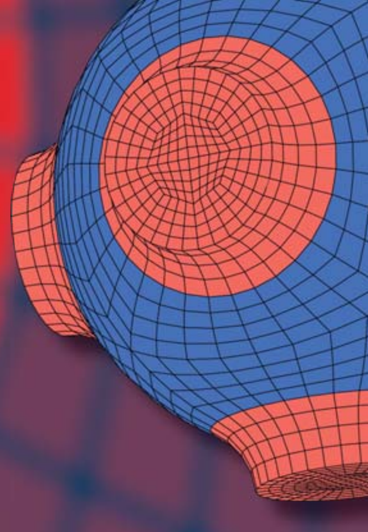

Advanced Structured Materials

Aleksander N. Guz
Viacheslav L. Bogdanov
Vladimir M. Nazarenko

Fracture of Materials Under Compression Along Cracks

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Preface

Fracture mechanics as an important branch of strength science, which studies the stage of exhaustion of the bearing capacity of materials and structural members, began to take shape after the publication of the seminal article by Alan Arnold Griffith [6] one hundred years ago. The intense development of fracture mechanics since the second half of the twentieth century until now has been due to the tremendous significance of the problems examined within its framework with a view to calculating the strength and ensuring the reliable operation of complex engineering facilities, machines and buildings. In terms of the problems covered, the number of studies conducted, its fundamental nature and the scope of practical application, this scientific area is now quite advanced, which is shown by numerous publications, including summarizing, reference and encyclopedic ones [3–5, 13, 15]. A vast majority of investigations in fracture mechanics have been based on the following key concepts and approaches: A. Griffith fundamental theory of brittle fracture; G. Irwin–E. Orovan concept of quasi-brittle fracture, which permitted A. Griffith theory to be generalized to inelastic structural materials; A. Griffith energy-based fracture criterion relying on the notion of specific surface energy or G. Irwin force-based criterion, which is equivalent to it; J. Eshelby–G.P. Cherepanov–J. Rice concept of invariant (independent of the integration contour) integral; A. Wells critical crack-opening criterion.

At the same time, a number of issues in fracture mechanics have not been studied exhaustively. Among them are, in particular, the investigations of new fracture mechanisms that cannot be described in the framework of the abovementioned five classical concepts and approaches, the formulation of new fracture criteria that would correspond to these mechanisms, as well as the investigation of specific classes of problems for materials and structural members as applied to the new fracture mechanisms under study. For example, the use of traditional approaches of classical fracture mechanics cannot ensure adequate investigations of the fracture of a body under compression along cracks situated in parallel to each other and the fracture of cracked materials when account is taken of initial (or residual, technological) stresses acting along crack surfaces. The non-classical characteristics of the problems mentioned stem from the fact that the solution of the corresponding problems of linear theory of elasticity on the stress–strain state of cracked materials shows that load components directed in parallel to the cracks are not involved in the expressions for stress intensity factors, *J*-integral and the values of crack-opening displacement. Hence, those load components cannot be taken into account in the Griffith–Irwin, Cherepanov–Rice classical criteria of crack opening or their generalizations, while in respect of the problems on the compression of bodies along preexisting cracks, the classical fracture criteria are not effective at all.

Here, it should be mentioned that the relevance of investigating the problems on material fracture under compression along parallel cracks, as well as those on the fracture of prestressed bodies containing cracks, is necessitated not only by the theoretical development of respective new approaches and models of fracture mechanics for their adequate description but also by practical needs in quite a number of branches of science and technology. For example, in the mechanics of composites and the mechanics of materials with coatings (heat insulating, anti-corrosive, etc.), quite common are the phenomena of delamination under the action of forces directed along near-surface defects. Such problems are also quite typical in geomechanics (a model of fissured and laminated rock massifs), biomechanics (in modeling blood vessels of live organisms), construction (in calculating various supports, evaluating the strength and durability of concrete structural members, etc.). Initial (residual) stresses and strains are nearly always present in actual structural materials and structural members as a result of the technological processes of their manufacturing. This is especially the characteristic of composite materials and polymers, machining, heat treatment and joining (in particular, by using welding technologies), which produces a significant effect on the processes of the fracture of cracked bodies.

As of today, a number of general approaches to investigating problems of the fracture mechanics of materials under the action of forces directed along cracks have been formed. In terms of the models of cracked bodies that are used, the rigor of mathematical formulations of the respective boundary problems and eigenvalue problems, as well as the methods of examining these problems, the approaches mentioned can be conditionally divided into two main groups.

As is known, in investigating the fracture of cracked bodies under compression along cracks the so-called *beam approach/beam approximation* is rather widely used [2, 14, 16]. This approach consists in conditional separation of a part of the material between two neighboring cracks in parallel planes or between a crack and the boundary surface of the body and using the applied theories of the mechanics of thin-walled systems with the involvement of flat-sections, Kirchhoff–Love, Timoshenko-type and other hypotheses to investigate the stability of the selected part of the material. The investigations for the selected parts of the material that are modeled with beams, plates or shells are carried out for the cases of boundary conditions of rigid fixing or hinged supporting on the "edges" of such one- or two-dimensional objects. Some authors also use similar approaches based on the applied theories of beams or plates to investigate problems of the fracture mechanics of bodies with initial (residual) stresses acting along cracks. At the same time, as shown in the present monograph and in several other publications (e.g., in [9, 10]), *beam approach* (*beam approximation*) is a purely approximate technique and leads to irreducible errors. That necessitates the evaluation of its applicability and the validity of its results depending on the value of the thin-wall parameter of the idealized thin-walled element chosen and the specified boundary conditions on its edges.

Radically different from the first general approach is the approach involving the three-dimensional linearized mechanics of deformable bodies to the investigation of the problems of the fracture mechanics of bodies under the action of forces directed along the cracks [7, 8, 11]. This *second general approach* is sufficiently rigorous due to the three-dimensional formulation of problems, which rules out the need for involving additional hypotheses, which are characteristic of beam approximation. Within this approach, the theoretical ultimate strength under body compression along the cracks it contains is determined as the critical stress value that corresponds to the local loss of equilibrium stability near cracks, which is obtained with the involvement of the approaches of the three-dimensional linearized theory of stability. The use of this approach in the investigations of problems on the fracture of bodies with initial (residual) stresses acting along cracks allows the qualitative and quantitative description (as opposed to classical approaches) of the main phenomenon related to the effect of force components acting along crack surfaces on the fracture parameters.

