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Road Map for Sliding Mode Control Design

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ISSN 2191-8198 ISSN 2191-8201 (electronic)
SpringerBriefs in Mathematics
ISBN 978-3-030-41708-6 ISBN 978-3-030-41709-3 (eBook)
https://doi.org/10.1007/978-3-030-41709-3

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Preface

This book reviews the basic ideas of sliding mode control (SMC) theory and demonstrates the deep interconnection of its different elements. The scope of the book is broad, encompassing the main design principles applied to a wide range of problems in control, observation, and information processing. The book is oriented toward engineers and theoreticians working in any area of control theory and applications.

In control systems, SMC is a nonlinear control method that alters the dynamics of a nonlinear system by application of a discontinuous control signal that forces the system to slide along manifolds with desired motions in the system state space. The state feedback control law is not a continuous function of time. Instead, such a system can switch from one continuous structure to another based on the current position in the state space. This explains why SMC stems from the so-called variable structure control method. In the context of modern control theory, any variable structure system, like a system under SMC, may be viewed as a special case of dynamical systems, supplied by switching logic.

Over the course of the development of the sliding modes theory during the past half-century, the range of both control problems and design methods to solve them has expanded significantly. Despite their great diversity, the basic principles of the SMC theory have remained unchanged since the creation of the necessary mathematical tools. The main control methods in sliding modes allow decomposition of the original system into independent subsystems of lower dimension and identification of the class of systems with sliding modes that are insensitive to various types of uncertainty in the description of the control object. These properties predetermine the success of application of the sliding modes theory for a wide class of dynamic objects: finite-dimensional models with scalar and vector inputs and outputs, models with delay, infinite-dimensional (described by partial differential equations) models, discrete-time models, stochastic models, etc.

The synthesis control procedure within the SMS is divided into two stages. In the first stage, it is necessary to choose the manifold in the state space of the system with the desired trajectories. This problem is equivalent to the design of a control in the systems of reduced dimension, for which any appropriate method of control theory is applicable. At the second stage, the control is selected to ensure the existence of a sliding mode on this preselected manifold. This is also a problem of reduced order with the same dimensions of the state and control vectors.

The task of providing a sliding mode is equivalent to the problem of stability of the zero solution of a nonlinear differential equation with a discontinuous right-hand side. The book provides a solution to this problem for systems of a fairly general form based on the Lyapunov function method. In particular, the described method allows us to reduce the well-known problems of root placement and quadratic optimization to solve similar problems of reduced dimension. All these properties are preserved when designing state observers for dynamical systems.

The main obstacle to the implementation of sliding modes is a phenomenon called chattering, which is caused by the mismatch between the ideal model and the real process. Within the power electronics literature, it is frequently referred to as ripple. In order to effectively suppress the chattering effect, the method of adapting the amplitude of the discontinuous signal and the harmonic cancellation method are used. These methods are applicable to both scalar and vector control systems. Note that in systems with scalar control, the sliding manifold is usually a surface in the state space. The high-order sliding mode method assumes that the sliding manifold can have a smaller dimension. Accordingly, the sliding mode equation has a smaller dimension. However, the problem of enforcing the sliding mode becomes more difficult as the dimension of control is less than the dimension of the system to be stabilized.

Methods developed for time-continuous systems have proved unsuitable for discrete-time systems because discontinuous control always leads to chattering even within the framework of an ideal model. It turns out that the main property of the SMC, namely, the achievement of the desired manifold in a finite time and the further motion in it, can be saved by using a control as a continuous function of the system state vector. This property has made it possible to develop methods for the synthesis of SMC for discrete systems.

SMC synthesis admits the presence of free parameters (such as the amplitude of discontinuous control and parameters of sliding manifold equations), which can be used to significantly improve the accuracy and dynamics of control processes. Such methods for designing adaptive control laws are applicable in systems with sliding modes.

The use of SMC for infinite-dimensional systems required a revision of the basic concepts of the theory. Questions of the mathematical description of solutions of partial differential equations with a discontinuous right-hand side had hardly been considered in the literature. Component-wise synthesis methods, developed for finite-dimensional systems, proved to be unacceptable. One of the chapters in this book is devoted to the solution of these problems, where distributed thermal processes and flexible mechanical systems are considered as applications.

An attempt to extend the SMC theory to the class of systems with random perturbations requires the use of the modern theory of stochastic systems. These studies are at an initial stage, and the first results are also reflected in this book.

