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Philine Schiewe

# Integrated Optimization in Public Transport Planning

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# Integrated Optimization in Public Transport Planning

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Kaiserslautern, Germany Philine Schiewe

# Chapter 1
# Introduction

In times of growing urban populations and increasing environmental awareness, the importance of public transport systems is increasing as well. Public transport provides an efficient way for commuting by bundling traffic flows with the same general direction, thus reducing the individual traffic and the resulting congestions in peak hours. A lot of research is concerned with providing good public transport systems, both from the passengers' and the operators' point of view. For an already existing infrastructure network, the problems *line planning*, *timetabling*, and *vehicle scheduling* are especially interesting.

The lines determined in the line planning stage are equally important for passengers and operators. As lines have to be covered by one vehicle end-to-end, they form an integral part of both the passenger routes and the vehicle schedule. A timetable appoints times to the departures and arrivals of lines at stations thus determining the journey times and the duration of potential transfers between different lines. The operational costs mainly depend on the vehicle schedule which determines the routes of the vehicles including the operation of lines and potential relocation trips.

Although these three problems are generally solved sequentially, they are highly dependent on one another. The line plan determines the passenger routes and the structure of the timetable as well as a large portion of the vehicle schedules because lines have to be covered by one vehicle end-to-end. The timetable also influences the vehicle schedule by determining the start and end times of trips. Here, trips model the operation of lines by vehicles and two trips can only be operated by the same vehicle if the time between the end of the first trip and the start of the second trip is long enough to facilitate the minimal turnover time as well as a potential relocation. Finding a line plan with corresponding timetable and vehicle schedule can therefore be interpreted as a *multi-stage problem*. The first stage, line planning, can be solved individually but its solution is not indicative of the solution quality of the overall system. The later stages, timetabling and vehicle scheduling, depend on



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the solutions of the former stages and the quality of the overall system can only be evaluated after all stages have been solved.

We therefore consider the following questions:

- Can several – or even all – of the stages line planning, timetabling and vehicle scheduling be considered as an *integrated* problem? How does this influence the solution quality and the computation times compared to the classical sequential solution approach?
- If the integrated problems cannot be solved to optimality, how can we adapt the sequential solution approach by incorporating the idea of integration?
- What is the general structure of abstract multi-stage problems? How much solution quality is lost when solving general multi-stage problems by a sequential approach instead of solving the overall problem?

## 1.1 Outline

The remainder of this book is structured as follows. A literature overview on line planning, timetabling, and vehicle scheduling as well as integrated approaches to public transport planning is given in Section 1.2. The basic concepts used in this book as well as the single stage problems line planning, timetabling, and vehicle scheduling are introduced in Section 1.3 while in Section 1.4 the data sets for the computational experiments in the subsequent chapters are presented.

In Chapters 2 to 5 we systematically build an integrated model for line planning, timetabling, and vehicle scheduling as depicted in Figure 1.1. We start in Chapter 2 by considering the importance of passenger routes and by stating an integrated model for timetabling and passenger routing. In order to reduce the computation