This monograph addresses problems of the fracture mechanics of materials loaded along cracks in them. Two non-classical fracture mechanisms are examined, namely the fracture of bodies compressed along cracks and the fracture of materials with initial (residual) stresses acting along cracks. In investigating these problems, the approaches developed in the framework of three-dimensional linearized mechanics of deformable bodies are employed. These approaches permit investigations to be carried out in a single form common for different material models (compressible and incompressible isotropic and anisotropic elastic and elastoplastic bodies) within both the theory of finite (large) initial strains and two variants of the theory of small initial strains. The fracture criteria that take into account the particularities of the loads acting along cracks have been formulated. For example, in examining the body compression along cracks, it is assumed that the onset (start) of the fracture process is determined by the local loss of equilibrium stability of the material surrounding cracks, while the numerical expressions of this criterion are the critical values of loading parameters (contractions, compressive stresses) that characterize the local instability of the material near cracks. For prestressed materials, approaches to the formulation of energy- and force-based fracture criteria are considered. They are analogs of the Griffith energy-based and Irwin force-based fracture criteria of materials free of initial stresses and turn into them when initial stresses tend to zero. The results of examining two- and three-dimensional problems for various configurations of isolated and interacting cracks in bodies are presented for isotropic and anisotropic materials. By relying on the results obtained in this work, we performed the critical evaluation of the precision and the applicability limits of the so-called *beam approximation*, which is rather widely used in the works by other authors investigating the fracture of bodies compressed along parallel cracks.

The monograph consists of five chapters; their contents are briefly presented below.

Chapter 1 provides general information regarding the presently available concepts of describing and analyzing the phenomena of the fracture of bodies loaded along preexisting cracks as non-classical problems of fracture mechanics. The general concept according to which the onset (start) of the process of material fracture under compression along cracks is determined by the local loss of stability of the equilibrium of the material surrounding the cracks has been corroborated. Two basically different approaches to investigating problems on the fracture of bodies compressed along cracks (the approximate approach within applied theories of thin-walled systems and the rigorous approach in the framework of three-dimensional linearized theory of the stability of deformable bodies) are described. Three basic problems that are characteristic of the subjects in question have been formulated. A brief review of the results obtained within two above-mentioned approaches is given, and the solutions of the respective problems are analyzed in terms of their reliability and compliance with the precision usually accepted in the mechanics of deformable solids. Also presented is the general approach to investigating problems on the fracture of materials with initial (residual) stresses acting along the cracks located in the material; it has been developed with the involvement of the relationships of the three-dimensional linearized mechanics of deformable bodies. Summarized results of the investigation of respective plane and spatial problems of the fracture mechanics of prestressed bodies are provided.

Chapter 2 gives an account of the approaches to constructing the fracture mechanics of materials under the action of forces directed along cracks. General principles of the theory being developed are demonstrated, and principal relationships are presented as applied to the models of solid bodies describing the deformation of nonlinearly elastic and elastoplastic materials as well as composite media. In the present monograph, the examination of the problems of the fracture mechanics of materials under the action of forces directed in parallel to the surfaces of cracks location relies on the use of the relationships of the three-dimensional linearized mechanics of deformable bodies. Therefore, it provides a brief review of the principal approaches to the linearization of the relationships of nonlinear mechanics of deformable bodies under finite and small strains and presents linearized geometric relations, expressions for stress and strain tensors, equilibrium equations and boundary conditions. Also considered are linearized constitutive stress–strain relationships (equations of state) for compressible and incompressible isotropic highly elastic and elastoplastic bodies, as well as anisotropic bodies, which model composite media in the continual approach. The general statement of the problems of fracture mechanics of materials under the action of forces directed along cracks has been formulated in the framework of the linearized theory and representations of the general solutions of two- and three-dimensional linearized problems for uniform initial states have been obtained. Within the three-dimensional linearized mechanics of deformable bodies, the unified approach to investigating problems of the fracture mechanics of cracked materials compressed along cracks and problems on the fracture of prestressed bodies has been validated [1, 12]. Processes of the fracture of bodies compressed along parallel cracks have been analyzed, and approaches have been proposed to formulating the criteria of materials fracture when the abovementioned non-classic fracture mechanisms are realized, namely the fracture of bodies compressed by forces directed in parallel to the planes of cracks location and brittle fracture of bodies with initial (residual) stresses acting along the cracks located in the bodies.

Chapter 3 is concerned with the examination of two-dimensional problems of the fracture mechanics of bodies compressed along cracks. It provides expressions for complex potentials of linearized plane problems and the representations of general solutions *via* harmonic potential functions. The employed procedure of investigating plane problems permits their solution to be constructed in a single form for different models of materials (nonlinearly elastic, elastoplastic, composite ones), while the specification of material model is only needed at the stage of numerical investigation of the resolving equations obtained in the general form. In the case of elastoplastic materials, the generalized concept of continuous loading is additionally engaged, which allows the changes of unloading zones during stability loss to be neglected. Problems on isolated cracks and arrays of coplanar (located in the same plane) cracks are investigated in this chapter. Also examined are problems for the most typical geometric configurations of crack arrays in the bodies in terms of assessing the effect of cracks interactions among themselves and with the boundary surfaces of bodies on the fracture parameters. Consistent patterns of the influence of the geometric parameters of problems and the physico-mechanical characteristics of materials on the fracture parameters in compression have been revealed. Specifically, it has been shown that the allowance for the mutual effect of the cracks located in neighboring parallel planes or the mutual influence of a near-surface crack and the boundary surface of the body could lead to a reduction in the value of compressive strength by an order of magnitude or more as compared to the corresponding compressive strength of the body with a single isolated crack.

