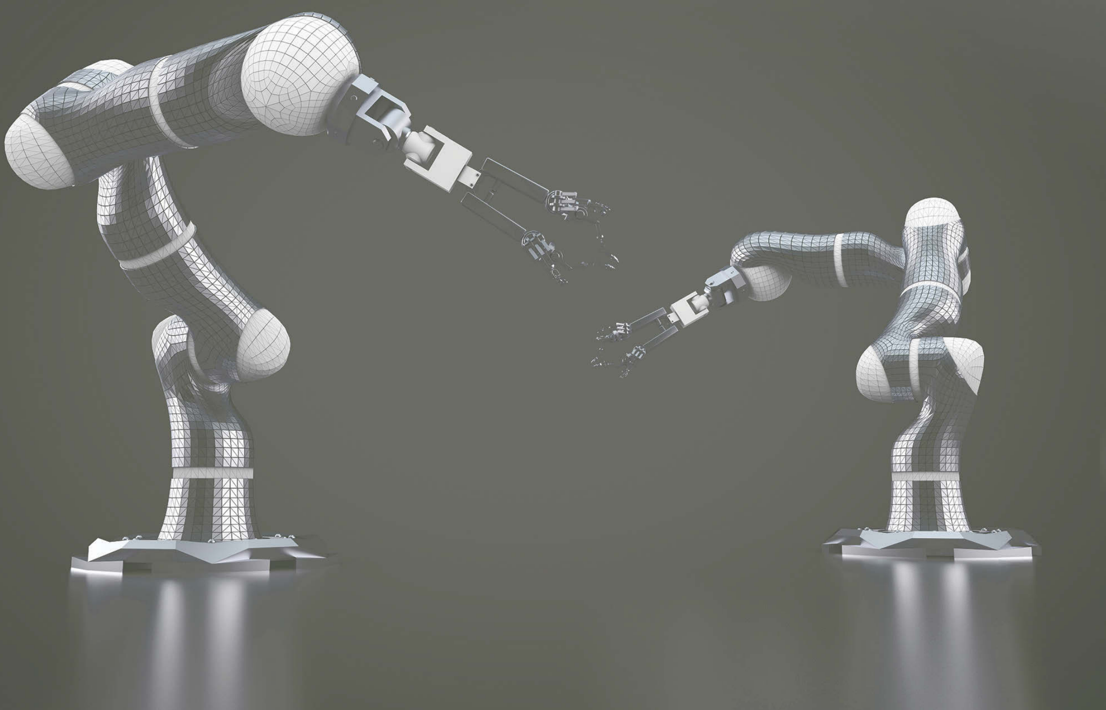
MARK W. SPONG | SETH HUTCHINSON
M. VIDYASAGAR

# ROBOT MODELING AND CONTROL

SECOND EDITION



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## Second Edition

Mark W. Spong

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# Preface

This text is a second edition of our book, *Robot Modeling and Control*, John Wiley & Sons, Inc., 2006, which grew out of the earlier text, M.W. Spong and M. Vidyasagar, *Robot Dynamics and Control*, John Wiley & Sons, Inc., 1989. The second edition reflects some of the changes that have occurred in robotics and robotics education in the past decade. In particular, many courses are now treating mobile robots on an equal footing with robot manipulators. As a result, we have expanded the discussion on mobile robots into a full chapter. In addition, we have added a new chapter on underactuated robots. We have also revised the material on vision, vision-based control, and motion planning to reflect changes in those topics.

## Organization of the Text

After the introductory first chapter, which introduces the terminology and history of robotics and discusses the most common robot design and applications, the text is organized into four parts. Part I consists of four chapters dealing with the geometry of rigid motions and the kinematics of manipulators.

Chapter 2 presents the mathematics of rigid motions; rotations, translations, and homogeneous transformations.

Chapter 3 presents solutions to the forward kinematics problem using the Denavit–Hartenberg representation, which gives a very straightforward and systematic way to describe the forward kinematics of manipulators.

