RONALD J. ANDERSON

THE PRACTICE OF ENGINEERING DYNAMICS

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The Practice of Engineering Dynamics

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This edition first published 2020

Registered Offices
John Wiley & Sons, Inc., 111 River Street, Hoboken, NJ 07030, USA
John Wiley & Sons Ltd, The Atrium, Southern Gate, Chichester, West Sussex, PO19 8SQ, UK

Editorial Office
The Atrium, Southern Gate, Chichester, West Sussex, PO19 8SQ, UK

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Library of Congress Cataloging-in-Publication data

Names: Anderson, Ron J. (Ron James), 1950- author.
Title: The practice of engineering dynamics / Ronald J Anderson, Queen's University, Kingston, Canada.
Description: First edition. | Hoboken, NJ, USA : John Wiley & Sons, Inc., [2020] | Includes bibliographical references and index.
Identifiers: LCCN 2020004354 (print) | LCCN 2020004355 (ebook) | ISBN 9781119053705 (hardback) | ISBN 9781119053682 (adobe pdf) | ISBN 9781119053699 (epub)
Subjects: LCSH: Machinery, Dynamics of. | Mechanics, Applied.
Classification: LCC TJ170 .A53 2020 (print) | LCC TJ170 (ebook) | DDC 620.1/04–dc23
LC record available at https://lccn.loc.gov/2020004354
LC ebook record available at https://lccn.loc.gov/2020004355

Cover Design: Wiley
Cover Image: © kovop58/Shutterstock

Set in 9.5/12.5pt STIXTwoText by SPi Global, Chennai, India

Printed and bound by CPI Group (UK) Ltd, Croydon, CR0 4YY

10 9 8 7 6 5 4 3 2 1

To June, Stacey, and Kate

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Preface

The design of a mechanical system very often includes a requirement for dynamic analysis. During the early concept design stages it is useful to create a mathematical model of the system by deriving the governing equations of motion. Then, simulations of the behavior of the system can be produced by solving the equations of motion. These simulations give guidance to the design engineers in choosing parameter values in their attempt to create a system that satisfies all of the performance criteria they have laid out for it.

There is a logical progression of analyses that are required during the design. The design engineer needs to determine, from the nonlinear differential equations of motion:

- The equilibrium states of the system – these are places where, once put there and not disturbed, the system will stay. The time varying terms are removed from the differential equations of motion, leaving a set of nonlinear algebraic equations. The solutions to these equations provide knowledge of all of the equilibrium states.
- The stability of the equilibrium states – the question here is: if the system is disturbed slightly from an equilibrium state, will it try to get back to that state or will it move farther away from it? It is usually not good practice to design systems around unstable equilibrium states since the system will always tend to move towards a stable equilibrium condition. Answering the stability question involves a linearization of the equations of motion for small perturbations away from the equilibrium states.
- How the system behaves around a stable equilibrium state – the study of small motions of a mechanical system around a stable equilibrium state lies in the realm of vibrations and leads to predictions of natural frequencies, mode shapes, and damping ratios, each of which is very useful during the design process. The linearized differential equations of motion are used.
- The response to harmonically applied external forces – systems, in stable equilibrium, are often subjected to harmonic external disturbances at known forcing frequencies and their response to these forces provides critical design information. The linearized equations of motion are used.
- The response of the system in the time domain – the fully nonlinear equations of motion are solved numerically to simulate the response of the system to known external forces. Large scale motions and the nonlinear characteristics of system elements are included. This is the numerical equivalent to conducting performance experiments with

a prototype of the system. The design information gleaned from these simulations is, perhaps surprisingly to some, not very useful in the early stages of the design. Accurate nonlinear simulations require precise knowledge of system parameters that simply isn't available in the early design phases of a project. The time domain simulations are best left to the prototype testing stage when they can be validated through comparison of predicted and measured system response. Validated time domain simulations are valuable tools to use when considering design changes aimed at improving the measured performance of the system.

The presentation of material in this book divides the practice of engineering dynamics into three parts.

