Cognitive Intelligence and Robotics

Abhijit Mahapatra
Shibendu Shekhar Roy
Dilip Kumar Pratihar

Multi-body Dynamic Modeling of Multi-legged Robots

Springer

Cognitive Intelligence and Robotics

Cognitive Intelligence refers to the natural intelligence of humans and animals, it is considered that the brain performs intelligent activities. While establishing a hard boundary that distinguishes intelligent activities from others remains controversial, most common behaviors and activities of living organisms that cannot be fully synthesized using artificial means are regarded as intelligent. Thus the acts of sensing and perception, understanding the environment, and voluntary control of muscles, which can be performed by lower-level mammals, are indeed intelligent. Besides the above, advanced mammals can perform more sophisticated cognitive tasks, including logical reasoning, learning, recognition, and complex planning and coordination, none of which can yet be realized artificially to the level of a baby, and thus are regarded as cognitively intelligent.

This book series covers two important aspects of brain science. First, it attempts to uncover the mystery behind the biological basis of cognition, with a special emphasis on the decoding of stimulated brain signals or images. Topics in this area include the neural basis of sensory perception, motor control, sensory-motor coordination, and understanding the biological basis of higher-level cognition, including memory, learning, reasoning, and complex planning. The second objective of the series is to publish consolidated research on brain-inspired models of learning, perception, memory, and coordination, including results that can be realized on robots, enabling them to mimic the cognitive activities performed by living creatures. These brain-inspired models of machine intelligence complement the behavioral counterparts studied in traditional artificial intelligence.

The series publishes textbooks, monographs, and contributed volumes.

More information about this series at http://www.springer.com/series/15488

Abhijit Mahapatra · Shibendu Shekhar Roy ·
Dilip Kumar Pratihar

Multi-body Dynamic Modeling of Multi-legged Robots

Springer

Abhijit Mahapatra
Advance Design and Analysis Group
Central Mechanical Engineering
Research Institute
Durgapur, West Bengal, India

Shibendu Shekhar Roy
Department of Mechanical Engineering
National Institute of Technology Durgapur
Durgapur, West Bengal, India

Dilip Kumar Pratihar
Department of Mechanical Engineering
Indian Institute of Technology Kharagpur
Kharagpur, West Bengal, India

ISSN 2520-1956 ISSN 2520-1964 (electronic)
Cognitive Intelligence and Robotics
ISBN 978-981-15-2952-8 ISBN 978-981-15-2953-5 (eBook)
https://doi.org/10.1007/978-981-15-2953-5

This Springer imprint is published by the registered company Springer Nature Singapore Pte Ltd.
The registered company address is: 152 Beach Road, #21-01/04 Gateway East, Singapore 189721, Singapore

Dedicated to Our Family Members

Preface

Over the past two decades, multi-body dynamics has emerged as one of the major tools in the design and analysis of complex mechanical system. However, in the field of robotics, its application is not so comprehensive. Further, multi-legged robotics is an emerging technology and a lot of research is yet to be done before a full proof robust system is developed to fulfill the human needs. Most of the available literature about multi-body dynamic modeling of multi-legged robots is in form of journal, conference proceedings, monographs, etc. However, only a handful of books are available online or published in this field. This book deals with constrained rigid multi-body dynamics. It presents a systematic approach to kinematics and coupled multi-body dynamic modeling along with the study of generating path and gait of a realistic six-legged robotic system locomoting in 3D Cartesian space on various terrains like sloping surface, staircase, and some user-defined rough terrains with straight-forward, crab, and turning motion capabilities. The book outlines the most effective way of handling the kinematic and dynamic constraints and discusses in detail the foot–ground interaction mechanics, energy efficiency, and stability (both dynamic and static) of a realistic six-legged robot. Further, the book deals with computer simulations carried out in MATLAB and subsequent validation using Virtual Prototyping (VP) tools and real experiment.