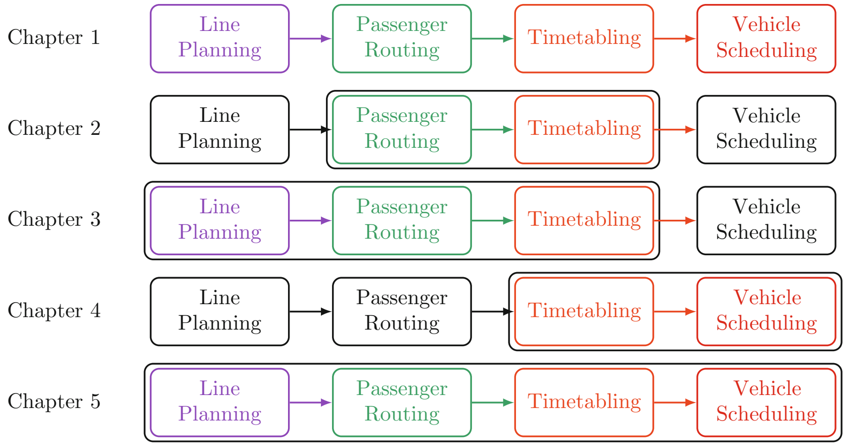


**Fig. 1.1** Overview of the problems in public transport planning considered in this book.

times and to make the model applicable to larger instances, we derive an exact preprocessing algorithm and two heuristics which yield lower and upper bounds on the objective value. This allows us to solve medium to large sized instances in reasonable time. Additionally, we derive a model for distributing the start times of the passenger routes. The model for timetabling and passenger routing is extended to the integrated line planning, timetabling, and passenger routing problem in Chapter 3. The heuristics and the preprocessing method can be adapted to the new challenges arising by integrating the selection of lines into the optimization process. Thus, even for the larger problem, good solutions can be found for medium sized instances. In Chapter 4 we consider the integration of (periodic) timetabling and (aperiodic) vehicle scheduling and discuss the resulting challenges. Again, we can find good solutions for medium to large sized instances.

The results of Chapter 2 to 4 are used to define the integrated line planning, timetabling, passenger routing, and vehicle scheduling problem in Chapter 5. In addition to the benefits of integrated solutions, we also consider the influence of different matrix decompositions on the computational performance of the problem. Due to increasing problem size, the integrated line planning, timetabling, passenger routing, and vehicle scheduling problem can only be solved for very small artificial instances. We therefore consider two heuristic approaches in Chapter 6, both incorporating ideas from integration. The look-ahead heuristic presented in Section 6.1 solves the planning problems sequentially—incorporating the solution quality of the next stages into each stage. In Section 6.2 an iterative re-optimization scheme is introduced where for a given line plan, timetable, and vehicle schedule iteratively one of the stages is re-optimized such that the solutions of the other two stages stay the same.

The concept of integrating several planning stages in public transport is generalized in Chapter 7 where abstract multi-stage problems are considered. To measure the difference between optimal solutions for the multi-stage problem and solutions that are found by solving the stages sequentially, the *price of sequentiality* is defined. In addition to a theoretical investigation, the price of sequentiality is calculated for a small example instance for the integrated public transport problems introduced in Chapter 2 to 5. We show that even for this very small example, solving two or more problems integratedly instead of sequentially leads to large benefits, emphasizing the importance of integration in public transport planning.

This book closes with a discussion of the results in Chapter 8 and an outlook to future work in Chapter 9.

## 1.2 Literature Overview

Public transport planning is a well researched topic in the operations research community. A general overview can be found in [BWZ97, HKLV05, DH07, GH08] while [BGJ10] focuses on recent success stories of applying operations research methods to public transport planning problems. Here, the planning process is

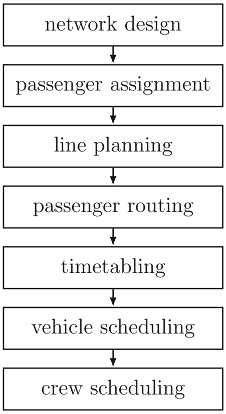


**Fig. 1.2** Classical sequential approach to public transport planning.

divided hierarchically into subproblems, most commonly network design, line planning, timetabling, vehicle scheduling, and crew scheduling, which are solved sequentially, see Figure 1.2. Passenger assignment and passenger routing are important intermediate steps, providing input data for line planning and timetabling, respectively.

Although this sequential approach is commonly applied, the potential benefits and difficulties of *integrating* several of these planning stages are already mentioned in [BWZ97]. While network design is an important problem, infrastructure is already present and fixed in most realistic settings. Although crew scheduling contributes an important component to the operational costs, it highly depends both on the legal circumstances and the institutional environment. In this book, we hence focus on integrating three of the aforementioned problems, namely line planning, timetabling, and vehicle scheduling as well as the intermediate step of passenger routing. We therefore provide a short literature review on these single stage problems in the next sections, Sections 1.2.1 to 1.2.3, and we introduce literature on integrated public transport problems in Section 1.2.4.

