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Preface

A system is said to have a delay when the rate of variation in the system state depends on past states. Such a system is called a time-delay system. Delays appear frequently in real-world engineering systems. They are often a source of instability and poor performance, and greatly increase the difficulty of stability analysis and control design. So, many researchers in the field of control theory and engineering study the robust control of time-delay systems. The study of such systems has been very active for the last 20 years; and new developments, such as fixed model transformations based on the Newton-Leibnitz formula and parameterized model transformations, are continually appearing. Although these methods are a great improvement over previous ones, they still have their limitations.

We recently devised a method called the free-weighting-matrix (FWM) approach for the stability analysis and control synthesis of various classes of time-delay systems; and we obtained a series of not so conservative delay-dependent stability criteria and controller design methods. This book is based primarily on our recent research. It focuses on the stability analysis and robust control of various time-delay systems, and includes such topics as stability analysis, stabilization, control design, and filtering. The main method employed is the FWM approach. The effectiveness of this method and its advantages over other existing ones are proven theoretically and illustrated by means of various examples. The book will give readers an overview of the latest advances in this active research area and equip them with a state-of-the-art method for studying time-delay systems.

This book is a useful reference for control theorists and mathematicians working with time-delay systems, engineering designing controllers for plants or systems with delays, and for graduate students interested in robust control theory and/or its application to time-delay systems.

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Contents

Abbreviations

inf	infimum
lim	limit
max	maximum
min	minimum
sup	supremum

BRL	bounded real lemma
CCL	cone complementarity linearization
DOF	dynamic output feedback
FWM	free weighting matrix
ICCL	improved cone complementarity linearization
IFWM	improved free weighting matrix
LFT	linear fractional transaction
LMI	linear matrix inequality
MADB	maximum allowable delay bound
MATI	maximum allowable transfer interval
NCS	networked control system
NFDE	neutral functional differential equation
NLMI	nonlinear matrix inequality
RFDE	retarded functional differential equation
SOF	static output feedback

Symbols

$\mathbb{R}$, $\mathbb{R}^n$, $\mathbb{R}^{n\times m}$	set of real numbers, set of n-dimensional real vectors, and set of $n \times m$ real matrices
$\mathbb{C}$, $\mathbb{C}^n$, $\mathbb{C}^{n\times m}$	set of complex numbers, set of n component complex vectors, and set of $n \times m$ complex matrices
$\bar{\mathbb{R}}_+$	set of non-negative real numbers
$\bar{\mathbb{Z}}_+$	set of non-negative integers
$\mathrm{Re}(s)$	real part of $s \in \mathbb{C}$

A^{T}	transpose of matrix A
A^{-1}	inverse of matrix A
$A^{-\mathrm{T}}$	shorthand for $(A^{-1})^{\mathrm{T}}$
I_n	$n \times n$ identity matrix (the subscript is omitted if no confusion will occur)
$\mathrm{diag}\{A_1, \cdots, A_n\}$	diagonal matrix with A_i as its ith diagonal element
$\begin{bmatrix} X & Y \\ * & Z \end{bmatrix}$	symmetric matrix $\begin{bmatrix} X & Y \\ Y^{\mathrm{T}} & Z \end{bmatrix}$
$A > 0$ (< 0)	symmetric positive (negative) definite matrix
$A \geqslant 0$ $(\leqslant 0)$	symmetric positive (negative) semi-definite matrix
$\det(A)$	determinant of matrix A
$\mathrm{Tr}\{A\}$	trace of matrix A
$\lambda(A)$	eigenvalue of matrix A
$\lambda_{\max}(A)$	largest eigenvalue of matrix A
$\lambda_{\min}(A)$	smallest eigenvalue of matrix A

$\sigma_{\max}(A)$	largest singular value of matrix A
$L_2[0,+\infty)$	set of square integrable functions on $[0,+\infty)$
$l_2[0,+\infty)$	set of square multipliable functions on $[0,+\infty)$
$\mathcal{C}\left([a,b],\mathbb{R}^n\right)$	family of continuous functions ϕ from $[a,b]$ to $\mathbb{R}^n$
$\mathcal{C}^b_{\mathcal{F}_0}\left([a,b],\mathbb{R}^n\right)$	family of all bounded $\mathcal{F}_0$-measurable $\mathcal{C}([a,b],\mathbb{R}^n)$-valued random variables
$L^2_{\mathcal{F}_0}\left([a,b],\mathbb{R}^n\right)$	family of all bounded $\mathcal{F}_0$-measurable $\mathcal{C}([a,b],\mathbb{R}^n)$-valued random variables $\xi = \left\{\xi(t): \sup_{a\leqslant t\leqslant b} \mathcal{E}\|\xi(t)\|^2 < \infty\right\}$
$\lvert\cdot\rvert$	absolute value (or modulus)
$\|\cdot\|$	Euclidean norm of a vector or spectral norm of a matrix
$\|\cdot\|_\infty$	induced l_∞-norm
$\|\phi\|_c$	continuous norm $\sup_{a\leqslant t\leqslant b} \|\phi(t)\|$ for $\phi \in \mathcal{C}\left([a,b],\mathbb{R}^n\right)$
$\mathcal{L}$	weak infinitesimal of a stochastic process
$\mathcal{D}x_t$	operator that maps $\mathcal{C}([-h,\ 0],\mathbb{R}^n) \to \mathbb{R}^n$; that is, $\mathcal{D}x_t = x(t) - Cx(t-h)$
$\mathcal{E}$	mathematical expectation
$\left[\begin{array}{c\|c} A & B \\ \hline C & D \end{array}\right]$	shorthand for state space realization $C(sI-A)^{-1}B+D$ for a continuous-time system or $C(zI-A)^{-1}B+D$ for a discrete-time system
$\forall$	for all
$\in$	belongs to
$\exists$	there exists
$\subseteq$	is a subset of
$\cup$	union
$\rightarrow$	tends toward or is mapped into (case sensitive)
$\Rightarrow$	implies
$:=$	is defined as
$\square$	end of proof

