

Milton Rosa

Cristiane Coppe de Oliveira *Editors*

Ethnomathematics in Action

Mathematical Practices in Brazilian
Indigenous, Urban and Afro
Communities



Springer

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Foreword

It was a great pleasure and honor when my colleagues and former students Milton Rosa and Cristiane Coppe invited me to write the preface for this marvelous collection of essays, dealing with the mathematical knowledge and beliefs of Brazilian communities, with special attention to Africa, Afro-Brazilians, and indigenous peoples, and not disregarding those Brazilians of European, and Asiatic ancestry, and mixed populations. Current Brazilian population is a dynamic encounter of several ancestries. The essays address these issues and the consequences in mathematical practices and conceptions.

This book is a very important contribution to the emerging area of ethnomathematics. The summary shows the breadth of this collection. It would be very difficult to comment on each essay. The titles are illustrative of their contents and each one has an extensive bibliography. A special characteristic of the book is its large geographic range. The book is very original in both the choice of contents and the treatment given to all of them. It adds relevant aspects to the area and also to the history of mathematics, to the history of sciences, and to cultural studies in general.

I will use this preface to comment on relevant aspects of ethnomathematics, which are treated with different nuances in this entire collection. As one of the proponents of ethnomathematics, more than four decades ago, I can appreciate the advances of this new research area as reflected in the essays of this collection. In a paper, published in 1977, I referred to ethnosciences and ethnomathematics as the study of scientific and, by extension, technological phenomena in direct relation to their social, economic, and cultural background. I claimed that the great advantage of working in this border zone lies in the possibility of combining academic dynamic treatment with historical studies to improve education. Indeed, this synthesizes the strength of this collection of essays.

Ethnomathematics, as a research area, is relatively new. It is a concept resulting from the recognition of the [biological kingdom](#) *Animalia*. With few exceptions, animals [consume organic material](#), [breathe oxygen](#), are [able to move](#), can [reproduce sexually](#), and grow from [embryonic development](#). Through a long evolutionary process, the genus *homo* evolved from animalia and gave origin to our species, *homo sapiens sapiens*, the humanity. From about 50,000, our species occupy the entire planet.

Our species aims at the survival of the individuals, through nourishment, and at the continuation of the species, through procreation. But the species *homo* developed another characteristic, unique to our species, which is to transcend the *pulsion of survival* of the individuals, and continuation of the species, with options that go beyond the anatomic-physiological mechanisms. This *pulsion of transcendence*, unique to the species *homo*, is difficult to explain. It gives rise to sophisticated and creative communication, and language, and to emotions, beliefs, and preferences. Some scholars call it will, others call it consciousness. There are other denominations, some related to myths and religions.

Consciousness is an elusive concept. Many scientists say that terms such as will, mind, consciousness, and other words describing subjective mental experiences cannot be defined. Is it located somewhere in the brain? It allows to predict what will happen in the future and to socially coordinate plans for the future. Consciousness helps to extract meaningful information from the sense organs. There are limitations, such as the recognition of will and of consciousness in anyone except oneself. Self-recognition is the ability to recognize oneself as distinct from another entity, as well as to plan, pay attention, recall memories of specific events, and take the perspective of another creature.

Environmental conditions require also activation of consciousness. Behavior may respond to instructions from the DNA, but also from learning and environmental influences within its own lifetime. To provide for all similar contingencies would require an impossible roster of instructions, wasteful volume of specific directions. In most of the contingencies, decisions must be *ad hoc*.

It is undeniable that the human species are characterized by the *pulsion of survival* of the individual and of the species, like all the living species, and by the *pulsion of transcendence*, unique to the human species. Both, survival and transcendence, are in mutual interaction. The acquisition of the *pulsion of transcendence* is the focus of mythologies and religions, a fascinating research theme.

Every individual (from birth to death), societies, and the human species in general develop strategies to cope with the ample reality. I clarify that every time I say ample reality I mean everything, the complex of natural and supernatural phenomena and facts, the physiological, sensorial, emotional, and psychic reactions to the environment in the broad sense and the social interactions. Indeed, everything, the *physis* (\approx nature) and the *nomos* (\approx social norms) are in permanent change. *Nomos* is as indispensable ground for *physis* as *physis* is an indispensable ground for *nomos*. They are in a symbiotic relation. *Physis* provides the potentialities, *nomos* the actuality of humanity.

The ensemble of the strategies to cope with ample reality is a complex system of knowledge and behavior, generated by individuals and socialized in a group of individuals with some affinity. This is the scenario discussed in all the essays of this book. The basic questions are how individuals and groups of individuals develop their means for surviving in their own natural and sociocultural environments, satisfying the *pulsion of survival* [body], and how they go beyond survival, acquiring the *pulsion of transcendence* [mind].

Survival and transcendence are the quintessence of human life and are dealt with a complex system of knowledge and behavior generated and organized by both each individual (from birth to death) and the affinity group. The way they are generated, and organized, and socialized relies in language and rhetoric in a broad sense.