### *1.2.1 Line Planning*

In the *line planning* stage, the set of operated lines is determined as well as their frequencies, i.e., how often the lines are operated in a planning period. Note that lines are paths in the infrastructure network that are covered by one vehicle end-to-end. For an overview of line planning problems we refer to [Sch12]. In the special case of *transit route design*, see survey [KK09], the generation of lines is part of the optimization problem. In [SBP74], a two-stage model is developed where first lines are created from skeleton lines and afterwards a fixed number of lines is chosen for operation. Iterative approaches to re-optimize lines are presented in [LS67, Son79, Man80] while in [CW86] a set of passenger-pleasing lines is created heuristically from which lines are chosen such that the travel time and the operational costs are minimized. A similar two-stage approach can be found in [GH08] where a bi-criterial objective is handled by a genetic algorithm which alternately optimizes travel time and operational costs. Another meta-heuristic approach, namely simulated annealing, is presented in [FM10]. In [Vig17] passenger-friendly bus lines are added to an existing multi-modal network by clustering the origin-destination data to identify corridors for lines.

However, in most line planning problems, a fixed line pool is given from which lines are chosen. It can either be provided by the operator or constructed in a separate optimization step, see [GHS17]. A special case is [BGP07] where column generation is used to create the lines during the optimization process instead of using an explicit line pool to choose lines from.

The most commonly used objectives in line planning are the minimization of operational costs and the maximization of passenger convenience. The minimization of operational costs is introduced in [CvDZ98] for which a fast variable fixing heuristic is given in [BLL04] and a branch-and-cut approach in [GvHK04]. An extension of the cost-minimization model where additionally the stopping patterns of the lines are determined is discussed in [Goo04, GvHK06]. In [BAE+18] a cost-minimal line planning model for multiple periods is considered where line plans for peak and off-peak hours are optimized simultaneously.

In contrast to cost-oriented line planning, which considers the operator's point of view, there are several models for optimizing lines for the passengers. Using the concept of direct travelers, i.e., passengers that have a transfer-free journey of acceptable length, as measurement of passenger convenience is introduced in [BKZ97, Bus98]. Another approach to define passenger convenience is to use the approximated generalized travel time of passengers, i.e., the travel time in the infrastructure network where transfers are penalized by a fixed amount. This concept is introduced in [Sch05b, SS06] and contains the routing of passengers on shortest paths. In order to model the transfers correctly, the infrastructure network is replaced with the *change-and-go graph* that additionally contains information on the lines. As the routing of the passengers is part of the optimization process for finding a line plan, we consider the travel time minimization in line planning as an integrated problem and we discuss further related literature in Section 1.2.4.

### *1.2.2 Timetabling*

In the *timetabling* stage, the arrivals and departures of lines at stations are assigned to time points. We distinguish *periodic* and *aperiodic* timetabling, i.e., whether the timetable is computed for a fixed period of time and then repeated multiple times or whether the timetable is planned for the complete planning horizon such that departures and arrivals do not have to be spaced out evenly. Recently, even the combination of periodic and aperiodic timetables has been investigated, see [RSAMB17], to harvest advantages of both systems, i.e., the regularity and memorability of the periodic case and the flexibility of the aperiodic one. Nevertheless, we here consider only periodic timetabling and refer to [LLER11] for an overview of both aperiodic and periodic timetabling.

