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Finite and Instantaneous Screw Theory in Robotic Mechanism

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Preface

In recent years, robotic system has attracted intensive attention and is widely applied in many areas, such as aviation, aerospace, transportation, medicine, and health. The successful application of robotic system depends on the robotic mechanism serving as the mechanical execution unit whose type synthesis and performance design are fundamental issues. This book presents a finite and instantaneous screw (FIS) theory and applies on robotic mechanism. The topological model can be analytically expressed and algebraically computed at the finite motion level. In addition, differential mapping between topological and performance models are formulated, with which an integrated analysis and design framework is proposed.

This book firstly introduces the FIS theory in Chap. 2 and proposes an integrated design framework in Chap. 3. Finite screw based type synthesis of robotic mechanism is presented in Chap. 4. It can explicitly analyze motion characteristics with analytical expression and algebraic derivation, which is exemplified by the mechanism with invariable and variable rotation axes in Chaps. 5 and 6. Kinematic, stiffness, and dynamic modeling based on FIS theory are discussed in Chaps. 7–9. In Chap. 10, optimal design method is proposed. Chap. 11 illustrates the synthesis, analysis, and design of two typical robotic mechanisms. Based on FIS theory, kinematic calibration is implemented in Chap. 12.

The features of this book include:

1. This book presents the FIS theory and proposes an integrated framework including type synthesis, performance modeling, optimal design, and kinematic calibration of robotic mechanism. It solves the long-term challenge on the development of robotic mechanism due to the disconnection between topological and performance models.
2. This book proposes a finite screw based type synthesis method based upon analytical expression and algebraic derivation. The proposed method is a generic one. In particular, motion characteristics of mechanisms with invariable and variable rotation axes can be analyzed, from which the type synthesis of these mechanisms can be performed at the finite motion level.

3. This book presents an optimal design method with the consideration of parameter uncertainty. Multiple performances, including kinematics, stiffness and dynamics, are included in the design process. The matching principle among multiple performances are defined, which addresses the difficulty in obtaining optimal result from the multi-objective optimization having objectives with conflict relations.
4. This book proposes a kinematic calibration method based on FIS theory. The complete, continuous, and minimal error model of robotic mechanism is formulated, on which basis the robust parameter identification method is presented. The accuracy of the robotic mechanism can be greatly improved in an efficient manner.

The book can be used as graduate textbook in mechanical engineering, or as a research monograph in robotics. It can also be used as a reference by engineers on the robot design and application, researchers on the robotic theory and technology, and students ranging from senior undergraduates to doctoral students. This book gives the readers a deep understanding on the invention, analysis, design, and application of the robotic mechanisms.

This book would not have been possible without the help and involvement of many people. In particular, the authors would like to thank Prof. Tian Huang, Prof. J.S. Dai, Prof. Yimin Song from Tianjin University for their useful suggestions, Xinming Huo who worked on the type synthesis, Meng Wang, Dongxing Yang and Ruifeng Guo contributed to the figure drawing and proof reading. The authors are also grateful for the financial support provided by National Key R&D Program of China under the grant 2018YFB1307800, and the National Natural Science Foundation of China (NSFC) under the grant 51875391, 51675366, 51475321, 51205278.

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Chapter 1
Introduction

Robots are widely applied in many areas, including but not limited to, automobile engineering, aerospace engineering, port engineering, electronic industry, food industry, surgical operation, housekeeping service. Compared with manual works, robots have advantages in the following perspectives [1, 2]:

(1) Robots are more powerful, more efficient, and more accurate in many machining, manufacturing, and assembling tasks;
(2) Robots bring no pollution into the process while executing tasks;
(3) Robots can perform tasks in extreme conditions, such as in the narrow spaces, and in dangerous environments;
(4) Robots are safer in man–robot interaction when curing and serving human because robots will never be affected by exhaustion and emotion;
(5) As manual costs increase, applications of robots lead to cost-saving.

Attracted by these merits, robot has been through a rapid increment in the last few decades. Robotics has become a key technology in improving the quality of human life. Robot is a complicated system consisting of five subsystems [3], including (1) a mechanical subsystem composed of either rigid bodies, deformable bodies or both, (2) an actuation subsystem acting as the input of the robotic system, (3) a control subsystem achieving the desired outputs by controlling the inputs of the robotic system, (4) a sensing subsystem monitoring the inputs or the outputs or both, (5) an information processing subsystem.

This book focuses on the mechanical subsystem that allows a rigid body, namely end-effector serving as the tool directly performing the task, to move with respect to a fixed base. The mechanical subsystem of a robot is called robotic mechanism. It is the execution unit connecting the inputs and outputs of the system. The performances of the robotic mechanism directly determine the behavior of the robot. Development of a robot meeting the application requirements, such as operational accuracy, load capacity, task flexibility, and reliability, relies on the development of a robotic mechanism.

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1.1 Classification of Robotic Mechanism

Motion is the property used to identify the robotic mechanism. The independent motion generated by a rigid body is referred to as its Degree of Freedom (DoF). Generally, the fixed base has zero DoF and a free rigid body in space has six DoFs, for instance, three rotations about mutually orthogonal axes and three translations along these axes. The output motion of a robotic mechanism is the result of a set of input motions. The motion transited from the input to the output is realized by connecting rigid bodies with joints. The motion of the joints can be produced from two basic types, namely the rotating pair called revolute (R) joint, and the sliding pair called prismatic (P) joint. From these two basic types of joints, helical (H) joint, cylindrical (C) joint, universal (U) joint, and spherical (S) joint that have practical interest are defined. Among them, R, P, and H joints are one-DoF joints. C and U joints have two DoFs, respectively. S joint has three DoFs.