Chapter 4 discuses velocity kinematics and the manipulator Jacobian. The geometric Jacobian is derived in the cross product form. We also introduce the so-called analytical Jacobian for later use in task space control. We have reversed the order of our treatment of velocity kinematics and inverse kinematics from the presentation in the first edition in order to include a new section in Chapter 5 on numerical inverse kinematics algorithms, which rely on the Jacobian for their implementation.

Chapter 5 deals with the inverse kinematics problem using the geometric approach, which is especially suited for manipulators with spherical wrists. We show how to solve the inverse kinematics in closed form for the most common manipulator designs. We also discuss numerical search algorithms for solving inverse kinematics. Numerical algorithms are increasingly popular because of both the increasing power of computers and the availability of open-source software for numerical algorithms.

Part II deals with dynamics and motion planning and consists of two chapters.

Chapter 6 is a detailed account of robot dynamics. The Euler–Lagrange equations are derived from first principles and their structural properties are discussed in detail. The recursive Newton–Euler formulation of robot dynamics is also presented.

Chapter 7 is an introduction to the problems of path and trajectory planning. Several of the most popular methods for motion planning and obstacle avoidance are presented, including the method of artificial potential fields, randomized algorithms, and probabilistic roadmap methods. The problem of trajectory generation is presented as essentially a problem of polynomial spline interpolation. Trajectory generation based on cubic and quintic polynomials as well as trapezoidal velocity trajectories are derived for interpolation in joint space.

Part III deals with the control of manipulators.

Chapter 8 is an introduction to independent joint control. Linear models and linear control methods based on PD, PID, and state space methods are presented for set-point regulation, trajectory tracking, and disturbance rejection. The concept of feedforward control, including the method of computed torque control, is introduced as a method for nonlinear disturbance rejection and for tracking of time-varying reference trajectories.

Chapter 9 discusses nonlinear and multivariable control. This chapter summarizes much of the research in robot control that took place in the late 1980s and early 1990s. Simple derivations of the most common robust and adaptive control algorithms are presented that prepare the reader for the extensive literature in robot control.

Chapter 10 treats the force control problem. Both impedance control and hybrid control are discussed. We also present the lesser known hybrid impedance control method, which allows one to control impedance and regulate motion and force at the same time. To our knowledge this is the first textbook that discusses the hybrid impedance control approach to robot force control.

Chapter 11 is an introduction to visual servo control, which is the problem of controlling robots using feedback from cameras mounted either on the robot or in the workspace. We present those aspects of vision that are most useful for vision-based control applications, such as imaging geometry and feature extraction. We then develop the differential kinematics that relate camera motion to changes in extracted features and we discuss the main concepts in visual servo control.

Chapter 12 is a tutorial overview of geometric nonlinear control and the method of feedback linearization of nonlinear systems. Feedback linearization generalizes the methods of computed torque and inverse dynamics control that are covered in Chapters 8 and 9. We derive and prove the necessary and sufficient conditions for local feedback linearization of single-input/single-output nonlinear systems, which we then apply to the flexible joint control problem. We also introduce the notion of nonlinear observers with output injection.

Part IV is a completely new addition to the second edition and treats the control problems for underactuated robots and nonholonomic systems.

Chapter 13 deals with underactuated serial-link robots. Underactuation arises in applications such as bipedal locomotion and gymnastic robots. In fact, the flexible-joint robot models presented in Chapters 8 and 12 are also examples of underactuated robots. We present the ideas of partial feedback linearization and transformation to normal forms, which are useful for controller design. We also discuss energy and passivity methods to control this class of systems.

Chapter 14 deals primarily with wheeled mobile robots, which are examples of systems subject to nonholonomic constraints. Many of the control design methods presented in the chapters leading up to Chapter 14 do not apply to nonholonomic systems. Thus, we cover some new techniques applicable to these systems. We present two fundamental results, namely Chow's theorem and Brockett's theorem, that provide conditions for controllability and stabilizability, respectively, of mobile robots.

Finally, the appendices have been expanded to give much of the necessary background mathematics to be able to follow the development of the concepts in the text.