These questions must be faced with *transcultural* and *transdisciplinary* strategies, which borrow methods of research from the sciences, from cognition, from mythology, from anthropology, from history, from sociology (politics, economics, and education), and from cultural studies in general. These strategies rely on the analyses of the history of ideas, and of the evolution of behavior, and knowledge of the human species, in every natural and sociocultural environment. All these aspects are present in the essays of this book.

The *Program Ethnomathematics* discusses the corpora of knowledge developed by humans to survive and to transcend. It is a transdisciplinary and transcultural research program supported by a new historiography. It is conceptually designed as a broad research program of the evolution of ideas, of knowledge, and of practices developed by the human species in different natural and sociocultural environments. Essentially, it implies in regard to the development of an analysis of how groups of humans generated ways, styles, arts, and techniques of *doing* and *knowing*, of learning and explaining, and of dealing with situations, and of solving problems of their natural and sociocultural environments.

I practiced an etymological abuse with the *free* appropriation of Greek roots: *techné* [*tics*] meaning ways, styles, arts, and techniques; *mathema* for *doing* and *knowing*, for learning and explaining, for dealing with situations and solving problems; and *ethno*, as distinct and specific natural and sociocultural environments. Thus, by using these Greek roots, the locution *tics+mathema+ethno* synthesizes the way groups of humans generated ways, styles, arts, and techniques of *doing* and *knowing*, of learning and explaining, and of dealing with situations, and of solving problems of their natural and sociocultural environments. Rearranging this conceptual etymological construction gave origin to the word *ethno+mathema+tics* or simply *Ethnomathematics*.

It is very important to clarify the meaning of the word ethnomathematics as I use it. It may be misleading. I do not use it as *mathematics of an ethnic group*. It is not *ethno+mathematics*. It is important for the readers of this collection of essays to be aware of this important concept. Although the essays of this book are assembled in parts dealing with Africanities, indigenous diversities, urban diversities, and cultural practices, which discuss specific cases, like case studies, they fit into the broader concept of *ethno+mathema+tics*.

Undeniably, each ethnic group, professional, and gremial group have their specific ways and modes of quantitative and qualitative practices, such as counting, weighing and measuring, comparing, sorting and classifying, inferring, and modeling, accumulated through generations in their natural, social, and cultural environments; the program ethnomathematics goes much beyond this.

Repeating what I wrote above, the word ethnomathematics was etymologically constructed as a broad theory of the evolution of ideas, of knowledge, and of practices developed differently by the human species in their different natural and sociocultural environments. Essentially, it implies of an analysis of how groups of humans generated ways, styles, arts, and techniques [*techné*] to *do* and to *know*, to learn and to explain, to deal with situations, and to solve problems [*mathema*] of their natural and sociocultural environments [*ethno*].

Although the words ethnobotany, ethnomusicology, ethnolinguistic, ethnomethodology, and other *ethno+disciplines* are used by anthropologists, by ethnographers, and by sociologists for research of specific disciplines in different ethnic and sociocultural contexts, they base their investigations on the views of an observer of *other* cultures, trying to find commonalities between the culture of the researchers (the observers) and the culture of the researched (the observed).

Obviously, the concept of mathematics of the observers, who studied it as an academic discipline, may have no meaning at all for the members of *other* cultures. Academic mathematics is a European construct springing out of specific styles and ways of counting, weighing and measuring, comparing, sorting and classifying, inferring, and modelling, which were organized by peoples of the Mediterranean Basin, since before the Christian Era, and organized by Euclid (c.300 B.C.) and several others academicians and scholars.

Different cultures have their own styles and ways of counting, weighing and measuring, comparing, sorting and classifying, inferring, and modelling, which have been historically generated and organized by their own people, sages, and scholars. Every human community developed cognitive strategies, ideas, language, myths, religions, social stratifications, governance, and other categories of social behavior to serve specific purposes.

The same occurs also for artistic representations and abstractions, figures, numerical tools meaning quantities, as well as ways of inferring. It may not make sense to address different cultural groups asking questions such as: “What is the meaning of a triangle?” or “How would you add 2 and 3?” or “What is the color of this flower?”. The categories: triangle, 2 plus 3, and color may be absolutely senseless in their culture. The conceptual locution *Ethno+mathema+tics* as I conceived it recognizes this important fact.

By drawing on the cultural experiences and practices of individuals and of communities, the pedagogical strand of the program ethnomathematics allows for an easier flow of scientific ideas with children, reducing the effects of cultural blocks that is so damaging in educational systems.

I recognize this in the collection of essays. Well-known specialists address different topics in school practices, which apply to different natural and sociocultural environments. Altogether, the essays mutually complement each other and the readers may draw conclusions that support the theoretical proposal of the program ethnomathematics.

This book is a valuable and necessary contribution to educational scholarship. I congratulate the editors and each individual author for this remarkable accomplishment.

