Springer Proceedings in Mathematics & Statistics

Vera Viana
Vítor Murtinho
João Pedro Xavier *Editors*

Thinking, Drawing, Modelling

GEOMETRIAS 2017, Coimbra, Portugal, June 16–18

Springer

Springer Proceedings in Mathematics & Statistics

Volume 326

Springer Proceedings in Mathematics & Statistics

Vera Viana · Vítor Murtinho · João Pedro Xavier
Editors

Thinking, Drawing, Modelling

GEOMETRIAS 2017, Coimbra, Portugal, June 16–18

Springer

Editors
Vera Viana
Faculdade de Arquitectura and Centro de Estudos de Arquitectura e Urbanismo
Universidade do Porto
Porto, Portugal

Associação dos Professores de Geometria e de Desenho
Aproged
Porto, Portugal

João Pedro Xavier
Faculdade de Arquitectura and Centro de Estudos de Arquitectura e Urbanismo
Universidade do Porto
Porto, Portugal

Vítor Murtinho
Departamento de Arquitectura and Centro de Estudos Sociais
Universidade de Coimbra
Coimbra, Portugal

ISSN 2194-1009 ISSN 2194-1017 (electronic)
Springer Proceedings in Mathematics & Statistics
ISBN 978-3-030-46803-3 ISBN 978-3-030-46804-0 (eBook)
https://doi.org/10.1007/978-3-030-46804-0

Mathematics Subject Classification (2010): M34000, M21006, M13003, K0000X, I23036

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Preface

The International Conference Geometrias'17, held in the Department of Architecture of the University of Coimbra, between the 16 and the 18 of June, 2017, can be regarded as (another) corollary and proof that geometry stands, to this day, as a subject of the utmost importance, through which scholars, researchers, specialists and students are continuously challenged and motivated in their professional procedures, teaching practices and scientific investigations. The prominence of digital technologies in every practice related to architecture, arts and engineering is an undeniable and welcomed fact, but one may recognize that there has never been, as much as today, such an awareness on the need for a well-informed reasoning on the representational procedures as an essential requirement to ensure the conscious developments in scientific and technological researches. Conceived as one more contribution to this discussion, the leitmotif *Thinking, Drawing, Modelling* for an International Conference with a call for contributions, revealed itself as a successful strategy to bring together many scholars and investigators that actively work upon these matters and hold geometry, in its broader sense, as common concern.

The conference was a firm testimony of the importance of form-finding traditional and innovative methodologies, as well as a moment for discussions on the procedures involved in the conceptualization of objects as creative outcomes of new materialities and artistic concepts. In fact, much of the production in architecture, arts or engineering is anchored in technologies that firmly entwine with the science of representation. It is precisely within this innovative milieu that new dynamics are being generated every day, with inspiring ground-breaking ideas to stimulate more challenges, new synergies, different frameworks and inventive forms. Challenged by these new energies, the impact that virtual environments outline not only in project methodologies, but also in its concretization in space should not to be undermined.

Geometrias'17 gathered keynote speakers and authors that have been producing some of the best scientific practices concerning geometry, drawing and digital knowledge, and this, by itself, was a notable starting point that settled the pace for the quality of the contributions presented in this unique event. This book combines a selection of the outcomes of this International Conference, including three papers authored by keynote speakers and nine others, authored by scholars, researchers and students from six European countries. The following paragraphs will try to briefly summarize the content of each research.

Maurizio Barberio, in "Prototyping Stereotomic Assemblies: Stone Polysphere", demonstrates the immense potential of digital fabrication for the exploration of stereotomic architecture. Barberio illustrates the geometrical transformations through which the stone installation *PolySphere* was conceived with the spherical icosidodecahedron as point of departure, and three-dimensional modelling and algorithmic software as recurring tools.

In "Geometry and Digital Technologies in the Architecture of Herzog & De Meuron. The Project for the Stamford Bridge Stadium in London", Alexandra Castro proposes an interesting perspective on the work of these noteworthy swiss architects, identifying how the exploration of digital technologies developed in their working procedures to the current status, that significantly recognize how modern technologies bring advantages for their architectural project methodologies.

In "The Dome as Minimal Housing Unit: "Ghibli" and "D-Home" Prototypes", Micaela Colella combines the potential of three-dimensional modelling with digital fabrication towards the development of prototypes for housing units, conceived from the discretization of domes, which, as shelter modules, may undergo severe climacteric conditions.

