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Seismic Inversion Methods: A Practical Approach

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Chapter 1
Fundamental of Seismic Inversion

Abstract Seismic inversion methods in geophysics is a technique used to transform seismic reflection data into quantitative subsurface rock properties. It is methods to integrate seismic reflection data along with well log data to extract a variety of petrophysical parameters. Geophysicists regularly perform seismic surveys to collect subsurface geological information in the exploration project. Such surveys record sound waves that have passed through the earth's layers of rock and fluid in the form of amplitude and time. The recorded seismic data can be interpreted on its own but this does not provide sufficient information of the subsurface and can be inaccurate under certain circumstances. Because of its efficiency and quality, most oil and gas companies now using seismic inversion methods to increase data resolution and reliability and improve rock properties estimation including porosity and net pay. In this chapter, the basics of forward modeling, convolution model, seismic inversions and their kinds are discussed.

1.1 Introduction

Seismic inversion is a procedure that helps extract underlying models of the physical characteristics of rocks and fluids from seismic and well-log data. In the absence of well data, the properties can also be inferred from the inversion of seismic data alone (Krebs et al. 2009). In the oil and gas industry, seismic inversion technique has been widely used as a tool to locate hydrocarbon-bearing strata in the subsurface from the seismic reflection data (Morozov and Ma 2009; Lindseth 1979). This method dramatically increases the resolution of seismic data and hence helps to interpret seismic data.

The physical parameters that are of interest to a modeler performing inversion are impedance (Z), P-wave (V_P) and S-wave (V_S) velocity and density (ρ). Lame parameters that are sensitive towards fluid and saturation in rocks (Clochard et al. 2009) can also be derived from inverted models of impedances. The petrophysical parameters like porosity, sand/shale ratio, gas saturation, etc. can be estimated further with the help of inverted volumes (Goodway 2001). These petrophysical parameters

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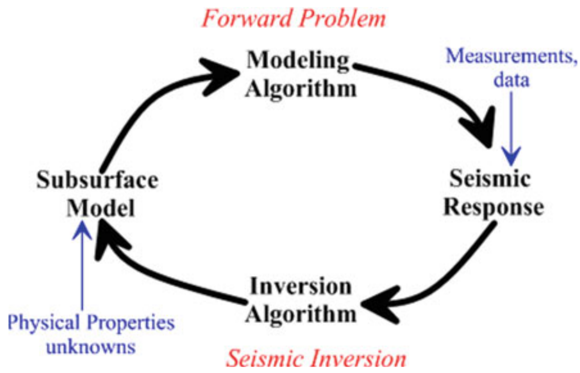


Fig. 1.1 A schematic diagram representing forward and inverse modeling processes

added strength to the seismic data interpretation which is a very crucial process for any exploration project.

To understand seismic inversion methods, one needs to first understand forward modeling. The seismic forward modeling uses the principle of convolution theory which states that seismic trace can be generated by the convolution of wavelet with earth reflectivity. The sound waves are sent to the subsurface that interacts with the earth's layer and returns on the surface which is recorded by the receivers. These recorded signals are called observations (Seismic signal) and the entire process is called forward modeling (Maurya et al. 2018). On the other hand, in seismic inversion methods, we have observation available and we seek to find the subsurface geological model that is a representation of observation. Both processes can be understood from Fig. 1.1.

1.2 Seismic Forward Modeling

There are many geophysical methods used to explore oil and gas from the subsurface but the most important technique is seismic imaging. The imaging means the visual representation of the earth's subsurface model. This is one of the ultimate goals of the geophysicist. Seismic forward modeling can be implemented by using both numerical studies in geoscience and computation technology. The forward modeling procedure uses an elastic impedance method that generates synthetic seismograms from velocities and densities of the subsurface layers (Connolly 1999). The elastic impedance at each interface is calculated as a function of the offset. The resulting impedance series is transformed into the reflectivity series and convolved with the source wavelet to get a stacked seismic gather. The impedance (Z) is computed from the product of velocity (v) and density ρ.

$$Z = V\rho \tag{1.1}$$

The zero offset reflectivity series can be calculated from the impedance as following.

$$R_j = \frac{Z_{j+1} - Z_j}{Z_{j+1} + Z_j} \tag{1.2}$$

where Z_j is the seismic impedance of jth layer, and R_j is seismic reflectivity of the interface between jth and $(j + 1)$th layer. The reflection coefficient for the angle-dependent incident wave is estimated using the following formula (Bachrach et al. 2014).