1. Introduction

In many physical and biological phenomena, the rate of variation in the system state depends on past states. This characteristic is called a delay or a time delay, and a system with a time delay is called a time-delay system. Time-delay phenomena were first discovered in biological systems and were later found in many engineering systems, such as mechanical transmissions, fluid transmissions, metallurgical processes, and networked control systems. They are often a source of instability and poor control performance. Time-delay systems have attracted the attention of many researchers [1–3] because of their importance and widespread occurrence. Basic theories describing such systems were established in the 1950s and 1960s; they covered topics such as the existence and uniqueness of solutions to dynamic equations, stability theory for trivial solutions, etc. That work laid the foundation for the later analysis and design of time-delay systems.

The robust control of time-delay systems has been a very active field for the last 20 years and has spawned many branches, for example, stability analysis, stabilization design, H_∞ control, passive and dissipative control, reliable control, guaranteed-cost control, H_∞ filtering, Kalman filtering, and stochastic control. Regardless of the branch, stability is the foundation. So, important developments in the field of time-delay systems that explore new directions have generally been launched from a consideration of stability as the starting point. This chapter reviews methods of studying the stability of time-delay systems and points out their limitations, and then goes on to describe a new method called the free-weighting-matrix (FWM) approach.

1.1 Review of Stability Analysis for Time-Delay Systems

Stability is a very basic issue in control theory and has been extensively discussed in many monographs [4–6]. Research on the stability of time-delay systems began in the 1950s, first using frequency-domain methods and later also using time-domain methods. Frequency-domain methods determine the stability of a system from the distribution of the roots of its characteristic equation [7] or from the solutions of a complex Lyapunov matrix function equation [8]. They are suitable only for systems with constant delays. The main time-domain methods are the Lyapunov-Krasovskii functional and Razumikhin function methods [1]. They are the most common approaches to the stability analysis of time-delay systems. Since it was very difficult to construct Lyapunov-Krasovskii functionals and Lyapunov functions until the 1990s, the stability criteria obtained were generally in the form of existence conditions; and it was impossible to derive a general solution. Then, Riccati equations, linear matrix inequalities (LMIs) [9], and Matlab toolboxes came into use; and the solutions they provided were used to construct Lyapunov-Krasovskii functionals and Lyapunov functions. These time-domain methods are now very important in the stability analysis of linear systems. This section reviews methods of examining stability and their limitations.

Consider the following linear system with a delay:

$$\begin{cases} \dot{x}(t) = Ax(t) + A_d x(t-h), \\ x(t) = \varphi(t), \quad t \in [-h, 0], \end{cases} \tag{1.1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector; $h > 0$ is a delay in the state of the system, that is, it is a discrete delay; $\varphi(t)$ is the initial condition; and $A \in \mathbb{R}^{n\times n}$ and $A_d \in \mathbb{R}^{n\times n}$ are the system matrices. The future evolution of this system depends not only on its present state, but also on its history. The main methods of examining its stability can be classified into two types: frequency-domain and time-domain.

Frequency-domain methods: Frequency-domain methods provide the most sophisticated approach to analyzing the stability of a system with no delay ($h = 0$). The necessary and sufficient condition for the stability of such a system is $\lambda(A+A_d) < 0$. When $h > 0$, frequency-domain methods yield the result that system (1.1) is stable if and only if all the roots of its characteristic function,

$$f(\lambda) = \det(\lambda I - A - A_d \mathrm{e}^{-h\lambda}) = 0, \tag{1.2}$$

have negative real parts. However, this equation is transcendental, which makes it difficult to solve. Moreover, if the system has uncertainties and a time-varying delay, the solution is even more complicated. So the use of a frequency-domain method to study time-delay systems has serious limitations.

