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Advances in Efficiency and Productivity II

Springer

International Series in Operations Research & Management Science

Volume 287

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ISSN 0884-8289 ISSN 2214-7934 (electronic)
International Series in Operations Research & Management Science
ISBN 978-3-030-41617-1 ISBN 978-3-030-41618-8 (eBook)
https://doi.org/10.1007/978-3-030-41618-8

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

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Part I
Background

Introduction

Juan Aparicio, C. A. Knox Lovell, Jesus T. Pastor, and Joe Zhu

Abstract We begin by providing contextual background of this book, the second in a series. We continue by proclaiming the importance of efficiency and productivity, for businesses, industries, and nations. We then summarize the chapters in the book, which consist of an equal number of advances in the analytical foundations of efficiency and productivity measurement and advances in empirical applications that illustrate the significance of efficiency and productivity.

Keywords Efficiency · Productivity · Analytical advances · Empirical applications

1 Background

The Santander Chair of Efficiency and Productivity was created at the Miguel Hernandez University (UMH) of Elche, Spain, at the end of year 2014. Its aim was, and continues to be, the promotion of specific research activities among the international academic community. This Research Chair was assigned to the UMH Institute CIO (Center of Operations Research). The funding of the Chair by *Santander Universidades* constitutes one more example of the generosity and the vision of this organization, which supports a network of over 1407 Ibero-American universities and over 19.9 million students and academicians. Professor Knox Lovell, Honorary Professor of Economics at the University of Queensland, Australia, was appointed Director of the Chair. The Advisory Board of the Chair consists of four members, two of them on behalf of *Santander Universidades*, Mr. José María García de los Ríos and Mr. Joaquín Manuel Molina, and the other two on behalf of the UMH, Ph.D. Juan Aparicio, appointed as Co-Director, and Ph.D. Lidia Ortiz, the Secretary of the Chair.

During 2015 and 2016, the Chair organized eight Efficiency/Productivity Seminars for starting new programs with researchers interested in a variety of topics such as education, municipalities, financial risks, regional cohesion, meta-heuristics, renewable energy production, food industry, and endogeneity. During 2015, an International Workshop on Efficiency and Productivity was organized. The Workshop contributions of 15 relevant researchers and research groups made it possible to conceive the first book in this series, *Advances in Efficiency and Productivity*, with the inestimable support of Professor Joe Zhu, the Associate Series Editor for the Springer International Series in Operations Research and Management Sciences.

During 2017–2019, the Chair organized ten seminars on topics such as financial efficiency, conditional efficiency applied to municipalities, analysis of the change in productivity through dynamic approaches, measures of teacher performance in education, meta-heuristic approaches, centralized reallocation of human resources, evaluation of the impact of public policies, and comparative evaluation of the performance of the innovation in the European Union. Additionally, in June 2018, a second International Workshop on Efficiency and Productivity was organized in the city of Alicante, and the contributions of the participants are collected in this volume, naturally entitled *Advances in Efficiency and Productivity II*. Professor Joe Zhu has graciously agreed to join the editors of this volume.

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J. Aparicio et al. (eds.), *Advances in Efficiency and Productivity II*, International Series in Operations Research & Management Science 287, https://doi.org/10.1007/978-3-030-41618-8_1

2 Advances in Efficiency and Productivity

The title of this book, like the title of the first book, is generic, but the two substantive components, efficiency and productivity, are of great concern to every business and economy, and to most regulators as well. The first word in the title is the theme of the book and of the 2018 International Workshop on Efficiency and Productivity that spawned it. The presentations at the workshop, and the chapters in this book, truly do advance our understanding of efficiency and productivity.

