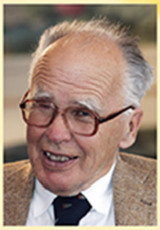
Helge Holden
Ragni Piene
Editors








The Abel Prize 2003–2007

The First Five Years

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ABEL PRISEN

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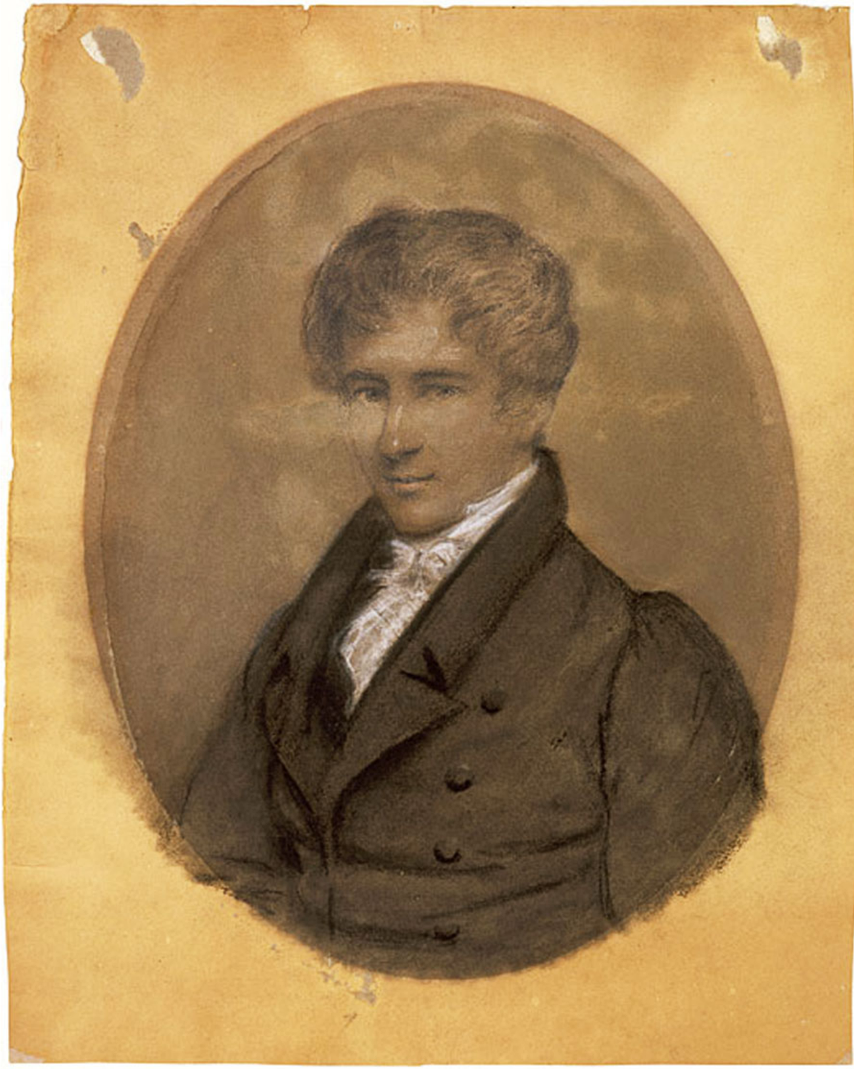
The Abel Prize



Niels Henrik Abel 1802–1829
The only contemporary portrait of Abel, painted by Johan Gørbitz in 1826

Helge Holden • Ragni Piene
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The Abel Prize

2003–2007 The First Five Years

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Preface

In 2002, the year marking the bicentennial of Abel's birth, the Norwegian Parliament established the Niels Henrik Abel Memorial Fund with the objective of creating an international prize for outstanding scientific work in the field of mathematics—the Abel Prize.

In this book we would like to present the Abel Prize and the Abel Laureates of the first five years. The book results from an initiative of the Mathematics section of the Norwegian Academy of Science and Letters. It is intended as the first volume in a series, each volume comprising five years.

The book starts with the history of the Abel Prize—a story that goes back more than a hundred years—written by Abel's biographer, Arild Stubhaug. It is followed by Nils A. Baas' biographical sketch of Atle Selberg; at the opening ceremony of the Abel Bicentennial Conference in Oslo in 2002, an Honorary Abel Prize was presented to Atle Selberg.

There is one part for each of the years 2003–2007. Each part starts with an autobiographical piece by the laureate(s). Then follows a text on the laureate's work: Pilar Bayer writes on the work of Jean-Pierre Serre, Nigel Hitchin on Atiyah–Singer's Index Theorem, Helge Holden and Peter Sarnak on the work of Peter Lax, Tom Körner on Lennart Carleson, and Terry Lyons on Srinivasa Varadhan. Each part contains a complete bibliography and a curriculum vitae, as well as photos—old and new.
Extra material can be found at https://extras.springer.com/2010/978-3-642-01373-7. It contains the interviews that Martin Raussen and Christian Skau made with each laureate in connection with the Prize ceremonies in the years 2003–2007.
Every year except the first, the interviews were broadcast on Norwegian national television. Transcripts of all interviews have been published in the *EMS Newsletter* and *Notices of the AMS*.