Chapter 4 addresses three-dimensional problems of the fracture mechanics of materials loaded along the cracks located in the same plane or in parallel planes. For the uniform initial stress–strain state, representations of the general solutions of spatial problems have been obtained in terms of potential harmonic functions, which allow the problems to be investigated by using integral transformation methods. The general solutions have been constructed in a single general form for compressible and incompressible bodies for the theory of large (finite) initial strains and for two variants of the theory of small initial strains. The focus is on the investigation of spatial problems on the compression of bodies with internal cracks by forces directed in parallel to the planes of cracks location. Here, examined are problems on the biaxial uniform compression of bodies along an array of cracks located in the same plane (in particular, for a single penny-shaped internal crack), two parallel coaxial cracks, and a periodic array of parallel coaxial cracks. For various models of materials (highly elastic, elastoplastic, composite ones), the values of critical compression parameters that correspond to the local loss of equilibrium stability of the material near cracks have been obtained. In the case of elastoplastic materials, as in the case of plane problems, the generalized concept of continuous loading is additionally engaged, which permits the changes in the unloading zones in the process of stability loss to be neglected. The continual approach is used in examining composites; the composite is modeled by a homogeneous transversely isotropic body with averaged values of mechanical parameters. The dependence of the critical parameters mentioned on the physico-mechanical characteristics of the materials and the problem geometry has been analyzed. In addition, as exemplified by the problems on the bodies containing arrays of parallel coaxial cracks, the employing of the unified approach in investigating spatial problems of the fracture mechanics of prestressed bodies and problems on the fracture of cracked materials compressed along cracks are considered. The influence of initial stresses and the geometric parameters of the problems on stress intensity factors has been analyzed for highly elastic bodies and composites with elastic components.

In Chap. 5, the applicability limits of beam approximation are analyzed with the reliance on the results of solving problems on the compression of bodies along interacting cracks in the rigorous linearized formulation. The approach is used that ensures the determination of the critical compressive contraction values and the theoretical ultimate strength for unbounded bodies compressed along flat cracks in closely spaced parallel planes and in semi-bounded bodies compressed along subsurface cracks that are close to the boundary surface. This approach allows actual passage to the limit to be determined when the separation distance between parallel coaxial cracks or the distance between a near-surface crack and the body boundary tends to zero. Due to that, the applicability limits of beam approximation for such problems have been found. Besides, for various design diagrams, boundary conditions have been determined on the edges of respective idealized two-dimensional objects that are as if separated off by cracks in the framework of beam approximation. Such boundary conditions, if specified, provide results that are the closest to those obtained with the rigorous three-dimensional approach.

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Contents

Chapter 1
Preliminary Consideration: Background, Approaches and Their Analysis

Abstract This chapter presents general information on the present-day concepts of the description and analysis of fracture phenomena in the bodies loaded along preexisting cracks as the non-classical problems of fracture mechanics. In particular, two essentially different approaches to investigating problems on the fracture of bodies compressed along cracks are described, and three basic problems typical of the topics mentioned are formulated. A brief review of the results obtained in the framework of those two approaches is given, and the solutions of respective problems are analyzed in terms of their validity and compliance with the precision usually accepted in the mechanics of a deformable solid. Besides, the general approach to the investigation of problems on the fracture of materials with initial (residual) stresses acting along the cracks present in the material is described, and the results of investigating the respective plane and spatial problems are considered in brief.

1.1 Fracture Phenomena Analyzed and Basic Design Diagrams

This section corroborates the division of fracture mechanics problems into classical and non-classical ones, provides examples of non-classical problems and focuses on the two of them that are of considerable theoretical and practical interest. These are the problems of the fracture mechanics of bodies compressed along cracks, and the fracture mechanics of materials with initial (residual) stresses acting in parallel to the surfaces of crack location. The basic design diagrams of such problems are presented for the cases of homogeneous materials and laminated composites.

1.1.1 Classical and Non-classical Problems of Fracture Mechanics

Since the second half of the twentieth century to the present day, fracture mechanics has been one of the most actively developed areas of the mechanics of deformable

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solid body. On the one hand, this has been due to the complex nature of the fracture process, which requires the advancement of new, increasingly refined, models and approaches to its analysis at the macro-, meso- and microlevels, as well as the formulation of the fracture criteria that would be adequate to this phenomenon. On the other hand, the results obtained in the framework of fracture mechanics are of crucial importance for evaluating and predicting the strength, durability and remaining life of vital structures, buildings, machines and mechanisms.

The origin of fracture mechanics as an individual important area in natural sciences and engineering is righteously associated with the publication of the fundamental work [87] nearly 100 years ago. In this publication, A. A. Griffith related a significant reduction in the experimentally determined strength of solid bodies as compared with their theoretical strength to the presence of pointed defects—cracks—in them. He suggested an approach to the mathematical investigation of the problems on cracked bodies, and proceeding from the principle of the minimum of potential energy of a deformable body with a crack, proposed an energy-based criterion of brittle fracture.

The next important developments in the advancement of fracture mechanics were:

1. the concept of quasi-brittle fracture [226, 268], which allowed the theory of Griffith [87, 88] to be generalized to inelastic structural materials without changing the analytical apparatus;
2. the force-based approach [224, 225], which is entirely equivalent to the energy-based approach but permits the computational difficulties inherent in A. A. Griffith approach to be reduced considerably;
3. the concept of invariant (independent of integration path) J-integral [63, 81, 279];
4. the criterion of critical crack opening displacement [291, 292].

Griffith energy-based approach and Irwin force-based approach, which is equivalent to it, are the basis of *linear elastic fracture mechanics, LEFM* (or *brittle fracture mechanics*), widely used for analyzing the brittle (quasi-brittle) fracture of many actual materials and engineering structures. Eshelby–Cherepanov–Rice approach and the model of a crack with a cohesive zone [76, 247, 269] are sufficiently effective in analyzing the fracture of cracked bodies in the cases when considerable plastic deformations develop in the vicinities of crack tips (which is observed in most of the actual structural materials, in particular, metals and alloys). The criterion of critical crack opening displacement is equally applicable to the analysis of brittle (quasi-brittle) and plastic fracture. Later, a number of generalizations of these approaches, models and fracture criteria to the cases of complex stress states and bodies of intricate shapes, non-stationary and cyclic loadings, the effects of corrosive environments, the influence of thermal, electric and electromagnetic fields, for viscoelastic, composite and other models of structural materials were proposed. Quite detailed reviews of crack models and fracture criteria are given, e.g., in [18, 60, 63, 66, 80, 237, 248, 274, 286].

Note here that the abovementioned criteria and approaches assume the fulfillment of several conditions, the basic ones being as follows:

1. the forces emerging in the vicinities of cracks are tensile or shear forces, *the action of compressive forces being ruled out*;

2. in the process of a cracked body deformation no sudden changes appear in the body configuration (in particular, in the process of a cracked body deformation, fracture is not preceded by stability loss);
3. in the process of a cracked body deformation no abrupt changes in deformation characteristics appear before fracture (e.g., the change of boundary conditions does not occur in the process of deformation).