Periodic timetabling is usually modeled by the *periodic event scheduling problem (PESP)* which is introduced and shown to be NP-complete in [SU89]. The objective is to minimize passengers' travel time on routes that are fixed a priori. We discuss the case with variable passenger routes in Section 1.2.4 as it represents an integrated problem. The modeling power of the periodic event scheduling problem corresponding to periodic timetabling is described in various publications. In [Odi96, Nac98], the driving of trains and their waiting at stations as well as transfers of passengers and headways between trains for security intervals are modeled by PESP constraints. In [KP03], variable drive times are added while in [Pee03], synchronization of departures, capacities at stations, and an approximation of the number of vehicles are added. This work also proposes alternative objectives to the minimization of the travel time, namely maximizing robustness, minimizing the number of vehicles needed, or minimizing the number of unsatisfied constraints in case of infeasibility. In [Lie06, LM07], further constraints are added such as fixed times for certain events, coupling and decoupling of train units as well as bundling of lines with similar speed.

Due to the inherent difficulty of PESP and the importance of periodic timetabling in public transport, there have been many different solution approaches. In [NV96, NV97] genetic algorithms are proposed while in [Odi96] a constraint generation algorithm and cuts for the integer programming formulation of PESP are introduced. In [Nac98, PK01], a cycle base IP formulation is introduced which is subject to a lot of further research. Different methods to derive good cycle bases are studied in [Lie03, LR05, Lie06, LP09] while in [BHK16] a pseudo-polynomial algorithm for deriving cutting planes is presented.

A modulo simplex heuristic for solving the periodic timetabling problem is introduced in [NO08], which is based on the fact that periodic timetables can be easily computed on trees. This heuristic is further investigated and improved in [GS13] and it leads to even better solutions when it is iteratively combined with mixed integer programs, see [GL17]. Another fast heuristic that is based on clustering lines is introduced in [PS16].

Additionally to IP based methods and heuristics, a satisfiability (SAT) based approach has been successfully pursued. In [GHM+12] the periodic event scheduling problem is modeled as a satisfiability problem and it is solved by specialized SAT solvers. The same approach is applied to large infeasible PESP instances to find a minimal set of constraints that have to be relaxed in order to get a feasible problem, see [KGN+15]. In [MASM18] a SAT formulation for periodic timetabling is combined with machine learning techniques to speed up the computations.

Periodic timetabling has even been successfully applied in practice. See [Lie06, Lie08a] for an application to the Berlin subway system and [KHA+09] for Netherlands Railways.

### 1.2.3 Vehicle Scheduling

During the *vehicle scheduling* stage the routes of the vehicles are determined, including the allocation of vehicles to line operations as well as potential relocations of vehicles between the operation of two lines. For a survey on vehicle scheduling, we refer to [DP95, BK09]. In [BKLL18], *periodic* and *aperiodic* vehicle scheduling models are compared. While periodic vehicle schedules are determined for one planning period and then repeated multiple times, aperiodic vehicle schedules are determined for the whole planning horizon. When the number of vehicles is minimized, the underlying timetable is periodic and the planning horizon is long enough, the aperiodic vehicle scheduling problem always has an optimal solution that is periodic, as shown in [BKLL18]. Thus, in this special case, considering period vehicle schedules is sufficient. But as we want to incorporate the relocation of vehicles to and from depots and a more sophisticated cost evaluation, we focus on the aperiodic case here.

Two properties are important when classifying vehicle scheduling problems: whether there are one or multiple vehicle types and whether one (or none) or multiple depots are considered. In [BCG87] it is shown that the single-type single-depot case can be solved in polynomial time and it remains polynomially solvable even for a restricted number of vehicles for "reasonable" definitions of operational costs although it is NP-hard for general cost functions. Additionally, NP-hardness is proven for the single-type, multi-depot case in [BCG87]. As we focus on the easier single-type single-depot case in this book, we refer to [DP95, BK09] for multiple vehicle types or multiple depots.

In [Sah70], a minimum decomposition formulation is given for the single-depot single-type vehicle scheduling problem where the number of vehicles is minimized but the relocation of vehicles between trips is not allowed. In [Orl76] both the relocation and the corresponding costs are added to the formulation. A formulation as a transportation model is given in [GS79] where in addition to the number of vehicles and the relocation costs also the costs for getting to and from the depot are considered and where a maximum number of vehicles is enforced. Additionally, a branch-and-bound procedure for solving the problem is provided. A similar model is given in [PB87] while network flow formulations are proposed in [BCG87, DP95].