Finally, further perspectives of the research are outlined in the concluding chapter.

The book reflects the consensus view of the authors regarding the current status of SMC theory, as it emerged from preceding discussions over a long period. We would like to underline that the authors are affiliated with different research centers and that their positions at the beginning of the discussions were far from being identical, as they are now. Finally, we would also like to point out that the book reflects the views of four generations of colleagues with rich experience in the area, the age difference between these generations being 10 years or more in each instance.

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Vadim Utkin graduated from Moscow Power Institute (Dipl. Eng.) and received Ph.D. and Doctor of Science degrees from the Institute of Control Sciences (Moscow, Russia). He worked at the Institute of Control Sciences from 1960 to 1994, and in 1973 was appointed as Head of the Discontinuous Control Systems Laboratory. Currently, he is a Professor at Ohio State University. He is one of the originators of the concepts of variable structure systems and sliding mode control. His application interests are control of power converters and electric drives, robotics, and automotive control. He is the author or co-author of five books and 350 papers.

Alex Poznyak graduated from Moscow Physical Technical Institute (MPhTI) in 1970. He earned Ph.D. and Doctor of Science degrees from the Institute of Control Sciences of the Russian Academy of Sciences in 1978 and 1989, respectively. From 1973 to 1993, he served first as a researcher and then as leading researcher at this institute, before accepting a post as Full Professor (3-F) at the Center for Research and Advanced Studies of the National Polytechnic Institute (CINVESTAV-IPN) in Mexico, where for 8 years he was Head of the Automatic Control Department. He has been the supervisor for 43 Ph.D. theses. He has published more than 240 papers in different international journals and 14 books. His areas of interest are robust nonlinear deterministic and stochastic control, identification theory, Markov processes, and game theory with economic applications.

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Andrey Polyakov received his Ph.D. in Systems Analysis and Control from Voronezh State University in 2005. Until 2010, he was an Associate Professor with this university. In 2007–2008, he worked at CINVESTAV-IPN in Mexico. From 2010 to 2013, he was a Lead Researcher of the Institute of Control Sciences of the Russian Academy of Sciences, and he then joined INRIA, Lille, France. His main research interests include robust and nonlinear control. He has authored or co-authored more than 150 papers.

Chapter 1
Introduction

Abstract The initial ideas for designing systems with a sliding mode were formulated half a century ago for dynamic models in canonical space with scalar input and output. The interest in the behavior analysis in the space of the output variable and its time derivatives was explained by the property of invariance with respect to perturbations and parametric variations. This invariance property is lost for arbitrary space in systems with vector input and output, which predetermined the need for new mathematical methods for the analysis of such systems and design of control methods with sliding modes. The set of these questions is the content of this book.

Keywords Sliding mode · Canonical space · Perturbations and parametric variations

The multi-branched tree of *Sliding Mode Control* (SMC) theory was seeded more than 50 years ago. At the very beginning, only SISO systems in the canonical space were studied. The interest in the space of output signal and its time derivatives can be explained easily: the sliding mode (SM) equation depends on neither parameter variations nor external disturbances (see Fig. 1.1).

The example in Fig. 1.1 demonstrated that the motion in sliding mode is of a reduced order and depends on the switching line equation only. It looks attractive to utilize these properties for control at the presence of disturbances. We compare the second-order systems with SMC and PID controller. As shown in Fig. 1.2, after a finite time interval, the sliding mode occurs (since the distance to the switching line s becomes equal to zero identically); the output does not depend on disturbance and tends to zero exponentially. Parameters of PID controller are selected such that the transient time for both processes are the same. The output of the system with PID controller cannot be reduced to zero (Fig. 1.3). An integral component in PID controller is able to reject constant disturbances only. The system with SMC is equivalent to the system with a high feedback gain, implemented by finite control actions. Therefore, an integral component is not needed for SMC, and steady-state error is equal to zero identically in contrast to systems with a linear PID controller.

SMC design method *can be generalized* easily for systems of an arbitrary order in canonical form

$$x^{(n)} + a_n x^{(n-1)} + \cdots + a_1 x = u + d,$$

V. Utkin et al., *Road Map for Sliding Mode Control Design*,
SpringerBriefs in Mathematics,
https://doi.org/10.1007/978-3-030-41709-3_1