The theoretical definition of *efficiency* involves a comparison of observed inputs (or resources) and outputs (or products) with what is optimal. Thus, technical efficiency is the ratio of observed to minimum input use, given the level of outputs produced, or as the ratio of observed to maximum output production, given the level of input use, or any of several combinations of the two, with or without additional constraints. Each type of technical efficiency is conditional on the technology in place, and each is independent of prices. In contrast, economic efficiency is price-dependent and typically is defined as the ratio of observed to minimum cost, given outputs produced and input prices paid, or as the ratio of observed to maximum revenue, given inputs used and output prices received. There are variants of these definitions based on different measures of value, such as the ratio of observed to maximum profit, but all take the form of a comparison of observed values with optimal values, conditional on the technology in place. It is also possible, and useful for conveying financial information to managements, to convert all ratios above, technical and economic, to differences expressed in units of the relevant currency. Indeed many important recent analytical advances, including some in this book, measure economic performance in terms of differences rather than ratios.

The principal challenge in efficiency measurement is the definition of minimum, maximum, or optimum, each of which is a representation of the unobserved production technology, which therefore must be estimated. The analytical techniques developed in this book provide alternative ways of defining optimum, typically as a (technical) production frontier or as an (economic) cost, revenue, or profit frontier, and alternative ways of measuring efficiency relative to an appropriate frontier. The concept of frontiers, and deficits or gaps or inefficiencies relative to them, plays a prominent role in the global warming and climate change literature, reflecting the fact that some nations are more at risk than others.

The theoretical definition of *productivity* coincides with the common sense notion of the ratio of observed output to observed input. This definition is straightforward on the rare occasion in which a producer uses a single input to produce a single output. The definition is more complicated otherwise, when multiple outputs in the numerator must be aggregated using weights that reflect their relative importance, and multiple inputs in the denominator must be aggregated in a similar fashion, so that productivity is again the ratio of two scalars, aggregate output and aggregate input. The time path of aggregate output is an output quantity index, and the time path of aggregate input is an input quantity index. Productivity growth is then defined as the rate of growth of the ratio of an output quantity index to an input quantity index or, equivalently, as the rate of growth of an output quantity index less the rate of growth of an input quantity index. Market prices are natural choices for weights in the two indexes, provided they exist and they are not distorted by market power, regulatory or other phenomena. As with efficiency measurement, productivity measurement can be expressed in terms of differences rather than ratios, in which case productivity indexes become productivity indicators. Also as with efficiency measurement, many important recent analytical advances, including some in this book, measure productivity performance with difference-based indicators rather than with ratio-based indexes.

Two challenges must be overcome, or at least addressed, in productivity measurement. The first involves the mathematical structure of the aggregator functions employed to convert individual outputs and inputs to output indexes or indicators and input indexes or indicators. This challenge is mentioned frequently in this book. The second is an extension of the first and occurs when prices do not exist or are unreliable indicators of the relative importance of corresponding quantities. In this case, productivity and its rate of growth must be estimated, rather than calculated from observed quantities and prices. The analytical techniques developed in this book provide alternative methods for estimating productivity and productivity change through time or productivity variation across producers.

The concepts of efficiency and productivity are significant beyond academe and characterize two important dimensions of the performance of businesses and economies, as evidenced by the following examples:

- The financial performance of businesses depends on the efficiency with which they conduct their operations. Statista, a financial statistics provider, has ranked the banking systems in selected European countries by their cost-income ratio, the ratio of the cost of running operations to operating income, a traditional measure of operating efficiency in banking. In 2018 it found Belgian banks to be the most cost-efficient, with a cost-income ratio of 38.1%, and German banks to be the least cost-efficient, with a cost-income ratio of 81.8%. The five most efficient national banking systems had an average cost-income ratio of 44.3%, while the five least efficient national banking systems had an average ratio of 69.4%. It seems worthwhile to search for an explanation for the variation in national operating efficiency, if only in an effort to shore up the financial performance of the laggard banking systems, some of which are very large.

