

Solid Mechanics and Its Applications

Alberto Carpinteri

Fracture and Complexity

One Century since Griffith's Milestone

 Springer

Solid Mechanics and Its Applications

Volume 237

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ISSN 0925-0042 ISSN 2214-7764 (electronic)
Solid Mechanics and Its Applications
ISBN 978-94-024-2024-1 ISBN 978-94-024-2026-5 (eBook)
<https://doi.org/10.1007/978-94-024-2026-5>

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The registered company address is: Van Godewijckstraat 30, 3311 GX Dordrecht, The Netherlands

Divergence Syndromes

“Almost every case study we perform involves a divergence syndrome. That is, some quantity that is commonly expected to be positive and finite turns out either to be infinite or to vanish. At first blush, such misbehavior looks most bizarre and even terrifying, but a careful re-examination shows it to be quite acceptable (...), as long as one is willing to use new methods of thought”

Benoit B. Mandelbrot (1982), “The Fractal Geometry of Nature”, W. H. Freeman & Company, New York, page 19.

Preface

Complexity, as a transversal scientific discipline, generally refers to the study of systems with many interacting components, in which the global behaviour is qualitatively different from (and not encoded in) the behaviour of the single components. Actually, researchers did not come to a univocal definition of *Complexity*, since it manifests itself in several different ways. *Complexity* can not represent a homogeneous and autonomous discipline. As a matter of fact, under the label of *Complexity Sciences* we comprehend a broad variety of phenomena, theories, approaches, and techniques: nonlinear dynamics, deterministic chaos, nonequilibrium thermodynamics, fractal geometry, fractional calculus, intermediate asymptotics (complete and incomplete self-similarity), renormalization group theory, catastrophe theory, self-organized criticality, neural networks, cellular automata, fuzzy logic, etc.

Complex Systems lie somehow in between perfect order and complete randomness –the two extreme conditions that occur only very seldom in nature– and exhibit several common features, such as: sensitivity to initial conditions, pattern formation, spontaneous self-organization, emergence of cooperation, emergence of hierarchical or multi-scale structures, collective properties emerging beyond those contained in the single parts, size-scale effects, etc.

Complexity Sciences are a subject of fast growing interest inside the international scientific community. In particular, the most interesting and even intriguing mechanical behaviours and failure phenomena in solids and structures can be synthetically captured and consistently interpreted through the use of innovative conceptual tools in the framework of *Complexity Sciences*. Aim of the present book is that of providing an insight into the role of *Complexity* in the fields of *Strength of Materials* and *Fracture Mechanics*. The book is divided into three parts. While the first deals with the traditional concepts of stress-concentration and stress-intensification at the tip of a notch or crack (local stress-singularity approach), and with the corresponding total potential energy release rate (global energy-based approach), the remaining two deal with the opposite natural trends in composite systems: catastrophe and chaos arising from simple nonlinear rules, as well as order and structure emerging from heterogeneity and randomness.

Part I (Chaps. 1–5) deals with the most classical concepts of *Linear Elastic Fracture Mechanics* (LEFM), at the same time preparing the ground for more advanced and sophisticated interpretations based on and inspired by *Complexity Sciences*. After introducing the concept of stress-concentration and its limit case of stress-intensification for curvature radii at the notch root tending to zero (Inglis paradox), the energy criterion by Griffith is recalled, for which a dimensional disparity between surface energy and elastic strain energy density appears. From this fundamental dimensional asymmetry descends the self-similarity loss in collinear crack propagation. On the other hand, an alternative manner to address the problem is that of considering the stress-singularity with its characteristic negative power in the radial distance from the crack tip or re-entrant corner vertex (exponent ranging between $-1/2$ and zero). The most severe stress-singularity of order $-1/2$ is achieved only for sharp cracks in linear elastic materials. Its attenuation is predicted for power-law hardening materials and/or for re-entrant corners, up to reach the value zero for rigid-perfectly plastic materials and/or for limit flat angles. Direct consequence of the singular stress field is the strength versus size-scale bi-log linear decreasing diagram with slope equal to the negative value of the exponent. Upper bound to the absolute value of the slope is the limit value $1/2$. The stress-intensity factor presents anomalous physical dimensions in order to establish dimensional homogeneity in the formulas representing the singular stress field. Therefore, the stress-intensity factor plays the role of a renormalized stress, making easier to accept the idea of a stress divergence at the crack tip. As Griffith solves the Inglis paradox by considering an energy balance between the differentials of two quantities respectively proportional to the crack length and to its square, so does Irwin by applying a renormalization group transformation to the singular stress field. In both cases, the dimensional disparity between tensile strength and surface energy or fracture toughness represents the fundamental reason for scale effects. In addition, Irwin himself discovers the deep connection between the two alternative approaches to *Fracture Mechanics* through his well-known theorem and quadratic relationship between stress-intensity factor and strain energy release rate. Also the cases of nonlinear elastic materials (*J-integral*) and of nonsymmetrical crack loading or noncollinear crack growth (*Mixed Mode*) are presented in the book. In the case of a statistical population of micro-cracks, the mixed mode locus associated to a frictional criterion can explain the nonconvexity of the resistance loci in the principal stress plane for cementitious materials.