An input motion of the robotic mechanism can be performed by a one-DoF joint while the output motion of the robotic mechanism is produced by different sets of joints. According to the arrangement of joints between the fixed base and the end-effector, the robotic mechanisms are mainly classified into three categories [4–6], i.e., open-loop mechanisms, closed-loop mechanisms, and hybrid mechanisms.

1.1.1 Open-loop Robotic Mechanism

An open-loop mechanism consists of several joints connected successively. As two- and three-DoF joints can be regarded as the combination of two and three one-DoF joints, an open-loop mechanism is the connection of several one-DoF joints in serial structure, as shown in Fig. 1.1. The open-loop mechanism can also be denoted by open-loop kinematic chain or open-loop limb.

Till now, many open-loop mechanisms have been invented and successfully designed as commercial products by companies like ABB [7], KUKA [8], and FANUC [9]. Among them, the most commonly used ones are those consisting of six R joints and having six DoFs. Sometimes, it is said that they have six controlled

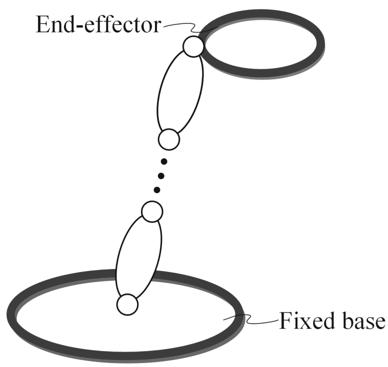


Fig. 1.1 The open-loop mechanism

(a) ABB IRB 120 (b) KUKA KR 120 R2700-2 (c) FANUC M-10*i*D

Fig. 1.2 Six-DoF open-loop mechanism

axes. Designed with different dimensions or actuated in different ways, these six-DoF open-loop mechanisms have been applied in different industry scenarios, from automotive to medical, electronic, and so on. The most typical robots developed upon them are ABB articulated robots [7], KUKA KR QUANTEC robots [8] and FUNUC ARC Mate robots [9], as shown in Fig. 1.2.

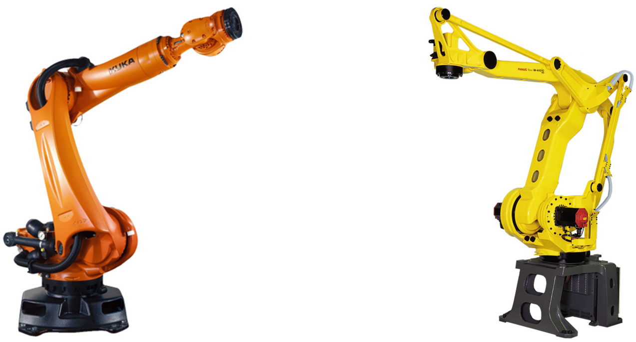
Another type of open-loop mechanism that is widely used has four DoFs. The four-DoF open-loop mechanisms can realize three-DoF translations and one-DoF rotation. They usually consist of three R joints and one H joint, of which joint axes and directions are parallel. Having less actuated joints than the six-DoF ones, the four-DoF mechanisms have simpler control systems. Thus, they are more cost-effective, more accurate, and faster in most cases. The most typical robots developed upon these mechanisms are ABB SCARA robots [7] and FUNUC SR robots [9], as shown in Fig. 1.3. The motion of these four-DoF open-loop mechanisms is sometimes called SCARA (Schoenfiles) motion. The mechanisms having SCARA motion are named as SCARA mechanisms.

Generally, the DoF of a spatial mechanism is no more than six, because the maximum mobility of a mechanism contains at most three translations and three rotations. The mechanisms with less than six DoFs are called lower-mobility mechanisms. It is obvious that the SCARA mechanisms are lower-mobility ones. Some other typical lower-mobility open-loop mechanisms are KUKA KR 40 PA robot [8] and FANUC M-410*i*C robot [9], which also have four DoFs. As shown in Fig. 1.4, these four-DoF mechanisms consist of two pairs of R joints. The two R joints in each pair have parallel axes at the initial pose. Thus, the mechanisms can realize two-DoF translations and two-DoF rotations.

The above six-DoF and lower-mobility robots are mainly used for industrial applications. They don't have redundant actuation for the purpose of cost-saving. However, for the robots applied in curing and serving human or some other tasks require for flexibility, compliance, and great agility, redundant actuation are assigned to mimic

(a) ABB IRB 910SC (b) FANUC SR-3*i*A

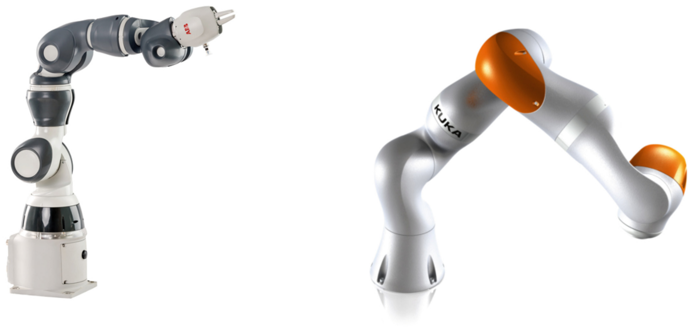
Fig. 1.3 Four-DoF SCARA open-loop mechanism



(a) KUKA KR 40 PA (b) FANUC M-410*i*C

Fig. 1.4 Other four-DoF open-loop mechanism

human-like movements. For instance, seven R joints are utilized to constitute such a mechanism. The two representatives for this kind of open-loop mechanisms are ABB IRB 14050 single-arm YuMi collaborative robot [4] and KUKA LBR iiwa robot [5], as shown in Fig. 1.5.