Here, the first of these conditions is crucial, since in the case of compression along cracks all of the abovementioned concepts and approaches of fracture mechanics are not applicable. In the case when the second condition is not satisfied, these concepts and approaches work in theory, but previously it is necessary to analyze the stress-strain state of the body, taking into account the dramatic change in its configuration in the process of deformation (at present, in the majority of studies in fracture mechanics this analysis is not performed). In the situation when a third of these conditions is not satisfied, the above concepts and approaches used in fracture mechanics can still be valid. Yet again, the analysis of the stress-strain state which would allow for the dramatic change in the nature of deformation before fracture (e.g., an investigation taking into account the change of the boundary conditions in the process of body deformation) is necessary. However, at present this is not carried out in the great majority of studies in fracture mechanics.

Taking into account the potentials of those criteria and approaches (it is assumed that in the vicinity of the crack tip tensile or shear stresses appear and this is a characteristic and rather common situation), the construction of the fracture mechanics of bodies compressed along cracks, which is the focus of this monograph, is a separate problem of fracture mechanics that requires the development of relevant concepts and approaches. This situation takes place in homogeneous materials under compression along the cracks located in a single plane and under compression along the cracks located in parallel planes or surfaces. Such phenomena also appear in the laminated composite materials compressed along layers, since, as a rule, a rather large number of cracks or other adhesion disturbances emerge in their interfaces. Besides, in the framework of A. Griffith theory and the four abovementioned concepts and approaches of fracture mechanics, it is not possible to analyze adequately the problem of the brittle fracture of cracked materials, with the account being taken of the action of initial (residual) stresses along cracks. This results from the fact that the singular part of the well-known Inglis–Muskhelishvili solution for a cracked material, involving the relationships of the classical linear elasticity theory, does not include stress components acting along the crack.

It should be noted that at present a great majority of studies rely on the abovementioned five main concepts of and approaches to the formulation of fracture criteria, viz.: the fundamental A. Griffith theory of brittle fracture; G. Irwin–E. Orovan concept of quasi-brittle fracture; A. Griffith energy-based fracture criterion or the equivalent G. Irwin force-based criterion; J. Eshelby–G.P. Cherepanov–J. Rice concept of invariant integral; the cohesive zone model and the criterion of critical crack opening displacement. When these concepts and approaches are used, the above basic conditions 1–3 are assumed to be satisfied. In this connection, the problems analyzed in the investigations mentioned can be conditionally referred to as the ***classical problems of fracture mechanics***.

At present, the following research areas within the classical problems of fracture mechanics can be considered as the major ones:

1. The determination of stress intensity factors, J-integrals and crack opening displacements for the bodies of intricate shapes containing cracks when they are exposed to various forces, thermal and electromagnetic factors. Here, diversified methods—analytical, numerical, experimental, and those combining theory and experiment—are employed to obtain the results. The outcomes of these studies, in conjunction with the fracture criteria mentioned, provide the necessary information on the failure of materials and structural members in those cases when these criteria are applicable.
2. Experimental investigations of complex cases of the fracture of materials and structural members, which usually produce descriptive results without their proper analysis and any attempts to formulate new fracture criteria that would be relevant to the phenomena under consideration.

At the same time, a number of problems in this area need to be examined more thoroughly. In particular, they concern new fracture mechanisms that *cannot be described* in the framework of the abovementioned five concepts and approaches, the formulation of new fracture criteria, as well as the investigation of certain classes of problems for materials and structural members in respect of the new fracture mechanisms under study. Those problems can be conditionally referred to as ***non-classical problems of fracture mechanics***.

Note that the subdivision of the problems of fracture mechanics into classical and non-classical ones was first proposed by A. N. Guz in the introduction to the multi-volume monograph [168]. Apparently, it is rather relative and not infrequently ambiguous. Yet, the classification proposed allows a comparatively distinct assessment of research trends in the area of fracture mechanics both from the viewpoint of their novelty and the relevance of the models and approaches employed there to the new fracture mechanisms that are studied. Relying on that classification, new research results concerning non-classical fracture problems were generalized and analyzed in the following works [138, 144, 149–151, 167, 170, 221].

The examples of the problems of fracture mechanics that cannot be investigated within the approaches classical in this sense are as follows: (1) fracture in composite materials compressed along reinforcing components when the start of fracture is determined by the loss of equilibrium stability in the internal structure of the composite; (2) the model of short fibers in the stability theory and in the fracture mechanics of composite materials under compression, describing experimentally found phenomena of stability loss in the composite structure with the forms of stability loss that are not periodic along the fiber axis and that are characteristic of short fibers in a matrix; (3) fracture as the crumpling of edges in the compression of composite materials due to the local loss of near-surface stability close to loaded composite edges; (4) the brittle fracture of cracked materials, with the account of the action of initial (residual) stresses; (5) brittle fracture as “fraying” in stretching or compression of composite materials with curvatures in their internal structures; (6) brittle and plastic fracture of the bodies compressed along the cracks located in parallel planes, when the onset (start) of the fracture process is determined by the compressive loads that achieve the values corresponding to the local loss of the stability of material equilibrium in the vicinities of cracks; (7) brittle fracture of cracked materials under the action of dynamic loads, with the account of contact interactions of crack faces; (8) fracture of cracked thin-walled bodies under tension in the case when prior stability loss is realized. These problems were investigated by the Department of Continuum Dynamics and Stability of S.P. Timoshenko Institute of Mechanics of the National Academy of Sciences of Ukraine, and detailed reviews of the results are presented in [149–151]. The non-classical problems of fracture mechanics also include problems of the sustained fracture of viscoelastic bodies in which subcritical crack growth occurs. Studies of those were carried out by the Department of Fracture Mechanics of S. P. Timoshenko Institute of Mechanics of the National Academy of Sciences of Ukraine [233–235]; they relied on the generalized model of a crack with a cohesive zone [76, 269].

The number of results obtained in the investigations of the non-classical problems of fracture mechanics is much smaller than that obtained with reliance on classical approaches, the exclusive employment of inherently approximate design diagrams and models being rather characteristic of such studies. In particular, approximate design diagrams and models are widely used to investigate the fracture of homogeneous and composite materials compressed along cracks. Note that the employment of approximate design diagrams and models leads to considerable quantitative errors and in many cases even causes qualitative deviations from experimental data. Hence, it is rather difficult to carry out a reliable analysis of the non-classical problems and mechanisms of fracture mechanics. Therefore, of considerable importance in it are the results of investigating non-classical problems and fracture mechanisms, obtained with reliance on sufficiently exact formulations of problems, mathematical models and research methods. Researchers analyzed the abovementioned eight non-classical problems of fracture mechanics in the following works [5, 42, 43, 115, 119, 122, 123, 134, 137–139, 142–145, 148, 152, 168, 170, 172, 173, 202, 203, 218, 219, 221, 222] with the involvement of the most rigorous and exact formulations, in particular, within the three-dimensional linearized theory of the stability of deformable bodies. They were presented, e.g., in monographs [91–94, 119, 137].