We would like to express our gratitude to the laureates for collaborating with us on this project, especially for providing the autobiographical pieces and the photos. We would like to thank the mathematicians who agreed to write about the laureates, and thus are helping us in making the laureates' work known to a broader audience.

Thanks go to Martin Raussen and Christian Skau for letting us use the interviews, to David Pauksztello for his translations, to Marius Thaule for his LaTeX expertise and the preparation of the bibliographies, and to Anne-Marie Astad of the

Norwegian Academy for Science and Letters for her help with the interviews and photos. HH gratefully acknowledges support from the Centre for Advanced Study at the Norwegian Academy of Science and Letters in Oslo during the academic year 2008–09.

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Oslo
July 15, 2009

Helge Holden and Ragni Piene

Contents

The History of the Abel Prize and the Honorary Abel Prize

The History of the Abel Prize

Arild Stubhaug

On the bicentennial of Niels Henrik Abel's birth in 2002, the Norwegian Government decided to establish a memorial fund of NOK 200 million. The chief purpose of the fund was to lay the financial groundwork for an annual international prize of NOK 6 million to one or more mathematicians for outstanding scientific work. The prize was awarded for the first time in 2003.

That is the history in brief of the Abel Prize as we know it today. Behind this government decision to commemorate and honor the country's great mathematician, however, lies a more than hundred year old wish and a short and intense period of activity.

Volumes of Abel's collected works were published in 1839 and 1881. The first was edited by Bernt Michael Holmboe (Abel's teacher), the second by Sophus Lie and Ludvig Sylow. Both editions were paid for with public funds and published to honor the famous scientist. The first time that there was a discussion in a broader context about honoring Niels Henrik Abel's memory, was at the meeting of Scandinavian natural scientists in Norway's capital in 1886. These meetings of natural scientists, which were held alternately in each of the Scandinavian capitals (with the exception of the very first meeting in 1839, which took place in Gothenburg, Sweden), were the most important fora for Scandinavian natural scientists. The meeting in 1886 in Oslo (called Christiania at the time) was the 13th in the series. At the meeting's farewell dinner, the Swedish mathematician Gösta Mittag-Leffler gave a toast in honor of Niels Henrik Abel, and he proposed starting a collection with the goal that in 16 years—in 1902, on the centennial of Abel's birth—a statue of the young genius could be erected. Money was collected during the meeting and national committees were appointed, but eventually the whole effort ran out of steam.

Mittag-Leffler, who had been publishing the Swedish mathematics journal, *Acta Mathematica*, since 1882, worked during these years to arrange and gather support for an international mathematics prize, namely King Oscar II's Mathematics Prize,

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a competition in which an answer was sought to one of four given questions. The prize was awarded on the King's 60th birthday in January 1889, and it was a tremendous success in every way. The prize winner was Henri Poincaré, who submitted a work that described chaos in space: a discovery that was only understood in its full breadth much later and that gradually developed into a major interdisciplinary research area. On the jury for the prize sat Charles Hermite and Karl Weierstrass together with Mittag-Leffler, and the latter discussed the possibility of establishing a permanent mathematics prize with King Oscar and various patrons and donors. Due to insufficient support, however, Mittag-Leffler initially tried to establish a smaller fund, and he proposed that money from this fund should be used for gold medals, which should be awarded to mathematicians who had published an exceptionally important work in *Acta Mathematica*. The gold medals were to be stamped with portraits of the greatest mathematicians, and he was of the opinion that it was suitable to begin with the greatest mathematician in the Nordic countries: Niels Henrik Abel.

These plans also came to naught. Instead, Mittag-Leffler managed to set up a fund that supported the editing of articles submitted to *Acta Mathematica* and that made it possible to invite great foreign mathematicians to Stockholm. When the content of Alfred Nobel's last will and testament became known in 1897, rumors abounded that Mittag-Leffler's financial antics and scientific plans and ideas might have dissuaded Nobel from providing funds for a prize in mathematics, in addition to those in physics, chemistry and medicine as well as literature and efforts to promote peace. It is true that Mittag-Leffler and Nobel discussed financial support for both Stockholm University College (now Stockholm University) and an extraordinary professorship for Sonja Kovalevsky and that they were in strong disagreement, but the reason why there was not any Nobel Prize in mathematics seems to clearly lie in Nobel's attitude to science and technology. He was a practical man and regarded mathematics in general as much too theoretical and having no practical applications.

The annual Nobel Prizes, awarded for the first time in 1901, quickly overshadowed other scientific prizes. At the academies of science in Paris and Berlin, mathematics prizes based on various problems, often in astronomy and navigation, had been awarded ever since the middle of the eighteenth century, and new prizes came into being in the nineteenth century [1]. Prizes were also announced in Leipzig, Göttingen and at other centers of learning. In 1897, the international Lobachevsky Prize was established at the University of Kazan. This prize was supposed to go to outstanding works in geometry, especially non-Euclidean geometry, and the first winner was Sophus Lie.