Time-domain methods: Time-domain methods are based primarily on two famous theorems: the Lyapunov-Krasovskii stability theorem and the Razumikhin theorem. They were established in the 1950s by the Russian mathematicians Krasovskii and Razumikhin, respectively. The main idea is to obtain a sufficient condition for the stability of system (1.1) by constructing an appropriate Lyapunov-Krasovskii functional or an appropriate Lyapunov function. This idea is theoretically very important; but until the 1990s, there was no good way to implement it. Then the Matlab toolboxes appeared and made it easy to construct Lyapunov-Krasovskii functionals and Lyapunov functions, thus greatly promoting the development and application of these methods. Since then, significant results have continued to appear one after another (see [10] and references therein). Among them, two classes of sufficient conditions have received a great deal of attention. One class is independent of the length of the delay, and its members are called delay-independent conditions. The other class makes use of information on the length of the delay, and its members are called delay-dependent conditions.

The Lyapunov-Krasovskii functional candidate is generally chosen to be

$$V_1(x_t) = x^{\mathrm{T}}(t)Px(t) + \int_{t-h}^{t} x^{\mathrm{T}}(s)Qx(s)\mathrm{d}s, \tag{1.3}$$

where $P > 0$ and $Q > 0$ are to be determined and are called Lyapunov matrices; and x_t denotes the translation operator acting on the trajectory: $x_t(\theta) = x(t+\theta)$ for some (non-zero) interval $[-h, 0]$ ($\theta \in [-h, 0]$). Calculating the derivative of $V_1(x_t)$ along the solutions of system (1.1) and restricting it to less than zero yield the delay-independent stability condition of the system:

$$\begin{bmatrix} PA + A^{\mathrm{T}}P + Q & PA_d \\ * & -Q \end{bmatrix} < 0. \tag{1.4}$$

Since this inequality is linear with respect to the matrix variables P and Q, it is called an LMI. If the LMI toolbox of Matlab yields solutions to LMI (1.4) for these variables, then according to the Lyapunov-Krasovskii stability theorem, system (1.1) is asymptotically stable for all $h \geqslant 0$; and furthermore, an appropriate Lyapunov-Krasovskii functional is obtained.

Since delay-independent conditions contain no information on a delay, they are overly conservative, especially when the delay is small. This consideration has given rise to another important class of stability conditions, namely, delay-dependent conditions, which do contain information on the length of a delay. First of all, they assume that system (1.1) is stable when $h = 0$. Since the solutions of the system are continuous functions of h, there must exist an upper bound, $\bar{h}$, on the delay such that system (1.1) is stable for all $h \in [0, \bar{h}]$. Thus, the maximum possible upper bound on the delay is the main criterion for judging the conservativeness of a delay-dependent condition.

The hot topics in control theory are delay-dependent problems in stability analysis, robust control, H_∞ control, reliable control, guaranteed-cost control, saturation input control, and chaotic-system control.

Since the 1990s, the main approach to the study of delay-dependent stability has involved the addition of a quadratic double-integral term to the Lyapunov-Krasovskii functional (1.3):

$$V(x_t) = V_1(x_t) + V_2(x_t), \tag{1.5}$$

where

$$V_2(x_t) = \int_{-h}^{0} \int_{t+\theta}^{t} x^{\mathrm{T}}(s) Z x(s) \mathrm{d}s \mathrm{d}\theta.$$

The derivative of $V_2(x_t)$ is

$$\dot{V}_2(x_t) = h x^{\mathrm{T}}(t) Z x(t) - \int_{t-h}^{t} x^{\mathrm{T}}(s) Z x(s) \mathrm{d}s. \tag{1.6}$$

Delay-dependent conditions can be obtained from the Lyapunov-Krasovskii stability theorem. However, how to deal with the integral term on the right side of (1.6) is a problem. So far, three methods of studying delay-dependent problems have been devised: the discretized Lyapunov-Krasovskii functional method, fixed model transformations, and parameterized model transformations.

The main use of the discretized Lyapunov-Krasovskii functional method is to study the stability of linear systems and neutral systems with a constant delay. It discretizes the Lyapunov-Krasovskii functional, and the results can be written in the form of LMIs [11–15]. The advantage of doing this is that the estimate of the maximum allowable delay that guarantees the stability of the system is very close to the actual value. The drawbacks are that it is computationally expensive and that it cannot easily handle systems with a time-varying delay. Consequently, this method has not been widely studied or used since it was first proposed by Gu in 1997 [11].