## A Note to the Instructor

This text is suitable for several quarter-long or semester-long courses in robotics, either as a two- or three- course sequence or as stand-alone courses. The first five chapters can be used for a junior/senior-level introduction to robotics for students with at least a minimal background in linear algebra. Chapter 8 may also be included in an introductory course for students with some exposure to linear control systems. The independent joint control problem largely involves the control of actuator and drive-train dynamics; hence most of the subject can be taught without prior knowledge of Euler–Lagrange dynamics.

A graduate-level course on robot dynamics and control can be taught using all or parts of Chapters 6 through 12.

Finally, one or more special topics courses can be taught using Chapters 9 through 14. Below we outline several possible courses that can be taught from this book:

**Course 1: Introduction to Robotics**
Level: Junior/Senior undergraduate
For a one quarter course (10 weeks):
- **Chapter 1:** Introduction
- **Chapter 2:** Rigid Motions and Homogeneous Transformations
- **Chapter 3:** Forward Kinematics
- **Chapter 4:** Velocity Kinematics and Jacobians
- **Chapter 5:** Inverse Kinematics

For a one semester course (16 weeks) add:
- **Chapter 7:** Motion Planning and Trajectory Generation
- **Chapter 8:** Independent Joint Control

**Course 2: Robot Dynamics and Control**
Level: Senior undergraduate/graduate
For a one quarter course (10 weeks):
- **Chapters 1–5:** Rapid Review of Kinematics (selected sections)
- **Chapter 6:** Dynamics
- **Chapter 7:** Path and Trajectory Planning
- **Chapter 9:** Nonlinear and Multivariable Control
- **Chapter 10:** Force Control

For a one semester course (16 weeks) add:
- **Chapter 11:** Vision-Based Control
- **Chapter 12:** Feedback Linearization

**Course 3: Advanced Topics in Robot Control**
Level: Graduate
For a one semester course (16 weeks):
- **Chapter 6:** Dynamics
- **Chapter 7:** Motion Planning and Trajectory Generation
- **Chapter 9:** Nonlinear and Multivariable Control
- **Chapter 11:** Vision-Based Control
- **Chapter 12:** Feedback Linearization

**Chapter 13:** Underactuated Robots
**Chapter 14:** Mobile Robots

The instructor may wish to supplement the material in any of these courses with additional material to delve deeper into a particular topic. Also, either of the last two chapters can be covered in Course 2 by eliminating the Force Control chapter or the Vision-Based Control chapter.

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Mark W. Spong
Seth Hutchinson
M. Vidyasagar

*To my wife Lila – MWS*
*To my wife Wendy – SH*
*To my grandson Niyuddh Anand – MV*

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# Chapter 1

# INTRODUCTION

Robotics is a relatively young field of modern technology that crosses traditional engineering boundaries. Understanding the complexity of robots and their application requires knowledge of electrical engineering, mechanical engineering, systems and industrial engineering, computer science, economics, and mathematics. New disciplines of engineering, such as manufacturing engineering, applications engineering, and knowledge engineering have emerged to deal with the complexity of the field of robotics and factory automation. More recently, mobile robots are increasingly important for applications like autonomous vehicles and planetary exploration.

This book is concerned with fundamentals of robotics, including **kinematics**, **dynamics**, **motion planning**, **computer vision**, and **control**. Our goal is to provide an introduction to the most important concepts in these subjects as applied to industrial robot manipulators, mobile robots and other mechanical systems.

The term **robot** was first introduced by the Czech playwright Karel Čapek in his 1920 play Rossum's Universal Robots, the word **robota** being the Czech word for worker. Since then the term has been applied to a great variety of mechanical devices, such as teleoperators, underwater vehicles, autonomous cars, drones, etc. Virtually anything that operates with some degree of autonomy under computer control has at some point been called a robot. In this text we will focus on two types of robots, namely industrial manipulators and mobile robots.

### Industrial Manipulators

An industrial manipulator of the type shown in Figure 1.1 is essentially a mechanical arm operating under computer control. Such devices, though

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Figure 1.1: A six-axis industrial manipulator, the KUKA 500 FORTEC robot. (Photo courtesy of KUKA Robotics.)

far from the robots of science fiction, are nevertheless extremely complex electromechanical systems whose analytical description requires advanced methods, presenting many challenging and interesting research problems.