Part II (Chaps. 6–10) deals with *Nonlinear Fracture Mechanics* models (in particular, the *Cohesive Crack Model* to describe strain localization both in tension and in compression) and their peculiar consequences: *fold catastrophes* (post-peak strain-softening and snap-through instabilities) or *cusp catastrophes* (snap-back instabilities) in plain or reinforced structural elements. Important applications to civil structural engineering are shown: evaluation of the minimum and maximum reinforcement percentages for concrete elements in flexure, in order to avoid brittle phenomena of cracking (edge in tension) or crushing (edge in compression); plastic rotation capacity of reinforced concrete beams; brittle debonding or delamination between concrete and external retro-fitting plates; detachment and slippage between

matrix and fibre or re-bar. In such a context, *superplasticity* in micro-crystalline materials can be interpreted as a snap-back condition. Also dynamic systems showing fractal attractors as well as transitions to chaos and period doubling are considered in the book. How can a relatively simple nonlinear constitutive law, which is scale-independent and does not contain built-in randomness, generate a size-scale dependent ductile-to-brittle transition, or a completely unpredictable dynamic behaviour? Constant reference is made to Dimensional Analysis and to the definition of suitable nondimensional *brittleness numbers* that govern the transition. These numbers can be defined in different ways, according to the selected theoretical model. The simplest way is that of directly comparing critical LEFM conditions and plastic limit analysis results. This is an equivalent way –although more effective for finite-sized cracked plates– to describe the ductile-to-brittle size-scale transition, if compared to the traditional evaluation of the crack tip plastic-zone extension in an infinite plate. In extremely brittle cases, the plastic zone or process zone tends to disappear and the cusp catastrophe conditions prevail over the strain-softening ones and tend to coincide with the LEFM critical conditions in the case of initially cracked plates.

Part III (Chaps. 11–15) deals with the occurrence of self-similar and fractal patterns in the deformation, damage, fracture, and fragmentation of heterogeneous disordered materials, and with the consequent apparent scaling in the nominal mechanical properties of the same materials. Such a scaling is negative (lacunar fractality) for tensile strength and fatigue limit, whereas it is positive (invasive fractality) for fracture energy, fracture toughness, and fatigue threshold. At the same time, corresponding fractal (or renormalized) quantities emerge, which are the true scale-invariant properties of the material. They appear to be the constant factor (the universal property) in the power-law relating the nominal canonical quantity to the size-scale of observation. When the reference sets from self-similar become self-affine, we obtain *Multi-fractal Scaling Laws*, which are asymptotic and present a decreasing fractality for increasing structural sizes. They reproduce the experimental data very consistently. Another relevant aspect of *Complexity* concerns *Criticality* in the acoustic (and, more generally, energy) emissions from damaging structures, as well as in the seismic activity of the Earth's Crust. The emergence of scaling and fractality is emphasized for the crack or fault distributions in both space and time. On the other hand, *Critical Phenomena* are always associated to the emergence of self-similar or self-affine patterns, to fractal (renormalized) or multi-fractal quantities, and to spontaneous self-organization. Typical examples are represented by: phase transformations, laminar-to-turbulent fluid flow transitions, avalanches in granular media, earthquakes, micro-cracking and fracture in structural materials. In this context, the *defect size distribution of self-similarity* can be defined, for which the largest defect is statistically proportional to the characteristic structural size. It intrinsically represents a critical condition by increasing the structural size-scale. In a fractal framework, it is then possible to define a scale-invariant constitutive law: the so-called *Fractal Cohesive Crack Model*, in which stress and strain are defined over lacunar fractal sets and the fracture energy in an invasive fractal set, which is the Cartesian product of the two previous sets. The slopes of the related bi-log scaling

laws are represented by the difference between the fractal dimension and the entire canonical dimension of the reference sets. In the case of stress or strength, the upper bound to the absolute value of the slope is equal to $1/2$ again, and represents the condition of maximum disorder at the smallest scales. It is given by the condition of self-similarity in the defect size distribution, i.e., by the LEFM stress-singularity, or, alternatively, by the invasive fractal dimension of the most disordered *Brownian surface* (allowing macro-crack kinematics of opening and closure), which is equal to 2.5. It is not difficult to prove that the corresponding most disordered ligament or resisting cross-section has a lacunar fractal dimension equal to 1.5. This means that its dimensional decrement with respect to the entire canonical dimension 2 is equal to 0.5, i.e., the slope is just the same as that deriving from the LEFM application. Such a convergence of values originating from different assumptions is an evidence of the validity and consistency of the scientific framework where this subject has been developed. Another relevant issue, which is tackled in the book, is the evolution of the fractal dimension of a micro-crack network. Initially, it could be assumed to be between 2 and 3. Increasing the load, it increases up to reach its maximum value 3 (*criticality condition* of smeared volumetric damage), and then decreases down to the value 2 (localization to a fracture surface). At the maximum, micro-cracks start to coalesce while a macro-crack starts to form. The *b-value of the Gutenberg-Richter distribution* is equal to one-half of the fractal dimension of the micro-crack network, and therefore it is demonstrated to evolve from 1.5 to 1.0, going from criticality to final fracture. In the case of a macro-crack propagation, the critical condition may be preceded by a condition of *Sub-critical Crack Growth*, which is generally due to *fatigue* and/or *creep* loading conditions. The well-known fatigue curves of Wöhler and Paris can be renormalized in both axes, including the crack growth rate and excluding the number of cycles, so that a collapse of the scale-dependent curves occurs onto a single renormalization curve that approximates the experimental results very satisfactorily. In this context, the *fatigue threshold* tends to zero for the crack length tending to zero, so providing a very consistent solution to the still open *short crack problem*. Analogously, the *fatigue limit* tends to zero for the specimen size tending to infinity.