Below, Sect. 1.1.2 as well as Sects. 1.2, 1.3, describe basic design diagrams, investigation approaches, and provide brief reviews of the works on two theoretically and practically important non-classical problems of fracture mechanics presented in this monograph. Those are the problems of the fracture mechanics of bodies compressed along cracks and the fracture mechanics of materials with the initial (residual) stresses acting in parallel to the surfaces of cracks location.

1.1.2 Basic Design Diagrams for Problems on the Fracture of Bodies Loaded Along Cracks

Figure 1.1 provides an example of the diagram of the problem on a body with cracks located in parallel planes, which is compressed along the planes of cracks location. Figure 1.2 shows the diagram of the problem on a body with initial (tensile or compressive) stresses σ_{11}^{0}, which contains parallel cracks (in the figures the cracks are shown in bold lines). It should be pointed out that although in respect of the investigated phenomena the problem on the materials compressed along the cracks they contain (Fig. 1.1) and the problem on the fracture of prestressed bodies under the action of the initial stresses directed along cracks (Fig. 1.2) are different, yet there is an essential common element in their formulations. This is the presence of

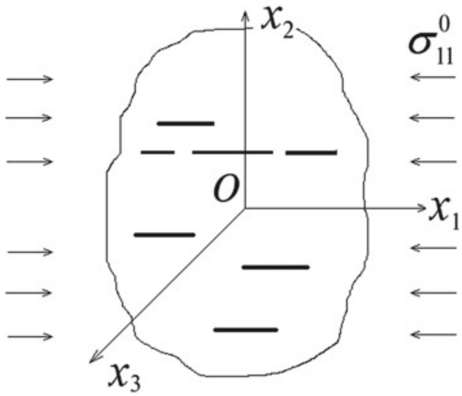

Fig. 1.1 Cracked body under compression along cracks

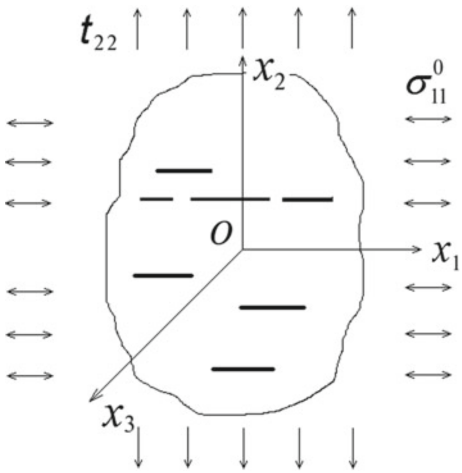

Fig. 1.2 Cracked body with initial stresses

load components σ_{11}^{0} parallel to cracks, whose effects, actually, cannot be taken into account by the methods of classical fracture mechanics.

The importance of examining the ***phenomenon of the fracture of compressed cracked bodies*** is determined by the need to solve practical and engineering problems in a number of areas of science and technology. E.g., in the mechanics of composites (especially laminated ones) and the mechanics of materials with coatings (heat insulating, anticorrosive etc.), the phenomena of near-surface buckling are very common in the vicinities of delaminations under the action of compressive stresses of various origins. Numerous engineering problems related to the analysis of products with design defects are reduced to the force schematic of compression along crack-like defects. Problems of this kind are quite typical in geomechanics when modeling the action of tectonic forces in mountain relief (the model of fissured laminated massif), in construction (while calculating various supports), etc.

The non-classical nature of fracture mechanics problems on body compression along cracks is related to the situation that under uniform compression strictly along the planes of cracks location (Fig. 1.1) in isotropic and orthotropic materials (for orthotropic bodies it is assumed that cracks are located in the planes parallel to one of the planes of symmetry of material properties), a uniform stress-strain state emerges in arbitrary elasto-viscous-plastic models. This results in the absence of singularities at crack tips in respective solutions. Hence, under such loading scheme, stress intensity factors, J-integral and crack tip opening displacement are equal to zero, therefore the approaches based on the classic Griffith–Irvin theory, Cherepanov–Rice J-integral and the criterion of critical crack opening displacements or their generalizations are not applicable here.

In the problem considered, the most likely mechanism of the onset (start) of the fracture process, by analogy with the problem on the compression of structural members along the symmetry axes, is the local loss of equilibrium stability of the material surrounding cracks. A more detailed argumentation for the applicability of this fracture mechanism in examining the phenomenon of body fracture under compression along the planes of cracks location is given in Sect. 1.2.

Initial (or residual, internal, technological) stresses appear in practice due to the inhomogeneity of linear or spatial deformations in the adjacent areas of the material. Such stresses are nearly always present in actual structural materials and members due to the technological processes of their production. That is especially characteristic of composite materials [77, 283] and polymers [65]), joining (in particular, with the use of welding technologies), mechanical and thermal surface treatment, exploitation, as they produce significant impacts on fracture processes in the bodies containing cracks. Such problems are also quite typical in biomechanics (in modeling blood vessels and live biological tissues [64, 278]), in geophysics, seismology and other basic and applied scientific areas.

Of special interest in this respect are the problems where initial (residual) stresses in prestressed materials act along the surfaces of cracks location [in Fig. 1.2 σ_{11}^{0} are initial stresses acting along the crack, while t_{22} are additional (acting or operational) stresses (here normal stresses are taken as an example)]. *The non-classical nature of the problems on brittle fracture of prestressed materials*, when initial (residual) stresses are directed in parallel to the surfaces of cracks location, is due to the situation that the solution of respective problems of linear elasticity theory (see, e.g., [63, 238]) implies that the initial stresses directed along cracks are not involved in the expressions for stress intensity factors, J-integral and crack opening displacements. Therefore, the stress components directed along cracks cannot be taken into account in Griffith–Irwin and Cherepanov–Rice classical models and fracture criteria, critical crack opening displacements or their generalizations. Hence, fracture characteristics calculated with reliance on those classical models *do not depend* on the stress components mentioned [1, 78, 138]. Note that the effect of initial stresses can be taken into account indirectly, e.g., in Griffith–Irwin energy-based criterion this can be done via the value of specific surface energy γ [87]. However, such an approach is difficult to implement, since in that case the value of γ must depend both on the values of initial stresses and on the types of the problems examined. At the same time, this is contrary to the data of numerous experimental studies, which reveal the effect of the load component directed along cracks on fracture parameters under both static and cyclic loads (see, e.g., [3, 21, 236, 245, 246]).