An official definition of such a robot comes from the **Robot Institute of America** (RIA):

*A robot is a reprogrammable, multifunctional manipulator designed to move material, parts, tools, or specialized devices through variable programmed motions for the performance of a variety of tasks.*

The key element in the above definition is the reprogrammability, which gives a robot its utility and adaptability. The so-called robotics revolution is, in fact, part of the larger computer revolution.

Even this restricted definition of a robot has several features that make it attractive in an industrial environment. Among the advantages often cited in favor of the introduction of robots are decreased labor costs, increased precision and productivity, increased flexibility compared with specialized machines, and more humane working conditions as dull, repetitive, or hazardous jobs are performed by robots.

The industrial manipulator was born out of the marriage of two earlier technologies: **teleoperators** and **numerically controlled milling machines**. Teleoperators, or master-slave devices, were developed during the second world war to handle radioactive materials. Computer numerical control (CNC) was developed because of the high precision required in the ma-

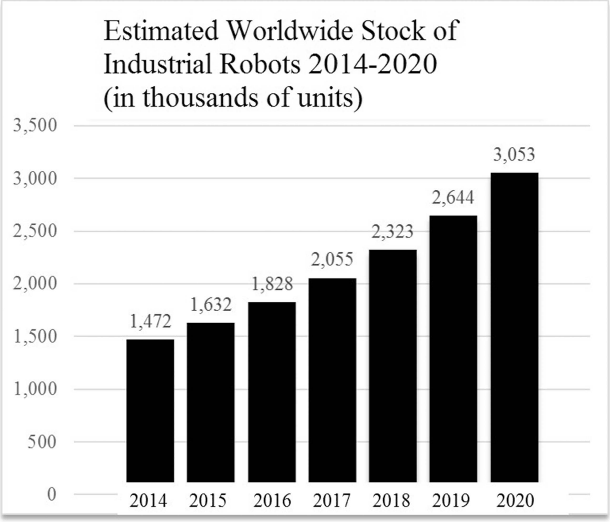

Figure 1.2: Estimated number of industrial robots worldwide 2014–2020. The industrial robot market has been growing around 14% per year. (Source: International Federation of Robotics 2018.)

chining of certain items, such as components of high performance aircraft. The first industrial robots essentially combined the mechanical linkages of the teleoperator with the autonomy and programmability of CNC machines.

The first successful applications of robot manipulators generally involved some sort of material transfer, such as injection molding or stamping, in which the robot merely attended a press to unload and either transfer or stack the finished parts. These first robots could be programmed to execute a sequence of movements, such as moving to a location A, closing a gripper, moving to a location B, etc., but had no external sensor capability. More complex applications, such as welding, grinding, deburring, and assembly, require not only more complex motion but also some form of external sensing such as vision, tactile, or force sensing, due to the increased interaction of the robot with its environment.

Figure 1.2 shows the estimated number of industrial robots worldwide between 2014 and 2020. In the future the market for service and medical robots will likely be even greater than the market for industrial robots. Service robots are defined as robots outside the manufacturing sector, such as robot vacuum cleaners, lawn mowers, window cleaners, delivery robots, etc. In 2018 alone, more than 30 million service robots were sold worldwide. The future market for robot assistants for elderly care and other medical robots will also be strong as populations continue to age.

### Mobile Robots

Mobile robots encompass wheel and track driven robots, walking robots, climbing, swimming, crawling and flying robots. A typical wheeled mobile robot is shown in Figure 1.3. Mobile robots are used as household robots for vacuum cleaning and lawn mowing robots, as field robots for surveillance, search and rescue, environmental monitoring, forestry and agriculture, and other applications. Autonomous vehicles, for example self-driving cars and trucks, is an emerging area of robotics with great interest and promise.

There are many other applications of robotics in areas where the use of humans is impractical or undesirable. Among these are undersea and planetary exploration, satellite retrieval and repair, the defusing of explosive devices, and work in radioactive environments. Finally, prostheses, such as artificial limbs, are themselves robotic devices requiring methods of analysis and design similar to those of industrial manipulators.