$$R(\theta) = \frac{1}{2}\left(1 - \tan^2\theta\right)\frac{\Delta I_p}{I_p} - 4\frac{V_s^2}{V_p^2}\sin^2\theta\frac{\Delta I_s}{I_s} - \left[\frac{1}{2}\tan^2\theta - 2\frac{V_s^2}{V_p^2}\sin^2\theta\right]\frac{\Delta\rho}{\rho} \tag{1.3}$$

The synthetic seismogram is calculated from the reflection coefficient using the following equation.

$$S(t) = W(t) * R(t) + N(t) \tag{1.4}$$

where $S(t)$ is synthetic seismogram, $W(t)$ is source wavelet, $R(t)$ is the reflection coefficient of the subsurface and $N(t)$ is additive noise and generally assumed to be zero for simplicity. The seismic forward modeling method gives the understanding of seismic travel time, elastic impedance, arrival time, earth's reflectivity, seismic amplitude generated by the seismic wave and the other aspects.

Figure 1.2 demonstrates the use of forward modeling in faulted and anticline geological models. Figure 1.2a shows geological model of the subsurface whereas Fig. 1.2b depicts the corresponding seismic section generated using the forward modeling technique. From the figure, we can see that the seismic gather (Fig. 1.2b) shows an approximately same geological structure which is in the geological model (Fig. 1.2a). Now our aim is to find this geological model from the observation i.e. seismic section which can be achieved by the seismic inversion methods.

1.3 Seismic Inversion

Seismic inversion methods involve mapping rock and fluid properties of the subsurface of the earth using seismic measurements made on the surface of the earth as input. In fact, all inversion methods aim to estimate the geophysical properties of the subsurface from the measurement made on the surface. In the seismic inversion process, there are three main issues that need to be addressed carefully to get a broadband spectrum with high-resolution images of the subsurface. The first problem arises due to the band-limited nature of the seismic data which means seismic

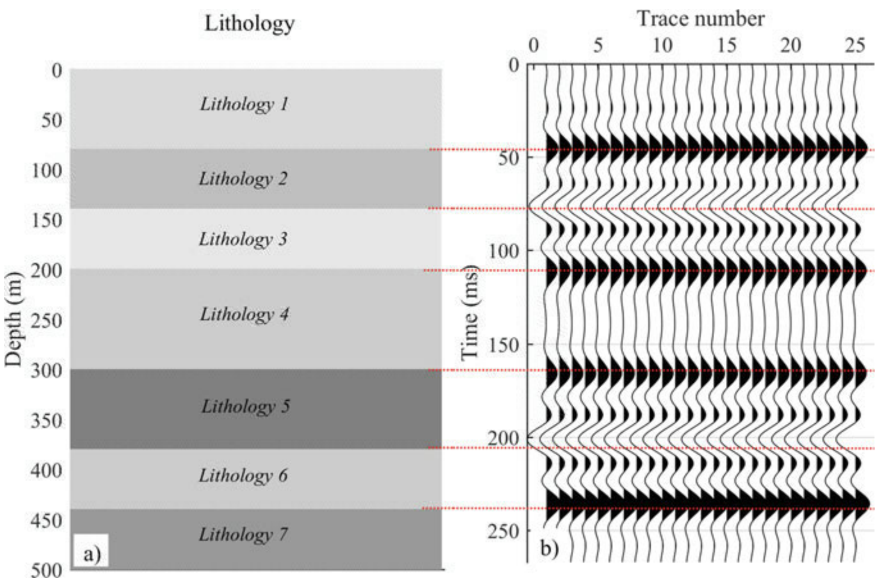


Fig. 1.2 **a** Shows geological model and **b** depicts seismic section expressed geological model

data does not have a low and high-frequency component. The seismic data generally have 10–80 Hz frequency and hence does not contains frequency less than 10 Hz and greater than 80 Hz however these frequencies are very important for the interpretation. On the other hand, well log data has both low and high frequencies and can be integrated with the seismic data to compensate for this. The second problem is to use a seismic wavelet. From the convolution theory, we have seen that the seismic wavelet is used to generate synthetic data and hence accurate wavelet estimation is critical for successful inversion results (Russell 1988; Maurya and Singh 2018). The third and most important problem is that the seismic inversion is non-unique. There may be possibly more than one solution to the same problem. To reduce the number of solution one needs other constraints to bind the inversion results. These constraints can be prior to geological information, well log data, etc.

Figure 1.3 shows an example of seismic inversion methods. Figure 1.3a shows seismic data from the Blackfoot field, Canada whereas Fig. 1.3b shows seismic inversion results. From Fig. (1.3) one can notice how dramatically the image quality has been increased. From the seismic section, one has an only amplitude and hence cannot interpret much. However from the inverted section, one can classify the sand formation, shale formation and hence can identify the productive zone.