This discrepancy between theoretical analysis and experimental data results, evidently, from the fact that classical fracture mechanics relies on the relationships of the classical linear theory of elasticity and, in fact, does not take into account non-linear effects in a body containing cracks. Thus, to investigate fracture mechanics problems for materials with initial (residual) stresses acting along cracks, the development and application of other approaches is needed. Brief information on such approaches will be provided in Sect. 1.2.

Hereinafter we consider the *basic design diagrams* of the investigated problems in respect of *homogeneous isotropic* and *orthotropic materials* (in the case of orthotropic materials we assume that the axes of symmetry of material properties coincide with coordinate axes). It is evident that there can be many configurations of cracks, even if their fixed number is small (Figs. 1.1 and 1.2 show the general cases of flat cracks location in parallel planes). Those arrays can vary with differences in the distances between the planes of cracks location, with possible different sizes and shapes of crack contours, as well as with the location of the projections of the areas occupied by cracks.

From numerous design diagrams of the fracture mechanics of materials under the action of forces directed along cracks, one can conditionally single out four diagrams with the most characteristic (reference) geometries of defects location in a body (Fig. 1.3a–d). These design diagrams permit the allegedly "pure" choice and allow the investigation of the main patterns in the interaction (mutual influence) of the cracks themselves and that of the cracks and body boundaries under the action of forces directed along the planes of cracks location. To make the presentation more compact, design diagrams in Fig. 1.3a–d are only given for problems on body compression along the planes of cracks location (cracks are shown in bold lines); there are also similar design diagrams for problems on bodies with the initial (residual) stresses acting in parallel to the cracks.

In the case of three-dimensional problems, cracks that are circular (penny-shaped) in plan are considered, since such configuration of cracks is the most common in

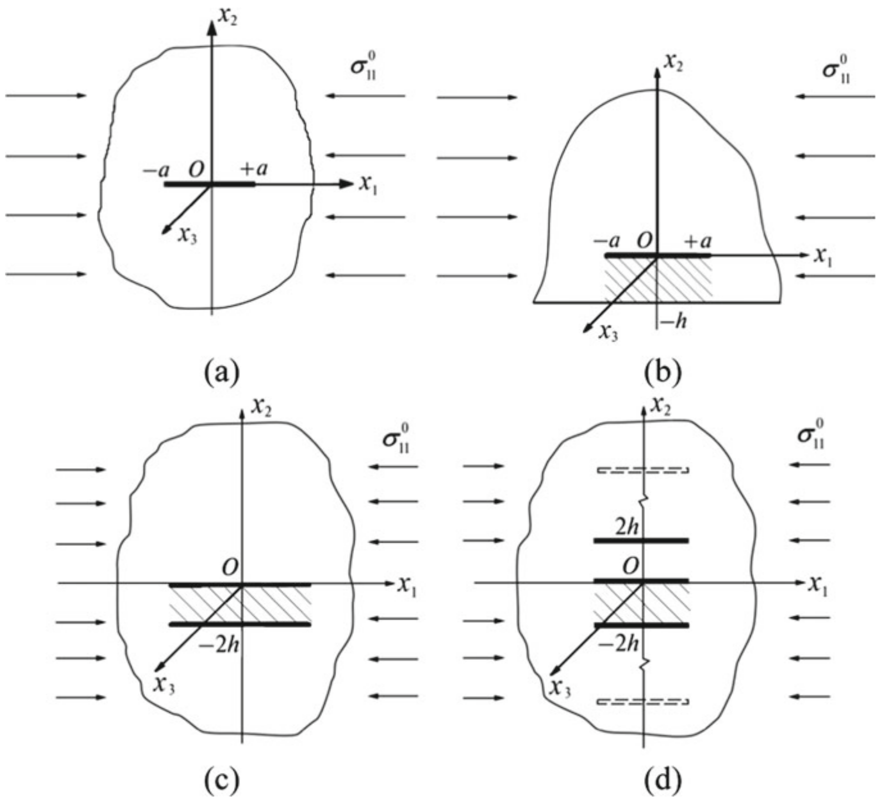

Fig. 1.3 Typical design diagrams for the problems of compression of isotropic bodies along cracks: an unbounded body with an isolated crack (**a**), a body with a near-surface crack (**b**), an unbounded body with two parallel cracks (**c**) and an unbounded body with a periodic array of parallel cracks (**d**)

structural materials [238] (in some cases elliptical cracks are examined). In the case of two-dimensional problems (for the state of plane strain) we analyze cracks that are infinite in one direction (in the direction of axis Ox_3) and are of finite length in the direction of axis Ox_1. Also note that for the arrays of parallel cracks we analyze the cases when the projections of the areas occupied by the cracks coincide, i.e., in two-dimensional problems all cracks are of the same length and are not displaced relative to one another in the direction of initial forces action, while in three-dimensional problems we consider coaxial cracks of the same radius.

It should be pointed out that Fig. 1.3a, c, d presents design diagrams for investigating the fracture of a material with cracks located inside it, which is shown by wavy lines marking the volume chosen inside the material. In this connection, the cracks presented in Fig. 1.3a, c, d can be referred to as internal cracks. Alongside internal cracks, there are also cracks near the surface of the material, which during fracture interact with the surface (Fig. 1.3b); due to that, these cracks can be named as near-surface cracks.

Figure 1.3a shows the simplest case of a single isolated crack of the length $2a$ (in the case of a plane problem) or of the same diameter (in the case of a spatial problem), which is located in the plane x_1Ox_3 and compressed along that plane by forces $\sigma_{11}^0 = const$. (In the case of a spatial problem, in this and other design diagrams, the presence of stress tensor component $\sigma_{22}^0 = const$ is also assumed; generally $\sigma_{11}^0 \neq \sigma_{22}^0$). This situation is the limit case for the chosen basic special cases of the geometries of interacting cracks location when the distance between the cracks or between a crack and the body boundary tends to infinity. Thus, the findings for a single isolated crack can be considered as test results for large distances between the cracks and for large distances between the cracks and the body boundary.