The primary way of dealing with the integral term on the right side of equation (1.6) is by using a fixed model transformation. It transforms a system with a discrete delay into a new system with a distributed delay (the integral term in (1.10)). The following inequalities play an important role in deriving the stability conditions:

Basic inequality: $\forall a, b \in \mathbb{R}^n$ and $\forall R > 0$,

$$-2a^{\mathrm{T}}b \leqslant a^{\mathrm{T}}Ra + b^{\mathrm{T}}R^{-1}b. \tag{1.7}$$

Park's inequality [16]: $\forall a, b \in \mathbb{R}^n$, $\forall R > 0$, and $\forall M \in \mathbb{R}^{n\times n}$,

$$-2a^{\mathrm{T}}b \leqslant \begin{bmatrix} a \\ b \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} R & RM \\ * & (M^{\mathrm{T}}R + I)R^{-1}(RM + I) \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}. \tag{1.8}$$

Moon *et al.*'s inequality [17]: $\forall a \in \mathbb{R}^{n_a}$, $\forall b \in \mathbb{R}^{n_b}$, $\forall N \in \mathbb{R}^{n_a\times n_b}$, and for $X \in \mathbb{R}^{n_a\times n_a}$, $Y \in \mathbb{R}^{n_a\times n_b}$, and $Z \in \mathbb{R}^{n_b\times n_b}$, if $\begin{bmatrix} X & Y \\ * & Z \end{bmatrix} \geqslant 0$, then

$$-2a^{\mathrm{T}}Nb \leqslant \begin{bmatrix} a \\ b \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} X & Y - N \\ * & Z \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}. \tag{1.9}$$

The basic features of the typical model transformations discussed in [18] are described below.

Model transformation I

$$\dot{x}(t) = (A + A_d)x(t) - A_d \int_{t-h}^{t} [Ax(s) + A_d x(s-h)]\mathrm{d}s. \tag{1.10}$$

The following Lyapunov-Krasovskii functional is used to determine a delay-dependent stability condition:

$$V(x_t) = V_1(x_t) + V_2(x_t) + V_3(x_t), \tag{1.11}$$

where

$$V_3(x_t) = \int_{-2h}^{-h} \int_{t+\theta}^{t} x^{\mathrm{T}}(s) Z_1 x(s)\mathrm{d}s\mathrm{d}\theta.$$

The derivative of $V(x_t)$ along the solutions of system (1.10) is

$$\dot{V}(x_t) = \Psi + \eta_1 + \eta_2 - \int_{t-h}^{t} x^{\mathrm{T}}(s)Zx(s)\mathrm{d}s - \int_{t-2h}^{t-h} x^{\mathrm{T}}(s)Z_1x(s)\mathrm{d}s, \tag{1.12}$$

where

$$\begin{aligned}
\Psi &= x^{\mathrm{T}}(t)[2P(A+A_d)+Q+h(Z+Z_1)]x(t) - x^{\mathrm{T}}(t-h)Qx(t-h),\\
\eta_1 &= -2\int_{t-h}^{t} x^{\mathrm{T}}(t)PA_dAx(s)\mathrm{d}s,\\
\eta_2 &= -2\int_{t-2h}^{t-h} x^{\mathrm{T}}(t)PA_dA_dx(s)\mathrm{d}s.
\end{aligned}$$

η_1 and η_2 are called cross terms.

Using the basic inequality (1.7) yields

$$\begin{aligned}
\eta_1 &\leqslant hx^{\mathrm{T}}(t)PA_dAZ^{-1}A^{\mathrm{T}}A_d^{\mathrm{T}}Px(t) + \int_{t-h}^{t} x^{\mathrm{T}}(s)Zx(s)\mathrm{d}s,\\
\eta_2 &\leqslant hx^{\mathrm{T}}(t)PA_dA_dZ_1^{-1}A_d^{\mathrm{T}}A_d^{\mathrm{T}}Px(t) + \int_{t-2h}^{t-h} x^{\mathrm{T}}(s)Z_1x(s)\mathrm{d}s.
\end{aligned}$$

Applying these two inequalities to (1.12) eliminates the quadratic integral terms, and a delay-dependent condition is established.

This process has two key points:

(1) The purpose of a model transformation is to bring the integral term into the system equation so as to produce both cross terms and quadratic integral terms in the derivative of a Lyapunov-Krasovskii functional along the solutions of the system.

(2) The bounding of the cross terms, η_1 and η_2, eliminates the quadratic integral terms in the derivative of the Lyapunov-Krasovskii functional, thereby yielding a delay-dependent condition.

Model transformation II

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[x(t) + A_d\int_{t-h}^{t} x(s)\mathrm{d}s\right] = (A+A_d)x(t). \tag{1.13}$$

In 2000 and 2001, Prof. Gu [19, 20] pointed out that, since model transformations I and II introduce additional dynamics into the transformed system, the transformed system is not equivalent to the original one. Thus, these transformations were soon replaced by others.