Durgapur, India — Abhijit Mahapatra
Durgapur, India — Shibendu Shekhar Roy
Kharagpur, India — Dilip Kumar Pratihar

Acknowledgements

This book would not have been possible without the help of few people around us who mattered the most and without whom this work would not have been possible. The authors express their heartfelt thanks to Indra Bahadur Biswakarma, Hemant Agarwal, Sandip Tiwari, Kondalarao Bhavanibhatla, and Kaustav Biswas for their kind help and cooperation.

The first author would like to thank all his colleagues and friends at CSIR-Central Mechanical Engineering Research Institute, Durgapur, India, for their encouragement, cooperation, and assistance rendered during the course of the work. The second author is also thankful to all present colleagues of Mechanical Engineering Department at NIT, Durgapur, India. The third author is grateful to his present colleagues of Mechanical Engineering Department at IIT, Kharagpur, India.

The authors would like to especially thanks to Dr. Manuel Ferre, Dr. P. M. Pathak, as well as others who have contributed directly or indirectly to the content of the book.

The authors also thank the unknown copy-editor and the staff of Springer Nature for helpful cooperation and thorough publication of the book.

On a personal note, the authors would like to express deepest gratitude to their family members for their invaluable love, affection, and support.

Last but not least, authors would like to thank everybody who directly and indirectly helped them for the successful completion of this work.

Durgapur, India — Abhijit Mahapatra
Durgapur, India — Shibendu Shekhar Roy
Kharagpur, India — Dilip Kumar Pratihar

Contents

About the Authors

Dr. Abhijit Mahapatra received B.E. and M.Tech. degrees in Mechanical Engineering from B.E. College (now, BESU), Shibpur, India, and NIT Durgapur, India, in 2002 and 2008, respectively. He received his Ph.D. from NIT Durgapur, India, in 2018. He is currently working as a Principal Scientist in the Advanced Design and Analysis Group at CSIR- Central Mechanical Engineering Research Institute, Durgapur, India.

Dr. Mahapatra has published a number of research papers in national and international journals and conference proceedings, and has filed a number of patents in the area of product development. His current research interests include design & analysis, multi-body dynamics, and modelling and simulation of legged robots.

Dr. Shibendu Shekhar Roy received B.E. and M.Tech. degrees in Mechanical Engineering from NIT, Durgapur. He also holds a Ph.D. from IIT, Kharagpur, India. He is currently working as a Professor at the Department of Mechanical Engineering and Associate Dean (Alumni Affairs & Outreach) at the National Institute of Technology, Durgapur.

Dr. Roy has published more than 68 papers in national and international journals and conference proceedings, as well as 4 book chapters, and has filed a number of patents in the area of product development. His current research interests include modelling and simulating legged robots, soft robotics, rehabilitation robotics, additive manufacturing and 3D printing on macro- and micro-scales.

Dr. Dilip Kumar Pratihar completed his B.E. and M. Tech. in Mechanical Engineering at NIT, Durgapur, India, in 1988 and 1995, respectively. He received his Ph.D. from IIT Kanpur in 2000. Dr. Pratihar pursued postdoctoral studies in Japan and then in Germany under the Alexander von Humboldt Fellowship Program. He is currently working as a Professor at IIT Kharagpur, India. His research areas include robotics, soft computing and manufacturing science.

He has made significant contributions in the development of intelligent autonomous systems in various fields, including robotics, and manufacturing science. He has published more than 230 papers, mostly in international journals, and is on the editorial board of 12 international journals. He is a member of the FIE, MASME and SMIEEE. He has completed a number of sponsored (funded by DST, DAE, MHRD, DBT) and consultancy projects and is a member of Expert Committee of Advanced Manufacturing Technology, DST, Government of India.