With a long and fruitful activity in research and pedagogy concerning drawing and representation, Lino Cabezas Gelabert addresses the procedures and methods of representation explored since Middle Ages, with the lecture "Geometry and Art". The fact that, in medieval methods of representation, geometry, in its instrumental component, was recurrently explored as much as in its conceptual counterpart, might be justified by the strong tendency to mysticism of this period of humankind's history. In fact, the methods of representation of religious architecture in that era, with many geometrized forms and rigorous metrical systems, developed into its known form, because of the scientific procedures chosen for the representation and control of space.

Soraya M. Genin's lecture, "The Vaults of Arronches Nossa Senhora da Assunção and Misericordia Churches. Geometric and Constructive Comparison with the Nave and Refectory Vaults of Jerónimos Monastery", describes hypotheses for the complex systems that might have been used to constructively resolve the enlargement of space in certain religious buildings of the first half of the sixteenth century. Regarding probable methods of drawing for the concretization of different kinds of vaults, Genin establishes interrelations between constructive methodologies for arched structures and ribbed solutions. Illustrating an historical journey with a number of examples with different degrees of complexity, the author theorizes about the methods of drawing known and the geometry that, in its essence, allowed its materialization in space.

In "Perspective Transformations for Architectural Design", Cornelie Leopold addresses the relations between space and image from the logic of perception of the architectonic conception procedures. This paper establishes numerous connections between architecture and mathematics, examining situations where perspective drawing precedes and mediates creative workflows in which space is created and manipulated.

Joana Maia and Vitor Murtinho expose some of the the procedures adopted by a Portuguese architect, in "Ordered Creativity: The Sense of Proportion in João Álvaro Rocha's Architecture". Starting with the analysis of his creative process and working methodologies, this paper provides an understanding of the peculiarity of his working process, in which grids, as well as metric and proportional systems are intentional recurring resources with a significant impact in the architect's creativity.

Andrés Martín-Pastor and Alicia López-Martínez present the paper "Developable Ruled Surfaces from a Cylindrical Helix and their Applications as Architectural Surfaces", to demonstrate how a set of surfaces, with certain curvature and geometrical definition, are extremely appropriate to perform as light architectural structures.

Hannah Müller, Christoph Nething, Anja Schalk, Daria Kovaleva, Olivier Gericke and Werner Sobek, develop the theme "Porous Spatial Concrete Structures Generated Using Frozen Sand Formwork", a research still at an experimental stage, that focuses on the exploration of hydroplotting to conceive complex concrete forms as spatial structures or architectural objects.

José Pedro Sousa, a renowned specialist in generative geometries, through "Calculated Geometries. Experiments in Architectural Education and Research", depicts a series of examples in which geometry and technology were crucial for the development of form. Combining traditional methodologies with robotic fabrication, his research projects highlight the great potential of these innovative methods in the act of thinking and materializing architecture. For educational and research purposes, these *Calculated Geometries* emerge as powerful instruments for the exploration of new forms in architecture and reveal themselves essential to expand, into unforeseen outcomes, the magnificent art of manipulating space.

Monika Sroka-Bizoń, in "How to Construct the Red Sea?", reflects upon the concepts that led to the materialization of a successfully accomplished example of the structural importance of geometry in free-form architecture, the Museum of the History of Polish Jews, in Warsaw. The author presents the methodology explored by the Finish architects Rainer Mahlamäki and Ilmari Lahdelma who, through architectural effects of intrinsic complexity, symbolically materialize *Yum Suf,* the parting of the *Red Sea* that led to the escape of the Jews from Egypt, combining a steady regular exterior shape with interior curved surfaces, structurally intertwined with Bezier, B-spline and NURBS surfaces.

With "How to Improve the Education of Engineers—Visualization of String Construction Bridges", Jolanta Tofil presents some examples on how the exploration of CAD software and its visualization features may assist engineering students to combine their knowledge and practice for conceiving string construction bridges, with the input of architectural design procedures. In reality, the inherent potential of the possibility of previewing any structural and architectonic solution unequivocally allows us to predict better solutions, thanks to a more complete and well-informed understanding of the project, which, in itself, fosters the enhancement of the knowledge of both structure and form.

In conclusion, as all the papers combined in this book aimed to demonstrate, *Thinking, Drawing and Modelling* remain still as fundamental activities that stand in the origin of every act leading to the creation of form and its materialization. As such, geometry, drawing and the sciences of representation prevail as pertinent matters whose discussion has a great future in sight. Three-dimensional modelling and algorithmic software are magnificent proposals that are paving the way to great challenges in our world, but one cannot ignore that stereotomy, drawing, the science of representation and geometric literacy remain still as limitless inspiring sources of knowledge, so fundamental as much as inevitable, for every professional and scholar committed to the investigation and innovation in geometry-related settings.