(a) ABB IRB 14050 (b) KUKA LBR iiwa

Fig. 1.5 Seven-axis open-loop mechanism

1.1.2 Closed-loop Robotic Mechanism

A closed-loop mechanism has at least two kinematic chains forming at least one closed-loop between fixed base and end-effector [6]. The kinematic chain is composed of a set of rigid bodies connected by joints. For distinguishing the output body of open-loop and closed-loop mechanisms, the output motion of the closed-loop mechanism is described by the shared body, named as moving platform, of the multiple kinematic chains. The closed-loop mechanism can be further divided into single closed-loop mechanism and multi-closed-loop mechanism according to the number of closed-loops.

1.1.2.1 Single Closed-loop Robotic Mechanism

The single closed-loop mechanism has one closed-loop. As shown in Fig. 1.6, the loop can be separated into two limbs connecting the fixed base to the moving platform. Each limb can be regarded as an open-loop mechanism.

The single closed-loop mechanism is usually regarded as the sub-structure of more complicated mechanisms such as multi-closed-loop mechanisms or hybrid mechanisms that will be introduced in the following sections. There are two basic types of single closed-loop mechanisms. The first type is the one-DoF mechanism whose limb is constituted by several R joints with specific geometric relationships among their axes. The second type can be one-DoF to six-DoF mechanisms and each limb consists of several one-DoF joints.

The commonly used one-DoF single closed-loop mechanisms are planar 4R mechanism, spherical 4R mechanism, Bennett mechanism, Myard mechanism, Goldberg mechanism, and Bricard mechanism.

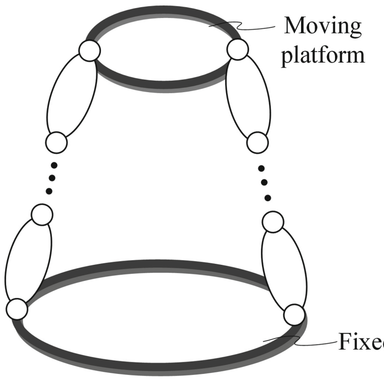
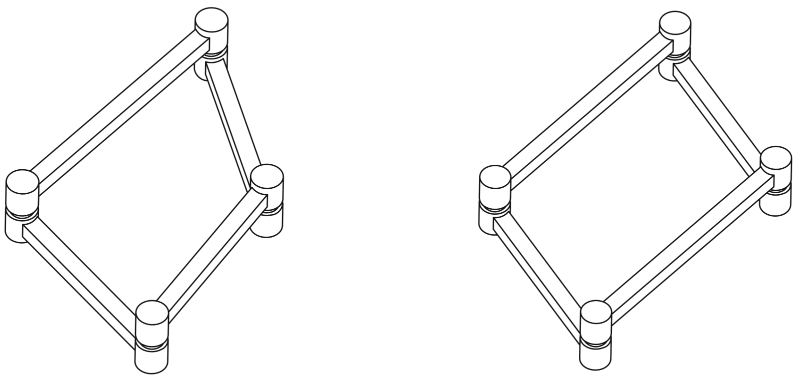


Fig. 1.6 Single closed-loop mechanism

A planar 4R mechanism consists of four R joints having parallel axes. As shown in Fig. 1.7a, the mechanism generates a one-DoF rotation that has fixed direction and varying position. When the four links of a planar 4R mechanism constitute a parallelogram, it can generate a bifurcated motion which is the union of a one-DoF translation along a circle and a one-DoF rotation that has fixed direction and varying position, as shown in Fig. 1.7b.

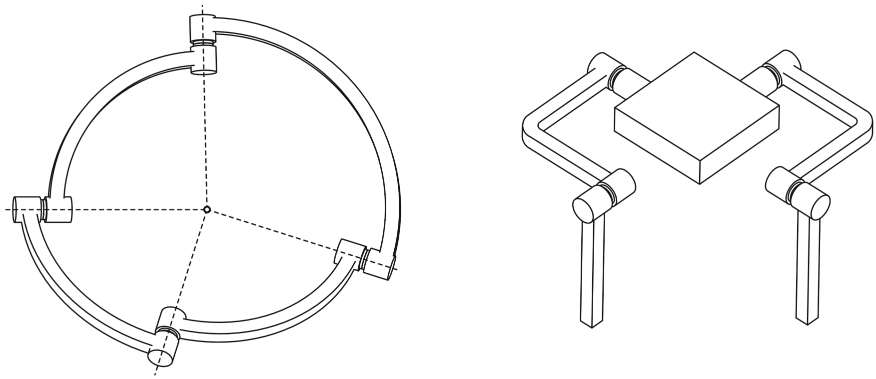
In a spherical 4R mechanism [10], the four R joints have intersecting axes that share the same rotation center, as shown in Fig. 1.8a. The output motion of the mechanism is a one-DoF rotation whose axis passes through the rotation center and has varying directions. When every two opposite R joints share the same axis, the mechanism generates bifurcated rotation [11] around two axes, as shown in Fig. 1.8b.