Model transformation Ⅲ

$$\dot{x}(t) = (A + A_d)x(t) - A_d \int_{t-h}^{t} \dot{x}(s)\mathrm{d}s. \tag{1.14}$$

In this case, the Lyapunov-Krasovskii functional is

$$V(x_t) = V_1(x_t) + V_4(x_t), \tag{1.15}$$

where

$$V_4(x_t) = \int_{-h}^{0} \int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s) Z \dot{x}(s)\mathrm{d}s\mathrm{d}\theta.$$

The derivative of $V(x_t)$ is

$$\dot{V}(x_t) = \Phi + \eta_3 - \int_{t-h}^{t} \dot{x}^{\mathrm{T}}(s) Z \dot{x}(s)\mathrm{d}s, \tag{1.16}$$

where

$$\Phi = x^{\mathrm{T}}(t)[2P(A + A_d) + Q]x(t) - x^{\mathrm{T}}(t-h)Qx(t-h) + h\dot{x}^{\mathrm{T}}(t)Z\dot{x}(t),$$

$$\eta_3 = -2\int_{t-h}^{t} x^{\mathrm{T}}(t)PA_d\dot{x}(s)\mathrm{d}s.$$

Just as for model transformation I, the bounding of the cross term, η_3, eliminates the quadratic integral terms in the derivative of Lyapunov-Krasovskii functional (1.16), thereby producing a delay-dependent condition.

Model transformation Ⅲ was presented in [16]. The basic idea is the same as that of model transformation I, with the difference being that, after model transformation Ⅲ, the transformed system is equivalent to the original one. In addition, after the transformation of system (1.1) into (1.14), when dealing with the term $h\dot{x}^{\mathrm{T}}(t)R\dot{x}(t)$ in the derivative of $V(x_t)$, system (1.1) is used as a substitute for system (1.14). That is, to obtain system (1.14), the state-delay term $x(t-h)$ in system (1.1) is replaced by using the Newton-Leibnitz formula; but $x(t-h)$ is not replaced in the derivative of $V(x_t)$. This inconsistency in the elimination of the integral terms leads to conservativeness.

In 2001, Fridman devised the following descriptor model transformation [21], which attracted a great deal of attention in subsequent years.

Model transformation Ⅳ

$$\begin{cases} \dot{x}(t) = y(t), \\ y(t) = (A + A_d)x(t) - A_d \int_{t-h}^{t} y(s)\mathrm{d}s. \end{cases} \tag{1.17}$$

Fridman employed the following generalized Lyapunov-Krasovskii functional:

$$V(x_t) = \xi^{\mathrm{T}}(t)EP\xi(t) + \int_{t-h}^{t} x^{\mathrm{T}}(s)Qx(s)\mathrm{d}s + \int_{-h}^{0}\int_{t+\theta}^{t} y^{\mathrm{T}}(s)Zy(s)\mathrm{d}s\mathrm{d}\theta, \tag{1.18}$$

where

$$\xi(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}, \quad E = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \quad P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}.$$

The derivative of $V(x_t)$ along the solutions of system (1.17) is

$$\dot{V}(x_t) = \Sigma + \eta_4 - \int_{t-h}^{t} y^{\mathrm{T}}(s)Zy(s)\mathrm{d}s, \tag{1.19}$$

where

$$\Sigma = \xi^{\mathrm{T}}(t)\left\{2P^{\mathrm{T}}\begin{bmatrix} 0 & I \\ A + A_d & -I \end{bmatrix} + \begin{bmatrix} Q & 0 \\ 0 & hZ \end{bmatrix}\right\}\xi(t) - x^{\mathrm{T}}(t-h)Qx(t-h),$$

$$\eta_4 = -2\int_{t-h}^{t} \xi^{\mathrm{T}}(t)P^{\mathrm{T}}\begin{bmatrix} 0 \\ A_d \end{bmatrix} y(s)\mathrm{d}s.$$

As before, the bounding of the cross term, η_4, eliminates the quadratic integral terms in the derivative of Lyapunov-Krasovskii functional (1.19), thereby producing a delay-dependent condition.

There are four important points regarding the development of model transformations.

(1) When double-integral terms are introduced into the Lyapunov-Krasovskii functional to produce a delay-dependent stability condition, it results in quadratic integral terms appearing in the derivative of that functional.
(2) Model transformations emerged as a way of dealing with those quadratic integral terms.
(3) More specifically, the purpose of a model transformation is to bring the integral terms into the system equation so as to produce cross terms and quadratic integral terms in the derivative of the Lyapunov-Krasovskii functional.
(4) Then, the bounding of the cross terms eliminates the quadratic integral terms.

The basic feature of all model transformations is that they produce cross terms in the derivative of the Lyapunov-Krasovskii functional. However, since no suitable bounding methods have yet been discovered, the bounding of cross terms results in conservativeness; and attempts to reduce the conservativeness have naturally focused on this point. For example, in 1999, Park extended the basic inequality (1.7) to produce Park's inequality [16]. In 2001, Moon *et al.* explored ideas in the proof of Park's inequality to extend it, resulting in Moon *et al.*'s inequality [17], which has greater generality.