Nomenclature

Latin Symbols

$\mathbf{A}^{G_0 G}$	3×3 Transformation matrix that maps *dynamic* reference frame (***G***) into *static* global reference frame ($\boldsymbol{G}_0$)
$\mathbf{A}^{G_0 N_{i3}}$	3×3 Transformation matrix that maps reference frame ($\boldsymbol{N}_{i3}$) into reference frame ($\boldsymbol{G}_0$)
$\mathbf{A}^{L_0 G}$	3×3 Transformation matrix that maps *dynamic* reference frame (***G***) into TB body reference frame ($\boldsymbol{L}_\mathbf{0}$)
$\mathbf{A}^{L_0 L'_{ij}}$	3×3 Transformation matrix that maps reference frame ($\boldsymbol{L}'_{ij}$) into reference frame ($\boldsymbol{L}_\mathbf{0}$)
$\mathbf{A}^{L''_{i3} L_{i3}}$, $\mathbf{A}^{L'''_i L_i}$	Transformation matrices that map reference frames ($\boldsymbol{L}_{i3}$) into reference frame ($\boldsymbol{L}''_{i3}$) and ($\boldsymbol{L}'''_{i3}$) respectively
$\mathbf{A}_i$	is a square matrix, $\in \mathbb{R}^{3,3}$
$\mathbf{A}_e$, $\mathbf{B}_e$, $\mathbf{A}_u$, $\mathbf{B}_u$	are combined matrices
$\mathbf{B}_i$	is a square matrix, $\in \mathbb{R}^{3,3}$
c	An auxiliary variable, $\in \mathbb{R}^{18}$
$\mathbf{C}_B$, $\mathbf{C}_L$, $\mathbf{C}_T$	COM of the trunk body, COM of the payload, aggregate center of mass of the system respectively
$\mathbf{C}_k$	Center of mass of the rigid body 'k' of the hexapod (here, $k = 0$ designates combination of trunk body and payload; $k = ij$ designates link ij, for $i = 1$ to 6, $j = 1$ to 3)
$\mathbf{C}_i$	COM of the interactive contact volume of the foot tip with the terrain
$\mathbf{C}p_\mathrm{i}$	COM of footpad

$C_{\max}$	Maximum damping
$\mathbf{D}_i$	is a square matrix, $\in \mathbb{R}^{3,3}$
$\mathbf{D}(q)$	Coupled mass and inertia matrix of the system in terms of generalized coordinates
d	Diameter of footpad
d_{ij}	Joint offset distance
e	Force exponent (mostly material property and $e > 1$ for stiff spring)
E_s	Specific energy consumption
$\mathbf{f}_K$, $\mathbf{f}_U$	Total summation of known and unknown forces and moments, respectively
${}^c\mathbf{f}$	Vector of constraint reaction forces and torques of the joints in terms of Cartesian coordinates, $\in \mathbb{R}^{114}$
$\mathbf{f}(\mathbf{p}^{G_0}, \mathbf{v}^{G_0})$	Vector of applied forces and torques
$\mathbf{F}_i^G$, $\mathbf{F}_i^{G_0}$	Vector of the ground reaction forces at the foot of leg i with respect to $\boldsymbol{G}$ and $\boldsymbol{G}_0$, respectively
$\mathbf{F}_e^{G_0}$	Vectors representing the centrifugal, Coriolis, gyroscopic, and gravitational forces acting on the combined mass of the trunk body and payload with respect to frame $\boldsymbol{G}_0$
${}^m\mathbf{F}_i^G$	Vector of resultant compliant impact force with respect to frame $\boldsymbol{G}$
$\mathbf{F}_{C_m}^{G_0}$	Vector of gravitational forces acting on the system with respect to frame $\boldsymbol{G}_0$
$\boldsymbol{G}$	*Dynamic* reference frame corresponding to XYZ axes system
$\boldsymbol{G}_\mathbf{o}$	*Static* global reference frame corresponding to XYZ axes system
g	Acceleration due to gravity
$\bar{\mathbf{H}}$	An auxiliary variable known as Hessian matrix, $\in \mathbb{R}^{18\times 18}$
$\mathbf{H}^{G_0}$	Total angular momentum of the system expressed in $\boldsymbol{G}_0$
h_c	Depth of the centroid of the interactive contact volume from the plane XY
h'_{in}	Height of the terrain at the point of touch by swing leg i with the ground at *nth* cycle
h_l	Vertical height that gives the measure of energy stability level
Hm_{in}	Maximum height of the topography on the path of the trajectory of swing leg i at *nth* cycle