The science of robotics has grown tremendously over the past twenty years, fueled by rapid advances in computer and sensor technology as well as theoretical advances in control and computer vision. In addition to the topics listed above, robotics encompasses several areas not covered in this text such as legged robots, flying and swimming robots, grasping, artificial intelligence, computer architectures, programming languages, and computer-aided design. In fact, the new subject of **mechatronics** has emerged over the past four decades and, in a sense, includes robotics as a subdiscipline.

Figure 1.3: Example of a typical mobile robot, the Fetch series. The figure on the right shows the mobile robot base with an attached manipulator arm. (Photo courtesy of Fetch Robotics.)

Mechatronics has been defined as the synergistic integration of mechanics, electronics, computer science, and control, and includes not only robotics, but many other areas such as automotive control systems.

## 1.1 Mathematical Modeling of Robots

In this text we will be primarily concerned with developing and analyzing mathematical models for robots. In particular, we will develop methods to represent basic geometric aspects of robotic manipulation and locomotion. Equipped with these mathematical models, we will develop methods for planning and controlling robot motions to perform specified tasks. We begin here by describing some of the basic notation and terminology that we will use in later chapters to develop mathematical models for robot manipulators and mobile robots.

### 1.1.1 Symbolic Representation of Robot Manipulators

Robot manipulators are composed of **links** connected by **joints** to form a **kinematic chain**. Joints are typically rotary (revolute) or linear (prismatic). A **revolute** joint is like a hinge and allows relative rotation between two links. A **prismatic** joint allows a linear relative motion between two links. We denote revolute joints by $R$ and prismatic joints by $P$, and draw them as shown in Figure 1.4. For example, a three-link arm with three revolute joints will be referred to as an RRR arm.

Each joint represents the interconnection between two links. We denote the axis of rotation of a revolute joint, or the axis along which a prismatic joint translates by $z_i$ if the joint is the interconnection of links $i$ and $i+1$. The **joint variables**, denoted by $\theta$ for a revolute joint and $d$ for the prismatic joint, represent the relative displacement between adjacent links. We will make this precise in Chapter 3.

### 1.1.2 The Configuration Space

A **configuration** of a manipulator is a complete specification of the location of every point on the manipulator. The set of all configurations is called the **configuration space**. In the case of a manipulator arm, if we know the values for the joint variables (i.e., the joint angle for revolute joints, or the joint offset for prismatic joints), then it is straightforward to infer the position of any point on the manipulator, since the individual links of the manipulator are assumed to be rigid and the base of the manipulator is

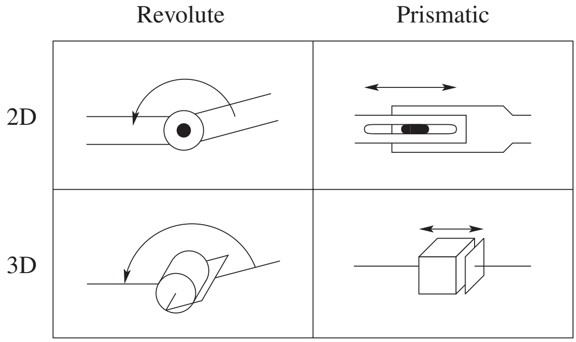

Figure 1.4: Symbolic representation of robot joints. Each joint allows a single degree of freedom of motion between adjacent links of the manipulator. The revolute joint (shown in 2D and 3D on the left) produces a relative rotation between adjacent links. The prismatic joint (shown in 2D and 3D on the right) produces a linear or telescoping motion between adjacent links.

assumed to be fixed. Therefore, we will represent a configuration by a set of values for the joint variables. We will denote this vector of values by $q$, and say that the robot is in configuration $q$ when the joint variables take on the values $q_1, \ldots, q_n$, with $q_i = \theta_i$ for a revolute joint and $q_i = d_i$ for a prismatic joint.