The Editors

Porto, Portugal — Vera Viana
vlopes@arq.up.pt

Coimbra, Portugal — Vítor Murtinho
vmurtinho@uc.pt

Porto, Portugal — João Pedro Xavier
jxavier@arq.up.pt

Acknowledgements

The Editors, Vera Viana, Vítor Murtinho and João Pedro Xavier, express their immense gratitude for the contribution of the reviewers listed below which, through their acute experience and knowledge, certified the scientific quality, not only of the Geometrias'17 Conference, but also of its published outcomes: the book "Thinking, Drawing, Modelling" and Aproged's Bulletin #34.[1]

Giuseppe Amoruso, Dipartimento di Design—Politecnico Milano, Italy
Antonio Luis Ampliato, Universidad de Sevilla, Spain
Javier Barrallo, University of the Basque Country, Spain
Maria Francisca Blanco, Universidad de Valladolid, Spain
Vasco Cardoso, Faculdade de Belas Artes da Universidade do Porto, Portugal
Graciela Colagreco, Universidad Nacional de la Plata, Argentina
Manuel Couceiro da Costa, Faculdade de Arquitectura da Universidade de Lisboa, Portugal
Giuseppe Fallacara, Politecnico di Bari, Italy
Lino Cabezas Gelabert, Facultat de Belles Arts, Universitat de Barcelona, Spain
Soraya Genin, ISCTE-Instituto Universitário de Lisboa, Portugal
Filipe Gonzalez, Escola de Arquitectura e Artes da Universidade Lusíada de Lisboa, Portugal
Encarnación Reyes Iglesias, Universidad de Valladolid, Spain
Andreas Kretzer, Technical University of Kaiserslautern, Germany
Cornelie Leopold, Technical University of Kaiserslautern, Germany
João Ventura Lopes, ISCTE—Instituto Universitário de Lisboa
Pedro Martins, Faculdade de Arquitectura da Universidade do Porto, Portugal
Luís Mateus, Faculdade de Arquitectura da Universidade de Lisboa, Portugal
Helena Mena Matos, Faculdade de Ciências da Universidade do Porto, Portugal
Riccardo Migliari, La Sapienza, Universitá di Roma, Italy

[1]The digital version of this publication is available in https://www.aproged.pt/geometrias17/boletim34digital.pdf.

Gonçalo Canto Moniz, Faculdade de Ciências e Tecnologia da Universidade de Coimbra, Portugal
Vítor Murtinho, Faculdade de Ciências e Tecnologia da Universidade de Coimbra, Portugal
Ratko Obradovic, University of Novi Sad, Serbia
Alexandra Paio, ISCTE—Instituto Universitário de Lisboa Portugal
Teresa Pais, Faculdade de Ciências e Tecnologia da Universidade de Coimbra, Portugal
Eliana Manuel Pinho, CEAU—Faculdade de Arquitectura da Universidade do Porto, Portugal
Luís Marques Pinto, Universidade Lusíada do Porto, Portugal
Francisco González Quintial, Escola Técnica Superior de Arquitectura Universidad del País Vasco, Spain
Rinus Roelofs, Netherlands
José Ignacio Rojas Sola, University of Jaen, Spain
José Pedro Sousa, Faculdade de Arquitectura da Universidade do Porto, Portugal
Vera de Spinadel, Mathematics and Design Association's, Argentina
Hellmuth Stachel, Technische Universität Wien, Austria
António Oriol Trindade, Faculdade de Belas Artes da Universidade de Lisboa, Portugal
Pedro de Azambuja Varela, Faculdade de Arquitectura da Universidade do Porto, Portugal
Gunter Weiss, Technische Universitat Dresden, Germany
João Pedro Xavier, Faculdade de Arquitectura da Universidade do Porto, Portugal

Contents

Prototyping Stereotomic Assemblies: Stone Polysphere

Maurizio Barberio

Abstract The paper describes the parametric design, fabrication and construction of the Stone PolySphere, an installation that investigates the potential of digital fabrication applied to the stone industry. It is a lithic sphere with a diameter of 1.4 m, composed by a massive hemisphere below and a stereotomic hemisphere above. The prototype ideally summarizes, in a single object, the two big trends in stone architecture: the megaliths (below) and the stereotomic assemblies (above). The upper part is a micro-architecture that represents a domed space. The research also shows a new workflow for the realisation of non-reciprocal stereotomic geodesic assemblies, in which are used complex holed voussoirs generated by means of *simple* morphing operations and recursive subdivisions. The fabrication of the voussoirs consists in milling of several layers of stones glued together. The results are critically discussed, and the implications of the parametric design process are pointed out.