(a) The general one

(b) The one in parallelogram structure

Fig. 1.7 Planar 4R mechanism



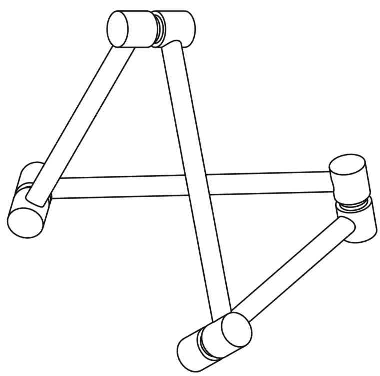
(a) The general one (b) The one in symmetrical structure

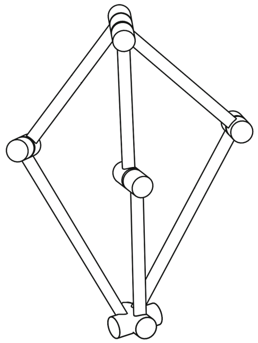
Fig. 1.8 Spherical 4R mechanism

Similar to planar and spherical 4R mechanisms, a Bennett mechanism [12] also consists of four R joints. However, the four R joints in the Bennett mechanism are neither parallel nor intersecting to each other. They have skew axes in space as shown in Fig. 1.9. In each pair of opposite links of the mechanism, the two links have the same length. The intersection angle of the two R joints connected to one link is the same as the intersection angle of the two R joints connected to the other link. The ratio between the length and sine of the intersection angle in this pair equals to the ratio in the other pair. The Bennett mechanism generates a one-DoF rotation about the axis with varying direction and position.

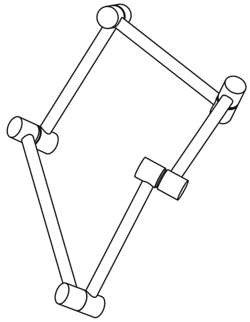
Both Myard mechanism [13] and Goldberg mechanism [14] are composed of five R joints. They are the combinations of two Bennett mechanisms. As in Fig. 1.10a, the two Bennett mechanisms share two common links and two common R joints, the common links and one common R joint is removed to form the Myard mechanism. As shown in Fig. 1.10b, one common link, and two common R joints are shared

Fig. 1.9 Bennett mechanism





(a) Myard mechanism



(b) Goldberg mechanism

Fig. 1.10 Myard and Goldberg mechanism

by the two Bennett mechanisms. The Goldberg mechanism is obtained by removing the common link and one common R joint. With the same geometric conditions as the Bennett mechanisms, one-DoF rotation about the axis with varying direction and position is generated by the Myard and Goldberg mechanisms.

A threefold-symmetric Bricard mechanism [14] contains six R joints and six links, as shown in Fig. 1.11. All the links have the same length. The intersection angle of a link is defined as the intersection angle of the axes of the two R joints connected to the link. The sum of the intersection angles of any two adjacent links in the mechanism is 2π. The Bricard mechanism generates a one-DoF rotation about the axis with varying direction and position.