Part 1. Modeling: Deriving Equations of Motion

Dynamic analysis is based on the use of accurate nonlinear equations of motion for a system. Deriving these complicated equations is a task that is prone to error. Because of this, it is important to derive the governing equations twice, using two different methods of analysis, and then prove to yourself that the two sets of equations are the same. This is a time consuming activity but is vital because predictions made from the equations of motion are critical in the design process. Predictions made using equations with errors are not of any use. The first part of the book discusses the generation of nonlinear equations of motion using, firstly, Newton's laws and, secondly, Lagrange's equation. Only when the two methods give the same equations can the analyst proceed to part 2 with confidence.

Part 2. Simulation: Using the Equations of Motion

The second part presents a logical progression of analysis techniques and methods applied to the governing equations of motion for systems. The progression is from equilibrium solutions that find in what states the system would like to be, to analyzing the stability of these equilibrium states (stability is usually considered only in textbooks on control systems but it is vitally important to dynamic systems), to considering small motions about the stable equilibrium states (this topic is covered in textbooks on vibrations but is, again, vital to engineers doing dynamic analysis), to frequency domain analysis (vibrations again), and finally to time domain solutions (these are rarely covered in textbooks).

Part 3. Working with Experimental Data

While not usually considered a part of the design process, analysis of experimental data measured on dynamic systems is critical to creating a successful product. To assist engineers in developing capabilities in this area, part 3 covers the practical use of discrete fourier transforms in analyzing experimental data.

In order to emphasize the idea that any dynamic mechanical system can be analyzed using the sequence of steps presented here, all the exercises at the ends of the chapters are based on 23 mechanical systems defined in an appendix. Any one of these systems could be used as an example of all of the types of dynamic analysis.

This book is based on course notes that I have developed while teaching a one-semester graduate course on dynamics over more than two decades. It could just as well be used in a senior undergraduate dynamics course.

November, 2019

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About the Companion Website

This book is accompanied by a companion website:

www.wiley.com/go/anderson/engineeringdynamics

The website includes:

- Animations
- Fully worked examples
- Software

Scan this QR code to visit the companion website.

Part I

Modeling: Deriving Equations of Motion

1

Kinematics

Kinematics is defined as the study of motion without reference to the forces that cause the motion. A proper kinematic analysis is an essential first step in any dynamics problem. This is where the analyst defines the degrees of freedom and develops expressions for the absolute velocities and accelerations of the bodies in the system that satisfy all of the physical constraints. The ability to differentiate vectors with respect to time is a critical skill in kinematic analysis.

1.1 Derivatives of Vectors

Vectors have two distinct properties – magnitude and direction. Either or both of these properties may change with time and the time derivative of a vector must account for both.

The rate of change of a vector $\vec{r}$ with respect to time is therefore formed from,

1. The rate of change of magnitude $\left(\frac{d\vec{r}}{dt}\right)_m$.
2. The rate of change of direction $\left(\frac{d\vec{r}}{dt}\right)_d$.

Figure 1.1 shows the vector $\vec{r}(t)$ that changes after a time increment, Δt, to $\vec{r}(t+\Delta t)$.

The difference between $\vec{r}(t)$ and $\vec{r}(t+\Delta t)$ can be defined as the vector $\vec{q}(t)$ shown in Figure 1.1 and, by the rules of vector addition,

$$\vec{r}(t) + \vec{q}(t) = \vec{r}(t+\Delta t) \tag{1.1}$$

or,

$$\vec{q}(t) = \vec{r}(t+\Delta t) - \vec{r}(t). \tag{1.2}$$

Then, using the definition of the time derivative,

$$\frac{d\vec{r}}{dt} = \lim_{\Delta t\to 0} \frac{\vec{r}(t+\Delta t) - \vec{r}(t)}{\Delta t} = \lim_{\Delta t\to 0} \frac{\vec{q}(t)}{\Delta t}. \tag{1.3}$$

Imagine now that Figure 1.1 is compressed to show only an infinitesimally small time interval, Δt. The components of $\vec{q}(t)$ for the interval Δt are shown in Figure 1.1. They are,

1. A component $d\vec{q}_m$ aligned with the vector $\vec{r}$. This is a component that is strictly due to the rate of change of magnitude of $\vec{r}$. The magnitude of $d\vec{q}_m$ is $\dot{r}\Delta t$ where $\dot{r}$ is the rate of