São Paulo, Brazil
February 2020

Ubiratan D'Ambrosio

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Part I

Introduction

The main objective of this introduction is to show that ethnomathematics as a research program has an agenda that offers a broad view of mathematics, which embraces the ideas, processes, methods, and practices that are related to the diverse mathematical practices dealt in the Brazilian context.

Chapter 1

An Overview of Diverse Mathematical Practices in Brazil: An Ethnomathematical Perspective in Action



Milton Rosa

1.1 Initial Considerations

It is necessary to address the interrelations of local mathematical knowledge sources with broader universal forms of mathematics in order for us to understand ideas, procedures, and techniques found in distinct cultural groups. The main objective of this approach is to link or bridge mathematical practices developed by the members of distinct cultures in order to clarify understanding and mutual respect among the members of all communities. In this regard, an ethnomathematics program aims to stimulate broader reflections about the nature of mathematical thinking in cognitive, historical, social, and cultural environments that helps us to understanding the development of *knowing* and *doing* mathematics created worldwide by mankind in distinct cultural contexts.

Thus, there is a need to create a new role in relation to mathematics instruction that empowers people to understand power and oppression more critically by considering the effect of culture and language on mathematical knowledge by working with students to uncover the often distorted and hidden history of mathematical knowledge. In this context, any study of mathematics and its connection to culture represents a powerful means for valuing and validating students' real-life experiences and gives them tools to become critical and reflective participants in society.

This perspective forms the basis for significant contributions of Dambrosian-based ethnomathematical perspectives in re-conceiving the discipline of mathematics and its pedagogical practices. Thus, the use of a Freirean dialogical methodology is essential in developing a pedagogical praxis of ethnomathematics by investigating cultures and languages in order to develop a mathematics curriculum that shows

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the contributions of people from other cultural groups that seeks the enrichment of mathematical knowledge.

In the context of national and international dialogues, it is important to broaden the discussion for the possibilities of the inclusion of cultural polysemy of ethnomathematics in the mathematics curriculum in order to provoke discussions about social justice and respect, which is related to the sociocultural diversity of people in their search for peace. This approach guarantees the development of understanding of differences through dialogical interactions and respect that enables us to avoid domination and oppression.

In this regard, related to ethnomathematical research conducted in the Brazilian context, the authors of the chapters of this books share the necessity of discussing issues regarding mathematics education, classroom practices, and the knowledge of the members of specific cultural groups in order to explore mathematical knowledge, which has a role in helping us to clarify the nature of mathematical knowledge and of knowledge in general.

According to this context, there are hundreds of reasons for teaching mathematics. One of the most relevant motives involves the consideration of mathematics as an expression of human endeavor, development, culture, and thought that forms an integral part of the cultural heritage of humankind (Rosa and Orey 2016a). Contemporary society places great value on a capitalistic scientific Western oriented science and mathematics. Conversely, ethnomathematics demonstrates that mathematics is composed of many diverse and distinct cultural traditions, not just those emerging from the Mediterranean region (D'Ambrosio 1985).

For example, local knowledge inter-relationships are related to varied academic knowledge areas and are important in enabling a more precise understanding about a particular study field and its use of mathematical processes. Here, these inter-relations must be clearly addressed, especially, wherever one needs to understand specific concepts acquired through academic knowledge broadly based on biology, ecology, and mathematics, which may be supported by ethnosciences in order to understand its ethnomathematical interactions (Rosa and Orey 2017).

The main goal in this discussion is to further advance the research and understanding on the context of ethnomathematics research programs in diverse contexts in Brazil. As part of this, the authors are invited to discuss how they research ethnomathematical relations with other knowledge fields in their own contexts. In this regard, Rosa and Orey (2015) state that mathematical thinking in regard with science has been influenced by a diversity of human characteristics such as our languages, religions, morals, and economical-social-political activities. In concert with these, humanity has developed logical processes related to our universal need to pattern, quantify, measure, model, and explain, all shaped and operating within different social and historical contexts.

Because each cultural group has its own way of doing mathematics, these connections have come to represent, and are embedded in, a given cultural system, especially in the way that diverse peoples quantify and use numbers, apply geometric forms and relationships, measure or classify objects in their own environments (D'Ambrosio 2011). In concert with these characteristics, humanity has developed

logical processes related to the universal need to pattern, quantify, measure, model, and explain, all shaped and operating within different social and historical contexts.

As part of this approach, the members of each cultural group have developed their own way to *mathematize* their own realities. Western scientific arrogance often presents a disrespect of and outright refusal to acknowledge diverse cultural identities puts processes of understanding and comprehension of the many non-Western cultural systems at risk. Because this aspect gives a sense of confidence and dignity to students when their own *tacit knowledge*¹ is acknowledged, these particularities should not be ignored and they should be respected when people enter school (D'Ambrosio 2011).