The book presents also the applications of *Fractional Calculus* to fractal and nonlocal media, as well as an insight into the phenomena that might occur artificially or naturally after and beyond the material compression failure: *fragmentation* and *comminution*. They describe properly what happens in rocks during drilling perforations or the superficial wear in technological materials (see the well-known case of MIR space station). The phenomena of impact and explosion are also contemplated in the book.

My personal studies and investigations on the topics considered in the book started in 1975 at the Institute of Structural Mechanics of the University of Bologna in Italy, under the supervision of Professor Angelo Di Tommaso. During the following years, I had the opportunity to collaborate and to have a scientific debate with an increasing number of people and within a very diversified set of contexts. Very important and influential was for me the sabbatical academic year 1982–1983

that I spent at the Institute of Fracture and Solid Mechanics of Lehigh University in Pennsylvania (USA), under the supervision of Professor George Sih. I am deeply indebted to both my mentors, who, with their experience, guided me in my early steps and helped me in the selection of the initial basic research subjects. In 1986, I moved to the Department of Structural Engineering of the Politecnico di Torino in Italy as a Chair Professor of Structural Mechanics. Since that time on, my research work from individual became team-supported and I had the possibility to collaborate with many different people on various subjects and with different methodologies. Most of the subjects were enriched by effective experimental investigations. In particular, the evaluation of the minimum reinforcement in concrete beams, a major subject of research in my new academic context, and, later, the plastic rotation capacity of reinforced concrete beams were studied on the basis of my previous fracture mechanics models. Professor Franco Levi, through his master competence and open-minded attitude, convinced me that my previous studies on the bridging action of reinforcing fibres and bars could be particularly useful. He is the third person who contributed to the formation of my scientific profile during those early years of my career.

Besides the three above-mentioned Universities and my three Senior Colleagues, I feel extremely grateful also to the Research Institutions and Industrial Companies that have contributed, sharing their expertise and/or with their financial support, to increase my knowledge in the field. They are gratefully acknowledged together with the respective scientific responsible persons: Montedison (Aurelio Savadori) for the size effects on polypropylene; Italcementi (Gianpietro Tognon) for the brittle behaviour of high-strength concrete; ENEL-CRIS (Michele Fanelli and Gerardo Ferrara) for the fracture mechanics analysis of gravity dams; HILTI (Werner Sack and Klaus-Peter Bohn) for drilling perforation of rocks and tool wear; Metal-Work (Erminio Bonatti and Fausto Rodella) for the fracto-emissions as seismic precursors.

More in particular, I am mostly grateful to the co-authors of the single chapters of this book, for their deep knowledge of the related subjects and for their remarkable patience in the realization of the manuscript: Giuseppe Lacidogna (Chaps. 10, 14, 15), Pietro Cornetti (Chaps. 3, 11, 12), Alberto Sapora (Chaps. 1, 4, 5), Mauro Corrado (Chaps. 6, 7, 8).

In addition, I wish to thank two of my best former co-workers very sincerely, for their outstanding contributions to the papers on which some parts of the book are based: Nicola Pugno (Chaps. 10 and 15) and Marco Paggi (Chaps. 9 and 13). A significant support in the final writing of some sections of the book has come from junior scholars working presently in my research group: Federico Accornero, Gianni Niccolini, Oscar Borla, Francesco Montagnoli, Alessio Rubino.

Going through the references of the different chapters, it is possible to observe the impressive number of collaborations, each one relevant for the final success of the research project. First of all, I would like to recall Erasmo Viola, my colleague at the University of Bologna, who, together with Professor Angelo Di Tommaso, introduced me to Structural Mechanics. Then, I am pleased to recall all the fruitful collaborations that I have had since my early years in Torino: Giovanni

Alpa, Pietro Bocca, Silvio Valente, Enrico Ballatore, Crescentino Bosco, Pier Giorgio Debernardi, Vincenzo Ilario Carbone, Giuseppe Mancini, Alessandro De Stefano, Rosario Ceravolo, Francesco Maradei, Antonino Quattrone, Claudio Scavia, Mauro Borri-Brunetto, Giorgio Zavarise, Laura De Lorenzis, Giulio Ventura, Francesco Mainardi, Michele Ciavarella, Pier Paolo Delsanto, Renato Orta, Riccardo Sandrone, Umberto Lucia, Massimo Zucchetti, Orazio Baglieri, Ezio Santagata, Giuseppe Surace, Anna Anzani, Luigia Binda, Massimiliano Zingales, Mario Di Paola, Roberto Ballarini, David Taylor, Elias Aifantis, George Frantziskonis, Panagouli Panagiotopoulos, Tulio Bittencourt, Olek Naimark, Carlos Brebbia, Ferri Aliabadi, Victor Panin.