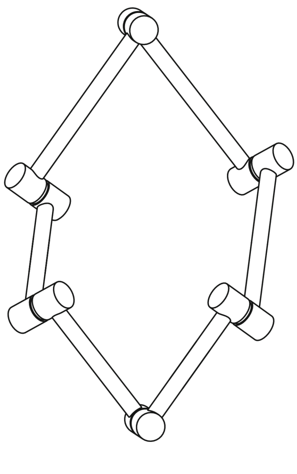


Fig. 1.11 Bricard mechanism

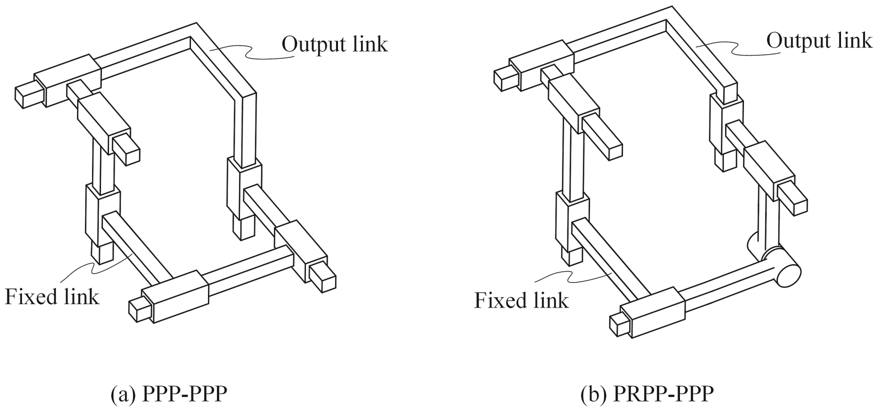


Fig. 1.12 Single closed-loop mechanisms having three-DoF translations

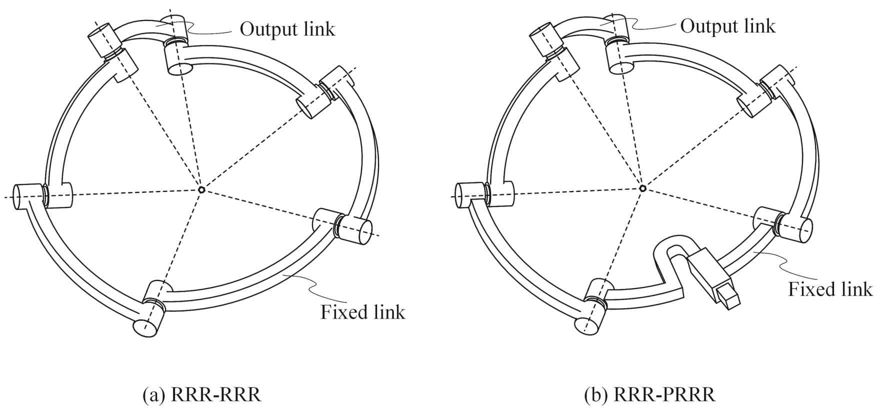


Fig. 1.13 Single closed-loop mechanisms having three-DoF rotations

Besides the above single closed-loop mechanisms with one-DoF and constituted only by R joints, there are many other single closed-loop mechanisms having one to six DoFs, which belong to the second type of single closed-loop mechanism. These mechanisms have fixed translational directions, which are systematically studied by Kong and Gosselin [15]. Here, we list some examples of the second type of single closed mechanisms, as shown in Figs. 1.12, 1.13 and 1.14.

1.1.2.2 Multi-closed-loop Robotic Mechanism

A multi-loop mechanism is composed of more than one closed-loop, as shown in Fig. 1.15. The number of limbs is twice the number of independent closed-loops.

The multi-closed-loop mechanism can be divided into two types. As in Fig. 1.15a, the first type has interconnected kinematic chains. For example, Ding et al. [16]

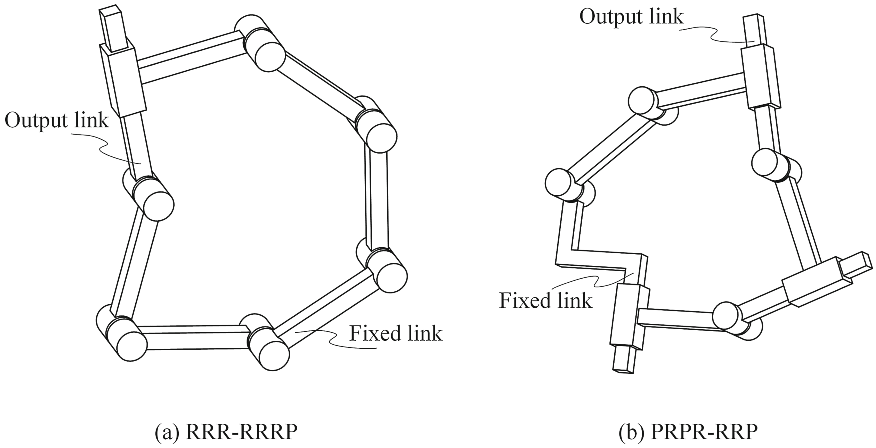


Fig. 1.14 Single closed-loop mechanisms having two-DoF translations and one-DoF rotation

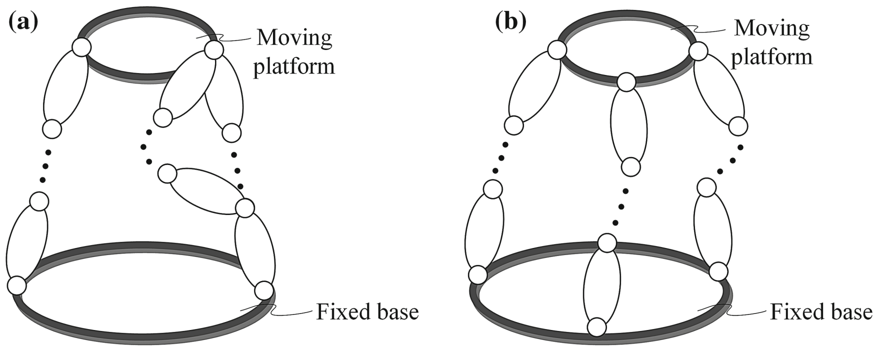
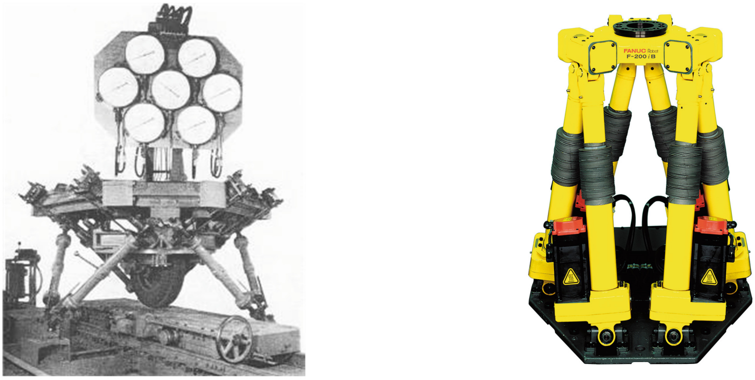


Fig. 1.15 Multi-closed-loop mechanism

investigated the kinematic chains consisting of a large number of R joints. All the R joints have parallel axes. Following given adjacency matrix and contracted graphs, a lot of mechanisms with coupled loops were invented. Based on Bennett mechanisms, the multi-loop mobile assemblies were invented by You and Chen [17]. Similarly, more mobile assemblies were presented based on Myard mechanisms [18].

The second type of multi-closed-loop mechanism has no interconnected kinematic chains. It is known as parallel mechanism. There have been many parallel mechanisms designed to be industrial robots and applied in manufacturing. The typical ones are listed in the following.