In this regard, a search for new methodological approaches is necessary to share, record, and include diverse forms of mathematical thinking, ideas, procedures, and techniques developed in different cultural contexts. These connections have come to represent and are embedded in distinct cultural systems in Brazil, especially in the way that people quantify and use numbers, use geometric forms and relationships, measure or classify objects in their own environment.

For example, Fantinato and Mafra (2017) studied the techniques, processes, and tools involved in the preparation of ornamental patterns on the curved surfaces of vegetable gourds called *cuias*, which are crafted by a group of craftswomen in a riverbank community of the Northern region of Brazil. The results of this investigation are related to the contributions to the research field of ethnomathematics by bringing empirical reflections that aim at the implementation of local mathematical practices for the development of contextualized educational practices.

Even though there are many different perspectives in conducting ethnomathematical research, we agree in regard to the importance of ideas of Brazilian mathematician and philosopher Ubiratan D'Ambrosio in relation to the development and evolution of the research field of ethnomathematics. He is also one of the most important theoreticians in regard to the connections between mathematics and culture and offers encouragement, leadership, and dissemination of new ideas, concepts, and perspectives involved in ethnomathematics around the world and its applications in mathematics education. He is without a doubt our primary leader.

In accordance to Powell and Frankenstein (1997), D'Ambrosio's broader view of ethnomathematics accounts for the dialectical transformation of knowledge within and among societies. Moreover, his epistemology is consistent with Freire's perspective in that D'Ambrosio's standpoint in regard to mathematical knowledge is not static and ordained, but dynamic and the result of human activity.

Because teaching becomes an activity that introduces the creation of knowledge, it is much more than the mere transference of knowledge or information. This

¹This type of knowledge is embedded in personal experience, it is subjective, contextualized and analogous (Nonaka and Takeuchi 1995). For example, individuals do not learn how to ride a bicycle by reading a manual, as they need personal experimentation and practice to acquire the skills necessary to learn this action. Therefore, this knowledge is acquired and accumulated through individual experience, as it involves intangible factors such as beliefs, perspectives, perceptions, value systems, ideas, emotions, norms, presentiments and intuitions (Rosa and Orey 2012).

approach in mathematics education is the antithesis of turning students into containers to be filled with information (Freire 1972). This perspective forms the basis for significant contributions of a Freirean-based ethnomathematical perspective in reconceiving the discipline of mathematics and in a pedagogical practice.

In this approach, Freire (1998) states that students actively participate in the design of their own education in which the content is developed to strengthen their critical consciousness by searching for experiences that provide meaning to their lives, and gives them direct tools to resolve problems stemming from the conditions of their own communities.

Therefore, these experiences are chosen in a collaboration of both students and teachers in a dialogical relation that fosters the development of a critical consciousness. Learners eventually get to see the beauty of formal or abstract mathematics, but they must first learn by using it to examine and solve problems in their own communities. The use of Freire's (1972) dialogical methodology is an essential pedagogical action in developing the curricular praxis of ethnomathematics by investigating mathematical ideas, procedures, and practices developed by the members of diverse contexts and cultures in constructing curricula with people from other cultures versus imposing norms and regulations on them.

According to this approach, Freire (1998) affirms that the importance of philosophical principles of learning and life allows us to develop notions like freedom, power, individuality, and success from the logic of the dominant society, moving them into the spheres of democratic authority, social justice, and political action in order to emancipate individuals from oppression and colonization. We seek to do this in Brazil by developing mathematical abilities in educators and learners alike by applying an ethnomathematical perspective in the development of mathematics curriculum.

It is necessary to highlight that D'Ambrosio's studies in the area of sociopolitical issues established a strong relationship between mathematics, anthropology, culture, and society. For example, in 1983, D'Ambrosio was honored with the title of *Fellow of the American Association for the Advancement of Science* (AAAS) for his imaginative and effective leadership in *Latin American Mathematics Education* and his efforts towards international cooperation. Because of his development in this research area, Gerdes (1997) and Powel and Frankenstein (1997) have considered D'Ambrosio as the "intellectual father of the ethnomathematics program" (p. 13).

It is also important to point out that D'Ambrosio was also selected as one of the most important mathematicians of the twentieth century in the area of sociopolitical issues and ethnomathematics (Shirley 2000). In 2001, D'Ambrosio was the recipient of the *Kenneth O. May Medal of History of Mathematics* granted by the *International Commission of History of Mathematics* (ICHM). According to Andersen (2002), the "ICHM has awarded the May Medal to D'Ambrosio for his never-ending efforts through writing and lectures to promote Ethnomathematics and thereby contributing intensely to make the field established" (p. 1). In 2005, D'Ambrosio was awarded with the second *Felix Klein Medal of the International Commission on Mathematical Instruction* (ICMI) that acknowledges his role in the development of mathematics education as a research field.