Sophus Lie, Norway's second world-class mathematician, died in 1899. One of the last things he used his international contact network for was to gather support for establishing a fund that would award an Abel Prize every fifth year for outstanding work in pure mathematics. Apparently, an inspiration in Lie's work was precisely the fact that Nobel's plans included no prize in mathematics. From leading centers of mathematical learning, Sophus Lie received overwhelming support for such an Abel Prize in the spring of 1898. From Rome and Pisa came assurances of support from Luigi Cremona and Luigi Bianchi; from Paris Émile Picard wrote that both he and Hermite would donate money to the fund, and Picard, who otherwise would like to see a more frequent awarding of the prize than once every fifth year, reported that through its universities and lyceums France would also probably be able to contribute large sums; Gaston Darboux followed up with similar positive reactions and thought that all mathematicians in the Academy of Science in Paris would support an Abel Fund; Sophus Lie also received a warm declaration of support from A.R. Forsyth at Cambridge, who thought that Lord Kelvin would certainly lend his support to the fund; Felix Klein at Göttingen reported that he would obviously support the work, and he believed that David Hilbert would do so as well; Lazarus Fuchs was also supportive. The only mathematicians who expressed skepticism were Georg Frobenius and H.A. Schwarz in Berlin; they thought prizes in general often diverted younger talents away from the true scientific path.

Sophus Lie's contacts and promises of support, however, were related to him personally. When Sophus Lie died, there was no one else who could carry on the work.

At the celebration of the centennial of Abel's birth in 1902, three main tasks were formulated in Norwegian political and scientific circles: first, to arrange a broad cultural commemoration, second, to erect a worthy monument to the genius, and third, to establish an international Abel Prize. The first two tasks were achieved. The Abel commemoration in September 1902 was held with pomp and circumstance, and students, citizens, scientists, artists, the national assembly, the government and the Royal House all took part. A number of foreign mathematicians were present and were awarded honorary doctorates. Gustav Vigeland's great Abel Monument on the Royal Palace grounds (in Oslo) was unveiled six years later, but the plans for an Abel Prize were put on ice for reasons of national politics.

Gustav Vigeland's Abel Monument, Oslo

In the Norwegian capital it was regarded as important that the commemoration of Abel should put Norway on the map as a cultural nation, not least with a view to the conflict over the union (with Sweden), which many realized was imminent. However, King Oscar still sat on the Swedish–Norwegian throne, and after his mathematics prize (in 1889) and his support for *Acta Mathematica* (in 1882), he was regarded as having a special fondness for mathematics. The King himself also took active part in the Abel celebrations and arranged a big festivity at the Palace. Just after the conclusion of the official celebrations, Norwegian politicians and scientists were informed that King Oscar was considering having a gold medal created in memory of Niels Henrik Abel. The idea was that the medal should be awarded once every three years by the University of Oslo for top-flight mathematical work.

Two Norwegian scientists, Waldemar C. Brøgger and Fridtjof Nansen, and a representative from the Royal Court were delegated to draw up statutes, and Gustav Vigeland drew sketches for an Abel medal. When the proposal was presented on the King's birthday in January 1903, it was recommended that the prize be awarded every fifth year by the Scientific Society of Christiania (now the Norwegian Academy of Science and Letters in Oslo), and that the prize should go to the best mathematical work published during the last five years. However, decisions about the procedure, the prize committee, etc. were to be announced later.

In the ongoing work, many people were consulted for advice. Mittag-Leffler, who was well-informed about the establishment of the Bolyai Prize in Budapest, sent a copy of the statutes for that prize to Brøgger [5]. (The Bolyai Prize was awarded for the first time in 1905 to Henri Poincaré and the next time five years later to David Hilbert.) At that time, Mittag-Leffler was afraid that an Abel Prize, if there were to be one, would be overshadowed by the Nobel Prize. He did not think it was possible to find a new patron who could elevate an Abel Prize to the Nobel Prize level, and he was also of the opinion that it would be easier for a jury to make an irreproachable selection if there was a prize competition focused on a given problem or question, preferably related to Abel's work.

The mathematicians Ludvig Sylow and Carl Størmer were the key members of a committee that was supposed to draw up a set of rules for an Abel Prize. In the autumn of 1904, they submitted a memo, but the work had not been completed when the dramatic political events of June 1905 resulted in the dissolution of the union between Sweden and Norway. All further plans for an Abel Prize were set aside. The realities of the matter were expressed by Nansen in a letter to the mathematician Elling Holst: "The Abel Prize that we had been promised by good King Oscar went to heaven with the union."

In international circles of mathematicians, however, the lack of a prize in mathematics on the same level as the Nobel Prize was a frequent topic of discussion. This lack was a prime motivation for John Charles Fields in his efforts to establish the prize medal that would come to bear his name. The Fields Medal was awarded for the first time at the International Congress of Mathematicians in Oslo in 1936. Even though no money is awarded with the Fields Medal, and it is only awarded every fourth year at the International Congress of Mathematicians to two to four mathematicians under age 40, the Fields Medal rapidly gained the status of the most eminent prize in mathematics, a kind of “Nobel Prize” in mathematics; a position it has held until the Abel Prize finally became a reality.

