of the contact. The friction torque is neglected.

The size of the contact is calculated according to the following formula

$$a_r = \left(\frac{3}{4} N \left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}\right) R_b\right)^{1/3}. \tag{20}$$

3 Numerical Simulations

During the presented in this section numerical simulations the following set of geometrical and mass parameters (corresponding to the experimental stand under construction) remain constant: $m_1 = 4.59$ kg, $m_2 = 2.41$ kg, $I_{x_1} = I_{y_1} = 0.0315$ kg m^2, $I_{z_1} = 0.0078$ kg m^2, $I_{x_2} = 0.0084$ kg m^2, $I_{y_2} = 0.0055$ kg m^2, $I_{z_2} = 0.0038$ kg m^2, $L_1 = 0.228$ m, $L_2 = 0.175$ m, $e_1 = 0.122$ m, $e_2 = 0.0586$ m and $R_b = 0.025$ m. Moreover, we assume $g = 9.81$ m/s^2. The axes of the Cardan joints are assumed to be massless, since their masses are partially taken into account in the mass parameters of the links, i.e. the corresponding masses of the axes are assumed to move together with the link in which they are mounted. The test simulations performed for two different versions of the pendulum (for the full model with mass joints and for reduced model with massless swivels) exhibited no significant differences.

Moreover, we assume the following parameters of the resistance in the joints and properties of the contact: $M_b = 0.04$ N m, $\varepsilon_b = 0.4$ 1/s, $E_1 = E_2 = 0.01$ GPa, $\nu_1 = \nu_2 = 0.3$, $b = 0.5\,\mathrm{m}^{-1}\mathrm{s}$, $\mu = 0.5$, $\varepsilon = 10^{-3}$ 1/s (regularization parameter) and $b_T = 0.681$. The last value was obtained based on optimizing the adjustment to the integral model of friction for circular contact area and Hertzian contact stress distribution. The parameters of the kinematic driving applied in the subsequent simulations are as follows: $\omega_0 = 0$ rad/s, $q = 3$ rad/s and $\Omega = 3$ rad/s. The obstacle rotates with constant angular velocity $\omega_0 = 5$ rad/s.

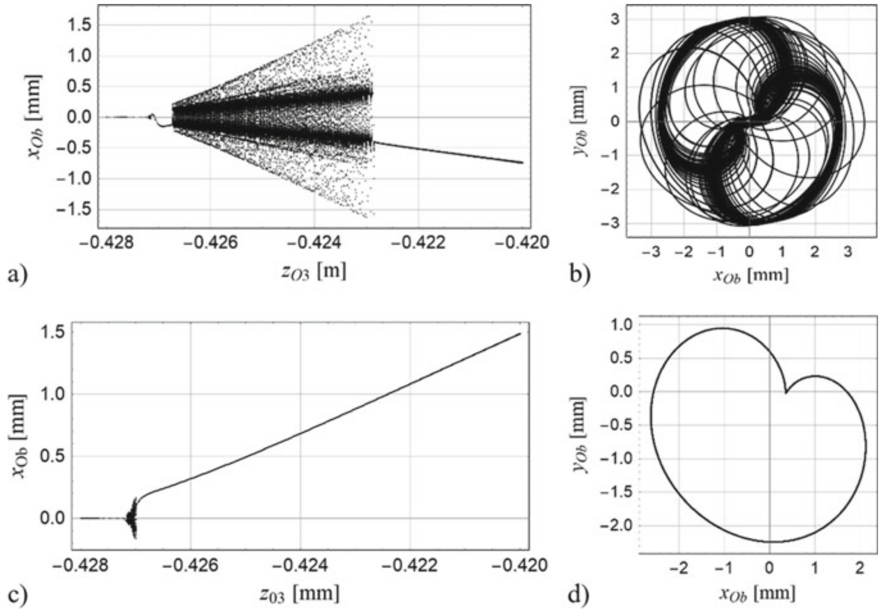

Fig. 2 Bifurcation diagrams with position of the obstacle z_{O_3} as a control parameter for $b_T = 0.681$ (**a**) and $b_T = 0$ (**c**), along with the corresponding orbits for $z_{O_3} = -0.424$ m (**b**—for $b_T = 0.681$, **d**—$b_T = 0$)

Figure 2 exhibits two bifurcation diagrams with (increasing) position of the obstacle z_{O_3} as a control parameter for $b_T = 0.681$ (a) and $b_T = 0$ (c). The first case corresponds to model of friction force depending on local translational and angular sliding relative velocities (with optimized fitting to the integral model). The second case corresponds to classical friction model assuming a point contact. As one can observe, the introduced elements of modelling of the contact are crucial for bifurcation dynamics of the system. In the panels (b) and (d), there are presented the corresponding orbits for $z_{O_3} = -0.424$ m: irregular orbit for $b_T = 0.681$ (b) and periodic attractor for $b_T = 0$ (d). Figure 3 exhibits the corresponding Poincaré section (a) and the process of computation of the largest Lyapunov exponents (b) of the attractor presented in Fig. 2b. One can conclude that the attractor is quasiperiodic. Note that since the system is non-autonomous with periodic forcing, there exists the second Lyapunov exponent equal to zero.

4 Concluding Remarks

In the works [16, 17] and the present paper, to our knowledge for the first time in the literature, there is presented an application of special class of reduced models of tangent contact forces based on approximations of the integral Contensou model

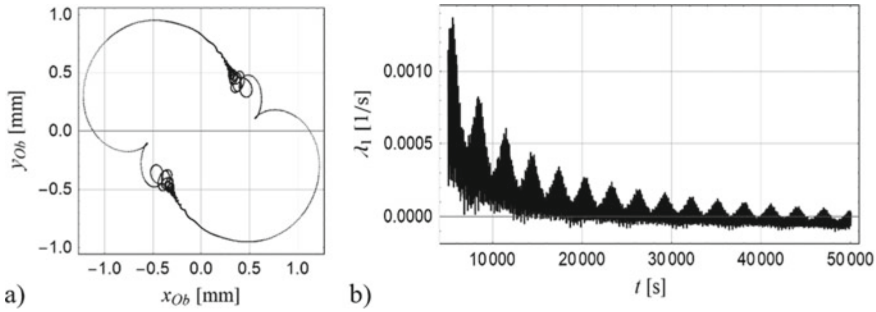

Fig. 3 Poincaré section of the orbit presented in Fig. 2b (**a**) and its largest Lyapunov exponent (**b**)

of friction, in modelling of impact dynamics of a rigid body in 3D space. Some components of the contact model, like rolling friction and spin friction, have been neglected. The authors of this work expect that influence of these elements on the system dynamics is negligible; however, it can be checked during further numerical and experimental investigations.

The investigated system can exhibit rich nonlinear dynamics and potentially new or rare bifurcation scenarios. The presented example of bifurcation dynamics shows the differences in simulations results, when applying the contact model for finite size of the contact and the model of friction force assuming a point contact, i.e. no relation between resultant friction force and rotational relative sliding motion of contacting surfaces. It proves the necessity of the use of the proposed models, when one can obtain fast and reliable numerical simulation of a rigid body dynamic with 3D frictional contacts.

The paper present results which can be treated as a preparation stage before further joint numerical and experimental investigations. We are going to confirm experimentally the bifurcation scenarios exhibited by the model and usefulness of the developed models.

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