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Michele Sce's Works in Hypercomplex Analysis

A Translation with Commentaries

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1.1 Foreword

The richness of the theory of functions in one complex variable stimulated, at the end of the nineteenth century and beginning of twentieth century, the interest in the study of functions of several complex variables but also of hypercomplex variables. Some references about these studies can be found in Dickson's book [29], p. 78 and in the later paper by Ketchum [31]. Some earlier references are [27, 28].

In this context, the Italian school played a significant role in the twenties and thirties and in fact there are several works which range from the study of algebras of hypercomplex numbers, see e.g. the works of Scorza [35–39], to the analysis of functions of hypercomplex variables, see [40, 42, 43]. When the theory of monogenic (also called analytic or regular) functions of a quaternionic variable emerged with the works of Moisil [33] and Fueter [30] it was rather clear that the setting of quaternions, being a skew field, was a convenient one. The importance of these functions is witnessed by the fact that they appear in the monograph written by Segre [41]. But at that time, it seemed that there were different possible approaches, see for example the work of Sobrero [42] where a specific algebra is introduced to deal with equations arising from the theory of elasticity. These studies continued, see e.g. [44], and in the fifties attracted the attention of mathematicians like Rizza and Sce. Rizza was more interested in the case of Clifford algebras, see [34], whereas Sce was working in the context of more general algebras with a contribution to the octonionic case, together with Dentoni.

Most of the works of the Italian school have been essentially forgotten. It is difficult to understand the reason, but certainly one possible explanation is that many articles were published in Italian journals and they were written in Italian. Even today, and despite the existence of powerful search engines, it would be difficult to retrieve these papers, since the keywords one may use in the search are English words, not Italian ones.

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On the other hand, it is remarkable to note that Sce, in his works in hypercomplex algebras and analysis was very accurate about the existing literature and seemed to be aware of all relevant published works. As the reader will discover in his biography, he was a passionate bibliophile and this passion probably lead him to check very carefully the available references, most of them in foreign languages like English, German, Romanian.

The inspiration for this volume was given by the event "A Scientific Day in Honor of Michele Sce" held in Milano on October 11th, 2018. Michele Sce was professor for long time at the University of Milano where the three of us have studied. His interests in hypercomplex algebras and analysis constitute the common ground with our own research interests. The fact that the only results of Sce that are quoted in the modern literature were the one related to Fueter mapping theorem and, more marginally, the paper with Dentoni on octonions, has stimulated our willingness to translate his works in hypercomplex analysis to make them accessible. In doing so, we added some comments since sometimes it is necessary to adapt the notation and the terminology to the modern language, as well as some examples to clarify the results of Sce. Where appropriate, we provided the developments of the theory thus offering a deeper sense of the importance of Sce's work, and the visionary role he played in the theory of functions of hypercomplex variables.

We kept the original text and formulas of the various papers, and where we corrected some typos we point this out in the Editors' Notes. For typographical reasons, in Chap. 6 we used the modern Latex for Definitions, Theorems, etc. combined with the style used by the authors D_1, T_1, etc.

The book is organized in five chapters, besides this Introduction. Chapter 2 contains the translation of the three parts of a same paper dealing with the notions of monogenicity and total derivability in real and complex algebras. Chapter 3 deals with Sce's paper on systems of partial differential equations related to real algebras. Chapter 4 contains the paper on the variety of zero divisors in algebras. A central role is played by Chap. 5 which contains the celebrated theorem nowadays referred to as the Fueter-Sce-Qian mapping theorem which gave rise to several modern results discussed in the comments to this chapter. Chapter 6 contains the translation of the paper by Dentoni and Sce which deals with octonionic analysis, another topic which has led to many interesting modern developments. Each chapter has its own list of references and may stand alone.

Our hope is that this book will make Sce's work accessible to a larger audience and, possibly, will provide inspiration for future works.

1.2 Biography

Michele Sce was born in 1929 in Tirano in Northern Italy and he graduated "cum laude" at the Scuola Normale Superiore in Pisa in 1951. He became assistant professor in Geometry at the University of Milano where he remained officially until 1963, but already on leave at the end of 1962. During this period he lectured on

function theory at the University of Parma, and on number theory at the University of Rome. He was the recipient, in 1959, of the Bonavera Prize awarded by the Academy of Sciences of Torino.

During this period Sce, among his many interests, was working on finite geometries and the enumerative problems connected to them but, when he tried to build examples or significant counterexamples, he faced difficult calculations. Thus he sought the help of his friend Lorenzo Lunelli who was among the first in Italy to work with an electronic processor at the Politecnico di Milano and had the "machine", namely the electronic calculator CRC-102A/P, capable of performing such calculations. The computations led them to the paper [2].

The CRC-102A/P, a machine with reduced computing capacity compared to the mathematical problems that can be resolved or at least clarified by automatic procedures, assisted Lunelli and Sce in writing four other papers. Given these interests, it is not surprising that Sce, at the end of 1962, left the university to take the opportunity to work at the Laboratorio di Ricerche Elettroniche of Olivetti. Sce then worked as a mathematical consultant at the Office of Electronic-Mechanical Equipment Projects directed by Pier Giorgio Perotto which was carrying out highly innovative projects such as the Olivetti Programma 101, the first personal computer in history, all designed in Italy.