The Gough-Stewart multi-closed-loop mechanism has six DoFs. It consists of six SPS open-loop limbs and can realize any spatial motion. The six limbs are distributed systematically between the fixed base and moving platform. Due to its compact structure, the mechanism does not have a large workspace, but it has high stiffness



(a) Dunlop tire testing machine (b) FANUC F-200*i*B

Fig. 1.16 Gough-Stewart multi-closed-loop mechanism

and accuracy. These multi-closed-loop mechanisms have been applied in industry since 1960s for the tasks like astronaut training, radar pointing, and tire testing [19]. A typical commercial robot based upon this mechanism is FANUC F-200*i*B [9], as shown in Fig. 1.16.

The Delta multi-closed-loop mechanism has three DoFs. It consists of three $R(SS)^2$ limbs and can translate along any direction. Herein, $(SS)^2$ is composed of four S joints which form an inner loop in each limb. The $R(SS)^2$ limbs are systematically distributed between fixed base and moving platform. Because of its specific structure, all the actuations can be placed on the fixed base and the moving platform can reach very high speed. The mechanism is mainly applied for fast picking and placing. The typical robots based upon this mechanism are ABB IRB 360 FlexPicker [7] and FANUC M-2*i*A [9] shown in Fig. 1.17.

(a) ABB IRB 360 (b) FANUC M-2*i*A

Fig. 1.17 Delta multi-closed-loop mechanism

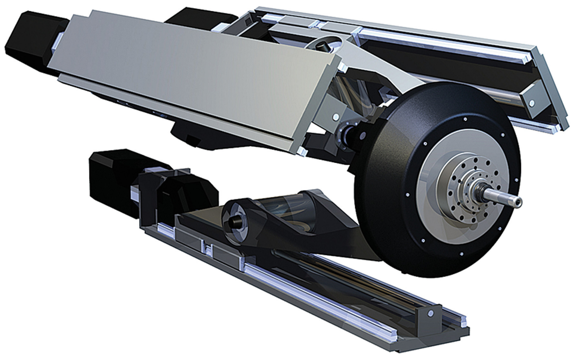


Fig. 1.18 Sprint Z3 head multi-closed-loop mechanism

The Sprint Z3 Head multi-closed-loop mechanism [20] has three DoFs, as shown in Fig. 1.18. It is composed of three PRS limbs and described as 3PRS mechanism. It was Hunt [21] who invented the 3PRS and 3RPS mechanisms in 1970s. Both of the 3PRS and 3RPS mechanisms generate the same motions, a translation along the direction perpendicular to the fixed base and two rotations with varying directions and positions. The Sprint Z3 Head has been employed for the machining tasks in the area such as automotive and aviation.

There are other two well-known three-DoF multi-closed-loop mechanisms, i.e., 3UPS-UP and 2UPR-SPR mechanisms [22, 23], invented by Neumann [24]. Because the UPS limbs have six DoFs, the motion of the 3UPS-UP mechanism is the same as the UP limb. Thus, it can realize one-DoF translation along with a fixed direction and two rotations about fixed axes. In a 2UPR-SPR mechanism, the axes connecting to the fixed base in the U joint of the UPR limbs are collinear. Hence, the two UPR limbs form a plane. Together with the SPR limb, the mechanism is structured in "T" shape. The 2UPR-SPR mechanism generates one-DOF translation along the fixed direction perpendicular to the fixed base together with two-DoF rotations. Both the axes of the two rotations have fixed positions. However, one of them has fixed direction, while the other has varying direction that is determined by the rotational angle of the first rotation. As shown in Fig. 1.19, these two parallel mechanisms have been developed to be industrial robots by PKM Tricept S. L. [25] and Exechon Enterprises L.L.C. [23], which are called Tricept and Exechon, respectively. They are widely used in machining, welding, and drilling tasks.

The Omni-Wrist VI developed by Ross-Hime Designs Inc [26] has four RSR limbs and one SS limb. The SS limb connects to the centers of the fixed base and the moving platform. The four RSR limbs are symmetrically placed with respect to the SS limb as shown in Fig. 1.20. The Omni-Wrist VI has two-DoF rotations about two axes with varying directions and positions. Due to the flexible rotational capability, it has wide applications in target tracking. For instance, it can be applied to track the satellite from the earth or the space station.

(a) Tricept (3UPS-UP)

(b) Exechon (2UPR-SPR)

Fig. 1.19 Tricept and Exechon multi-closed-loop mechanisms

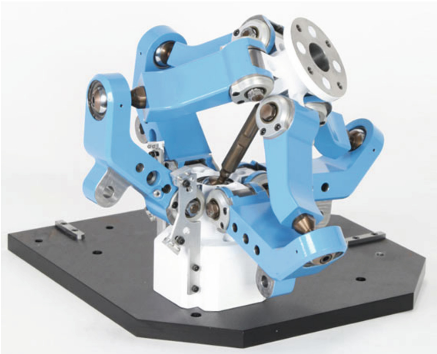


Fig. 1.20 Omni-wrist VI multi-closed-loop mechanism

Besides the above well-known ones, many multi-closed-loop mechanisms, that generate three-translational and two-rotational (3T2R) five-DoF motions, 3T1R four-DoF motions, 2T1R three-DoF motions, etc., have been synthesized and analyzed in academia and have great potentials to be applied as commercial robots.

1.1.3 Hybrid Robotic Mechanism

A hybrid mechanism consists of several parts in serial structure. Each part is either an open-loop mechanism or a multi-closed-loop mechanism, as shown in Fig. 1.21. In most cases, the multi-closed-loop parts are parallel mechanisms.