In Brazil, ethnomathematics has been a research field, important for its own development because of its rich sociocultural environment and history. Many native peoples were surprised by the arrival of European conquerors, then had their cultural roots changed by political repression, which has ranged from a paternalistic reduction into folkloric forms to its own unique forms of suppressive as of late. Some of these cultural roots had their practices excluded from society, even criminalized (D'Ambrosio 2006). Many immigrants, women, LGBTQ's, volunteers, or non-volunteers are currently recipients of the repressive politics. Everyone, minorities, and native and foreign peoples participate in the sociocultural dynamics that is responsible for modern Brazil.

In 1550, Brazil became a major importer of enslaved African peoples. **This pattern continued** during the Brazilian colonial era. From 1650 to 1850, about 4.5 million enslaved people from Africa were taken to Brazil, which is well over ten times as many as were trafficked to North America and more than the total number of Africans who were transported to all of the Caribbean and North America combined (Curto and Soulodre-La France 2005).

The colonial era in Brazil ended in 1822 with independence from Portugal. During this period, enslaved Africans brought many mathematical, scientific, social, and cultural contributions to the Brazilian culture, indeed they were both bought and sold for their knowledge in these areas. For example, Benjamin (2006) points out that the African presence in the Brazilian culture is shown in the cooking, dance, religiosity, music, handicrafts, children's games and toys, and language.

Similarly, Rosa and Orey (2016b) affirm that an evidence of the development of mathematical knowledge of enslaved African people brought to Brazil is related to the use of their scientific and mathematical knowledge in the construction of gold mines, in the expansion of baroque art, architecture, and in the confection of musical instruments. Investigations in ethnomathematics demystify the Eurocentric view that society developed regarding African and Afro-descendants by revealing the reasons for racism and prejudice and distinguishing the different cultural traits that exist in Brazilian culture.

The enormity of the slave trade's foothold in Brazil up to the end of nineteenth century was so overpowering that the nation failed to develop an effective anti-slavery movement, even while many other nations around the world were making revolutionary reforms to end it. It was not until the late 1800s that Brazilian reformist activities began to foment in institutions of higher learning. Thus, young lawyers, students, and journalists started to urge their fellow Brazilians to follow the example of the liberation of the slaves in North America. The struggle for total abolition kept moving forward, and on May 13, 1888, Brazil was one of the last nations in the world to formally abolish slavery.

These contexts enabled the development of the holistic understanding of the nature of mathematics in the Brazilian context. However, a dilemma related to this issue is to show how to reconstruct mathematical traditions, when, probably, many of them have been, as a consequence of slavery and colonialism, wiped out. Unfortunately, much of the art, and scientific and mathematical knowledge was lost

when colonialism and the slave trade caused the disappearance and/or the *freezing*² of many local traditions.

From an ethnomathematical perspective, educators and teachers learn to integrate contributions of diverse African cultures into the school curriculum by defrosting frozen mathematical thinking in order to stimulate a reflection on the impact of colonialism on the historical and political dimensions of mathematics education (Gerdes 2014). This context enables educators, teachers, and learners to study African numbers systems, geometrical thinking, oral history, and the *Orixás*³, who are the Candomblé deities to Brazil by, especially the Yoruba people (D'Ambrosio and Rosa 2008).

The recognition of these dynamics in relation to the mathematics of *knowing* and the mathematics of *making and doing* contributes to the organization of an educational model that corresponds to the aspirations of its people. This is one of the main objectives of the ethnomathematics program in Brazil. It is crucial to ensure that mathematics is contextualized and grounded in the needs and expectations of the community that utilizes it (Rosa and Orey 2016b). For example, teaching for social justice in Brazil focuses on the context of the understanding of mathematical and scientific ideas, practices, and processes, which force confrontations in relation to assumptions about *truth* and *knowledge*, which can easily be confused with the *right* and *wrong* of science and mathematics (D'Ambrosio and Rosa 2017).

In a similar way, teaching using ethnomathematical perspectives reminds us that information may be meaningless unless it is embedded in appropriate contextual understandings. It relies on relevant political and cultural aspects of mathematics in order to develop its pedagogical action. This perspective encourages exploration, interpretation, and reconsideration regarding to what is understood about mathematics and science.

²Probably, the majority of mathematical knowledge of colonized peoples has been lost, hidden, or frozen. There is a necessity to attempt to reconstruct or unfreeze the mathematical thinking that is hidden or frozen in local techniques and strategies developed by the members of distinct cultural groups. Unfreezing frozen mathematical ideas, procedures and practices forces mathematicians and philosophers to reflect on the relationship between geometrical thinking and material production, between doing mathematics and technology (Gerdes 2014).

³In Africa, *Orixás* were the kings, queens, mythical heroes and other ancestors rose to the status of gods. In Brazil and other nations of the Americas, as in Cuba with the Santería, African deities were disguised through their association with Catholic saints in order for them to practice their own religion in spite of it being forbidden. In Brazilian Candomblé, for example, Xangô corresponds with Saint Hieronymus and Oxossi with Saint George (D'Ambrosio and Rosa 2008).