Δh	Clearance between the maximum height of trajectory (along Z-direction) of swing leg i and Hm_{in} at nth cycle
i	Leg number (here i = 1 to 6)
j	Joint number (here j = 1 to 3)
$J^L_{C_k x}$, $J^L_{C_k y}$, $J^L_{C_k z}$, $J^L_{C_k xy}$, $J^L_{C_k yz}$, $J^L_{C_k zx}$	Mass moment of inertia of the rigid body 'k' of the system about x, y, z, xy, yz, and zx with respect to frame $\boldsymbol{L}_0$ respectively [here, $\boldsymbol{L} = \boldsymbol{L}_0$ for $k = 0$; $\boldsymbol{L} = \boldsymbol{L}'_{ij}$ for $k = ij$]
$\mathbf{J}^L_{C_k}$, $\mathbf{J}^L_{P_k}$	Mass moment of inertia matrix of the rigid body 'k' of the system with respect to the points $\mathbf{C}_k$ and $\mathbf{P}_k$, respectively, represented in the local (body-fixed) frame $\boldsymbol{L}$ [here, $\boldsymbol{L} = \boldsymbol{L}_0$ for $k = 0$; $\boldsymbol{L} = \boldsymbol{L}'_{ij}$ for $k = ij$]
$\mathbf{J}$, $\dot{\mathbf{J}}$	Jacobian matrix and its derivative, respectively, for the kinematically transformed system in terms of generalized coordinates
$\mathbf{J}_{r_{ij}}$, $\mathbf{J}'_{r_{ij}}$	3×6 Jacobian matrices of the *inverse kinematics*
$\dot{\mathbf{J}}_{r_{ij}}$, $\dot{\mathbf{J}}'_{r_{ij}}$	3×6 Time derivative of the Jacobian matrices
K	Characteristic stiffness of the boundary surface considered in MSC.ADAMS®
k	Body part number (here, $k = 0$ designates combined trunk body and payload; $k = ij$ designates link ij)
l	Edge or segment of support polygon considered as the rotation axis
l_i	Leg offset distance, that is, minimum projective distance between the points at $\mathbf{S}_i$ and $\mathbf{P}_{i3}$
l_{ij}	Kinematic link length of link ij
l'_{i3}	Minimum distance between rotating axis of joint i3 and foot-pad center
$\boldsymbol{L}_0$	Body-fixed reference frame located at point $\mathbf{P}_0$ on the trunk body
$\mathbf{L}^{G_0}$	Total linear momentum of the system expressed in $\boldsymbol{G}_0$
$\boldsymbol{L}'_i$, $\boldsymbol{L}''_i$	Reference frames on joint i1 indicating successive joint states respectively
$\boldsymbol{L}'_{i1}$, $\boldsymbol{L}''_{i1}$	Reference frames on joint i2 indicating successive joint states respectively
$\boldsymbol{L}'_{i2}$, $\boldsymbol{L}''_{i2}$	Reference frames on joint i3 indicating successive joint states respectively