For a more application-based approach we refer to [Mar06] where tactical, maintenance, and operational routing are distinguished. Also, in [RS18], a realistic vehicle scheduling problem is described and solved by coarse-to-fine column generation.

### 1.2.4 Integration

In this section we review literature where two or more planning stages are handled in an integrated way instead of sequentially. We start by considering the integration of passenger routing into one of the planning stages.

As mentioned in Section 1.2.1, the travel time model of line planning presented in [Sch05b, SS06] constitutes an integrated problem as line frequencies and passenger routes are determined simultaneously instead of fixing passenger routes a priori. A similar model is presented in [NJ08] and handled by a column generation approach, while the complexity of the problem is discussed in depth in [Sch14, SS15a]. The model introduced in [BRLL16] uses frequency based transfer penalties instead of a fixed penalty in order to better approximate the realistic travel time. Different routing models are discussed in [PB06], especially the difference between letting all passengers with the same origin and destination use the same route, splitting them to different routes of different lengths to better utilize the capacity of the lines or fixing the route in the infrastructure network and thus restricting the choice to the lines that are used. For artificial examples it is shown that compared to routing all passengers with the same origin and destination on the same shortest path, the other two models can be arbitrarily bad. A similar comparison is made in [GS17] where routing all passengers on shortest routes is compared to routing passengers such that a system optimal solution concerning the capacities is found and a genetic algorithm is presented. In [SSS19], a game theoretical approach to line planning is presented where line plans are constructed according to equilibrium solutions found by determining passenger routes. Note though that even integrating passenger routes into line planning can only result in approximated travel times and that the optimal passenger routes may shift when a timetable is introduced.

Although in timetabling often the travel time of passengers is minimized, it is usually assumed that passengers travel on routes that have been fixed a priori. However, in [BHK17] it is shown that this fixed routing can be arbitrarily bad compared to an integrated routing where passenger travel on shortest routes according to the timetable. This motivates studying the integration of passenger routing into timetabling. In [Sie11, SG13], routing is added to the periodic timetabling problem in form of passenger flow constraints and a re-timetabling heuristic is introduced where timetable optimization and passenger routing are iterated alternately leading to shorter travel times for the passengers. A similar iteration scheme is discussed in [Kin08], while the aperiodic case and its complexity are considered in [Sch14, SS15a, SS15b]. In [RSAMB17] passenger routing is integrated into a hybrid model of periodic and aperiodic timetabling and the resulting problem is solved by a simulated annealing heuristic.

In Chapter 2 of this book, we consider a model for integrating passenger routing into periodic timetabling and we propose an exact preprocessing method as well as two heuristics for reducing the problem size in Section 2.2. In Section 2.4, we propose a way to distribute the passenger demand within the planning period and we present a satisfiability model for the integrated problem in Section 2.5. The corresponding results are additionally published in [SS20] and [GGNS16], respectively.

The integration of line planning and timetabling is often approached heuristically. In [Sch05a] line segments are connected to lines during the timetabling stage. A similar heuristic is described in [Lie08b]. In [BBVL17], an iterative heuristic is proposed where line plans and timetables are re-optimized alternately. However, in [RN09] an integer programming model for integrating line planning, periodic timetabling, and passenger routing is proposed and a corresponding column generation method is discussed where a weighted sum of travel time and operational costs is minimized. A similar model is considered in [Kas10, KR13] and solved by a cross entropy heuristic.

In Chapter 3 of this book, we consider the integrated line planning, timetabling, and passenger routing problem. Similar to the integrated timetabling and passenger routing problem, we present an exact preprocessing method as well as heuristics for reducing the problem size in Section 3.3.