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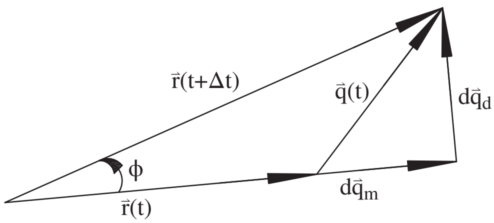


Figure 1.1 A vector changing with time.

change of length (or magnitude) of the vector $\vec{r}$. The direction of $d\vec{q}_m$ is the same as the direction of $\vec{r}$. Let $d\vec{q}_m$ be designated[1] as $\dot{\vec{r}}\Delta t$.

2. A component $d\vec{q}_d$ that is perpendicular to the vector $\vec{r}$. That is, a component due to the rate of change of direction of the vector. Terms of this type arise only when there is an angular velocity. The rate of change of direction term arises from the time rate of change of the angle ϕ in Figure 1.1 and $\dot{\phi}$ is the magnitude of the angular velocity of the vector. *The rate of change of direction therefore arises from the angular velocity of the vector*. The magnitude of $d\vec{q}_d$ is $\dot{\phi} r\Delta t$ where r is the length of $\vec{r}$. By definition the rate of change of the angle ϕ (i.e. $\dot{\phi}$) has the same positive sense as the angle itself. It is clear that $\dot{\phi} r$ is the "tip speed" one would expect from an object of length r rotating with angular speed $\dot{\phi}$.

The angular velocity is itself a vector quantity since it must specify both the angular speed (i.e. magnitude) and the axis of rotation (i.e. direction). In Figure 1.1, the speed of rotation is $\dot{\phi}$ and the axis of rotation is perpendicular to the page. This results in an angular velocity vector,

$$\vec{\omega} = \dot{\phi}\,\vec{k} \tag{1.4}$$

where the right handed set of unit vectors, $(\vec{\imath}, \vec{\jmath}, \vec{k})$, is defined in Figure 1.2. Note that it is essential that right handed coordinate systems be used for dynamic analysis because of the extensive use of the cross product and the directions of vectors arising from it. If there is a right handed coordinate system (x, y, z), with respective unit vectors $(\vec{\imath}, \vec{\jmath}, \vec{k})$, then the cross products are such that,

$$\vec{\imath} \times \vec{\jmath} = \vec{k}$$

$$\vec{\jmath} \times \vec{k} = \vec{\imath}$$

$$\vec{k} \times \vec{\imath} = \vec{\jmath}.$$

Using this definition of the angular velocity, the motion of the tip of vector $\vec{r}$, resulting from the angular change in time Δt, can be determined from the cross product

$$(\vec{\omega} \times \vec{r})\Delta t$$

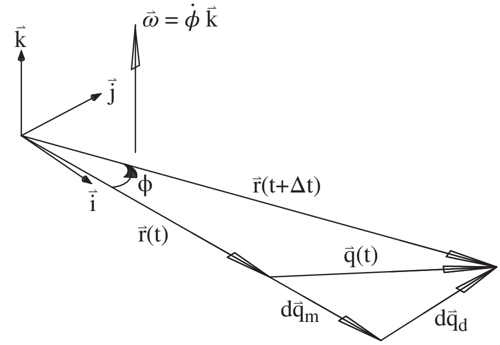
which, by the rules of the vector cross product, has magnitude,

$$|\vec{\omega}||\vec{r}|\Delta t = \dot{\phi} r\Delta t$$

---

1 The convention used here is that a vector with an *overdot* such as $\dot{\vec{r}}$ is used to represent the rate of change of magnitude of the vector and the overdot is not to be interpreted as a shorthand method of signifying the total derivative of the vector. For a scalar function there is only a magnitude and the overdot will represent its rate of change.

Figure 1.2 Even 2D problems are 3D.



and a direction that, according to the right hand rule[2] used for cross products, is perpendicular to both $\vec{\omega}$ and $\vec{r}$ and, in fact, lies in the direction of $d\vec{q}_d$.