To understand the seismic inversion technique, one must first understand the physical processes which are involved in generating these data. The seismic data is generated using a forward modeling technique that uses the convolution model for its implementation. Initially, therefore one should look at the basic convolutional model of the seismic trace in the time and frequency domains. This model has three

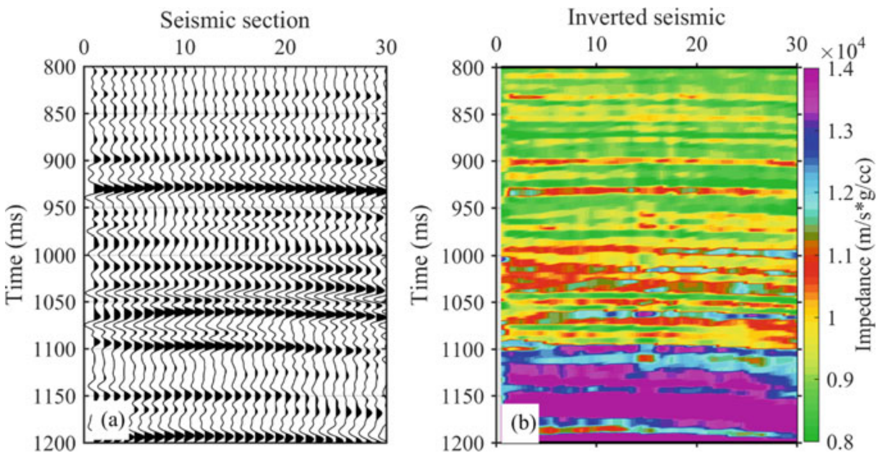


Fig. 1.3 **a** Depicts seismic section and **b** depicts inverted section

components, first is earth's reflectivity, second is seismic wavelet, and the third is the noise. After understanding the concepts and the problems which can occur, one is in a position to look at the methods which are currently used to invert seismic data.

1.4 The Convolution Model

The convolution model is the most common one-dimensional model for the seismic trace. The convolution model states that the seismic trace can be generated by the convolution of seismic wavelet with the earth's reflectivity series along with the addition of noises. It can be written mathematically as follows.

$$S(t) = R(t) * W(t) + N(t) \tag{1.5}$$

where * implies convolution process, $S(t)$ is a seismic trace, $R(t)$ is earth reflectivity, $W(t)$ is wavelet and $N(t)$ is the noise component. By considering perfect case one can consider noise component to be zero and hence Eq. 1.5 can be written simply as

$$S(t) = R(t) * W(t) \tag{1.6}$$

Equations 1.5 and 1.6 represents the convolution model in the time domain. An alternate form of the above equation is frequency domain, can be obtained by taking the Fourier transform of the above equations which results in

$$S(f) = W(f) \times R(f) \tag{1.7}$$

where $S(f)$ is seismic trace and presented in the frequency domain, $W(f)$ is wavelet in the frequency domain and $R(f)$ is the earth's reflectivity in the frequency domain.

From the above equation, we can see that the convolution in time domain becomes multiplication in the frequency domain. However, working in frequency domain is very complex, yet it is normal to consider the amplitude and phase spectra of each individual component (Russell 1988). The Eq. 1.7 can be simplified in the amplitude and phase spectrum as follows.

$$|S(f)| = |W(f)| \times |R(f)| \tag{1.8}$$

$$\theta_S(f) = \theta_W(f) + \theta_R(f) \tag{1.9}$$

where || indicates amplitude spectrum and θ indicates phase spectrum.

In other words, one can say that the convolution process involves multiplying the amplitude spectra of wavelet and reflectivity and adding their phase spectra individually. If one is able to suppress the noise component from the data, and then deconvolve with the wavelet give the earth's reflectivity series. This reflectivity series can be transformed into acoustic impedance which is the ultimate goal of any seismic inversion methods. Figure 1.4 shows the convolution process graphically. The first track of Fig. 1.4 shows reflectivity series generated randomly, track 2 and track 3 show minimum phase wavelet and seismic trace which are generated by the convolution of reflectivity with the source wavelet.