1.2 Ethnomathematics as Diverse Ways of *Knowing* and Doing Mathematics

The field of ethnomathematics links students' diverse ways of *knowing*, *doing*, and learning in culturally embedded contexts in order to bridge school mathematics. It explores academic and culturally rich ways to provide inclusive developmental programs for the diverse populations served at educational institutions. Ethnomathematics is a program that includes curricular relevance and builds curricula around the local interests and culture of the students (D'Ambrosio and Rosa 2017).

According to Rosa and Orey (2013), teaching mathematics through cultural relevance and personal experiences helps students to know more about reality, culture, society, environmental issues, and themselves by providing them with mathematical content that enable them to successfully master academic mathematics. An important change in mathematical instruction needs to take place in order to accommodate continuous and ongoing changes in the demographics of students in mathematics classrooms. It is necessary to integrate culturally relevant pedagogies into the existing mathematics curriculum.

This perspective is an essential component of culturally relevant education because it proposes that educators contextualize mathematics learning by relating mathematical content to the real-life experiences of learners (Torres-Velásquez and Lobo 2004). It is also necessary to highlight the importance of building connections between mathematics and students' personal lives and cultures. In accordance to this approach, when practical or culturally based problems are examined in a proper social context, the practical mathematics of social groups is not trivial because they reflect themes that are linked to the daily lives of students.

Ethnomathematics should focus on the role of mathematics in a sociocultural context that involves the ideas and concepts associated with ethnomathematics by using its perspective for solving real-life problems (Rosa and Orey 2008). In this regard, mathematics was for a long time regarded as a neutral and culturally free discipline removed from social values (D'Ambrosio 2006), and it was always taught in schools as a culturally free subject that involve learning supposedly universally accepted facts, concepts, and contents.

Western or academic mathematics consists of a body of knowledge of facts, algorithms, axioms, and theorems. Frequently, mathematics is referred to as a universal language. However, when people speak of universals, it is important to recognize that often something thought of as universal is merely universal to those who share the same cultural and historical backgrounds. In this context, many educators who operate under the assumption that mathematics is acultural, that it is a discipline without cultural significance, fail to see the connection between mathematics and culture (D'Ambrosio 2006).

Because mathematics in any culture has been based upon certain values and needs, students' cultural and linguistic references may interfere in the learning process of mathematical concepts in the classrooms. For example, the results of a study

conducted by Rosa (2010) show that most teachers believe that cultural background of students does not influence their performance on standardized assessments. The data reveal that these teachers do not seem to be aware of the impact of the cultural backgrounds of students on their performance on curricular activities. In their opinion, culture does not play an important role in the mathematical academic success of the students because learning is about their attitude towards mathematics. Thus, mathematics is not influenced by the students' cultural and linguistic backgrounds. It is important that educators use culturally specific contexts in teaching and learning mathematics by exposing students to a variety of cultural contexts and mathematical practices.

The pervasive view of mathematics as Eurocentric and value-free misrepresents the evolution of modern mathematics (Joseph 2000). This perception is also reinforced by students' experiences of the way mathematics is taught in schools. Educators' view of mathematics is transmitted to the students in their instruction and this fact helps to shape students' views about the nature of mathematics (Brown et al. 1990). Even though the universality of mathematical truths is not in question, it is only in the last two decades that the perception of mathematics as culture free has been challenged. In this context, "there is no sense in regarding mathematics learning as abstract and culture free" (Bishop et al. 1993, p. 1). Then, the learning process cannot be decontextualized because it cannot be free of societal influence.

The contextualization of mathematics has been described as the identification of mathematical practices developed in different cultural groups (Orey 2000). In Brazil, if mathematics is considered as a cultural construct, then it is a product of cultural development. For example, Zaslavsky (1996) affirms that this claim of mathematics as a cultural construct contradicts the aims that modern mathematics is universal, objective, and culturally neutral. This leads to the development and inquiries of culturally relevant mathematics (Nasir and Cobb 2007). Frequently teachers are unaware of the norms that govern their behavior until students do not follow their rules because they are unfamiliar with the expectations of the teachers' culture (Rosa 2010).

One goal of the ethnomathematics program in Brazil is to acknowledge that there are different ways of doing mathematics by considering the appropriation of the academic mathematical knowledge developed by different sectors of the society as well as by considering different modes in which different cultures negotiate their own mathematical practices. Hence, ethnomathematics is a program that investigates ways in which the members of distinct cultural groups comprehend, articulate, and apply ideas, procedures, and techniques that can be identified as mathematical practices.

Moreover, ethnomathematics may be described as a way in which the members from a particular culture use mathematical ideas and procedures for dealing with quantitative, relational, and spatial aspects of their lives (Barton 1996). This way of viewing mathematics validates and affirms that the experiences of the members of distinct cultural groups regarding to mathematics demonstrates that mathematical thinking is inherent to their lives. Further evidence of this assertion is offered by Orey (2000) who stated that the "paradigm that diverse cultures use or work within

evolves out of unique interactions between their language, culture, and environment” (p. 248).