Combining these two terms to get $\vec{q}(t)$ and substituting into Equation 1.3 results in,

$$\frac{d\vec{r}}{dt} = \lim_{\Delta t \to 0} \frac{\dot{\vec{r}}\Delta t + (\vec{\omega} \times \vec{r})\Delta t}{\Delta t} = \dot{\vec{r}} + \vec{\omega} \times \vec{r}. \tag{1.5}$$

The time derivative of any vector, $\vec{r}$, can therefore be written as,

$$\frac{d\vec{r}}{dt} = \underbrace{\dot{\vec{r}}}_{\substack{\text{rate of change}\\ \text{of magnitude}\\ \text{of the vector}}} + \underbrace{\vec{\omega} \times \vec{r}}_{\substack{\text{rate of change}\\ \text{of direction}\\ \text{of the vector}}}. \tag{1.6}$$

It is important to understand that the angular velocity vector, $\vec{\omega}$, is the angular velocity of the coordinate system in which the vector, $\vec{r}$, is expressed. There is a danger that the rate of change of direction terms will be included twice if the angular velocity of the vector with respect to the coordinate system in which it measured is used instead. The example presented in Section 1.3 shows a number of different ways to arrive at the derivative of a vector which rotates in a plane.

1.2 Performing Kinematic Analysis

Before proceeding with examples of kinematic analyses we state here the steps that are necessary in achieving a successful result. This first step in any dynamic analysis is vitally important. The goal is to derive expressions for the *absolute velocities and accelerations of the centers of mass* of the bodies making up the system being analyzed. In addition, expressions for the *absolute angular velocities and angular accelerations* of the bodies will be required.

2 Let the cross product of two vectors, $\vec{A}$ and $\vec{B}$, be the vector, $\vec{C}$

$$\vec{A} \times \vec{B} = \vec{C}.$$

According to the right hand rule, the direction of $\vec{C}$ is the direction aligned with the thumb of your right hand if you point that hand in the direction of $\vec{A}$ and curl your fingers towards $\vec{B}$.

It is at this first step of the analysis that degrees of freedom are defined and constraints on relative motion between bodies are satisfied.

For this general description of kinematic analysis, we assume that we are analyzing a system that has multiple bodies connected to each other by joints and that we are attempting to derive an expression for the acceleration of the center of mass of a body that is not the first in the assembly.

The procedure is as follows.

1. Find a fixed point (i.e. one having no velocity or acceleration) in the system from which you can begin to write relative position vectors that will lead to the centers of mass of bodies in the system.
2. Define a position vector that goes from the fixed point, through the first body, to the next joint in the system. This is the position of the joint *relative* to the fixed point.
3. Determine how many degrees of freedom, both translational and rotational, are required to define the motion of the relative position vector just defined. The degrees of freedom must be chosen to satisfy the constraints imposed by the joint that connects this body to ground.
4. Define a coordinate system in which the relative position vector will be written and determine the angular velocity of the coordinate system.
5. Repeat the previous three steps as you go from joint to joint in the system, always being careful to satisfy the joint constraints by defining appropriate degrees of freedom.
6. When the desired body is reached, define a final relative position vector from the joint to the center of mass.
7. The sum of all the relative position vectors will be the absolute position of the center of mass and the derivatives of the sum of vectors will yield the absolute velocity and acceleration of the center of mass.

1.3 Two Dimensional Motion with Constant Length

Figure 1.3 shows a rigid rod of length, ℓ, rotating about a fixed point, O, in a plane. An expression for the velocity of the free end of the rod, P, relative to point O is desired.

By definition, the velocity of P relative to O is the time derivative of the position of P relative to O. This position vector is designated $\vec{p}_{P/O}$ and is shown in the figure.