Several important collaborations were then originated from Ph.D. Dissertations and from the contribution of visiting scientists. I wish to acknowledge the relevant and pertinent contributions by: Roberta Massabò, Giuseppe Ferro, Bernardino Chiaia, Fabrizio Barpi, Stefano Invernizzi, Antonio Brencich, Ilenia Monetto, Francesco Ciola, Moh'd El-Khatieb, Luciano Dimastrogiovanni, Simone Puzzi, Amedeo Manuello, Davide Masera, Alessandro Grazzini, Erica Cadamuro, Diego Veneziano, Jie Xu, Baoming Gong, Guo Ping Yang, Fan Ping Zhou, Kamran Nemati, Kiran Kolwankar, Jacinto Ruiz Carmona, Ignacio Iturrioz, Olek Plekhov.

Another crucial moment for my scientific growth is represented by my interactions with the International Scientific Societies on Fracture Mechanics. Since the early 1990s, I have been very active inside the International Association of Fracture Mechanics for Concrete and Concrete Structures (IA-FraMCoS), founded by Zdenek Bazant in 1992. That scientific community had already started its informal activities even earlier, at the beginning of the 1980s. At that time, I co-edited two books on the subject, one with Anthony Ingrassia, and the other with Surendra Shah. Other colleagues, with whom I had scientific and human interactions, were: Hubert Hilsdorf, Bhushan Karihaloo, Folker Wittmann, Hans-Wolf Reinhardt, Manuel Elices, Jan Rots, Jan van Mier, René de Borst, Jacky Mazars, Gonzalo Ruiz, Victor Li, Hirozo Mhashi, Keitetsu Rokugo, Buyang Oh.

Since the mid 1990s, my activity inside the European Structural Integrity Society (ESIS) became very intense. In that context, I had the opportunity to work with several outstanding European Colleagues, among whom: Keith Miller, Karl-Heinz Schwalbe, André Pineau, Dominique Francois, Gordon Williams, Nikita Morozov, Volodymyr Panasiuk, Dragoslav Sumarac, Andrzej Neimitz, Jaroslav Pokluda, Liviu Marsavina, Donato Firrao, Francesco Iacoviello.

Since 1981, I have attended the quadrennial conferences of the International Congress on Fracture (ICF), founded by Takeo Yokobori in 1965. I had the great pleasure and privilege of organizing ICF11 in Torino in 2005. That has been and still is a community of significant and harmonious collaboration under different viewpoints. I have herein the possibility to report a list of very good friends and esteemed colleagues: David Taplin, Emmanuel Gdoutos, Ashok Saxena, Robert Goldstein, Ludmila Botvina, Palle Rama Rao, Shouwen Yu, Robert Ritchie, Krishnaswamy Ravi-Chandar, You-Wing Mai, Toshimitsu Yokobori, Mimoun El-Boujdaini, Yukitaka Murakami, Rhys Jones, Ares Rosakis, Guruswamy Ravichandran, John Hutchinson, James Rice.

As a last tribute, I like to recall the names of those great scientists who mostly inspired the project of this book with their seminal contributions. I have had the venture to personally meet and be acquainted with some of them: George Irwin, Paul Paris, Arne Hillerborg, Benoit Mandelbrot, Grigory Barenblatt. Eminent scientists who deserve to be recalled and honoured are also: Alan Griffith, Nikoloz Muskhelishvili, Alan Cottrell, René Thom, Mitchell Feigenbaum, Kenneth Wilson, Claude Shannon, Andrej Kolmogorov, Per Bak, Donald Turcotte.

Sincere thanks are eventually due to the Publisher Springer for inviting me, already in 2009, to write such an encyclopedic book, and for the great care and ability in realizing it.

Turin, Italy
June 2021

Alberto Carpinteri

Contents

**Part I From Stress Singularity to Strain Energy Release Rate:
Local Versus Global Approach to Fracture Mechanics**

1	Stress Concentration at the Notch Root	3
	Alberto Carpinteri and Alberto Sapora	
1.1	Preliminary Remarks	3
1.2	Plane Stress Condition	5
1.3	Plane Strain Condition	8
1.4	Thick-Walled Cylinder	10
1.5	Circular Hole in a Plate Subjected to Tension	14
1.6	Analytical Functions	18
1.7	Kolosoff–Muskhelishvili Method	22
1.8	Elliptical Hole in a Plate Subjected to Tension	27
1.9	Griffith’s Fracture Energy Criterion	35
1.10	Experimental Confirmations	38
	References	44
2	Stress Intensification at the Crack Tip	47
	Alberto Carpinteri	
2.1	Preliminary Remarks	47
2.2	Westergaard’s Method	48
2.3	Mode II and Mixed Modes	57
2.4	Stability of Crack Propagation	62
2.5	Elasto-Plastic Material	66
2.6	Isoparametric Finite Elements	73
2.7	Quarter Point Element	79
2.8	Degenerate Triangular Element	84
2.9	Estimation of Error in the Numerical Evaluation of Stress-Intensity Factors	89