$\boldsymbol{L}_{i3}$, $\boldsymbol{L}'_{i3}$, $\boldsymbol{L}''_{i3}$, $\boldsymbol{L}'''_{i3}$	Reference frames located at tip point $\mathbf{P}_{i3}$ and parallel to reference frame $\boldsymbol{G}$; parallel to reference frame $\boldsymbol{L}''_{i2}$; signify reference frame for crab angle during crab motion; signify reference frame for turning angle during turning motion respectively
m_{B}, m_{L}, m_{ij}	Masses of trunk body, payload, and links ij, respectively
m	Number of divisions of a gait cycle
m_b	Mobility of the system
$M_{ij}^{G_0}$	Torque at the jth joint of ith leg,
$M_{ij,\min}$ and $M_{ij,\max}$	Torque limitations at the jth joint of ith leg
$\mathbf{M}_k(\mathbf{p}_k^{G_0})$	Mass matrix of the rigid body 'k' in terms of Cartesian coordinates, $\in \mathbb{R}^{6,6}$
$\mathbf{M}\ (\mathbf{p}^{G_0})$	Combined mass matrix of the system in terms of Cartesian coordinates, $\in \mathbb{R}^{114,114}$
$\mathbf{M}_0^{G_0}$	Vector of joint torques acting on the trunk body and payload (combined)
$\mathbf{M}_e^{G_0}$, $\mathbf{M}_{ei}^{G_0}$	Vectors representing the centrifugal, Coriolis, gyroscopic, and gravitational moments acting on the trunk body, payload, and leg i with respect to frame $\boldsymbol{G}_0$ respectively
$\mathbf{M}_i^{G_0}$	Vector of joint torques of leg i, $\in \mathbb{R}^{18}$
$\mathbf{M}^{G_0}$	Overall joint torque vector, $\in \mathbb{R}^{18}$
n	Number of gait cycles
n_c	Number of kinematic joint constraints
n_g	Number of grounded legs
$\mathbf{O}$	Global origin or point of coincident of the reference frames $\boldsymbol{G}_0$ and $\boldsymbol{G}$
p	Boundary penetration ($\ll z_1$) at which MSC.ADAMS® solver applies full damping
P_{av}	Average power consumption
$\mathbf{p}_k^G$	Vectors of Cartesian coordinates of point $\mathbf{P}_k$ with respect to $\boldsymbol{G}_0$
$\mathbf{p}^G$	Overall vectors of Cartesian coordinates of the system with respect to $\boldsymbol{G}$
$\mathbf{p}^{G_0}$	Overall vectors of Cartesian coordinates of the system with respect to $\boldsymbol{G}_0$
$\mathbf{P}_k$	Point representing the origin of the local reference frame located on the rigid body 'k' of the hexapod
$\mathbf{P}_r^T(x,y)$, $\mathbf{P}_r^T(x,z)$	2×3 matrix projectors