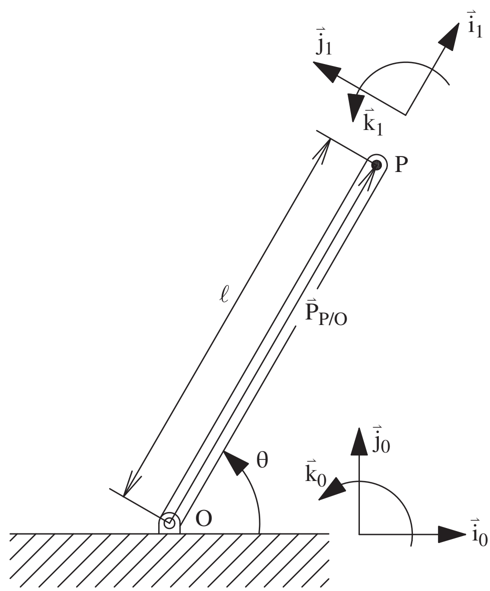
In order to differentiate the position vector, we must have an expression for it and this means we must first choose a coordinate system[3] in which to work. For a start, we can choose a right handed coordinate system fixed in the ground. The set of unit vectors $(\vec{\imath}_0, \vec{\jmath}_0, \vec{k}_0)$ is such a system. The angular velocity of this coordinate system is zero (i.e. $\vec{\omega}_0 = \vec{0}$) since it is fixed in the ground.

An expression for the position of P relative to O in this system is,

$$\vec{p}_{P/O} = \ell \cos\theta \vec{\imath}_0 + \ell \sin\theta \vec{\jmath}_0. \tag{1.7}$$

3 Three dimensional sets of unit vectors shown in two dimensional figures such as Figure 1.3 will be shown with the positive sense of the vector out of the plane represented by a curved arrow using the right hand rule (e.g. $\vec{k}_0$ in Figure 1.3).

Figure 1.3 A rigid rod rotating about a fixed point.



We apply Equation 1.6 to $\vec{p}_{P/O}$ to get,

$$\vec{v}_{P/O} = \frac{\mathrm{d}}{\mathrm{d}t}(\vec{p}_{P/O}) = \dot{\vec{p}}_{P/O} + \vec{\omega}_0 \times \vec{p}_{P/O}$$
$$= \frac{\mathrm{d}}{\mathrm{d}t}(\ell \cos\theta \vec{\imath}_0 + \ell \sin\theta \vec{\jmath}_0) + \vec{0} \times (\ell \cos\theta \vec{\imath}_0 + \ell \sin\theta \vec{\jmath}_0). \tag{1.8}$$

In this coordinate system, it is clear that there is a rate of change of magnitude of the vector only and the velocity of point P relative to O after performing the simple differentiation is,

$$\vec{v}_{P/O} = -\ell\dot{\theta} \sin\theta \vec{\imath}_0 + \ell\dot{\theta} \cos\theta \vec{\jmath}_0 = \ell\dot{\theta}(-\sin\theta \vec{\imath}_0 + \cos\theta_0 \vec{\jmath}_0). \tag{1.9}$$

Another derivation of the velocity of P relative to O might use the system of unit vectors $(\vec{\imath}_1, \vec{\jmath}_1, \vec{k}_1)$ that are fixed in the rod. The advantage of using this system is that the position vector is easily expressed as,

$$\vec{p}_{P/O} = \ell \vec{\imath}_1. \tag{1.10}$$

Note that the length of this vector is a constant so that the total derivative must come from its rate of change of direction. The angular velocity of the coordinate system is equal to the angular velocity of the rod since the coordinate system is fixed in the rod. That is,

$$\vec{\omega}_1 = \dot{\theta}\vec{k}_1 \tag{1.11}$$

and the velocity of P relative to O is therefore[4],

$$\vec{v}_{P/O} = \frac{\mathrm{d}}{\mathrm{d}t}(\vec{p}_{P/O}) = \dot{\vec{p}}_{P/O} + \vec{\omega}_1 \times \vec{p}_{P/O} = (\dot{\ell}\vec{\imath}_1) + (\dot{\theta}\vec{k}_1) \times (\ell \vec{\imath}_1). \tag{1.12}$$

Since ℓ is constant, $\dot{\ell} = 0$, and the final result is,

$$\vec{v}_{P/O} = (\dot{\theta}\vec{k}_1) \times (\ell \vec{\imath}_1) = \ell\dot{\theta}\vec{\jmath}_1. \tag{1.13}$$

4 Readers are encouraged to review the rules for cross multiplication of vectors.