In Norway, Abel’s name and memory were kept alive in various ways on into the twentieth century. On the occasion of the centennial of his death in 1929, Abel was commemorated on Norwegian stamps; aside from the royal family, only the playwright Henrik Ibsen had previously been so honored. In 1948, Norges Bank printed Abel’s portrait on the obverse of the 500-kroner banknote. Abel has also been used in later banknote and stamp issues, and books have been written about his life and scientific efforts. When the International Mathematical Union, with UNESCO support, designated the year 2000 as the “World Mathematical Year”, Abel was Norway’s leading logo. Abel’s international position and his life and work were also at the heart of the efforts leading up to the bicentennial of Abel’s birth. The objective of a number of national and international efforts aimed at the profession, schools and society at large was to create a broader appreciation of the importance of mathematics and science for today’s society.

In 1996, I published a biography of Niels Henrik Abel (an English edition was published in 2000 [3]), and in response to an initiative from the Department of Mathematics at the University of Oslo, I subsequently worked on a biography of Sophus Lie [4]. I was very familiar with Lie’s contact network and efforts on behalf of an Abel Prize, and in lectures and conversations in academic circles of mathematicians, I brought up the old idea of such a prize. Most of the people I talked to thought the idea was fascinating, but extremely unrealistic. At a book signing in August of 2000, I met Tormod Hermansen, the President and CEO of Telenor at the time and a prominent Labor Party supporter. Hermansen showed immediate interest in an Abel Prize and argued in his political circles that funds should be allocated for such a prize. The reactions were positive, and at the Department of Mathematics at the University of Oslo, a working group was formed: the Working Group for the Abel Prize, consisting of Professors Jens Erik Fenstad, Arnfinn Laudal and Ragni Piene together with Administrative Head of Department Yngvar Reichelt, Assistant Professor Nils Voje Johansen and myself. With support from key figures in university, business and cultural circles, this working group had talks with the relevant Ministries and members of the Storting [the Norwegian Parliament]. Declarations of support were also received from the major international mathematics organizations—the *International Mathematical Union* and the *European Mathematical Society*. In May 2001, the working group submitted a proposal to the Prime Minister to establish an Abel Prize, and in August 2001, Prime Minister Jens Stoltenberg announced that the Norwegian Government would establish an Abel Fund worth NOK 200 million: a greater amount than the working group had proposed [2]. The Prime Minister emphasized the broad political consensus that the proposal had aroused and the hope that an annual Abel Prize would strengthen the research in and recruitment to mathematics and the natural sciences and raise international awareness of Norway as a knowledge-based nation.

The *Niels Henrik Abel Memorial Fund* is administered by the Norwegian Ministry of Education and Research, and the annual return on the fund is allocated to the Norwegian Academy of Science and Letters, which is entrusted with awarding the prize and the management of other matters related to the funds. The Norwegian Academy of Science and Letters has established a board and a committee of mathematicians for the Abel Prize. The Abel Board shall be responsible for distributing the return on the fund and for events associated with the award ceremony, whereas the Abel Committee is responsible for reviewing candidates for the prize and make a recommendation to the Academy. This international committee consists of five persons who are outstanding researchers in the field of mathematics; both the International Mathematical Union and the European Mathematical Society nominate committee members.

As it is laid down in the statutes, the annual Abel Prize is a recognition of a scientific contribution of exceptional depth in and significance for the field of mathematics, including mathematical aspects of information technology, mathematical physics, probability theory, numerical analysis and computational science, statistics, and applications of mathematics in other sciences. One of the objectives for the prize is that it shall be awarded over the years in a broad range of areas in the field of mathematics.

As is also laid down in the statutes, the prize should contribute towards raising the status of mathematics in society and stimulate the interest of young people and children in mathematics. This objective was a very important argument for the creation of the prize, it was explicitly mentioned by the Prime Minister when he announced the establishment of the Fund, and it was most likely decisive for the Government's and the Parliament's acceptance.

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2002—an Honorary Abel Prize to Atle Selberg

Nils A. Baas

When the Abel Prize was established in 2002 it was decided to award an honorary prize to the renowned Norwegian mathematician Atle Selberg in recognition of his status as one of the world's leading mathematicians. His contributions to mathematics are so deep and original that his name will always be an important part of the history of mathematics. His special field in mathematics was number theory in a broad sense.

Selberg was born on June 14, 1917 in Langesund, Norway. He grew up near Bergen and went to high school at Gjøvik. His father was a high school teacher with a doctoral degree in mathematics, and two of his older brothers—Henrik and Sigmund—became professors of mathematics in Norway. He was studying mathematics at the university level at the age of 12. When he was 15 he published a little note in *Norsk Matematisk Tidsskrift*.