Figure 1.3b shows design diagrams for the problem on body compression along a single near-surface crack located in the plane parallel to the body surface (in Fig. 1.3b the material occupies the upper half-space and the surface of the material is marked with a straight line) when it is compressed along the crack and along the material surface. It should be noted that in the design diagrams in Fig. 1.3b the crack of the length (diameter) $2a$ is located at a sufficiently small distance h from the body surface. Due to that, the mutual influence (interaction) of the near-surface crack and the body surface should be taken into account when analyzing this fracture phenomenon. In Fig. 1.3b the part of the material located between the crack and the body boundary [with geometric parameters ($2a$, h)] is shaded.

Figure 1.3c presents the simplest case of the interaction of two identical cracks of the length (diameter) $2a$ located in two neighboring parallel planes that are at the distance $2h$ from each other. Here the part of the material that is between cracks [with geometric parameters ($2a$, $2h$)] is shaded. It should be noted that Fig. 1.3c corresponds to the design diagram that only takes into account the interaction of two neighboring cracks; hence, from the viewpoint of the design diagrams shown in Fig. 1.1, according to Fig. 1.3c, for a larger number of cracks there is no mutual influence of different "crack pairs".

Figure 1.3d shows the case of a large number of interacting cracks located in parallel planes. This interaction of cracks is modeled by the interaction of a periodic array (along axis Ox_2) of identical cracks. In Fig. 1.3d, the part of the material located between the neighboring cracks [with geometric parameters ($2a$, $2h$)] is also shaded similarly to Fig. 1.3c.

Note that the cases of two parallel cracks and a periodic array of parallel cracks permit an assessment of the effect of the interaction between cracks on fracture parameters and are limit cases for problems on the fracture of materials with an arbitrary finite number of parallel cracks. Here it should also be noted that, mathematically, the problem on a body containing a periodic array of parallel cracks is reduced to the problem on a layer (in the plane case—on a strip) with a single crack parallel to the layer faces (the strip boundaries). Therefore, the procedure of its solution can be employed in the investigation of problems on a crack in a layer (strip) with free or reinforced faces (boundaries) under compression along the plane of crack location and along the faces of the layer (strip).

It should be pointed out that, to shorten the discussion, a single crack is shown in each of the parallel planes in Fig. 1.3a–d. Given the general case of flat crack location shown in Fig. 1.1, the generalization of design diagrams presented in Fig. 1.3a–d seems evident. Besides, the generalization of the design diagrams shown in Fig. 1.3b for a single near-surface crack to the case of several near-surface cracks located in parallel planes, taking into account the situations presented in Fig. 1.3c, d, seems evident as well.

We will confine ourselves to the above information when discussing the analyzed phenomena in the fracture mechanics of homogeneous isotropic and orthotropic materials under the action of forces along the arrays of flat cracks located in parallel planes.

Hereinafter we consider *laminated* (*layered*) *composite materials* made up of the layers of fixed thickness. The materials of the layers are assumed isotropic or orthotropic. In the case of an orthotropic material of any layer, we assume that one of the planes of symmetry of material properties is parallel to the plane of layers interface and analyze fracture phenomena in such laminated composite materials where flat cracks are located in interfaces under compression along cracks and along interfaces. Hence, in the situation considered, as in that mentioned above in this section and for homogeneous materials, compression along an array of flat cracks located in parallel planes is investigated. In addition, for composites (in contrast to homogeneous materials) we assume that compression "at infinity" is realized in the manner ensuring the same contraction along all layers.

In all Fig. 1.4a–d, interface cracks are shown in bold lines, interfaces are presented in thin lines, while the layers containing different materials are marked by broad slanting shading. In Fig. 1.4b–d, the same as in Fig. 1.3b–d, a rather dense slanting shading marks the part of the material between the cracks in the neighboring parallel planes. Figure 1.4a–d, the same as Fig. 1.3a–d, presents the basic design diagrams for investigating typical problems in respect of the analyzed phenomena in laminated composites.

It is appropriate to note that all the information and considerations presented following Fig. 1.3 for homogeneous materials are equally applicable to design diagrams in Fig. 1.4 in respect of laminated composite materials, otherwise those considerations should be somewhat specified. Due to that, this information and these considerations will not be mentioned specifically for laminated composites with interfacial cracks.

We will confine ourselves to the above information in the discussion of the phenomena analyzed in the fracture mechanics of laminated composite materials made up of the layers of constant thickness, those with flat interfacial cracks, under compression along the cracks and, naturally, along the interfaces of laminated composite materials.

Earlier in this section, we discussed the analyzed phenomena in the fracture mechanics of homogeneous bodies and laminated composites. Related phenomena also take place in the fracture mechanics of structural members (beams, rods, plates, shells etc.) made of such materials, when cracks are located in the planes parallel to the face surfaces, in the case of compression along cracks. Figure 1.5a, b presents, as