2.10	Experimental Determination of the Critical Stress-Intensity Factor K_{IC} for Metallic Materials (ASTM E399 Standard) . . .	93
2.10.1	Characteristics and size of Test Specimens	93
2.10.2	Test procedure	96
2.10.3	Testing Apparatus	98
2.10.4	Analysis of the Load–Displacement Diagrams	98
2.11	Determination of the Fracture Toughness of Rocks	101
2.11.1	Calculation of the Fracture Toughness K_{CB}	104
2.11.2	Correction of Fracture Toughness for Nonlinearity	104
	References	104
3	Stress Intensification at the Vertex of a Re-entrant Corner	107
	Alberto Carpinteri and Pietro Cornetti	
3.1	Preliminary Remarks	107
3.2	Singular Stress Field in the Case of Linear Elastic Material	108
3.3	Singular Stress Field in the Case of Strain-Hardening Material	114
3.4	Extension of the Plastic Zone Around a Re-entrant Corner . . .	117
3.5	Generalized Fracture Toughness	118
3.6	Critical Amplitude of a Re-entrant Corner	125
3.7	Size-scale Effects in Structural Elements with Re-entrant Corners	129
3.8	The Notch Blunting Effect	134
	References	140
4	Energy Approach to Fracture Mechanics	143
	Alberto Carpinteri and Alberto Sapora	
4.1	Preliminary Remarks	143
4.2	Relation Between Energy and Stress-Singularity Treatments: Irwin’s Theorem	144
4.3	Local Compliance of a Cracked Structural Element	151
4.4	J -Integral	154
4.4.1	Independence of the J -Integral from the Integration Curve	155
4.4.2	Variations of Energy	157
4.4.3	Identity of the J -Integral with the Strain Energy Release Rate $\mathcal{G}_I$	161
4.5	Experimental Investigations	163
4.5.1	Experimental Determination of Fracture Toughness Parameters	164
4.5.2	Comparison Between the Fracture Parameters Obtained	170

4.6	Experimental Determination of Fracture Energy for Mortar and Concrete (Rilem Recommendation)	170
4.6.1	The Proposed Testing Method	171
4.6.2	Testing Procedure and Characteristics of the Test Specimens	173
	References	175
5	Mixed-Mode Crack Propagation	179
	Alberto Carpinteri and Alberto Sapora	
5.1	Preliminary Remarks	179
5.2	Criterion of Maximum Hoop Stress	180
5.3	Criterion of Minimum Strain Energy Density	184
5.4	Criterion of Maximum Released Energy	188
5.5	<i>J</i> -Vector Criterion	193
5.6	Experimental Tests and Empirical Criteria	201
5.7	Scale Effects in Relation to Crack Size	205
5.8	Effect of Stress Parallel to the Crack	208
5.9	Plastic Effects at the Crack Tip	217
5.10	Directional Stability in Crack Propagation	220
5.11	Loci of Resistance in the Principal Stress Plane	221
	5.11.1 Mohr–Coulomb Criterion	221
	5.11.2 Griffith’s Macroscopic Criterion	225
	5.11.3 Friction on Griffith Cracks	228
	5.11.4 Microcrack Population Model	230
	References	235
 Part II From Simple Nonlinear Constitutive Laws to Complex Mechanical Behaviour: Catastrophe and Chaos		
6	Nonlinear Crack Models	239
	Alberto Carpinteri and Mauro Corrado	
6.1	Preliminary Remarks	239
6.2	Plastic Zone at the Crack Tip	240
6.3	Strain Energy Density Criterion—Strain-Hardening Materials	244
	6.3.1 Material Behaviour	246
	6.3.2 Isotropic Versus Kinematic Hardening	249
	6.3.3 Effect of Loading Step	250
6.4	Strain Energy Density Criterion—Strain-Softening Materials	250
	6.4.1 Material Behaviour	250
	6.4.2 Variation in the σ – ε Softening Slope	254
	6.4.3 Effect of Loading Step	256
	6.4.4 Size Effects on Strength and Ductility	258
	6.4.5 Centre-Cracked Slab in Tension	263

6.4.6	Three-Point Bending of a Reinforced Beam with Edge Crack	266
6.4.7	Eccentric Compression of Wall with Edge Crack . . .	266
6.5	Cohesive Crack Model—Mode I	267
6.5.1	Localized Strain	269
6.5.2	Three-Point Bending Test	272
6.5.3	Numerical Procedure	274
6.6	Ductile–Brittle Transition and Snap-Back Instability	277
6.6.1	Influence of Initial Crack Depth and Beam Slenderness	280
6.7	Cohesive Crack Model—Mixed Mode	295
6.7.1	Experimental Program	300
6.7.2	Directional Stability of the Crack Trajectory	304
6.8	Loss of Symmetry and Bifurcations	314
6.8.1	Crack Length Control Scheme	314
6.8.2	Solution of the Single Crack Growth Step	316
6.9	Nonlinear Crack Concepts Applied to Compression: The Overlapping Crack Model	322
6.10	Overlapping Crack Model for Eccentric Compression	330
6.10.1	Numerical Algorithm	330
6.10.2	Comparison Between Model Predictions and Experimental Results	333
6.10.3	Size-Scale and Slenderness Effects in Eccentric Compression Tests	335
	References	337
7	Size-Scale Transition from Ductile to Brittle Failure	343
	Alberto Carpinteri and Mauro Corrado	
7.1	Preliminary Remarks	343
7.2	Dimensional Analysis	344
7.2.1	Buckingham’s Theorem	347
7.3	Different Structural Geometries	349
7.4	Size-Scale Effects on Apparent Fracture Toughness	358
7.4.1	Metals	358
7.4.2	Concrete and Rocks	360
7.4.3	Cohesive Crack Model	367
7.4.4	Influence of the Shape of the Cohesive Diagram σ – w	367
7.4.5	Damage Model Versus Cohesive Model	370
7.5	Dugdale Plastic Zone Correction	370
7.6	BCS Crack Model	373
7.7	Virtual Crack Propagation Model	377