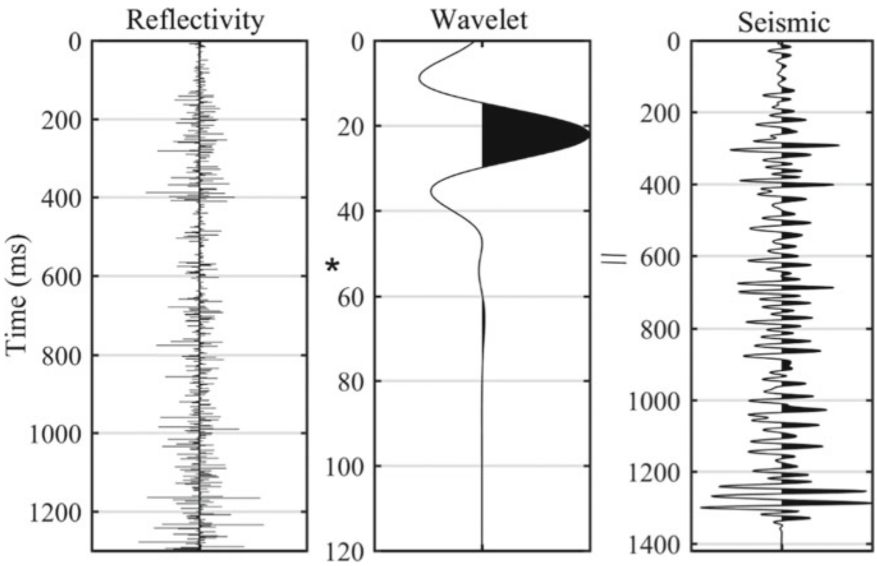


Fig. 1.4 Generation of seismic data by using the convolution model. Track 1 shows the earth's reflectivity, track 2 shows minimum phase wavelet and track 3 shows seismic trace

Now, after having a good understanding of the basic convolution model which is used by most of the seismic inversion methods. We are now in a position to look into the seismic inversion methods and their classifications. The seismic inversion methods are summarized in the following sections.

1.5 Classification of Seismic Inversion

The seismic inversion techniques can be divided into two broad categories, Post-stack, and Pre-stack inversion methods. The first approach is the most commonly used where the effect of the wavelet is removed from the seismic data and a high-resolution image of the subsurface is produced (Chen and Sidney 1997). The second approach relies on model building from well log, seismic and geological data (Downton 2005). This also generates a high-resolution image of the subsurface from which reservoir properties are calculated. A reliable estimate of the reservoir properties is critical in the decision-making process during the development phase (Pendrel 2006). These inversion methods are further divided into subparts which are discussed in the following sections.

1.5.1 Post-stack Seismic Inversion

Post-stack seismic inversion techniques are categorized as the first type. This type of inversion results in acoustic impedance volume utilizing seismic data through the integration of the well data and a basic stratigraphic interpretation. This impedance volume can be used to estimate reservoir properties away from well (Russell and Hampson 1991; Morozov and Ma 2009). Some of the advantages of post-stack inversion are mentioned below.

1. As the acoustic impedance is a layer property; hence stratigraphic interpretation is easier on impedance data than seismic data.
2. The reduction of wavelet effects, side lobes, and tuning enhance the resolution of subsurface layers.
3. Acoustic impedance can be directly computed and compared to well log measurements that serve as a link to reservoir properties.
4. Porosity can be related to the acoustic impedance. Using geostatistical methods these impedance volume can be transformed into the porosity volume within the reservoir.
5. Acoustic impedance can be utilized to locate individual reservoir regions.
6. It takes very less time than pre-stack inversion.
7. It does not give shear wave information to discriminate against the fluid effects.

Further, post-stack inversion can be divided into two parts namely deterministic inversion which includes model-based inversion methods and second is Stochastic inversion methods which include Band-limited inversion, Colored inversion, and sparse spike inversion methods. All these methods utilize post-stack seismic data and are inverted for the impedance section. All these methods have a different operating principle and they need proper understanding before applying to the data.

We will start by looking at the most common methods of post-stack inversion i.e. band-limited inversion methods. The Band-Limited recursive (iterative) inversion method was developed by Lindseth (1979). It is the most basic type of inversion which assumes that seismic amplitude is proportional to the earth reflectivities and transform the input seismic trace to acoustic impedance traces. The equation used by BLI is given below.

$$Z_{j+1} = Z_1 exp\left(\gamma \sum_{k=1}^{j} S_k\right) \tag{1.10}$$

where $S_k = 2r_k/\gamma$, Z is impedance and r is earth's reflectivity. It used to integrate the seismic trace and then exponentiates the result to provide an impedance trace. The input seismic data is usually wavelet processed. This does not however fully satisfy the basic assumptions since the wavelets are not completely removed from the data. Consequently, the tuning and the wavelet side lobe's effects remain. Moreover, the results are produced within the seismic bandwidth and errors in calculating the acoustic impedance (AI) for the subsequent layers get integrated (Maurya and Singh 2018; Maurya et al., 2019)

Thereafter, the second type of post-stack inversion methods i.e. model-based inversion is utilized. This method uses post-stack seismic data and computes acoustic impedance. The method is based on the convolutional theory which states that the seismic trace can be generated from the convolution of wavelet with the reflectivity function (Leite 2010; Maurya and Singh 2015a, b). This method model the subsurface as layers or blocks in terms of acoustic impedance and time. The initial impedance model is built from the interpolation of the impedance logs obtained from the wells in the area. The impedance of each layer may vary laterally and vertically. The impedance bonds are set to keep the optimized model laterally smooth within the given limits.