- The relative prosperity of economies depends in large part on their productivity growth. According to the OECD, labor productivity (real GDP per hour worked) has grown faster in Germany than Italy since the turn of the century. Not coincidentally, according to the World Bank, GDP per capita also has grown faster in Germany than in Italy during the same time period. Many of the sources of the productivity gaps are well known, but a decade of experience suggests that they are difficult to rectify. As an aside to ponder, Italy lags Germany by far less on the 2019 World Happiness Index than it does by GDP per capita.

To be useful, efficiency and productivity must be not just well defined and satisfy a reasonable set of axioms, but they must be capable of measurement using quantitative techniques. Many popular concepts, such as cost-income ratios, labor productivity, and GDP per capita, can be calculated directly from company reports and country national accounts. When the data constraint prohibits direct calculation, empirical efficiency and/or productivity analysis is required. Such analyses typically are based on either a mathematical programming technique known as data envelopment analysis (DEA) or an econometric technique known as stochastic frontier analysis (SFA). Both types of estimation construct best practice frontiers, technical or economic, that bound the data from above or below, more or less tightly, and these frontiers provide empirical approximations to the theoretical optima referred to above. Both types of estimation, but especially DEA, are analyzed and employed throughout this book.

3 The Contents of the Book

Fortuitously the contributions divide themselves evenly between theoretical advances in the analysis of efficiency and productivity and empirical applications of efficiency and productivity analysis. The theoretical advances appearing in Sect. 1.3.1 address a wide range of modeling issues, and the empirical applications appearing in Sect. 1.3.2 address a similarly wide range of policy-relevant topics.

3.1 Analytical Advances

The analytical advances in the study of efficiency and productivity considered in the studies contained in this section are wide-ranging and explore a variety of model specification issues with important empirical ramifications. The three studies in Sect. 1.3.1.1 refer to production activities in general, whether modeled parametrically or nonparametrically. The four studies in Sect. 1.3.1.2 characterize production activities modeled nonparametrically, with DEA and its numerous extensions.

3.1.1 Modeling Advances

The concept of cross-efficiency uses the mean of the optimal weights of all DMUs in a sample, rather than those of a target DMU, to evaluate the efficiency of a target DMU in a DEA exercise. Originally introduced to estimate technical cross-efficiency, the concept recently has been extended to the estimation of economic, cost and profit, cross-efficiency, both of which decompose into technical and allocative cross-efficiencies. In chapter "New Definitions of Economic Cross-Efficiency" Aparicio and Zofio exploit duality theory to provide further economic extensions of the cross-efficiency concept to revenue, profit, and profitability to reflect alternative managerial objectives. They then place economic cross-efficiency in a panel data context, which leads naturally to extensions of a cost Malmquist productivity index and a profit Luenberger productivity indicator, which provide alternative frameworks for the evaluation of economic cross-efficiency change and its decomposition into technical and economic sources. The authors provide an empirical application to a sample of Iranian branch banks.

Theoretical and empirical production analysis incorporating the generation of undesirable outputs within a nonparametric framework has traditionally treated undesirable outputs as being weakly disposable. However recent research has discredited this influential approach, since it is incompatible with the materials balance principle. Alternative nonparametric approaches have been developed that do incorporate the materials balance principle, and in chapter "Modelling Pollution-Generating Technologies: A Numerical Comparison of Non-parametric Approaches" Dakpo, Jeanneaux and Latruffe consider the properties of four approaches to production analysis incorporating undesirable outputs, the family of weak disposability models, the family of eco-efficiency models, and a pair of models that incorporate the materials balance principle. Their theoretical and numerical analyses lead to a preference for a category of multi-equation models in which the global production technology is contained in the intersection of an intended outputs sub-technology and an undesirable outputs sub-technology. Further analysis is required to distinguish among models within this category.