Sce worked on projects concerning character recognition, working on projects on OCR-B (OCR stands for Optical Character Recognition), see [23], for which he was using a machine, the Elea 9003, to assist the computations.

Meanwhile, he married Paola Maria Manacorda and they eventually had three sons: Giovanni, Simone and Jacopo.

The Electronic Division of Olivetti was sold to General Electric in 1965 and Sce underlined in a newspaper that doing so Italy was losing a possible supremacy in the newly born computer industry. He continued to work for Olivetti as Head of the Research and Development Division, and he was still working for Olivetti when he was called to be a member of the Committee of Mathematics of the CNR (Consiglio Nazionale delle Ricerche) for the period 1968–1972. There he had the opportunity to illustrate his point of view on the state and the possible development of applied mathematics in Italy. He promoted scientific computing, also facilitating the creation of laboratories equipped with computers. He also envisioned and promoted the computer assisted teaching, an idea which was absolutely new and revolutionary at that time.

During the period he spent at Olivetti Sce was continuing his research and his teaching activity at the university. After he left Olivetti in 1971, he had various roles at the CNR where he was tasked to provide an impulse to the diffusion of computers in some Italian universities and to the preparation of curricula in computer science.

The next phase of his career led him to become Director of the Statistical Division of A.C. Nielsen in Milano, where he proposed a strategic plan based on data analysis.

He finally returned to academia in 1976 as Full Professor first at the University of Lecce, then in Torino and eventually, in 1980, back to the University of Milano

where he taught several courses. As a teacher, he was available and very helpful to students, most of which still remember him for these qualities.

He had a strong commitment to mathematical libraries, initiating innovations in their management, and he took care of the Italian version of the Universal Decimal Classification of Mathematical Sciences on behalf of the CNR. He promoted the automation of Italian libraries for classification and management. In particular when he was Director of the library of the University of Milano, he adopted the system Aleph.

He also devoted himself to the dissemination of mathematics, both collaborating with magazines in the field and curating the edition of a large Dictionary of Mathematics published in 1989 by Rizzoli.

Sce has had many interests, a vast culture, and a personal library of about ten thousands volumes. This collection started when he was young and contains not only scientific books but also literary, historical, philosophical, anthropological books, as well as science fiction works and comics. A conspicuous part of the scientific books has been donated to the University of Milano Bicocca.

Despite his mild, shy and reserved nature, Sce always showed a great willingness to work with others in projects that aimed to strengthen the role of mathematics in Italian culture and society. As his wife recalled in her recollection he had few friends in academy, because of his reserved personality. Among them the late Carlo Pucci, Edoardo Vesentini and Gianfranco Capriz, and in Milano Stefano Kasangian, Stefania De Stefano and Alberto Marini. At Olivetti his best friends were Filippo Demonte and Mario Prennushi, with whom Sce collaborated while working at the project for the Programma 101.

Michele Sce passed away in Milano in 1993.

His ability to collaborate with various colleagues, his participation to numerous research projects and his dedication to teaching, left a trace of esteem and affection in his students, colleagues and friends, evidenced by the Scientific Day in his honor that the University of Milano organized on the 11th October 2018.

During that celebration it was particularly impressive to realize Sce's humble attitude. Despite his many achievements and his bright way of thinking and envisioning the future, he has never shown off his work with his family and friends. His various activities are now collected in the website `www.michelesce.net` which is a form of acknowledgement of his vision and understanding.

In the list of references below, the items [1–26] correspond to Michele Sce's scientific production.

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The authors are also grateful to Paola Maria Manacorda, Michele Sce's wife, for sharing some memories and to Giovanni, Simone and Jacopo Sce, their sons, for the material which was the source for the biography in the next section.

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Chapter 2
Monogenicity and Total Derivability in Real and Complex Algebras

In this chapter we collect three papers that correspond to the translations of three parts of the same work originally published as:

M. Sce, *Monogeneità e totale derivabilità nelle algebre reali e complesse. I*, (Italian) Atti Accad. Naz. Lincei. Rend. Cl. Sci. Fis. Mat. Nat. (8) **16** (1954), 30–35.
M. Sce, *Monogeneità e totale derivabilità nelle algebre reali e complesse. II*, (Italian) Atti Accad. Naz. Lincei. Rend. Cl. Sci. Fis. Mat. Nat. (8) **16** (1954), 188–193.
M. Sce, *Monogeneità e totale derivabilità nelle algebre reali e complesse. III*, (Italian) Atti Accad. Naz. Lincei. Rend. Cl. Sci. Fis. Mat. Nat. (8) **16** (1954), 321–325.

2.1 Monogenicity and Total Derivability in Real and Complex Algebras, I

Article I by Michele Sce, presented during the meeting of 16 January 1954 by B. Segre, member of the Academy.

To construct a theory of functions of a hypercomplex variable, a natural way would be to generalize the function theory of a complex variable. However, to pass from functions of a complex variable (for which the uniqueness of the derivative follows from the monogenicity condition) to functions of a hypercomplex variable, there are two possibilities: one is to impose the uniqueness of the derivative, and

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