An object is said to have $n$ **degrees of freedom** (DOF) if its configuration can be minimally specified by $n$ parameters. Thus, the number of DOF is equal to the dimension of the configuration space. For a robot manipulator, the number of joints determines the number of DOF. A rigid object in three-dimensional space has six DOF: three for **positioning** and three for **orientation**. Therefore, a manipulator should typically possess at least six independent DOF. With fewer than six DOF the arm cannot reach every point in its work space with arbitrary orientation. Certain applications such as reaching around or behind obstacles may require more than six DOF. A manipulator having more than six DOF is referred to as a **kinematically redundant** manipulator.

### 1.1.3 The State Space

A configuration provides an instantaneous description of the geometry of a manipulator, but says nothing about its dynamic response. In contrast, the **state** of the manipulator is a set of variables that, together with a description of the manipulator's dynamics and future inputs, is sufficient to determine the future time response of the manipulator. The **state space** is

Figure 1.5: The Kinova® Gen3 Ultra lightweight arm, a 7-degree-of-freedom redundant manipulator. (Photo courtesy of Kinova, Inc.)

the set of all possible states. In the case of a manipulator arm, the dynamics are Newtonian, and can be specified by generalizing the familiar equation $F = ma$. Thus, a state of the manipulator can be specified by giving the values for the joint variables $q$ and for the joint velocities $\dot{q}$ (acceleration is related to the derivative of joint velocities). The dimension of the state space is thus $2n$ if the system has $n$ DOF.

### 1.1.4 The Workspace

The **workspace** of a manipulator is the total volume swept out by the end effector as the manipulator executes all possible motions. The workspace is constrained by the geometry of the manipulator as well as mechanical constraints on the joints. For example, a revolute joint may be limited to less than a full 360° of motion. The workspace is often broken down into a **reachable workspace** and a **dexterous workspace**. The reachable workspace is the entire set of points reachable by the manipulator, whereas the dexterous workspace consists of those points that the manipulator can reach with an arbitrary orientation of the end effector. Obviously the dexterous workspace is a subset of the reachable workspace. The workspaces of several robots are shown later in this chapter.

## 1.2 Robots as Mechanical Devices

There are a number of physical aspects of robotic manipulators that we will not necessarily consider when developing our mathematical models. These include mechanical aspects (e.g., how are the joints actually constructed), accuracy and repeatability, and the tooling attached at the end effector. In this section, we briefly describe some of these.

### 1.2.1 Classification of Robotic Manipulators

Robot manipulators can be classified by several criteria, such as their **power source**, meaning the way in which the joints are actuated; their **geometry**, or kinematic structure; their **method of control**; and their intended **application area**. Such classification is useful primarily in order to determine which robot is right for a given task. For example, an hydraulic robot would not be suitable for food handling or clean room applications whereas a SCARA robot would not be suitable for automobile spray painting. We explain this in more detail below.

#### Power Source

Most robots are either electrically, hydraulically, or pneumatically powered. Hydraulic actuators are unrivaled in their speed of response and torque producing capability. Therefore hydraulic robots are used primarily for lifting heavy loads. The drawbacks of hydraulic robots are that they tend to leak hydraulic fluid, require much more peripheral equipment (such as pumps, which require more maintenance), and they are noisy. Robots driven by DC or AC motors are increasingly popular since they are cheaper, cleaner and quieter. Pneumatic robots are inexpensive and simple but cannot be controlled precisely. As a result, pneumatic robots are limited in their range of applications and popularity.

#### Method of Control

Robots are classified by control method into **servo** and **nonservo** robots. The earliest robots were nonservo robots. These robots are essentially **open-loop** devices whose movements are limited to predetermined mechanical stops, and they are useful primarily for materials transfer. In fact, according to the definition given above, fixed stop robots hardly qualify as robots. Servo robots use **closed-loop** computer control to determine their motion and are thus capable of being truly multifunctional, reprogrammable devices.

Servo controlled robots are further classified according to the method that the controller uses to guide the end effector. The simplest type of robot in this class is the **point-to-point** robot. A point-to-point robot can be taught a discrete set of points but there is no control of the path of the end effector in between taught points. Such robots are usually taught