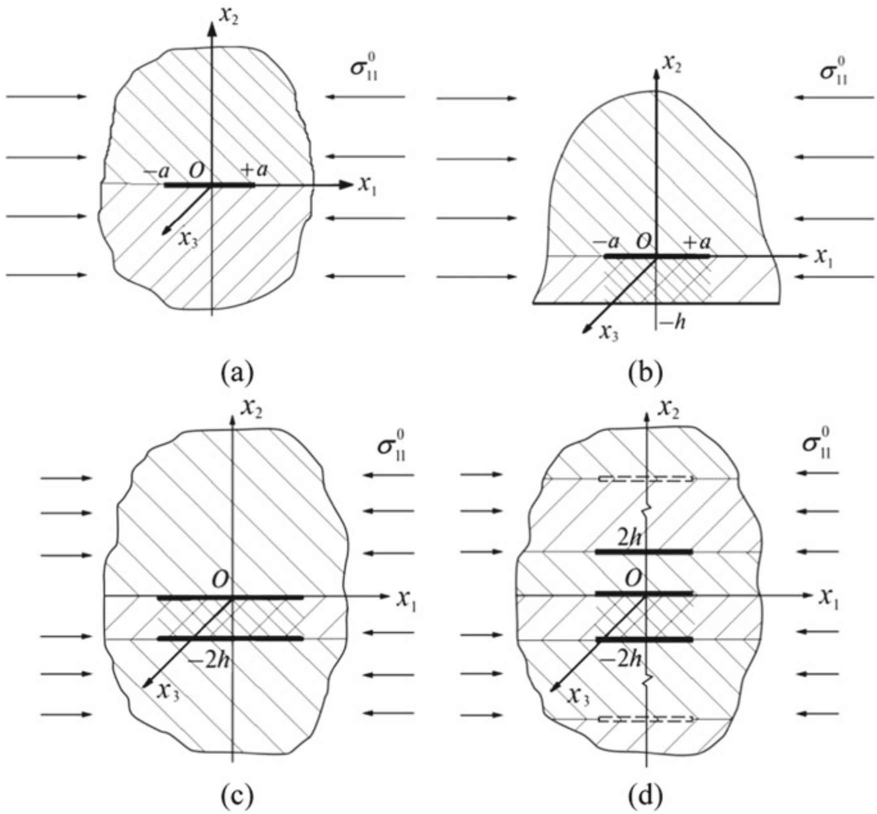

Fig. 1.4 Typical design diagrams for the problems on compression of laminated (layered) composites along cracks: an unbounded composite with an interfacial crack (**a**), a composite with a near-surface interfacial crack (**b**), an unbounded composite with two parallel interfacial cracks (**c**) and an unbounded composite with a periodic array of parallel interfacial cracks (**d**)

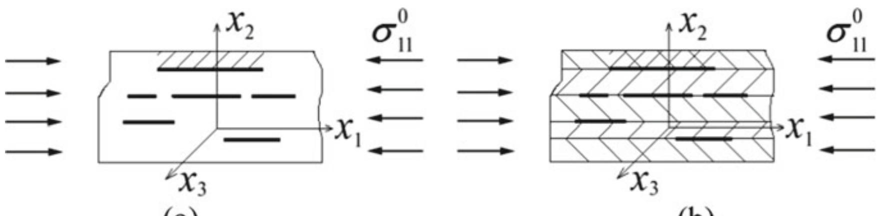

Fig. 1.5 Plates under compression along cracks: an isotropic plate containing parallel cracks (**a**) and a plate of a laminated composite containing parallel interfacial cracks (**b**)

examples, design diagrams for the problems on compression of plates of fixed thickness (in Fig. 1.5a—those made of a homogeneous material, while in Fig. 1.5b—those produced of a laminated composite) by the forces directed in parallel to the planes of cracks location (they are shown in bold lines) and the face surfaces of the plates. Note that boundary conditions on plate edges are not specified on the design diagrams presented in Fig. 1.5a, b, therefore the edges are shown by wavy lines.

It should be noted that similar phenomena are also present in some other loading diagrams of cracked bodies, e.g., in compressed zones under bending loads or near internal layers of the material in thick-walled structural elements (cylindrical tubes, spherical vessels) under external pressure.

1.2 General Concepts of Investigating Problems of Fracture Mechanics of Materials Under the Action of Forces Directed Along Cracks

It should be pointed out that in every situation and every design diagram analyzed in Sect. 1.1.2, in all cases, under the action of compressive forces (or initial stresses) directed along the cracks present in the material, *a uniform stress-strain state* emerges *in different models of deformable isotropic and orthotropic materials* (*elastic, plastic, and viscoelastic media*). (In the case of orthotropic materials, it is assumed that cracks are located in the planes of material properties symmetry, and loads are thus directed along the symmetry axes of material properties). This uniform stress-strain state is characterized by such expressions for the components of stress tensor $\tilde{\sigma}^0$ and displacement vector $\mathbf{u^0}$:

$$\sigma_{11}^0 = const, \sigma_{33}^0 = const, \sigma_{22}^0 = 0, \quad \text{when } i \neq j; \tag{1.1}$$

$$u_m^0 = (\lambda_m - 1)x_m; \lambda_1 \neq \lambda_2 \neq \lambda_3; \lambda_m = const, \tag{1.2}$$

where λ_m are ratios of elongation (contraction) along the coordinate axes.

1.2.1 Formulation of the General Concept of and Approaches to Investigating the Problems of the Fracture Mechanics of Bodies Compressed Along Cracks

1.2.1.1 The General Concept Based on the Phenomenon of the Local Loss of Stability of a Part of the Material in the Crack Vicinity

Owing to the satisfaction of conditions (1.1), (1.2) in the investigation of problems on the compression of bodies along the cracks they contain, the appropriate exact solutions lack the singular part of the solution and, hence, the following conditions take place

$$K_{\mathrm{I}} = 0,\, K_{\mathrm{II}} = 0,\, K_{\mathrm{III}} = 0. \tag{1.3}$$

In (1.3) and further, K_{I}, K_{II}, K_{III} are Irwin stress intensity factors [224]. It should be noted that all fracture criteria in the fracture mechanics of materials, based on Griffith [87]–Irwin [224] approach, are determined by the values of stress intensity factors or by other quantities calculated in terms of stress intensity factors. Since in the fracture mechanics of materials and structural members compressed along the arrays of flat cracks located in parallel planes conditions (1.3) are valid, *Griffith–Irwin approach cannot be used for constructing the basis of the fracture mechanics under consideration* (*in the case analyzed, this approach is not effective*).

Given the above considerations, we arrive at the following conclusion: as regards the fracture mechanics of the bodies *compressed along the arrays of two-dimensional cracks located in parallel planes, in homogeneous* isotropic and orthotropic materials (where cracks are located in the planes of properties symmetry) and in the *bodies compressed along the layers in whose interfaces two-dimensional cracks are present*, in laminated composite materials (where layers are of constant thickness, and in orthotropic layers the planes of properties symmetry are parallel to interfaces), it is necessary to employ concepts and approaches different from those of Griffith–Irwin.

In the cases analyzed, a general concept for investigating the phenomena at issue was developed by analogy with the related situation in the mechanics of thin-walled structural members (rods, beams, plates and shells). E.g., in the mechanics of thin-walled structural members *compressed along the axes of symmetry* (of material properties and geometric shape), *the phenomenon of the loss of equilibrium stability emerges*. In the majority of cases, later this leads to the exhaustion of their carrying capacity. It should be pointed out that in the investigated cases of the fracture mechanics of compressed materials (Figs. 1.1, 1.3, 1.4 and 1.5) the compression along the axes of symmetry is also considered. The information and considerations presented above corroborate the general concept developed to investigate problems of the fracture mechanics of materials compressed along cracks, which can be formulated in the following way.

General concept. *In the situations examined* (Figs. 1.1, 1.3a–d, 1.4a–d, 1.5a, b), *the onset (start) of the process of material fracture is determined by the local loss of*