Keywords Stereotomy · Stone · Parametric design · Digital fabrication · Sphere

1 Background

Throughout history, the domed space has always been the ideal field for more sophisticated and complex studies about the construction of architecture. This is particularly true for the stereotomic architecture. Stereotomy, from the Greek: $\sigma\tau\epsilon\rho\epsilon\acute{o}\varsigma$ (*stereós*) "solid" and $\tau o\mu\acute{\eta}$ (*tomē*) "cut", is the art and science of cutting three-dimensional solids into particular shapes [1]. The intrinsic quality of the domed architecture resides in its immediate ability to define measurable areas, which can serve as the endpoint for the indeterminacy of a generic space outside the building [2]. Historically, a stone dome is usually built laying voussoirs by rows. This building method is very common throughout the history of architecture and construction, and it has been widely used since ancient times [3]. More recently, several scholars have studied another way to build a stone dome, by using geodetic tessellations. Historically, they

M. Barberio (✉)
New Fundamentals Research Group, Politecnico di Bari, Bari, Italy
e-mail: mb@newfundamentals.it

V. Viana et al. (eds.), *Thinking, Drawing, Modelling*,
Springer Proceedings in Mathematics & Statistics 326,
https://doi.org/10.1007/978-3-030-46804-0_1

have been used in architecture since the last century, mostly thanks to the work of Buckminster Fuller. Regarding stereotomy, it is possible to use two types of geodesic tessellations: reciprocal and non-reciprocal. Reciprocal tessellations employ interlocking blocks with a specific geometry that are arranged such that the inner end of each block rests upon and is supported by the adjacent block. On the other hand, non-reciprocal tessellations ensure stability thanks to the friction between the contact faces of blocks and the overall shape of the structure itself. The first built example of a geodesic stereotomic dome made of interlocking reciprocal stone blocks is the *Bin Jassin Dome*, built in 2012 in Qatar, and designed by Giuseppe Fallacara [4]. An example of a geodesic non-reciprocal dome has been recently studied by Roberta Gadaleta [5] for her doctoral dissertation.

2 Research Topic

Stone PolySphere is an installation that investigates the potential of digital fabrication applied to the stone industry. It is a lithic sphere with a diameter of 1.4 m, composed by a massive hemisphere below and a stereotomic hemisphere above. The prototype ideally summarizes, in a single object, the two big trends in stone architecture: the megaliths (below) and the stereotomic assemblies (above). The upper part of the PolySphere is a micro-architecture that represents the domed space.[1] As such, this case-study is useful to investigate the following themes:

- Using not reciprocal geodesic tessellations to build domed spaces;
- Producing voussoirs through milling of several layers of stones glued together;
- Generating complex holed voussoirs through simple morphing operations and recursive subdivisions;
- Setting specific algorithms in order to automatically generate the files necessary for the fabrication by means of CNC machines.

3 Computational Workflow

The computational workflow is based on a parametric code which has a base polyhedron as input, and all the three-dimensional lithic elements to be fabricated as output. The software used to accomplish the algorithm is Grasshopper™, a parametric plug-in developed for the commercial software Rhinoceros™. From a geometric point of view, Stone PolySphere (Fig. 1) is generated by the geodetic projection of a polyhedron, the icosidodecahedron, whose faces are tessellated as follows:

[1] It is important to specify that the research intent was not to make a scaled-down model of a real dome, but a sphere divided into voussoirs like a real dome. For this reason, structural analysis and material tests were not carried out.

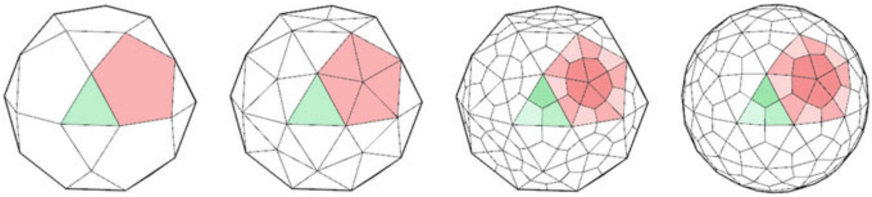

Fig. 1 Subdivision steps and geodesic projection of the base polyhedron

All the pentagonal faces were divided into triangles;

1. Each of these triangles, as well as the icosidodecahedron's triangular faces, were divided into three kites, by joining the middle of each side to the centroid of the triangle itself (Catmull–Clark subdivision, level 1), thus generating the quadrilateral meshes;
2. All vertices of the kite-shaped meshes are projected onto the surface of the circumsphere. These vertices moved along the direction of the vectors formed by connecting each vertex with the centroid of the sphere.