Another type of post-stack inversion technique used most commonly is the Colored inversion method. In this method, the inversion is represented as a convolutional process where an operator (O) in the frequency domain is used to transform the seismic traces (S) into impedance (Z): Z = O * S (Lancaster and Whitcombe 2000). This operator maps the seismic amplitude spectrum into the earth impedance spectrum. Spectra of Acoustic Impedance logs calculated from wells in the same area are used to derive the spectrum of the operator. The phase of this operator is 900 making it easy to integrate it with the reflectivity series to give impedance (Ansari 2014).

The sparse-spike method is another type of post-stack inversion methods used to estimate subsurface physical property. This method, unlike other techniques, gives an estimate of the reflectivity series that would approximate the seismic data with minimum number of (sparse) spikes. Non-uniqueness, in this case, is taken care of by applying the sparse reflectivity criterion. The maximum likelihood deconvolution and L_1 norms (Linear Programming) logarithms are used to achieve this (Banihasan et al. 2006).

1.5.2 Pre-stack Seismic Inversion

The Pre-stack inversion falls into the second category of the seismic inversion techniques. The estimate of the elastic properties of the subsurface such as the S-wave velocity of the subsurface layers which are sensitive to fluid saturation can be obtained from Pre-stack inversion (Moncayo et al. 2012).

Pre-stack inversion transforms seismic data (angle/offset gathers) into P-impedance, S-impedance and density volumes through the integration of well data and horizon information from seismic data. P-impedance and Vp/Vs ratio are reliable, depending on target depth and acquisition configuration, and can be used to predict reservoir properties away from well (Carrazzone et al. 1996). Pre-stack seismic inversion provides several benefits.

1. The P-impedance, S-impedance, and density give layer properties, whereas seismic data is an interface property.
2. Enhanced resolution of sub-surface layers due to the reduction of wavelet effects, tuning and side lobes.
3. Acoustic impedance can be directly compared to well log measurements which in turn are linked to reservoir properties.
4. Compared with other inversion techniques (e.g. post-stack inversion), the data offers additional information to distinguish between lithology and fluid effects.

The most common methods which fall in this category are simultaneous inversion, Elastic impedance inversion, and AVO inversion methods.

Simultaneous inversion is the first type of pre-stack seismic inversion. Pre-stack seismic gather contains additional information i.e. S-wave velocity which travels slowly in the subsurface and contains more information about the rock properties of the earth. This information can be estimated from the pre-stack gathers using several seismic inversion methods. A common approach is simultaneous inversion of pre-stack seismic data, which inverts for several rock property parameters simultaneously. The Aki and Richards (1980) formula gives access to the approximate reflectivity at the various offsets in the pre-stack domain and can be expressed as follows.

$$R(\theta) = a\frac{\Delta V_P}{V_P} + b\frac{\Delta\rho}{\rho} + c\frac{\Delta V_S}{V_S} \tag{1.11}$$

where $a = \frac{1}{2\cos^2\theta}$, $b = 0.5 - \left[2\left(\frac{V_S}{V_P}\right)^2 \sin^2\theta\right]$, $c = -4\left(\frac{V_S}{V_P}\right)^2 \sin^2\theta$,

$$\rho = \frac{\rho_1 + \rho_2}{2}, \Delta\rho = \rho_2 - \rho_1, V_P = \frac{V_{P1} + V_{P2}}{2},$$
$$\Delta V_P = V_{P2} - V_{P1}, V_S = \frac{V_{S1} + V_{S2}}{2},$$
$$\Delta V_S = V_{S2} - V_{S1}, \theta = \frac{\theta_1 + \theta_2}{2}$$

These functions are used to compute the earth's reflectivity. Thereafter, reflectivity is convolved with the wavelet to obtain the synthetic seismic gather. Further, this synthetic gather is compared with the original seismic gather of the area and the misfit is computed between them. The model is subsequently perturbed and a new comparison made to reduce the misfit and the least misfit model is selected as the final solution.