The conventional hybrid mechanisms are commonly constituted by two parts, i.e., a parallel part and an open-loop part (serial part). The parallel part is connected to

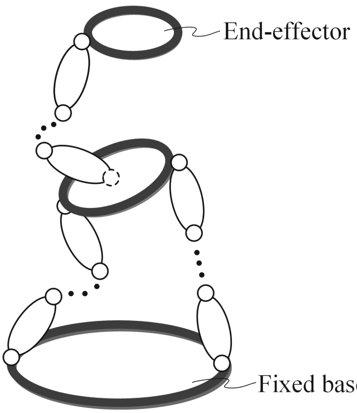


Fig. 1.21 Hybrid mechanism

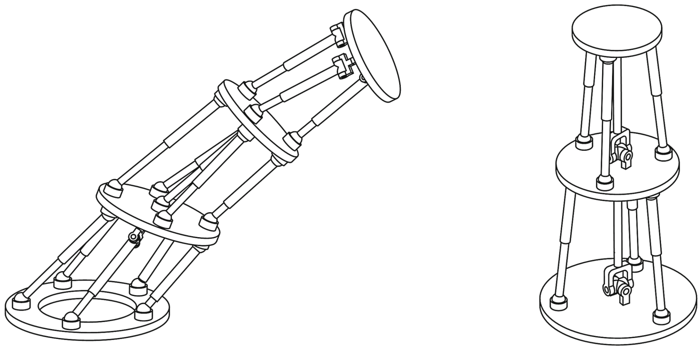
the fixed base while the serial part contains the end-effector. Usually, two- and three-DoF open-loop mechanisms are connected to the moving platforms of the three-DoF multi-closed-loop mechanisms, resulting in five- and six-DoF hybrid mechanisms. For example, the five-axis Exechon machine tool [23] is the combination of an Exechon multi-closed-loop mechanism and a two-DoF RR open-loop mechanism. The six-DoF FANUC M-3*i*A [9] is the combination of a Delta multi-closed-loop mechanism and a three-DoF RRR open-loop mechanism, as shown in Fig. 1.22. These hybrid mechanisms have the advantages of both the open-loop and closed-loop parts in terms of large workspace, flexible orientations, high stiffness, and high accuracy.

Recently, some general hybrid mechanisms were invented by Lu and Hu [27, 28]. These mechanisms are the combinations of several closed-loop mechanisms, as

(a) Execheon machine tool

(b) FANUC M-3*i*A

Fig. 1.22 Typical hybrid mechanism



(a) (4SPS-SPR)-(4SPS-SP)-3SPR hybrid mechanism (b) 2(3SPS-UP) hybrid mechanism

Fig. 1.23 The hybrid mechanisms consisting of several closed-loop mechanisms

shown in Fig. 1.23. Similarly, more hybrid mechanisms can be synthesized as the combinations of several open-loop and closed-loop mechanisms in serial structure following specific sequences.

1.2 Synthesis, Analysis, Design and Calibration of Robotic Mechanism

The development of a robotic mechanism undergoes type synthesis [15], performance analysis [4], optimal design [29], and kinematic calibration [30].

(1) *Type synthesis*: Type synthesis focuses on obtaining all possible robotic mechanisms that can realize the expected motion characteristics. The motion characteristics, including the number and types of motions, are given by the task, from which all the kinematic architectures of the robotic mechanism are generated.
(2) *Performance analysis*: Performance analysis involves kinematic, stiffness, and dynamic modeling of the obtained robotic mechanism from the previous study. The relations of inputs and outputs, motions and forces are thoroughly investigated. Performance analysis and modeling lead to a comprehensive understanding of the robotic mechanism.
(3) *Optimal design*: Optimal design determines the structural parameters, including dimensional and sectional parameters of the robotic mechanism, which lead to the desired performances. The expected performances are given by the application. Based on the performance models in the previous study, the optimal parameters can be obtained by the optimal design.
(4) *Kinematic calibration*: Kinematic calibration compensates the errors of the robotic mechanism to achieve high precision in task operation. The generation

of kinematic errors is inevitable during the construction of the physical prototype guided by the optimal design results. Kinematic calibration, including error modeling, measurement, identification, and compensation, can improve the accuracy of the robotic mechanism economically.

Above mentioned processes are necessary steps to the development of a robotic mechanism for specific application requirements. The key issue of carrying out the synthesis, analysis, design, and calibration of the robotic mechanism is the description and computation of its topological, motion, and performance characteristics and properties. Hence, mathematical modeling of the topology and performance models are the prerequisites for the development of robotic mechanism. It is the foundation of this book to select a mathematical tool that is rigorous and suitable for topology and performance modeling. Compared with matrix Lie group and Lie algebra, dual quaternions, finite and instantaneous screws have simpler and non-redundant formats. The composition algorithms of screws are easier to be implemented. In addition, the interrelations of the topology and performances of the robotic mechanism can be revealed and calculated. Therefore, finite and instantaneous screws are selected as the mathematical tool that is consistently employed throughout this book.

1.3 Screw Theory in Robotic Mechanism

The literature review on the screw theory applied in robotic mechanisms is given in this section. Based upon the authors' and other researchers' works, the sound and thorough finite and instantaneous screw theory will be proposed in this book (Chap. 2), which is named as FIS theory.

1.3.1 Instantaneous Screw

The most well-known and commonly used screw is an instantaneous screw. It originates from Mozzi's theorem that describes an angular velocity of a rigid body about a line followed by a linear velocity along that line. This line is referred to as instantaneous screw axis. The amplitude of instantaneous screw is defined as the value of angular velocity, while the pitch is defined as the ratio between values of linear and angular velocities. Hence, instantaneous motion can be expressed by an instantaneous screw in a six-dimensional vector form.