7.8	Cohesive Limit Analysis	382
7.8.1	Uniaxial Tensile Loading of Slabs	382
7.8.2	Three-Point Bending of Beams [34, 35]	386
7.8.3	Three-Point Bending of Deep Beams	390
7.9	Size-Scale Effects on Apparent Bending Strength	391
7.10	Brittleness Limit for Infinite Size-Scale	392
7.11	Structural Response Versus Crack Growth Resistance	
	Curve	395
7.11.1	Scale Effect on the Structural Response	395
7.11.2	Strain-Hardening Material	395
7.11.3	Linear-Elastic Material	398
7.11.4	Three-Point Bending Geometry	399
7.11.5	Scale Effect on the J-Resistance Curve	400
	References	402
8	Mechanical Behaviour of Reinforced Structural Elements	407
	Alberto Carpinteri and Mauro Corrado	
8.1	Preliminary Remarks	407
8.2	Crack Growth Stability in Steel-Bar Reinforced Concrete	
	Elements: Rotation Compatibility Condition	408
8.2.1	Statically Indeterminate Reaction	
	of Reinforcement	408
8.2.2	Bending Moment of Reinforcement Plastic Flow	412
8.2.3	Rigid-Hardening Behaviour of the Cracked Beam	
	Section	413
8.2.4	Bending Moment of Matrix Fracture	414
8.2.5	Stability of the Process of Matrix Fracture and Steel	
	Plastic Flow	417
8.2.6	Summary	419
8.3	Crack Growth Stability in Steel-Bar Reinforced Concrete	
	Elements: Crack Opening Displacement Compatibility	
	Condition	422
8.3.1	Displacement Compatibility Condition	
	and Statically Indeterminate Reaction	
	of Reinforcement	423
8.3.2	Combined Stress-Intensity Factor	427
8.3.3	Crack Propagation	428
8.3.4	Moment Versus Rotation Response	430
8.3.5	Comparison with Experimental Results	433
8.3.6	Experimental Confirmation of Snap-Back	
	Behaviour	435
8.3.7	Concluding Remarks	436

8.4	Crack Growth Stability in Fibrous Composites:	
	Discrete Model	436
8.4.1	Theoretical Model	437
8.4.2	Displacement Compatibility Conditions	439
8.4.3	Crack Propagation	441
8.4.4	Structural Response of the Cracked Element	442
8.4.5	Two Fibres	442
8.4.6	Large Number of Fibres	446
8.4.7	Concluding Remarks	449
8.5	Crack Growth Stability in Fibrous Composites:	
	Continuous Model	449
8.5.1	Continuous Model	450
8.5.2	Discrete Model Versus Continuous Model	456
8.5.3	Continuous Model Versus Experimental Results	458
8.5.4	Bridging Option Versus Cohesive Option	460
8.5.5	Concluding Remarks	461
8.6	Hysteretic Behaviour of Steel-Bar Reinforced Concrete	
	Elements: Rotation Compatibility Condition	462
8.6.1	Elastic–Plastic Shake-Down Under Repeated Loadings	462
8.6.2	Critical Crack Depth and Bending Moment	465
8.6.3	Fatigue Crack Growth and Energy Dissipation	467
8.7	Hysteretic Behaviour of Steel-Bar Reinforced Concrete	
	Elements: Crack Opening Displacement Compatibility Condition	471
8.7.1	Moment Versus Rotation Diagrams	472
8.7.2	Beam A	474
8.7.3	Beam B	474
8.7.4	Beam C	474
8.7.5	Experimental Comparisons	475
8.7.6	Concluding Remarks	479
8.8	Hysteretic Behaviour of Fibrous Composites	480
8.8.1	Concluding Remarks	485
8.9	Transitions of Reinforced Concrete Beams in Flexure:	
	Tensile, Shearing, Crushing Failures	490
8.9.1	Modelling Flexural and Shear Cracks	491
8.9.2	Modelling Concrete Crushing	494
8.9.3	Transition Between Different Failure Modes	496
8.9.4	Experimental Evidences	499
8.10	Cohesive/Overlapping Crack Model for Nonlinear Analysis	
	of Reinforced Concrete Beams	502
8.10.1	Mathematical Formulation	502
8.10.2	Numerical Algorithm	504
8.10.3	Computation of the Elastic Coefficients	506