This geometric construction is identical for the whole sphere, but is processed in two different ways for the lower part (massive) and the upper one (stereotomic). From a computational point of view, however, the parametric definition necessary to the creation of the three-dimensional model is conceptually identical. In fact, both the voussoirs of the upper part and the base-surfaces are obtained through the following operations: recursive subdivision of the base-pattern and morphing of the base-pattern used to "populate" the surface. Morphing operations are commonly used in parametric modelling, and they are generally employed to tessellate a given surface, dividing it into portions of square or rectangular-based prisms (cuboids or "boxes"). To each box, a generic geometric base-pattern is associated (a sphere, a cube, etc.). In other words, this operation is used to transfer a geometric base-pattern—usually three-dimensional—onto a tessellated surface. The process is well known among Grasshopper's users [6]. Figure 2 shows the typical workflow used to tessellate a surface through morphing operations.

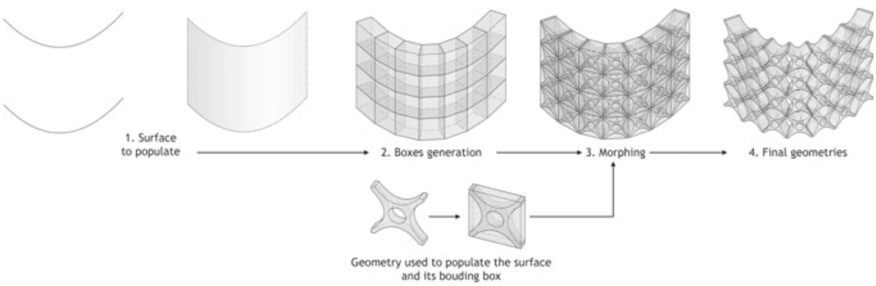

Fig. 2 Workflow for tessellating a generic surface by means of morphing operations

Most of the time, however, the process is used more on formal speculations than in constructive purposes. This happens because it is possible to tessellate the base-surface using only quadrangular-based prisms and the tessellation of the surface is not generally optimized by the geometric and constructive point of view. Consequently, this process of modelling leads to non-optimized tessellations of free-form surfaces, obtaining structural patterns deformed by morphing operations that not only are inaccurate from a formal point of view, but also not easily manufacturable.

However, this research aims to demonstrate that this *simple* modelling strategy can lead to interesting and accurate results, able to associate morphing operations with more complex spatial tessellations, as the kite-based prismatic explored in this research. As such, the workflow outlined for generating the voussoirs of the stone PolySphere was the following (Fig. 3):

1. The input mesh was analysed to check the flatness of the faces. In order to optimize the three-dimensional model for fabrication purposes by means of CNC machines, it is recommended that each voussoir has, at least, its inner face flat. This is fundamental to put the raw block on the CNC base platform and process it without rotations or the aid of supports;
2. If the intradosal faces are not flat, planarity is obtained if the distance between face diagonals is 0. This process was achieved by moving the points of each face until they were all in the same plan. The mesh was then processed using the add-on *Kangaroo Physics* [7];
3. Afterwards, the offset mesh was generated;
4. The boxes used for the morphing operations were generated;
5. The base voussoir, which will be subjected to morphing operations, had to be modelled in this step. In the specific case of the stone PolySphere, the basic voussoir was modelled starting from a squared mesh divided into four triangles, with two of them perforated. The basic voussoir was intentionally modelled with the minimum number of faces possible, in order to be gradual and recursively divided through an algorithm based on the Catmull-Clark algorithm [8]. This allowed us to obtain a uniformly smoothness for the voussoirs at the point where they are perforated, increasing the number of subdivisions of the coarse mesh.
6. Generation of the voussoirs to be produced.

It is important to point out that the use of this parametric workflow is justified when: the geometry (the voussoir) to be transferred to the tessellated surface is topologically equivalent to all the blocks or families of voussoirs; there is no need to locally manage the variation of one of the fundamental parameters of the geometry to be transferred (for example, in this case, the size of the openings); and the geometry to be transferred onto the tessellated surface has such a degree of complexity, that obtaining it with other modelling methods would be very difficult or even impossible.