Models for integrated timetabling and vehicle scheduling often use an aperiodic timetable as input which is modified during the vehicle scheduling stage to allow for better schedules. Examples are [vdHvdAvKN08] where a simulated annealing heuristic is used to modify the timetable, [GH10] where local search is applied iteratively and [PLM+13] where a large neighborhood search heuristic is used. In [SE15] a similar re-optimization approach is used and the integrated problem is solved by a hybrid metaheuristic that iteratively optimizes the operational costs and the balance of departure times which measures the deviation from ideal service intervals for the passengers. Another iterative matheuristic is presented in [FvdHRL18], where excess transfer times and operational costs are minimized. In [IRRS11], a model is presented where an aperiodic timetable and a vehicle schedule are computed simultaneously such that the number of vehicles is minimized while the number of the so-called synchronizations is maximized, i.e., the number of vehicles at the same stations such that transfers are possible. A similar model is considered in [CM12] with the objective of avoiding shunting operations which are prone to delays. A bi-level model is presented in [YHWL17] and solved by a simulated annealing heuristic. Here, the upper level corresponds to finding an aperiodic timetable and the lower level corresponds to finding a matching vehicle schedule. A periodic version for integrating timetabling and vehicle scheduling can be found in [DRB+17] where some vehicle scheduling constraints are added to a periodic event scheduling problem. In [Lin00] the periodic case is considered as well and vehicle scheduling is integrated indirectly by using an approximation of the operational costs as objective.

The model for integrating timetabling and vehicle scheduling presented in Chapter 4 of this book combines periodic timetabling and aperiodic vehicle scheduling with one depot and one vehicle type allowing us to determine the operational costs exactly.

In [Lie08b], a model for integrating timetabling and a simplified version of vehicle and crew scheduling is introduced as a modified periodic event scheduling problem. Additionally, some aspects of line panning can be integrated by re-matching line segments at stations. The publication [LHS18] can also be interpreted as an integrated approach as aspects of line planning and vehicle scheduling are added to an aperiodic timetabling formulation. This problem is considered for a single-track metro line and it is solved by a branch-and-bound method together with a rolling horizon heuristic. Another heuristic approach to the integration of line planning, timetabling, and vehicle scheduling is given in [MS09] where the order of solving the planning stages is changed. Here, the vehicle schedule is considered first and lines and timetables are fixed later. This concept of solving the planning stages in different orders is generalized in [Sch17], providing an iterative algorithmic scheme for integrated public transport problems.

In Chapter 5, we present a model for integrating line planning, timetabling, passenger routing, and vehicle scheduling for one vehicle type and one depot. The model as well as some analysis on its structure is additionally published in [LPSS18]. Two heuristics for solving the problem are presented in Chapter 6 of which one is additionally published in [PSSS17]. Both heuristics specify parts of the general algorithmic scheme introduced in [Sch17].

The integration of other planning stages is also a subject of current research, e.g., the integration of vehicle and crew scheduling in [MPR09]. Generating robust timetables, see, e.g., [Goe12, GS18], can also be considered as an integration of timetabling and aspects of delay management. In [LHZ18], the integration of real-time data on passenger behavior and vehicle position is suggested to improve the planning process in public transport planning.

Integration is not only considered in public transport planning but it has also gained in importance in other fields as the ability to solve larger problems expanded. Especially in process planning and scheduling, the benefits of integration are considered, see, e.g., [LK93, TK00, GVDHH02] and more recently [DC18]. The integration of several planning stages becomes more complicated and less promising when several parties with opposing interests are involved as demonstrated in [LPW97, BO01] for the example of integrated supply chain management.

## 1.3 Problem Definitions

In the following section we introduce the notations and basic concepts used in this book. Some problem-specific notation is introduced in the later chapters as well as problem-specific assumptions. We often use linear integer programs (abbreviated as IPs) to model the arising optimization problems. As an introduction to integer programming, we refer to [NW88] while for an introduction to complexity and especially NP-completeness and NP-hardness, we refer to [GJ79].