8.10.4	Parametric Analysis and Experimental Comparisons	508
8.10.5	Size-Scale Effects	508
8.10.6	Effect of the Tensile Steel Reinforcement Percentage	509
8.10.7	Effect of the Steel Reinforcement in Compression	511
8.10.8	Effect of the Concrete Compressive Strength	512
8.10.9	Effect of the Stirrups Confinement	513
8.11	Lower and Upper Reinforcement Limits to the Ductile Behaviour of Concrete Members: Minimum Reinforcement and Rotational Capacity	515
8.11.1	Minimum Reinforcement	515
8.11.2	Models for Computing Minimum Reinforcement	516
8.11.3	Application of Dimensional Analysis to Lightly RC Members	517
8.11.4	Comparison of Predictions and Experimental Results	519
8.11.5	Parametric Analysis and Discussion	521
8.11.6	Plastic Rotation Capacity	527
8.11.7	Concluding Remarks	529
	References	532
9	Debonding and Decohesion at the Interface Between Dissimilar Media	537
	Alberto Carpinteri	
9.1	Preliminary Remarks	537
9.2	Stress Singularities at Multi-material Interfaces	538
9.3	Generalized Stress-Intensity Factors and Computational Methods	546
9.4	Interface Crack Propagation Criteria	551
9.5	Nonlinear Interface Constitutive Laws and Interface-Contact Elements	554
9.6	Delamination of Plated Beams	560
9.7	Fibre-Matrix Debonding in Microstructured Composites	570
	References	576
10	Nonlinear and Chaotic Behaviour in the Vibration of Cracked Bodies	583
	Alberto Carpinteri and Giuseppe Lacidogna	
10.1	Preliminary Remarks	583
10.2	Theoretical Continuum Approach	585
10.3	Theoretical Discrete Approach	587
10.4	Period Doubling Cascade	590

10.5	Parametrical Simulations	594
10.6	General Discussion	615
	References	618

Part III From Complex Morphological Patterns to Simple Mechanical Models: Fractality and Critical Phenomena

11	Fractality of Scale-Invariant Cohesive Constitutive Laws	623
	Alberto Carpinteri and Pietro Cornetti	
11.1	Preliminary Remarks	623
11.2	Weibull's Theory	624
11.2.1	The Weakest-Link Concept	624
11.2.2	Safety Factor	629
11.3	Defect Size Distribution of Self-similarity	630
11.3.1	Weibull Parameters	636
11.3.2	Experimental Results	637
11.4	Fractal Geometry	638
11.4.1	Hausdorff Dimension	643
11.4.2	Box-Counting Dimension	650
11.4.3	Random Fractals	651
11.5	The Fractal (Scale-invariant) Cohesive Crack Model	654
11.5.1	The Effect of Microstructural Disorder	656
11.5.2	Scale-Invariant Cohesive Crack Model	659
11.5.3	Comparison with Experimental Data	664
11.6	Dimensional Transition from Order to Disorder: Multi-fractal Scaling Laws	668
11.6.1	Multi-fractal Scaling Laws	669
11.6.2	Comparison with Experimental Data	673
11.7	Fractal Overlapping Crack Model	676
11.8	Rotation Versus Curvature Fractal Scaling in Bending Failure	678
11.9	New Tools for New Challenges	682
	References	683
12	Fractional Calculus Applied to Fractal Media and Nonlocal Continua	689
	Alberto Carpinteri and Pietro Cornetti	
12.1	Preliminary Remarks	689
12.2	Fractional Calculus: A Brief Review	690
12.3	Fractional Calculus and Fractal Functions: The Local Fractional Derivative	694
12.4	Kinematic and Static Equations for Fractal Media	698
12.5	The Fractal Bar	702
12.6	Eringen's Approach and Nonlocal Fractional Elasticity	703

12.7	The Nonlocal Fractional Elastic Bar	713
12.8	Numerical Simulations	719
	References	726
13	Scaling and Fractality in Sub-critical Fatigue and Creep Crack Growth	729
	Alberto Carpinteri	
13.1	Preliminary Remarks	729
13.2	Analytical Correlations Between the Fatigue Properties of Engineering Materials	736
13.3	Generalized Cumulative Fatigue Damage Formulation	741
13.4	Generalized Fatigue Crack Propagation Formulation	743
13.5	Interpretation of Specimen-Size Effects on Paris' Law According to Incomplete Self-similarity	745
13.6	Interpretation of Crack-Size Effects on Paris' Law According to Fractal Geometry	747
13.7	Interpretation of Specimen-Size Effects on Wöhler's Curve According to Fractal Geometry	758
13.8	Fatigue Propagation of Cracks Subjected to Mixed-Mode Loading	765
13.9	Scaling Laws for Creep Deformation and Rupture Time	771
13.10	Specimen-Size Effects on Creep Rupture Time	778
13.11	Crack-Size Effects on Creep Crack Growth	783
	References	790
14	Critical Phenomena and Acoustic Emission in Structural Elements and the Earth's Crust	797
	Alberto Carpinteri and Giuseppe Lacidogna	
14.1	Preliminary Remarks	797
14.2	Critical Phenomena in Disordered Materials	798
14.3	Scaling Laws in Geophysics and Mechanics	801
14.4	Acoustic Emission and Detection of Crack Evolution in Damaging Structures	803
	14.4.1 Dissipated and Emitted Energies	804
14.5	AE Waves and Signal Waves	806
	14.5.1 Measuring System	808
	14.5.2 Signal Processing	809
	14.5.3 Event Counting	809
	14.5.4 Ring-Down Counting	810
	14.5.5 Parameter-Based AE Techniques	810
	14.5.6 Signal-Based AE Techniques	811
	14.5.7 Spectral Analysis	812
	14.5.8 Source Localization	812
	14.5.9 Amplitude Distribution Analysis	812