Similarly, mathematical thinking as developed by different cultures is in accordance to common phenomena encountered by diverse contexts. Therefore, in order to solve specific problems, *ad hoc*⁴ solutions are created, generalized methods are developed from those solutions created to solve similar problems, and theories are developed from these generalized methods (D’Ambrosio 2011). This tendency in the Brazilian context has been to consider *ad hoc* mathematical practices as non-systematic and nontheoretical constructs. In contrast, the study of ethnomathematics underlies a structure of inquiry in the development of *local* mathematical practices by considering how these practices and problem-solving techniques can be developed into methods and theories.

In the context of ethnomathematics, members of many culturally differentiated groups *know* mathematics in ways that are quite different from academic mathematics as taught in schools. Since different types of problems are common in different cultures, the kinds of solutions, methods, and theories developed in these contexts may differ from culture to culture (Orey 2017). In this regard, it is necessary to recognize that situations that are considered as problems in a specific culture may have no meaning in other cultures. Ethnomathematics refers to mathematical ideas and procedures embedded in cultural practices developed in traditional and nontraditional societies. It recognizes that members of distinct cultural groups develop unique methods and sophisticated explanations to understand, comprehend, and transform their own realities (D’Ambrosio and Rosa 2017).

It also recognizes that the accumulated methods developed in these cultures are engaged in a constant, dynamic, and natural process of evolution and growth through the process of *cultural dynamism*⁵. Ethnomathematics is referred to here as the study of how members of various cultural groups develop techniques and procedures that help them to explain, understand, and comprehend their own world in response to problems, struggles, and endeavors of human survival (D’Ambrosio 1985). This includes material needs, arts, and spirituality through the development of *cultural artifacts*⁶, which are objects created by members of a specific cultural group that inherently give cultural clues about the culture of its creator and users.

⁴Ad hoc is a Latin expression that means *for this purpose*. It generally means a solution designed for a specific problem or task, non-generalizable, and which cannot be adapted to other purposes (Rosa 2010).

⁵Cultural dynamism refers to the exchange of systems of knowledge that facilitate members of distinct cultures to exploit or adapt to the world around them. Thus, these cultural dynamics facilitates the incorporation of human invention, which is related to changing the world to create new abilities and institutionalizing these changes that serve as the basis for developing more competencies (Rosa and Orey 2016a).

⁶*Cultural artifacts* are objects created by the members of distinct cultural groups, which inherently give cultural clues and information about the culture of its creators and users (D’Ambrosio 2011). They are also the physical manifestations or expressions of a specific culture and they include but are not limited to food, clothing, tools, art, and architecture (Rosa and Orey 2012).

This perspective “provides an important opportunity for educators to link current events and the importance of these artifacts in the context of ethnomathematics, history, and culture” (Rosa and Orey 2008, p. 33). Cultural artifacts such as art works, language, myths, and literature influence the representational system of different cultures and civilizations and are more often than not related to some form of or use of mathematics. Another presupposition of ethnomathematics is that it values all forms of mathematical explaining and understanding formulated and accumulated by the members of distinct cultural groups (D’Ambrosio 2006).

This knowledge is regarded as part of an evolutionary process of cultural dynamism as the members of each cultural group come into contact with each other (Zaslavsky 1996). This approach is related to diverse forms of mathematics that vary because of being embedded in cultural activities whose purpose is other than doing mathematics. In this regard, “ethnomathematics might be characterized as a tool to act in the world” (Orey 2000, p. 250), and as such it provides insights into the sociocultural role and nature of mathematics in the Brazilian society.

1.3 Ethnomathematics and Mathematical Practices in the Brazilian Context

The field of ethnomathematics links students’ diverse ways of knowing and learning and culturally embedded knowledge with school mathematics because it explores academic and culturally rich ways to provide more inclusive developmental programs for the diverse populations served at educational institutions (D’Ambrosio 2006). An ethnomathematics as a program includes curricular relevance and builds a curriculum around the local interests and culture of the learners (Rosa 2010).

Teaching mathematics through cultural relevance helps learners know more about reality, culture, society, environmental issues, and themselves by providing them with mathematics content and approaches that enables them to successfully master mathematics. According to Rosa and Orey (2007), an ethnomathematics approach to the curriculum is considered a pedagogical vehicle for achieving such a goal.

The field of ethnomathematics in Brazil has offered much to mathematics education in that it opposes, and/or advocate for an alternative perspective to the kinds of mathematics taught from a traditional formal school, and test preparation and orientation that is not always related to sociocultural and political aspects of mathematics. As a program, ethnomathematics seeks to understand diverse processes of thinking and ways of explaining, as well creating the awareness to reflect on social and political dimensions of mathematics.