The use of Park's or Moon *et al.*'s inequality in combination with model transformation Ⅲ or Ⅳ brought forth a series of delay-dependent conditions with less conservativeness that are very useful in stability analysis and control synthesis.

However, model transformations Ⅲ and Ⅳ still have limitations: In a stability or performance analysis, they basically use the Newton-Leibnitz formula to replace delay terms in the derivative of the Lyapunov-Krasovskii functional; but not all the delay terms are necessarily replaced. For example, in [17], the derivative of the Lyapunov-Krasovskii functional is

$$\dot{V}(x_t) = 2x^{\mathrm{T}}(t)P\dot{x}(t) + \cdots + h\dot{x}^{\mathrm{T}}(t)Z\dot{x}(t) + \cdots , \tag{1.20}$$

where $P > 0$ and $Z > 0$ are matrices to be determined in the Lyapunov-Krasovskii functional. When dealing with the term $x(t-h)$ (which appears when $\dot{x}(t)$ is replaced with the system equation) in $\dot{V}(x_t)$, the $x(t-h)$ in $2x^{\mathrm{T}}(t)P\dot{x}(t)$ is replaced, but the $x(t-h)$ in $h\dot{x}^{\mathrm{T}}(t)Z\dot{x}(t)$ is not. This treatment is equivalent to adding the following zero-equivalent term to the derivative of the Lyapunov-Krasovskii functional:

$$2x^{\mathrm{T}}(t)PA_d\left[x(t) - x(t-h) - \int_{t-h}^{t}\dot{x}(s)\mathrm{d}s\right]. \tag{1.21}$$

Fixed weighting matrices are used to express the relationships among the terms of the Newton-Leibnitz formula in (1.21). That is, the weighting matrix of $x(t)$ is PA_d and that of $x(t-h)$ is zero. Similarly, in [18, 22–24], which employ the descriptor model transformation, the delay term $x(t-h)$ in

$$2\left[x^{\mathrm{T}}(t),\ \dot{x}^{\mathrm{T}}(t)\right]\begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}^{\mathrm{T}}\begin{bmatrix} 0 \\ A_d x(t-h) \end{bmatrix}$$

in the derivative of the Lyapunov-Krasovskii functional is replaced with $x(t) - \int_{t-h}^{t}\dot{x}(s)\mathrm{d}s$. This treatment is equivalent to adding the following zero-equivalent term to the derivative of the Lyapunov-Krasovskii functional:

$$2\left[x^{\mathrm{T}}(t)P_2^{\mathrm{T}}A_d + \dot{x}^{\mathrm{T}}(t)P_3^{\mathrm{T}}A_d\right]\left[x(t) - \int_{t-h}^{t}\dot{x}(s)\mathrm{d}s - x(t-h)\right]. \tag{1.22}$$

Here, fixed weighting matrices are also used to express the relationships among the terms of Newton-Leibnitz formula. (The weighting matrix of $x(t)$ is $P_2^{\mathrm{T}}A_d$, that of $\dot{x}(t)$ is $P_3^{\mathrm{T}}A_d$, and that of $x(t-h)$ is zero). This substitution method is currently used in model transformations III and IV to obtain a delay-dependent condition. Note that, when weighting matrices are used for the above purpose, optimal weights do exist and the values should not be chosen simply for convenience. However, no effective way of determining the weights has yet been devised.

The chief feature of a parameterized model transformation [25–28] is the division of the delay term of system (1.1) into two parts: a delay-independent one and one to which a fixed model transformation is applied. That transforms system (1.1) into

$$\dot{x}(t) = Ax(t) + (A_d - C)x(t-h) + Cx(t-h), \tag{1.23}$$

where C is a matrix parameter to be determined. In this way, a parameterized model transformation is combined with a fixed model transformation; so the limitations of the latter remain. On the other hand, although an effective approach to matrix decomposition was presented by Han in [28] (Remark 7 on page 378), three undetermined matrices have to be equal, which leads to unavoidable conservativeness.

The stabilization problem is closely related to stability. Stabilization involves finding a feedback controller that stabilizes the closed-loop system, with the main feedback schemes being state and output feedback. Methods of stability analysis include both frequency- and time-domain approaches, but the latter are more commonly used for stabilization problems because the former do not lend themselves readily to solving such problems. For synthesis problems (such as delay-dependent stabilization and control), there is no effective controller synthesis algorithm, even for simple state feedback; solutions are even more difficult for output feedback.

The main problem is that, even if model transformation I or II is used to derive an LMI-based controller synthesis algorithm, they both introduce additional eigenvalues into the original system, as mentioned above; so the transformed system is not equivalent to the original one. Moreover, they employ conservative vector inequalities. So, they have been replaced by model transformations III and IV. However, when using either of them to solve a synthesis problem, the design of the controller depends on one or more nonlinear matrix inequalities (NLMIs). There are two main methods of solving this type of inequality: One is the iterative algorithm of Moon *et al.* [17], who used it on a robust stabilization problem. [23,24] also used it on an H_∞ control problem. This method yields a small controller gain, which is easy to implement; but the solutions are suboptimal [17]. The other is the widely used parameter-tuning method of Fridman *et al.* [18,22,29–34]. It transforms the NLMI(s) into an LMI(s) by using scalar parameters to set one or more undetermined matrices in the NLMI(s) to specific forms; and then the tuning of those parameters produces a controller. This method also yields a suboptimal solution, and experience is required to properly tune the parameters.