He studied at the University of Oslo where he obtained the *cand. real.* degree in 1939, and in the autumn of 1943 he defended his thesis which was about the Riemann Hypothesis. At that time there was little numerical evidence supporting the Riemann Hypothesis. He got the idea of studying the zeros of the Riemann zeta-function as a kind of moment problem, and this led to his famous estimate of the number of zeros. From this it followed that a positive fraction of the zeros must lie on the critical line. This result led to great international recognition.

When Carl Ludwig Siegel, who had stayed in the USA, asked Harald Bohr what had happened in mathematics in Europe during the war, Bohr answered with one word: Selberg.

During the summer of 1946 Selberg realized that his work on the Riemann zeta function could be applied to estimate the number of primes in an interval. This was the beginning of the development leading to the famous Selberg sieve method.

Selberg's collected works were published in [1], and an extensive interview appeared in [2].

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Photo by Herman Landshoff. Courtesy of the Institute for Advanced Study

In 1947 Selberg went to the Institute for Advanced Study in Princeton in USA where he continued the work on his sieve method. In the spring of 1948 he proved the Selberg Fundamental Formula which later in 1948 led to an elementary proof of the Prime Number Theorem. This was a sensation since even the possibility of an elementary proof had been questioned by G.H. Hardy and other mathematicians.

For these results he was awarded the Fields Medal in 1950—at the time the highest award in mathematics.

He became a permanent member of the Institute for Advanced Study in 1949 and a professor in 1951—a position he held until he retired in 1987.

In the early 1950s Selberg again produced a new and very deep result, namely what is now called the Selberg Trace Formula. Selberg was inspired by a paper by H. Maass on differential operators, and he realized that in this connection he could use some ideas from his Master Thesis. This result has had many important implications in mathematics and theoretical physics, but Selberg was never interested in the wide range of applications. In the Trace Formula Selberg combines many mathematical areas like automorphic forms, group representations, spectral theory and harmonic analysis in an intricate and profound manner. Selberg's Trace Formula is by many mathematicians considered as one of the most important mathematical result in the 20th century. His later works on automorphic forms led to the rigidity results of lattices in higher rank Lie groups.

In his later years he continued to work on his favourite subjects: sieve methods, zeta-functions and the Trace Formula. In 2003 Selberg was asked whether he thought the Riemann Hypothesis was correct. His response was: "If anything at all in our universe is correct, it has to be the Riemann Hypothesis, if for no other reasons, so for purely esthetical reasons." He always emphasized the importance of simplicity in mathematics and that "the simple ideas are the ones that will survive". His style was to work alone at his own pace without interference from others.

In addition to the Fields Medal in 1950, Selberg received the Wolf Prize in 1986 and then in 2002 the honorary Abel Prize prior to the regular awards. He was also a member of numerous academies.

Atle Selberg was highly respected in the international mathematical community. He possessed a natural and impressive authority that made everyone listen to him with the greatest attention.

He loved his home country Norway and always spoke affectionately about the Norwegian nature, language and literature. In 1987 he was named Commander with Star of the Royal Norwegian Order of St. Olav.

Atle Selberg died on August 6, 2007 in his home in Princeton.

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2003

Jean-Pierre Serre

ABEL
PRISEN

Jean-Pierre Serre: Mon premier demi-siècle au Collège de France

Jean-Pierre Serre: My First Fifty Years at the Collège de France

Marc Kirsch

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Jean-Pierre Serre, Professeur au Collège de France, titulaire de la chaire d'*Algèbre et Géométrie* de 1956 à 1994.

Vous avez enseigné au Collège de France de 1956 à 1994, dans la chaire d'Algèbre et Géométrie. Quel souvenir en gardez-vous?

J'ai occupé cette chaire pendant 38 ans. C'est une longue période, mais il y a des précédents: si l'on en croit l'Annuaire du Collège de France, au XIX^e siècle, la chaire de physique n'a été occupée que par deux professeurs: l'un est resté 60 ans, l'autre 40. Il est vrai qu'il n'y avait pas de retraite à cette époque et que les professeurs avaient des suppléants (auxquels ils versaient une partie de leur salaire).

Quant à mon enseignement, voici ce que j'en disais dans une interview de 1986[1]: "Enseigner au Collège est un privilège merveilleux et redoutable. Merveilleux à cause de la liberté dans le choix des sujets et du haut niveau de l'auditoire: chercheurs au CNRS, visiteurs étrangers, collègues de Paris et d'Orsay — beaucoup sont des habitués qui viennent régulièrement depuis cinq, dix ou même vingt ans. Redoutable aussi: il faut chaque année un sujet de cours nouveau, soit sur ses propres recherches (ce que je préfère), soit sur celles des autres; comme un cours annuel dure environ vingt heures, cela fait beaucoup!"

Comment s'est passée votre leçon inaugurale?

À mon arrivée au Collège, j'étais un jeune homme de trente ans. La leçon inaugurale m'apparaissait presque comme un oral d'examen, devant professeurs, famille, collègues mathématiciens, journalistes, etc. J'ai essayé de la préparer. Au bout d'un mois, j'avais réussi à en écrire une demi-page.