The other inversion method used commonly is the elastic impedance inversion. Connolly (1999) introduced the concept of Elastic Impedance and defined a function R which is dependent on the incidence angle and is related to the relative variation of Elastic Impedance.

$$R(\theta) \approx \frac{1}{2}\frac{\Delta EI}{EI} \approx \frac{1}{2}\Delta \ln(EI)$$

This function R is now called the Elastic Impedance, in analog to the acoustic impedance concept. The angle-dependent P-wave reflectivity is also approximated by the well-known simplified description of the Zoeppritz equations (Aki-Richards):

$$R(\theta) = A + B\sin^2\theta + C\tan^2\theta\sin^2\theta \tag{1.13}$$

where $A = \frac{1}{2}\left[\frac{\Delta_P}{V_P} + \frac{\Delta\rho}{\rho}\right]$, $B = \frac{1}{2}\frac{\Delta V_P}{V_P} - 4\left[\frac{V_S}{V_P}\right]^2\frac{\Delta V_S}{V_S} - 2\left[\frac{V_S}{V_P}\right]^2\frac{\Delta\rho}{\rho}$, $C = \frac{1}{2}\frac{\Delta V_P}{V_P}$.

Combining the two expressions, doing $k = [V_S/V_P]^2$ constant results in the Elastic Impedance being equal to:

$$EI = V_P^{(1+\tan^2\theta)} V_S^{(8K\sin^2\theta)} \rho^{(1-8K\sin^2\theta)} \tag{1.14}$$

Other types of pre-stack inversion techniques, Amplitude-Variation-with-Offset (AVO), have been widely used in hydrocarbon exploration over the previous two decades. Traditional AVO analysis involves calculating the term AVO intercept, gradient, and higher-order AVO from the fit of the amplitude of P-wave reflection to the sine of the square of angle of incidence. This model is focused, among others, on Bortfeld (1961) and Shuey (1985), the estimated P-wave reflection coefficient model in intercept-gradient shape. The AVO intercept and gradient values can also be coupled under the assumption of a background PS velocity ratio to acquire extra AVO characteristics such as the pseudo-S-wave information and Poisson's contrast ratio. In a hybrid inversion system, AVO intercept and pseudo-S-wave information are also used together with pre-stack waveform reversal (PSWI). Hybrid inversion is

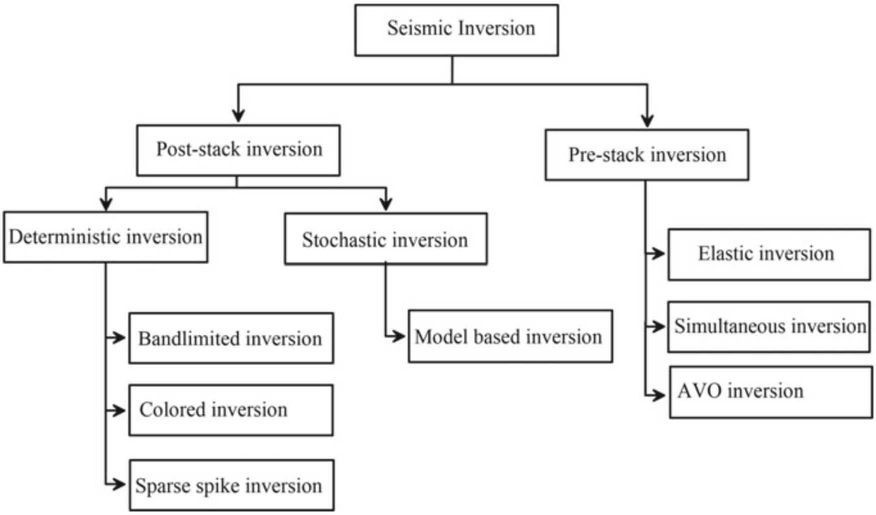


Fig. 1.5 Flowchart of classification of seismic inversion methods

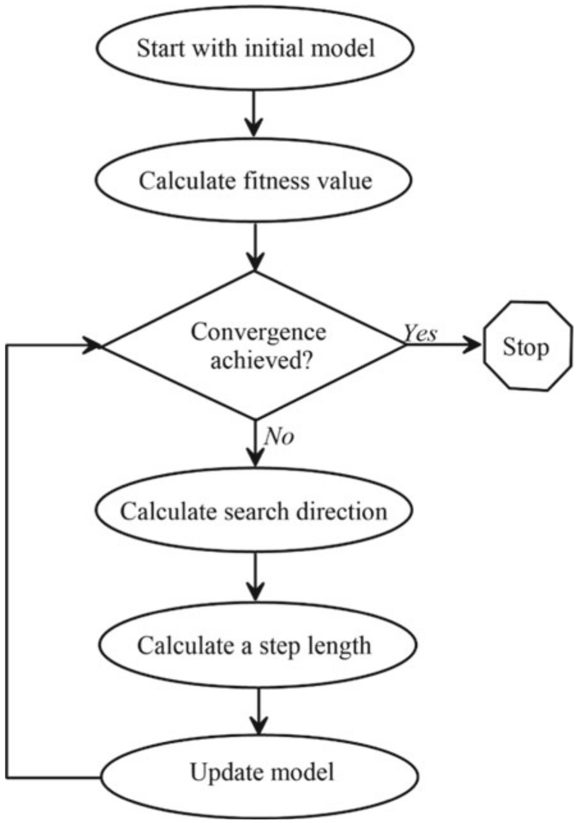
a mixture of methodologies for pre-stack and post-stack inversion. This mix allows big data amounts to be efficiently reversed in the lack of good information.