1.2 Introduction to FWMs

In Section 1.1, we saw that the method of Moon *et al.* [17] adds the equation (1.21) to $\dot{V}(x_t)$; and the descriptor model transformation [18, 22–24, 28–35] adds the term (1.22) to it. The difference is that the weighting matrices of terms such as $x(t)$ and $\dot{x}(t)$ are different, but they are all constant. For example, in Moon *et al.* [17], the weighting matrix of $x(t)$ is PA_d, where A_d is a coefficient matrix and P is a Lyapunov matrix. P is closely related to other matrices and cannot be freely chosen. For other terms, also, the weighting matrix is constant (for example, for $x(t-h)$ it is zero). Moreover, in the descriptor model transformation, they are also constant. This is where FWMs come in. In equations (1.21) and (1.22), the weighting matrices of $x(t)$, $\dot{x}(t)$, and $x(t-h)$ are replaced by unknown FWMs. From the Newton-Leibnitz formula, the following equation is true for any matrices N_1 and N_2 with appropriate dimensions:

$$2\left[x^{\mathrm{T}}(t)N_1 + x^{\mathrm{T}}(t-h)N_2\right]\left[x(t) - \int_{t-h}^{t}\dot{x}(s)\mathrm{d}s - x(t-h)\right] = 0. \quad (1.24)$$

Now, we add the left side of this equation to the derivative of the Lyapunov-Krasovskii functional. The fact that N_1 and N_2 are free and that their optimal values can be obtained by solving LMIs overcomes the conservativeness arising from the use of fixed weighting matrices [36–43].

On the other hand, since the two sides of the system equation are equal, FWMs thus express the relationships among the terms of that equation. That is, from system equation (1.1), the following equation is true for any matrices

T_1 and T_2 with appropriate dimensions:

$$2\left[x^{\mathrm{T}}(t)T_1 + \dot{x}^{\mathrm{T}}(t)T_2\right]\left[\dot{x}(t) - Ax(t) - A_d x(t-h)\right] = 0. \tag{1.25}$$

And from the Newton-Leibniz formula, the following equation is true for any matrices N_i, $i = 1, 2, 3$ with appropriate dimensions:

$$2\left[x^{\mathrm{T}}(t)N_1 + \dot{x}^{\mathrm{T}}(t)N_2 + x^{\mathrm{T}}(t-h)N_3\right]\left[x(t) - \int_{t-h}^{t} \dot{x}(s)\mathrm{d}s - x(t-h)\right] = 0. \tag{1.26}$$

Reserving the term $\dot{x}(t)$ in the derivative of the Lyapunov-Krasovskii functional and adding the left sides of these two equations to the derivative produce another type of result; Chapter 3 theoretically proves the equivalence of these two methods. This shows that the descriptor model transformation of Fridman *et al.* is a special case of the FWM approach. Furthermore, this treatment in combination with a parameter-dependent Lyapunov-Krasovskii functional is easily extended to deal with the delay-dependent stability of systems with polytopic-type uncertainties [44–47].

1.3 Outline of This Book

This book is organized as follows:

Chapter 1 reviews research on the stability of time-delay systems and describes the free-weighting-matrix approach.

Chapter 2 provides the basic knowledge and concepts on the stability of time-delay systems that are needed in later chapters.

Chapter 3 deals with linear systems with a time-varying delay. FWMs are used to express the relationships among the terms in the Newton-Leibniz formula, and delay-dependent stability conditions are derived. The criteria are then extended to delay-dependent and rate-independent stability conditions without any limitations on the derivative of the delay. Two classes of criteria are obtained for two different treatments of the term $\dot{x}(t)$ (retaining it or replacing it with the system equation) in the derivative of the Lyapunov-Krasovskii functional; and their equivalence is proved. On this basis, the criteria are extended to systems with time-varying structured uncertainties. Furthermore, since retaining the term $\dot{x}(t)$ allows the Lyapunov matrices and system matrices to readily be separated, this treatment in combination with a parameter-dependent Lyapunov-Krasovskii functional is easily extended to deal with the delay-dependent stability of systems with polytopic-type uncertainties. Finally, systems with a time-varying delay are investigated based on an improved FWM (IFWM) approach that yields less conservative results.

Chapter 4 focuses on systems with multiple constant delays. For a system with two delays, delay-dependent criteria are derived by using the FWM approach to take the relationship between the delays into account. When the delays are equal, the criteria are equivalent to those for a system with a single delay. This idea is extended to the derivation of delay-dependent stability criteria for a system with multiple delays.