14.5.10	Moment Tensor Analysis	812
14.5.11	Cracking Modes and Typical AE Signals	813
14.5.12	AE Localization Procedures	813
14.6	Size-scale Effects in AE Monitoring	819
14.6.1	Experimental Assessment	820
14.7	Time-scale Effects in AE Monitoring	822
14.7.1	Experimental Assessment	823
14.8	AE Frequency–Magnitude Statistics and b -Value Analysis	826
14.9	Fractal Dimension Evolution of Microcrack Networks in Disordered Materials	828
14.9.1	Self-similar and Self-affine Crack-Size Distributions	830
14.10	Experimental Observations on AE	834
14.10.1	In-situ Retrofitted RC Beam Test	834
14.10.2	Three-Point Bending Test	837
14.10.3	Concrete Specimen in Compression	838
14.11	From Criticality to Final Collapse: Evolution of the Fractal Dimension D and of the b -Value	840
14.11.1	Different Approaches for Damage Domain Characterization in Disordered Materials: Fractal Energy Density and b -Value Statistics	842
14.12	Acoustic and Electromagnetic Emissions from Quasi-brittle Materials	844
14.12.1	Models for EME	844
14.13	Experimental Observations on AE and EME	846
14.13.1	AE and EME Measurements	847
14.13.2	Test Results	849
14.14	Regional Seismicity and AE Structural Monitoring	852
14.14.1	From Acoustic Emission to Earthquakes	853
14.15	b -Value Analysis on Medieval Towers as a Seismic Precursor	856
14.15.1	Damage Detection in the Towers	856
14.15.2	The b -Value Analysis	859
14.15.3	Correlation Between AE Activity in the Towers and Regional Seismicity	860
14.15.4	Fractal Dimensions from Space–Time Correlation Integral	861
14.16	Universality in Fracture Systems	865
14.16.1	Gutenberg–Richter Law and Scale Invariance in the Timing of Ruptures	866
14.16.2	Scaling Laws for Waiting-Time Distributions of Italian Seismicity	867

14.16.3	Scaling Laws for Waiting-Time Distributions in Concrete Fracture	870
14.16.4	Discussion on the Results	872
14.17	Statistical Seismic Precursors and Preparation Time for the Latest Earthquakes in Italy	873
14.17.1	Methods and Data Set	873
14.17.2	The L'Aquila Earthquake	874
14.17.3	The Emilia Earthquake	875
14.17.4	The Amatrice–Norcia Earthquake	875
14.18	Fracto-Emissions as Seismic Precursors	881
14.18.1	Acoustic, Electromagnetic and Neutron Emissions: The Crack-Size Evolution in the Earthquake Preparation Zone	882
14.18.2	The Case Study of “San Pietro–Prato Nuovo” Gypsum Mine Station	885
	References	890
15	Scaling and Fractality in Fragmentation and Comminution: Applications to Drilling and Wear	899
	Alberto Carpinteri and Giuseppe Lacidogna	
15.1	Preliminary Remarks	899
15.2	Fractal Fragmentation Theory of Quasi-Brittle Materials in Compression	900
15.3	Energy Dissipation Under Fragmentation	901
15.4	Fractal Scaling Laws	903
15.4.1	Shape Effects	903
15.4.2	Size Effects	904
15.4.3	Geometrical Multi-fractal Extension	909
15.5	One-, Two-, and Three-Dimensional Universal Laws for Fragmentation Due to Impact and Explosion	913
15.5.1	Three-Dimensional Theory	913
15.5.2	Two-Dimensional Theory	915
15.5.3	One-Dimensional Theory	916
15.6	An Example of Application: The Asteroid Collision	916
15.7	Fractal Comminution Approach to Evaluate the Drilling Energy Dissipation	917
15.7.1	Experimental Fractal Assessment: The Single-Scratch Test	918
15.7.2	Experiments on Drilling Comminution	919
15.8	Power Balance for Drilling Comminution	922
15.9	Coupled Fractal Theory of Drilling and Wear	926
15.9.1	Coupled Law of Drilling and Wear Velocities	928
15.9.2	Fractal Coupled Law of Wear and Drilling Velocities	932

15.10	Evolutionary Fractal Theory of Erosion and Experimental Assessment on MIR Space Station	937
15.10.1	Classical Erosion	938
15.10.2	Classical Coupled Erosion	939
15.10.3	Fractal Erosion	940
15.10.4	Fractal Coupled Erosion	942
15.10.5	Experimental Comparison with In-Flight Erosion Due to Space Debris Impacts on MIR Orbital Space Station	942
References		945

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