A perspective that offers us important viewpoints for the development of dynamic and *glocalized*⁷ communities, which allows us to recognize that all cultures and all people develop unique methods and explanations that allow them to understand, act, and transform their own reality (Rosa and Orey 2019). Looking at mathematical ideas, procedures, and practices from different cultures enables learners to perceive mathematics in ways that are not always valued or distinguished in the traditional curriculum.

In this context, Rosa (2010) argued that educators need to be supportive in constructing understanding between mathematics and the language and culture of students in order to encourage the development of reflective and critical thinking abilities. Currently, both a greater and more sensitive understanding of mathematical ideas, procedures, and practices developed by the members of diverse cultural groups have become available through the growth of and study in the fields of multiculturalism, anthropology, linguistics, and ethnomathematics.

The term ethnomathematics was officially coined by D'Ambrosio (1985) to describe the mathematical practices of identifiable cultural groups and is regarded as the study of mathematical ideas found in any cultural context. The search for solutions for specific problems that help the development of mathematics are always imbedded in a cultural context because in order to understand how mathematics (*tics*) is created, it is necessary to understand the problems (*mathema*) that precipitate it in each diverse context.

In order to understand those problems (*mathema*), it is necessary to consider the cultural context (*ethnos*) that drives them. Ethnomathematics as the study of mathematical ideas developed by different sociocultural groups offers a contextualization of the curriculum that contributes to the elaboration of pedagogical practices in multicultural classrooms (Rosa and Orey 2007) towards social justice. One of the characteristics of ethnomathematics is to help in the development of concepts of what mathematics really is through an understanding of its deep connection with culture (D'Ambrosio 1985).

The purpose of ethnomathematics is to trace the development and transformation of mathematical ideas by developing research on how ethnomathematical perspectives in the mathematics curriculum contributes to a new approach on mathematics education. In the Brazilian context, an ethnomathematics program offers a very broad view of mathematics, a view that embraces tacit knowledge, ideas, processes, procedures, techniques, methods, and practices related to different cultural environments. This aspect leads to increased evidence of cognitive processes, learning capabilities, and attitudes that may direct learning processes occurring in many mathematics classrooms. For example, according to Masingila and King (1997), ethnomathematics becomes a workable tool that helps students to make connections and develop deeper mathematical understanding.

⁷*Glocalization* is the ability of a culture, when it encounters other cultures, to absorb influences that naturally fit into and can enrich that culture, to resist those things that are truly alien and to compartmentalize those things that, while different can nevertheless be enjoyed and celebrated as different (Rosa and Orey 2016a).

In this context, Masingila and King (1997) also argued that ethnomathematics helps students to learn about procedures of *other* peoples as well as develop deeper understandings, indeed come to value, their own practices. For Rosa and Orey (2007), as students learn about the culture of *other* peoples, they can also learn about their mathematical knowledge, since mathematics is an integral part of their own culture.

Ethnomathematics may be used as a tool to motivate disenfranchised students to pursue a study of mathematics (D'Ambrosio 2006). It enables the achievement of two objectives in mathematics teaching:

- (a) It can establish a multicultural context for the development of mathematical knowledge and skills.
- (b) It can help students in making connections among other disciplines (D'Ambrosio and Rosa 2008).

Consequently, learners begin to maximize their own possibilities for improving their mindset towards mathematics as they are improving their skills. It is very much connected to perspective and self-esteem. In this regard, Rosa (2010) argues that ethnomathematics is a program through which teachers may create lasting positive impact on affective and cognitive domains of students who are underachieving in academic mathematics.

Ethnomathematics investigates the ways in which different cultural groups comprehend, articulate, and apply ideas and concepts that can be identified as mathematical practices (Barton 1996). The very essence of this program is to acknowledge that there are different ways of doing mathematics by taking the time to consider the appropriation of the academic mathematical knowledge developed by different sectors of society. As well, it considers different modes in which diverse cultures negotiated and developed their own unique problem-solving techniques, mathematical ideas, procedures, and practices (D'Ambrosio 2011).

Moreover, ethnomathematics may be described as a way in which people from a particular culture use mathematical ideas and concepts for dealing with the quantitative, relational, and spatial aspects of their lives. In this context, mathematics values the diverse cultural experiences by demonstrating how mathematical thinking is inherent to all our lives, cultures, and places. Mathematics is very much identified in cultural activities and is found in both traditional and nontraditional societies. This refers to the mathematical concepts embedded in cultural practices and it recognizes that all cultures develop unique methods and sophisticated explications to understand, comprehend, and transform their own reality (Orey 2017).

Hence, ethnomathematics means the study of how people within various cultural groups come to develop techniques used to explain and understand the world in response to unique history, location, problems, struggles, and endeavors (D'Ambrosio and Rosa 2008). This approach includes the kinds of mathematics that assists in meeting material needs as well as art and spirituality through the use of the development of artifacts, which are objects created by members of a specific cultural group that inherently give cultural clues about the culture of its creator and users. This perspective provides an important opportunity for teachers to link