Chapter 5 investigates neutral systems. The FWM approach is used to analyze the discrete-delay-dependent and neutral-delay-independent stability of a neutral system with a time-varying discrete delay. Delay-dependent stability criteria for neutral systems are derived for identical discrete and neutral delays using the FWM approach and using that approach in combination with a parameterized model transformation and an augmented Lyapunov-Krasovskii functional, respectively. Again based on the FWM approach, discrete-delay- and neutral-delay-dependent stability criteria are obtained for a neutral system with different discrete and neutral delays. It is shown that these criteria include those for identical discrete and neutral delays as a special case.

Chapter 6 deals with the stabilization of linear systems with a time-varying delay. Based on the delay-dependent stability criteria obtained in Chapter 3, a static-state-feedback controller that stabilizes the system is designed by an iterative method that uses the cone complementarity linearization (CCL) algorithm or the improved CCL (ICCL) algorithm that we devised by using a new stop condition, along with a method of adjusting the parameters. In addition, an LMI-based method of controller design is developed from a delay-dependent and rate-independent stability condition.

Chapter 7 employs the IFWM approach to investigate the output-feedback control of a linear discrete-time system with a time-varying interval delay. The delay-dependent stability is first analyzed by a new method of estimating the upper bound on the difference of a Lyapunov function that does not ignore any terms; and based on the stability criterion, a design criterion for a static-output-feedback (SOF) controller is derived. Since the conditions thus obtained for the existence of admissible controllers are not expressed strictly in terms of LMIs, the ICCL algorithm is employed to solve the nonconvex feasibility SOF control problem. Furthermore, the problem of designing a dynamic-output-feedback (DOF) controller is formulated as one of designing an SOF controller, and a DOF controller is obtained by transforming the design problem into one for an SOF controller.

Chapter 8 concerns the design of an H_∞ controller for systems with a time-varying interval delay. The IFWM approach is used to devise an improved delay-dependent bounded real lemma (BRL). A method of designing an H_∞ controller is given that employs the ICCL algorithm.

Chapter 9 focuses on the design of an H_∞ filter for both continuous-time and discrete-time systems with a time-varying delay. The IFWM approach is used to carry out a delay-dependent H_∞ performance analysis for error systems. The resulting criteria are extended to systems with polytopic-type uncertainties. Based on the results of the analysis, H_∞ filters are designed in terms of LMIs.

Chapter 10 discusses stability problems for neural networks with time-varying delays. First, the stability of neural networks with multiple time-varying delays is considered; and the FWM approach is used to derive a delay-dependent stability criterion, from which both a delay-independent and rate-dependent criterion, and a delay-dependent and rate-independent criterion are obtained as special cases. Next, the IFWM approach is used to establish stability criteria for neural networks with a time-varying interval delay. Moreover, the FWM and IFWM approaches are used to investigate the exponential stability of neural networks with a time-varying delay. Finally, the IFWM approach is used to deal with the exponential stability of a class of discrete-time recurrent neural networks with a time-varying delay.

Chapter 11 shows how the IFWM approach can be used to study the asymptotic stability of a Takagi-Sugeno (T-S) fuzzy system with a time-varying delay. By considering the relationships among the time-varying delay, its upper bound, and their difference, and without ignoring any useful terms in the derivative of the Lyapunov-Krasovskii functional, an improved LMI-based asymptotic-stability criterion is obtained for a T-S fuzzy system with a time-varying delay. Then the criterion is extended to a T-S fuzzy system with time-varying structured uncertainties.

Chapter 12 investigates the problem of designing a controller for a networked control system (NCS). The IFWM approach is used to derive an improved stability criterion for a networked closed-loop system. This leads to the establishment of a method of designing a state-feedback controller based on the ICCL algorithm.

Chapter 13 concerns the delay-dependent stability of a stochastic system with a delay. The robust stability of an uncertain stochastic system with a time-varying delay is discussed; and the exponential stability of a stochastic Markovian jump system with nonlinearity and a time-varying delay is investigated. Less conservative results are established using the IFWM approach.

Chapter 14 investigates the stability of nonlinear systems with delays. First, for Lur'e control systems with multiple nonlinearities and a constant delay, LMI-based necessary and sufficient conditions for the existence of a Lyapunov-Krasovskii functional in the extended Lur'e form that ensures the absolute stability of the system are obtained and extended to systems with time-varying structured uncertainties. Then, the FWM approach is used to derive delay-dependent criteria for the absolute stability of a Lur'e control system with a time-varying delay. Finally, the IFWM approach is used to discuss the stability of a system with nonlinear perturbations and a time-varying interval delay. Less conservative delay-dependent stability criteria are established because the range of the delay is taken into account and an augmented Lyapunov-Krasovskii functional is used.

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