$\mathbf{P}_r(y)$, $\mathbf{P}_r(z)$	1×3 matrix projectors
$\mathbf{q}_0$, $\mathbf{q}_i$, $\mathbf{q}$	Generalized coordinates of the trunk body, leg i and overall system respectively
$\mathbf{q}_{GC}\ (\mathbf{p}^{G_0}, \mathbf{v}^{G_0})$	Vector of centrifugal forces and gyroscopic terms
$\mathbf{Q}_i$	Friction coefficient matrix, $\in \mathbb{R}^{4,3}$
$\mathbf{r}^{L_0}_{C_L P_0}$, $\mathbf{r}^{L_0}_{C_T P_0}$, $\mathbf{r}^{L_0}_{S_i P_0}$	Displacement vector from point $\mathbf{P}_0$ to $\mathbf{C}_L$, $\mathbf{C}_T$, and $\mathbf{S}_i$, respectively represented in reference frame $\boldsymbol{L}_0$
$\mathbf{r}^{L'_{i1}}_{P_{i1} S_i}$, $\mathbf{r}^{L'_{i2}}_{P_{i2} P_{i1}}$, $\mathbf{r}^{L'_{i3}}_{P_{i3} P_{i2}}$	Displacement vectors from point $\mathbf{S}_i$ to $\mathbf{P}_{i1}$, $\mathbf{P}_{i1}$ to $\mathbf{P}_{i2}$, $\mathbf{P}_{i2}$ to $\mathbf{P}_{i3}$, respectively, in their local reference frames $\boldsymbol{L}'_{i1}$, $\boldsymbol{L}'_{i2}$, and $\boldsymbol{L}'_{i3}$
$\mathbf{r}^{L'_{ij}}_{C_{ij} P_{ij}}$	Displacement vector from point $\mathbf{P}_{ij}$ to $\mathbf{C}_{ij}$ represented in reference frame L'_{ij}
$\mathbf{r}^{G}_{C_0 O}$, $\mathbf{r}^{G}_{C_{ij} O}$, $\mathbf{r}^{G}_{C_m O}$	Displacement vector from point $\mathbf{O}$ to $\mathbf{C}_0$, $\mathbf{C}_{ij}$, and $\mathbf{C}_m$ represented in reference frame $\boldsymbol{G}$ respectively
$\mathbf{r}^{G}_{P^s_{i3} O}$	Displacement vector of the tip point $\mathbf{P}_{i3}$ of leg i at any instant of time
$\mathbf{r}^{G}_{P_0 O}$	Displacement vectors represented in reference frame $\boldsymbol{G}$ from point $\mathbf{O}$ to $\mathbf{P}_0$
$\mathbf{r}^{G}_{P_{ij} O}$	Displacement vectors represented in dynamic reference frame $\boldsymbol{G}$ from point $\mathbf{O}$ to $\mathbf{P}_{ij}$ (for j = 1 to 3)
$\mathbf{r}^{G}_{S_i O}$	Displacement vectors represented in dynamic reference frame $\boldsymbol{G}$ from point $\mathbf{O}$ to $\mathbf{S}_i$
$\dot{\mathbf{r}}^{G}_{P_0 O}$, $\ddot{\mathbf{r}}^{G}_{P_0 O}$	Translational velocity and acceleration vectors of point $\mathbf{P}_0$ respectively, represented in reference frame $\boldsymbol{G}$
$\dot{\mathbf{r}}^{G}_{P_{ij} O}$, $\ddot{\mathbf{r}}^{G}_{P_{ij} O}$	Translational velocity and acceleration vectors of point $\mathbf{P}_{ij}$ respectively, represented in reference frame $\boldsymbol{G}$
$\mathbf{r}^{G_0}_{C_m O}$	Displacement vector from point $\mathbf{O}$ to $\mathbf{C}_m$ represented in reference frame $\boldsymbol{G}_0$
$\mathbf{r}^{G_0}_{C_m/l}$	Vector from line $\mathbf{P}_{13}\mathbf{P}_{53}$ to COG with respect to frame $\boldsymbol{G}_0$ and is orthogonal to line $\mathbf{P}_{13}\mathbf{P}_{53}$
$\mathbf{r}'^{G_0}_{C_m/l}$	Vector obtained by rotating vector $\mathbf{r}^{G_0}_{C_m/l}$ about line $\mathbf{P}_{13}\mathbf{P}_{53}$, until it lies in plane I (vertical plane which includes the line $\mathbf{P}_{13}\mathbf{P}_{53}$, that is edge l)
$\dot{\mathbf{r}}^{G_0}_{P_{ij} O}$, $\ddot{\mathbf{r}}^{G_0}_{P_{ij} O}$	Translational velocity and acceleration vectors of point $\mathbf{P}_{ij}$ respectively, represented in reference frame $\boldsymbol{G}_{\mathbf{0}}$