Figure 1.5 discusses the classification of seismic inversion methods graphically although some recent development has been done which added more classification but above discussed inversion methods are convention techniques that are still in use to estimate subsurface properties from seismic and well log data. These inverted sections strengthen to seismic data interpretation and hence help to characterize the reservoir.

1.6 Local Optimization Methods

Local optimization method is a heuristic technique to solve computationally difficult problems of optimization. Local optimization can be used on problems that can be formulated as a solution that maximizes a criterion among a number of candidate solution alternatives. A typical local optimization scheme's flow chart is displayed in Fig. 1.6. Most local optimization systems are iterative algorithms, and the primary objective of all these algorithms is to ensure that the objective function is reduced at each iteration. These algorithms always try to move downhill and are therefore referred to as greedy algorithms (Xu et al. 2012). It is quite evident that the local algorithms in the neighborhood of the starting point are searching for a local minimum. The selection of a starting solution is therefore of paramount importance, a poor selection of the starting solution will result in the algorithm being trapped in a local minimum.

Fig. 1.6 Flow chart of local optimization methods



The algorithm calculates a search path using a local property of the objective function, determining an update or increase to the present model, given a starting solution. Only a fraction of the measured update is applied, however, and a step-length factor is used to ensure that the new objective function value is always lower than the current value (Chunduru et al. 1997). Thus the critical factors are the update calculations and the step length. Methods of optimization vary on the basis that how these two parameters are calculated.

1.7 Global Optimization Methods

Global methods of optimization are grooving techniques for methods of seismic inversion. Optimization involves finding the optimal value of a multi-variable function. The feature we want to minimize (or maximize) is a misfit (or fitness) function which characterizes the distinctions (or similarities) between observed and synthetic

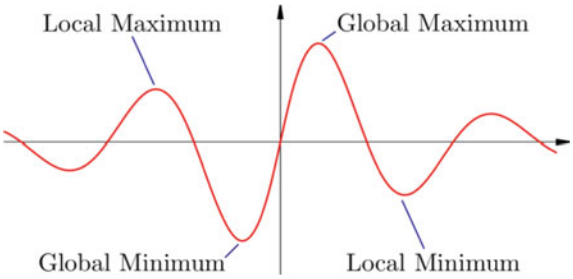


Fig. 1.7 Figure demonstrates the difference between local and global optimum

data calculated using an assumed earth model (Sen and Stoffa 2013). Physical parameters that characterize the characteristics of rock layers, such as compressional wave velocity, shear wave speed, resistivity, etc., describe the earth model.

The solution of a linear equation system is the same as a quadratic function minimization. Local minimization processes mentioned above are appropriate when there is a single minimum of the cost function. In particular, however, inverse geophysical problems are non-linear and extremely complicated. The cost function is probable to demonstrate multiple minima in such instances. In such cases, local optimization techniques are anticipated to fail as they tend to cover the closest local minima (Sen et al. 1995). Local minimization algorithms are anticipated to cover a fake solution and produce bad outcomes unless there is adequate a priori data to make an intelligent guess about the original model. Gradient-based optimization systems do not provide the means to jump from a local minimum to a worldwide solution (Maurya and Singh 2019). However, in most situations, computationally more expensive are the global optimization schemes compared to local optimization schemes. Figure 1.7 shows the difference between local and global solutions.

The primary motivation to develop efficient global optimization technique lies in the fact that the technique should be practicable in obtaining better results compared to the local optimization techniques in situations where the problem is complex and the model dimensionality is large. There are many global optimization methods available but Genetic algorithm and Simulated annealing are broadly used in the seismic inversion.

The technique of the genetic algorithm (GA) is based on the analogy that the genetic modifications that occur in the living species work towards making the species smarter and more adaptive to the changing natural environment. One of the powerful tools for global optimization is the genetic algorithm; it is also based on the principle of random walking in the space parameter (Sen and Oltz 2006; Maurya et al. 2018). GA has artificial intelligence that can handle issues that are highly nonlinear. GA needs no derivatives or information on curvature. Therefore, once the forward problem is solved, the reverse problem can be solved automatically because the whole exercise consists of selecting some models, calculating synthetic data, comparing d^{prs} with d^{obs}, calculating the cost function or error function and selecting better and better models through certain guidelines when deciding on the acceptance criterion (Sen and Stoffa 2013).