Arrive le jour de la leçon, un moment assez solennel. J'ai commencé par lire la demi-page en question, puis j'ai improvisé. Je ne sais plus très bien ce que j'ai dit (je me souviens seulement avoir parlé de l'Algèbre, et du rôle ancillaire qu'elle joue en Géométrie et en Théorie des Nombres). D'après le compte-rendu paru dans le journal *Combat*, j'ai passé mon temps à essuyer machinalement la table qui me séparait du public; je ne me suis senti à l'aise que lorsque j'ai pris en main un bâton de craie et que j'ai commencé à écrire sur le tableau noir, ce vieil ami des mathématiciens.

Quelques mois plus tard, le secrétariat m'a fait remarquer que toutes les leçons inaugurales étaient rédigées et que la mienne ne l'était pas. Comme elle avait été improvisée, j'ai proposé de la recommencer dans le même style, en me remettant mentalement dans la même situation. Un beau soir, on m'a ouvert un bureau du Collège et l'on m'a prêté un magnétophone. Je me suis efforcé de recréer l'atmosphère initiale, et j'ai refait une leçon sans doute à peu près semblable à l'originale. Le lendemain, j'ai apporté le magnétophone au secrétariat; on m'a dit que l'enregistrement était inaudible. J'ai estimé que j'avais fait tout mon possible et je m'en suis tenu là. Ma leçon inaugurale est restée la seule qui n'ait jamais été rédigée.

En règle générale, je n'écris pas mes exposés; je ne consulte pas mes notes (et, souvent, je n'en ai pas). J'aime réfléchir devant mes auditeurs. J'ai le sentiment, Jean-Pierre Serre, Professor at the Collège de France, held the Chair in *Algebra and Geometry* from 1956 to 1994.

You taught at the Collège de France from 1956 to 1994, holding the Chair in Algebra and Geometry. What are you memories of your time there?

I held the Chair for 38 years. That is a long time, but there were precedents. According to the Yearbook of the Collège de France, the Chair in Physics was held by just two professors in the 19th century: one remained in his post for 60 years, and the other for 40. It is true that there was no retirement in that era and that professors had deputies (to whom they paid part of their salaries).

As for my teaching career, this is what I said in an interview in 1986[1]: "Teaching at the Collège is both a marvelous and a challenging privilege. Marvelous because of the freedom of choice of subjects and the high level of the audience: CNRS [Centre national de la recherche scientifique] researchers, visiting foreign academics, colleagues from Paris and Orsay—many regulars who have been coming for 5, 10 or even 20 years. It is challenging too: new lectures have to be given each year, either on one's own research (which I prefer), or on the research of others. Since a series of lectures for a year's course is about 20 hours, that's quite a lot!"

Can you tell us about your inaugural lecture?

I was a young man, about 30, when I arrived at the Collège. The inaugural lecture was almost like an oral examination in front of professors, family, mathematician colleagues, journalists, etc. I tried to prepare it, but after a month I had only managed to write half a page.

When the day of the lecture came, it was quite a tense moment. I started by reading the half page I had prepared and then I improvised. I can no longer remember what I said (I only recall that I spoke about algebra and the ancillary role it plays in geometry and number theory). According to the report that appeared in the newspaper *Combat*, I spent most of the time mechanically wiping the table that separated me from my audience. I did not feel at ease until I had a piece of chalk in my hand and I started to write on the blackboard, the mathematician's old friend.

A few months later, the Secretary's Office told me that all inaugural lectures were written up, but they had not received the transcript of mine. As it had been improvised, I offered to repeat it in the same style, mentally putting myself back in the same situation. One evening, I was given a tape recorder and I went into an office at the Collège. I tried to recall the initial atmosphere, and to make up a lecture as close as possible to the original one. The next day I returned the tape recorder to the Secretary's Office. They told me that the recording was inaudible. I decided that I had done all I could and left it there. My inaugural lecture is still the only one that has not been written up.

As a rule, I don't write my lectures. I don't consult notes (and often I don't have any). I like to do my thinking in front of the audience. When I am explaining lorsque j'explique des mathématiques, de parler à un ami. Devant un ami, on n'a pas envie de lire un texte. Si l'on a oublié une formule, on en donne la structure; cela suffit. Pendant l'exposé j'ai en tête une quantité de choses qui me permettraient de parler bien plus longtemps que prévu. Je choisis suivant l'auditoire, et l'inspiration du moment.

Seule exception: le séminaire Bourbaki, où l'on doit fournir un texte suffisamment à l'avance pour qu'il puisse être distribué en séance. C'est d'ailleurs le seul séminaire qui applique une telle règle, très contraignante pour les conférenciers.

Quel est la place de Bourbaki dans les mathématiques françaises d'aujourd'hui?

C'est le séminaire qui est le plus intéressant. Il se réunit trois fois par an, en mars, mai et novembre. Il joue un rôle à la fois social (occasion de rencontres) et mathématique (exposé de résultats récents — souvent sous une forme plus claire que celle des auteurs); il couvre toutes les branches des mathématiques.