$\mathbf{S}_{\boldsymbol{i}}$	Origin or point of coincident of the reference frames $\boldsymbol{L}'_i$ and $\boldsymbol{L}''_i$ associated with joint i1
s_0, s_0^c	Linear and angular stroke of the trunk body respectively
s_T	Total distance travelled during locomotion
s_w^f	Full swing stroke of Leg i
s_0''	Distance covered by the trunk body per division per duty cycle
t_0	Start time of the trunk body motion
t_1	Time at which velocity of trunk body reaches maximum
t_2	Time at which the trunk body starts decelerating
t_3	End time of the trunk body motion
$t_3^s\|_n$	End time of the swing leg motion for the nth cycle
t_{start}^s	Start time of the swing leg motion
t_{end}^s	End time of the swing leg motion
$\mathbf{T}_i^{G_0}$	Vectors of moments at the foot of leg i with respect to $\boldsymbol{G}_0$
$\mathbf{T}_{C_m}^{G_0}$	Resultant moment acting at COG
$\mathbf{u}$	Velocity vector of the overall system in terms of generalized coordinates
$\dot{\mathbf{u}}$	Acceleration vector of the overall system in terms of generalized coordinates
$v_{xy}^{N_{i3}}$	Slip velocity
v_s	Stiction transition velocity
v_d	Friction transition velocity
$\mathbf{v}_k^{G_0}$	Velocity vector of the rigid body 'k' of the system measured with respect to $\boldsymbol{G}_0$ in the Cartesian system
v_{ref}	Reference velocity
v_{t_0}	Initial rate of change of displacement (linear or angular) of the trunk body with respect to point $\mathbf{P}_0$ is considered to be,
v_{t_1}	Maximum rate of change of displacement (linear or angular)
$\mathbf{W}$	Symmetric positive definite matrix, $\in \mathbb{R}^{18,18}$
z	Distance function of the impact
z_1	Trigger distance (vertical) between COM of foot-pad ($\mathbf{C}p_{\mathrm{i}}$) and edge of foot-pad ($\mathbf{H}$)
$\dot{z}$	Derivative of z to impact

Greek Symbols

α_G, β_G, θ_G	Angles representing slope, banking, initial position of the robot on the terrain, respectively
α_0, β_0, θ_0	Angular displacement of trunk body along X-, Y-, and Z- directions with respect to frame $\boldsymbol{L}_0$, respectively
θ_{i1}, β_{i2}, β_{i3}	Angular displacement of joint $i1$ located at $\mathbf{S}_i$, joint $i2$ located at $\mathbf{P}_{i1}$, joint $i3$ located at $\mathbf{P}_{i2}$, respectively
γ	Fixed angle between frames $\boldsymbol{L}_0$ and $\boldsymbol{L'}_i$
ϕ	Twisted angle of the coxa
ψ	Bounding angle between l_{i3} and l'_{i3}
Ψ_l	Angle between $\mathbf{r'}^{G_0}_{C_{m/l}}$ and unit vector $\hat{\mathbf{z}}$
Ω_l	Angle between $\mathbf{r}^{G_0}_{C_{m/l}}$ and $\mathbf{r'}^{G_0}_{C_{m/l}}$
θ_c	Crab angle
$\theta^{L_{i3}}_{i3}$	Turning angle
ε_s	Slip angle
μ_s	Coefficient of static friction
μ_d	Coefficient of dynamic friction
ζ	Friction angle
δ_i	Angle of inclination of the flat surface of the footpad with respect to reference frame $\boldsymbol{G}$
χ	Angle subtended by the impact force ${}^{m}\mathbf{F}^{G}_{i}$ with the point $\mathbf{C}_i$
$\boldsymbol{\tau}$	Vector of ground reaction forces/ moments and coupled joint torques
$\boldsymbol{\lambda}$	Vector of the Lagrange multipliers, $\in \mathbb{R}^{n_c}$
${}^{G_0}\boldsymbol{\omega}_{ij}$, ${}^{G_0}\dot{\boldsymbol{\omega}}_{ij}$	Vector of angular velocity and angular acceleration of the link ij represented in frame $\boldsymbol{G}_0$, respectively
Δ'_a and Δ'_d	First derivatives of Δ_a and Δ_d, respectively, with respect to time
Δt	Duration of acceleration and deceleration of the trunk body

Abbreviations

3D	Three-dimensional
ADAMS	Automated dynamic analysis of mechanical systems
CAD	Computer-aided design
CAE	Computer-aided engineering
CATIA	Computer-aided three-dimensional interactive application
COG	Center of gravity
COM	Center of mass
DF	Duty factor
DGSM	Dynamic gait stability measure
DH	Denavit–Hartenberg

DOF	Degrees of freedom
FBD	Free body diagram
NE	Newton–Euler
NESM	Normalized energy stability margin
QP	Quadratic programming
VP	Virtual prototyping
ZMP	Zero moment point

List of Figures