The simulated annealing (SA) uses techniques of random walking, but with some artificial intelligence. It brings the concept of temperature in this technique and which is controlling parameters even though the temperature has nothing to do with an inverse problem. The function of energy in thermodynamics is replaced by the function of error. This error function is also referred to in global optimization as cost function or energy function. In thermodynamics, the probability density functions of Gibbs can be expressed as follows.

$$P(E_j) = \frac{exp\left(\frac{-E_j}{KT}\right)}{\sum exp\left(\frac{-E_j}{KT}\right)} \tag{1.15}$$

where K is the constant of Boltzmann. K is assumed to be unity in global optimization because it has no role in iterative methods of inversion. Difference between the observed field data d^{obs} and the model's synthetic data, i.e. d^{pred} generates the error function, defined as follows.

$$E(m) = \left(d^{obs} - d^{pred}\right)^T \left(C_D^{-1}\right)\left(d^{obs} - d^{pred}\right) \tag{1.16}$$

where C_D is the data covariance. The inverse problem begins with a very high initial temperature of the controlling parameter. The temperature is progressively reduced in consecutive iteration to make Gibb's probability density function more and more sensitive (Vestergaard and Mosegaard 1991). In this manner, the cost function is minimized. This technique is discussed in detail in Chap. 6.

1.8 Geostatistical Inversion

Geostatistical methods are routinely followed to predict various geophysical parameters from seismic and well log data. The geostatistical methods use sample points taken at different locations and interpolate in the seismic section where log data are not available. These sample points are measurements of petrophysical parameters in the boreholes (Haas and Dubrule 1994). The geostatistics derives a surface using the values from the measured locations to estimate data points for each location in between the data points. Two groups of interpolation techniques are provided by Geostatistics namely deterministic and geostatistical (Russell et al. 1997). Mathematical functions are used in the deterministic techniques for interpolation whereas geostatistics uses both statistical and mathematical methods (Hampson et al. 2001).

There are four types of geostatistical method namely single attribute analysis, multi-attribute regression, Neural network which include multi-layer feed-forward and probabilistic neural network methods. The procedures of the geostatistical method are as follows.

1. The spatial continuity of the well log data is quantified using variograms.
2. A statistical relationship is derived between the log and seismic data at all well locations using cross-validation plots. The single attribute analysis, multivariate regression uses linear relationship whereas the Neural Network uses a non-linear relationship.
3. These linear and non-linear relationships are then used to estimate well log property away from the boreholes.
4. The predicted well log property is evaluated for its reliability.

In single attribute analysis, first, a variety of attributes are estimated directly or indirectly from the seismic data and are analyzed to get the best attribute. The best attribute selected on the basis of its correlation with the target log values. Further, the best attribute is cross plotted with target value from the well log data and a best fit straight line is chosen which gives the desired relationship. This desired relationship is then used to predict petrophysical parameters in the inter-well region.

Multi-attribute regression is the second type of geostatistical method used to predict petrophysical parameters. The basic principle is similar to the single attribute analysis. The multi-attribute analysis has only one difference from the single attribute analysis, in the sense that this method uses more than one attribute at a time. In this method, all the attributes are analyzed and the best combination of attributes (more than one) is selected and cross plotted with a target value to give linear relationship which is used for further analysis.

Till now the analysis is linear which have some limitation but now we are extending our analysis to nonlinear. The neural network falls in this category. It is divided into two parts namely multi-layer feed-forward neural network and probabilistic neural network. A multilayer feed-forward neural network is an interconnection of neurons in which data and calculations flow in a single direction, from the input data to the outputs with intermediate one or more hidden layers. Each layer consists of nodes, and the nodes are connected with a particular weight. These weights decide the results of the output layer (Dubrule 2003; Hampson et al. 2001). In this book, we have used as many input nodes as the number of attributes used for the analysis. Mostly, the output layer consists of one node and hence one output results, since our target is to predict one single petrophysical parameter at a time. Figure 1.8 demonstrates MLFN architecture in which four attributes are used as four-node in the input layer, one hidden layer with three nodes are used and finally one node of the output layer has been used. All nodes from the input layer are connected to the hidden layer with the weights. And all nodes of the hidden layer are connected to the output layer by the property called summation.

An alternative type of neural network is a probabilistic neural network (Masters 1995; Specht 1990, 1991). This method is actually based on a mathematical interpolation technique that uses neural network architecture for its implementation. This is the advantage of using PNN rather than MLFN since by studying the mathematical formulation one can understand the behavior in a better way.

Forgiven training datasets, the PNN technique assumes that the predicted log can be written as a linear combination of the log values in the training data. Let's we have