Les livres (*Topologie, Algèbre, Groupes de Lie,...*) sont encore lus, non seulement en France, mais aussi à l'étranger. Certains de ces livres sont devenus des classiques: je pense en particulier à celui sur les systèmes de racines. J'ai vu récemment (dans le *Citations Index* de l'AMS[2]) que Bourbaki venait au 6e rang (par nombre de citations) parmi les mathématiciens français (de plus, au niveau mondial, les nos 1 et 3 sont des Français, et s'appellent tous deux Lions: un bon point pour le Collège). J'ai gardé un très bon souvenir de ma collaboration à Bourbaki, entre 1949 et 1973. Elle m'a appris beaucoup de choses, à la fois sur le fond (en me forçant à rédiger des choses que je ne connaissais pas) et sur la forme (comment écrire de façon à être compris). Elle m'a appris aussi à ne pas trop me fier aux "spécialistes."

La méthode de travail de Bourbaki est bien connue: distribution des rédactions aux différents membres et critique des textes par lecture à haute voix (ligne à ligne: c'est lent mais efficace). Les réunions (les "congrès") avaient lieu 3 fois par an. Les discussions étaient très vives, parfois même passionnées. En fin de congrès, on distribuait les rédactions à de nouveaux rédacteurs. Et l'on recommençait. Le même chapitre était souvent rédigé quatre ou cinq fois. La lenteur du processus explique que Bourbaki n'ait publié finalement qu'assez peu d'ouvrages en quarante années d'existence, depuis les années 1930–1935 jusqu'à la fin des années 1970, où la production a décliné.

En ce qui concerne les livres eux-mêmes, on peut dire qu'ils ont rempli leur mission. Les gens ont souvent cru que ces livres traitaient des sujets que Bourbaki trouvait intéressants. La réalité est différente: ses livres traitent de ce qui est utile pour faire des choses intéressantes. Prenez l'exemple de la théorie des nombres. Les publications de Bourbaki en parlent très peu. Pourtant, ses membres l'appréciaient beaucoup, mais ils jugeaient que cela ne faisait pas partie des *Éléments*: il fallait d'abord avoir compris beaucoup d'algèbre, de géométrie et d'analyse.

Par ailleurs, on a souvent imputé à Bourbaki tout ce que l'on n'aimait pas en mathématiques. On lui a reproché notamment les excès des "maths modernes" dans les programmes scolaires. Il est vrai que certains responsables de ces programmes se mathematics, I feel I am speaking to a friend. You don't want to read a text out to a friend; if you have forgotten a formula, you give its structure; that's enough. During the lecture I have a lot of possible material in my mind—much more than possible in the allotted time. What I actually say depends on the audience and my inspiration.

Only exception: the Bourbaki seminar for which one has to provide a text sufficiently in advance so that it can be distributed during the meeting. This is the only seminar that applies this rule; it is very restrictive for lecturers.

What is Bourbaki's place in French mathematics now?

Its most interesting feature is the Bourbaki seminar. It is held three times a year, in March, May and November. It plays both a social role (an occasion for meeting other people) and a mathematical one (the presentation of recent results—often in a form that is clearer than that given by the authors). It covers all branches of mathematics.

Bourbaki's books (*Topology, Algebra, Lie Groups*, etc.) are still widely read, not just in France but also abroad. Some have become classics: I'm thinking in particular about the book on root systems. I recently saw (in the AMS *Citations Index*[2]) that Bourbaki ranked sixth (by number of citations) among French mathematicians. (What's more, at the world level, numbers 1 and 3 are French and both are called Lions: a good point for the Collège.) I have very good memories of my collaboration with Bourbaki from 1949 to 1973. Bourbaki taught me many things, both on background (making me write about things which I did not know very well) and on style (how to write in order to be understood). Bourbaki also taught me not to rely on "specialists".

Bourbaki's working method is well-known: the distribution of drafts to the various members and their criticism by reading them aloud (line by line: slow but effective). The meetings ("congrès") were held three times a year. The discussions were very lively, sometimes passionate. At the end of each congrès, the drafts were distributed to new writers. And so on. A chapter could often be written four or five times. The slow pace of the process explains why Bourbaki ended up publishing with relatively few books over the 40 years from 1930–1935 till the end of the 1970s when production faded away.

As for the books themselves, one may say that they have fulfilled their mission. People often believe that these books deal with subjects that Bourbaki found interesting. The reality is different: the books deal with what is useful in order to do interesting things. Take number theory for example. Bourbaki's publications hardly mention it. However, the Bourbaki members liked it very much it, but they considered that it was not part of the *Elements*: it needed too much algebra, geometry and analysis.