Similarly, a force exerted on a rigid body can also be described by an instantaneous screw defining its intensity as the value of force, and its pitch as the ratio between values of moment and force, respectively. It is based upon Poinsot's theorem in which a force can be equivalent to a force along and a moment about an axis. In order to distinguish the instantaneous screws describing instantaneous motion and force, the former is called twist, and the latter is named as wrench.

The systematic theory on screw has been proposed by Ball [31] in his treatise more than one hundred years ago. However, this theory had not been paid much attention for a quite long time. Its first application in robotic mechanisms can be traced back to 1978 by Hunt [32]. He put forward a new and more effective method to build the velocity Jacobian matrix of the robotic mechanism through writing the velocity of moving platform as the linear combination of the twists generated by all actuation joints. In this way, the Jacobian mapping between joint parameters and velocity of the mechanism is formulated clearly. The Jacobian mapping can be written into matrix form easily, which was followed by Angeles and Tsai [33, 34], resulting in force Jacobian and overall Jacobian, respectively.

Hunt's outstanding work sparked subsequent thorough and systematical researches on instantaneous screws. Hunt, Duffy, and Angeles [32, 35, 36] spent much efforts on the definitions of twists generated by and wrenches exerted on robotic mechanisms. Based on Ball's description, they gave clearly physical meanings of the concepts of instantaneous screws, screw systems, and reciprocal products. Their work made screws to be visual and concrete physical tools instead of purely abstract mathematical ones. The instantaneous screw can be directly used to describe velocities, forces, powers, and analyze performances of various robotic mechanisms including open-loop and closed-loop mechanisms as well as hybrid ones. Taking the orders of mechanisms with different DoFs into account, the classification of screw systems which are vector spaces spanned by no more than six screws was carried out by Gibson and Hunt, Martínez and Duffy [37], Dai and Jones [38]. They presented a comprehensive enumeration of possible linear combinations of given instantaneous screws that are central to the analysis of multi-DoF mechanisms and established normal form for each screw system in terms of base screws. Based upon these work, Huang [39], Dai [40], and their colleagues implemented mobility analysis of numerous mechanisms through determining the orders and characteristics of mechanisms' instantaneous screw systems and corresponding reciprocal systems.

By employing instantaneous screw systems and their reciprocal products to describe instantaneous motions of a multi-closed-loop mechanism, its limbs, joints and their relationships, type synthesis of multi-closed-loop mechanisms was carried out in an instantaneous motion level by Huang and Li [41, 42], Fang and Tsai [43, 44], Kong and Gosselin [45, 46], as well as the authors of this book [47–53]. This is actually the reverse process of mobility analysis.

As for any given robotic mechanism, the velocity, force, stiffness, accuracy, acceleration, and dynamic modeling and performance evaluation can be done using the instantaneous screw based Jacobian matrices or Hessian matrices. In this way, performance analysis, optimal design, and kinematic calibration of different categories of mechanisms can be carried out.

1.3.2 Finite Screw

As the counterpart of instantaneous motion, finite motion can be defined by Chasles' theorem that any rigid body finite motion can be produced by a translation along a line followed by a rotation about that line. This line is called finite screw axis in the whole book.

It has long been desired to describe and calculate finite motions by employing finite screw with non-redundant and the simplest form, just as instantaneous motions. This work can be dated back to 1965. It is Dimentberg and Dai [54, 55] who proposed a concept of finite displacement screw matrix to describe finite motion. Parkin [56] defined its pitch in 1992 as the ratio between half the translational distance and tangent of half the rotational angle, which is called quasi-pitch by Hunt and Parkin [57]. Two years later, Huang defined the amplitude as twice the tangent of half the angle. So far, the finite motion can be described by a six-dimensional quasi-vector. This quasi-vector is referred to as finite screw. Recently, the authors of this book strictly proved that the finite screw is derived from dual quaternion and gave a more general quasi-vector form by considering the two situations that the rotational angle equals zero and does not equal zero.

Utilizing finite screw, the finite motions generated by one-DoF joints could be described. Huang and Chen [58] rewrote the composition of two finite screws expressing finite motions of one-DoF joints as the screw triangle product that equals to linear sum of five terms. This leads to the easy acquisition of the finite motion expression of simple open-loop mechanisms, RR, PR, RP for instance. Huang's work was extended by the authors of this book. They proposed general procedures to do finite motion analysis and kinematics of robotic mechanisms in an algebraic manner.

Based on the finite motion described and calculated by finite screw, the authors proposed an analytical approach of type synthesis for parallel mechanisms by systematically investigating properties of screw triangle products [59–62]. Compared with the approach based on instantaneous screw, the finite screw-based approach can synthesize parallel mechanisms algebraically in finite motion level, resulting in that the full cycle DoFs of the obtained mechanisms do not need to be verified.

1.3.3 Relation Between Finite and Instantaneous Screws

Finite and instantaneous screws are proved to be powerful mathematical tools respectively in describing and calculating finite and instantaneous motions. The finite motion described by finite screw enables the type synthesis of robotic mechanism with analytical expression and algebraic derivation. The instantaneous motion denoted by the instantaneous screw is able to model the motions and forces that correspond to the kinematic, stiffness and dynamic of the robotic mechanism. In order to enable type synthesis and performance analysis of robotic mechanisms to be