Besides, Bourbaki is often blamed for everything that people do not like about mathematics, especially the excesses of "modern math" in school curricula. It is true that some of those responsible for these curricula claimed to follow Bourbaki. But sont réclamés de Bourbaki. Mais Bourbaki n'y était pour rien: ses écrits étaient destinés aux mathématiciens, pas aux étudiants, encore moins aux adolescents. Notez que Bourbaki a évité de se prononcer sur ce sujet. Sa doctrine était simple: on fait ce que l'on choisit de faire, on le fait du mieux que l'on peut, mais on n'explique pas pourquoi on le fait. J'aime beaucoup ce point de vue qui privilégie le travail par rapport au discours — tant pis s'il prête parfois à des malentendus.

Comment analysez-vous l'évolution de votre discipline depuis l'époque de vos débuts? Est-ce que l'on fait des mathématiques aujourd'hui comme on les faisait il y a cinquante ans?

Bien sûr, on fait des mathématiques aujourd'hui comme il y a cinquante ans! Évidemment, on comprend davantage de choses; l'arsenal de nos méthodes a augmenté. Il y a un progrès continu. (Ou parfois un progrès par à-coups: certaines branches restent stagnantes pendant une décade ou deux, puis brusquement se réveillent quand quelqu'un introduit une idée nouvelle.)

Si l'on voulait dater les mathématiques "modernes" (un terme bien dangereux), il faudrait sans doute remonter aux environs de 1800 avec Gauss.

Et en remontant plus loin, si vous rencontriez Euclide, qu'auriez-vous à vous dire?

Euclide me semble être plutôt quelqu'un qui a mis en ordre les mathématiques de son époque. Il a joué un rôle analogue à celui de Bourbaki il y a cinquante ans. Ce n'est pas par hasard que Bourbaki a choisi d'intituler ses ouvrages des *Éléments de Mathématique*: c'est par référence aux *Éléments* d'Euclide. (Notez aussi que "Mathématique" est écrit au singulier. Bourbaki nous enseigne qu'il n'y a pas plusieurs mathématiques distinctes, mais une seule mathématique. Et il nous l'enseigne à sa façon habituelle: pas par de grands discours, mais par l'omission d'une lettre à la fin d'un mot.)

Pour en revenir à Euclide, je ne pense pas qu'il ait produit des contributions réellement originales. Archimède serait un interlocuteur plus indiqué. C'est lui le grand mathématicien de l'Antiquité. Il a fait des choses extraordinaires, aussi bien en mathématique qu'en physique.

En philosophie des sciences, il y a un courant très fort en faveur d'une pensée de la rupture. N'y a-t-il pas de ruptures en mathématiques? On a décrit par exemple l'émergence de la probabilité comme une manière nouvelle de se représenter le monde. Quelle est sa signification en mathématiques?

Les philosophes aiment bien parler de "rupture." Je suppose que cela ajoute un peu de piment à leurs discours. Je ne vois rien de tel en mathématique: ni catastrophe, ni révolution. Des progrès, oui, je l'ai déjà dit; ce n'est pas la même chose. Nous travaillons tantôt à de vieilles questions, tantôt à des questions nouvelles. Il n'y a pas de frontière entre les deux. Il y a une grande continuité entre les mathématiques

Bourbaki had nothing to do with it: its books are meant for mathematicians, not for students, and even less for teen-agers. Note that Bourbaki was careful not to write anything on this topic. Its doctrine was simple: one does what one chooses to do, one does it the best one can, but one does not explain why. I very much like this attitude which favors work over discourse—too bad if it sometimes lead to misunderstandings.

How would you describe the development of your discipline since the time when you were starting out? Is mathematics conducted nowadays as it was 50 years ago?

Of course you do mathematics today like 50 years ago! Clearly more things are understood; the range of our methods has increased. There is continuous progress. (Or sometimes leaps forward: some branches remain stagnant for a decade or two and then suddenly there's a reawakening as someone introduces a new idea.)

If you want to put a date on "modern" mathematics (a very dangerous term), you would have to go back to about 1800 and Gauss.

Going back further, if you were to meet Euclid, what would you say to him?

Euclid seems to me like someone who just put the mathematics of his era into order. He played a role similar to Bourbaki's 50 years ago. It is no coincidence that Bourbaki decided to give its treatise the title *Éléments de Mathématique*. This is a reference to Euclid's *Éléments*. (Note that "Mathématique" is written in the singular. Bourbaki tells us that rather than several different mathematics there is one single mathematics. And he tells us in his usual way: not by a long discourse, but by the omission of one letter from the end of one word.)

Coming back to Euclid, I don't think that he came up with genuinely original contributions. Archimedes would be much more interesting to talk to. He was the great mathematician of antiquity. He did extraordinary things, both in mathematics and physics.

In the philosophy of science there is a very strong current in favor of the concept of rupture. Are there ruptures in mathematics? For example the emergence of probability as a new way in which to represent the world. What is its significance in mathematics?

Philosophers like to talk of "rupture". I suppose it adds a bit of spice to what they say. I do not see anything like that in mathematics: no catastrophe and no revolution. Progress, yes, as I've already said; but that is not the same. We work sometimes on old questions and sometimes on new ones. There is no boundary between the two. There is a deep continuity between the mathematics of two centuries ago and that