Conventional wisdom asserts that most index numbers do not satisfy the desirable circularity, or transitivity, property. In chapter "Local Circularity of Six Classic Price Indexes" Pastor and Lovell challenge conventional wisdom, by considering six popular index numbers commonly used to measure price change. They acknowledge the validity of conventional wisdom, provided the property must be satisfied *globally*, over all possible nonnegative prices, not just the prices in a data set, and through all time periods included in a data set. However they also show that, although each of these index numbers fails to satisfy circularity globally, each can satisfy circularity *locally*, on a restricted data domain involving both the observed price mix and three consecutive time periods. Although the conditions for local circularity are demanding, the probability that a data set satisfies local circularity is not zero. They also distinguish necessary and sufficient conditions for local circularity, satisfied by Laspeyres and Paasche indexes, and sufficient conditions, satisfied by the geometric Laspeyres and geometric Paasche indexes and the Fisher and Törnqvist indexes.

3.1.2 Extensions of DEA

DEA was developed to analyze the relative performance of a comparable group of DMUs. Comparability requires DMUs to use similar inputs to produce similar outputs in similar operating environments, a stringent requirement that is rarely satisfied. If heterogeneity is ignored, some DMUs are unfairly disadvantaged in the exercise. Alternative adjustments to the DEA methodology have been proposed to account for heterogeneity, particularly in the operating environment. In chapter "Evaluating Efficiency in Non-homogeneous Environments" Avilés-Sacoto, Cook, Güemes-Castorena and Zhu develop and empirically test an adjusted DEA model designed to provide a fair evaluation of DMUs operating in different environments or belonging to different groups. The adjustment compensates disadvantaged DMUs for their excessive use of inputs by assuming that a proportion of their inputs is used to cope with a difficult environment or group rather than to produce intended outputs. The trick is to estimate this proportion, which the authors do with their adjusted DEA model, which provides estimates of both production efficiency and compensation efficiency. A sample of Mexican economic activities is used to illustrate the workings of the model.

Endogeneity occurs when an explanatory variable is correlated with the error term. This is a common problem in stochastic frontier analysis (SFA), in which endogeneity involves correlation between input use and the inefficiency error component, which causes biased and inconsistent parameter estimates. Endogeneity has suffered from relative neglect in DEA, but it exists and has similar dire consequences, particularly when endogeneity involves strong positive correlation. In chapter "Testing Positive Endogeneity in Inputs in Data Envelopment Analysis" Aparicio, Ortiz, Santin and Sicilia provide a new robust statistical test of the presence of endogeneity resulting from either positive or negative correlation between each input and the efficiency level *prior to conducting an output-oriented DEA analysis*. A Monte Carlo analysis shows their test successfully detects endogenous inputs, the inclusion of which would otherwise adversely affect the performance of output-oriented DEA. An empirical application to a sample of Uruguayan public schools illustrates the ability of the model to detect positive endogeneity linked to students' socioeconomic status.

A sample may not be representative of the population from which it is drawn. This makes the use of sample weights important when using sample information to conduct inference about the population, and although the use of sample weights is common in econometrics, it is less so in nonparametric frontier analysis. This is particularly unfortunate if, for example, the sample is not representative of the population due to the exclusion of best performers from the sample. In chapter "On the Estimation of Educational Technical Efficiency from Sample Designs: A New Methodology Using Robust Nonparametric Models" Aparicio, González, Santin and Sicilia propose a methodological strategy for incorporating sample weight information when using robust nonparametric order-α methods to improve the estimation of nonparametric production frontiers. The authors conduct a Monte Carlo analysis to explore the consequences for inference about the population frontier and the population average efficiency of not incorporating sample weight information.

The selection of variables to be included in a relative performance evaluation is an old and important one in empirical research. The problem has attracted considerable attention in DEA, particularly when the Delphi method exploiting expert opinion is not available. Two recommended strategies involve assignment of unconditional probabilities of inclusion in a DEA model or assuming the Principle of Maximum Entropy as a measure of uncertainty. In chapter "Robust DEA Efficiency Scores: A Heuristic for the Combinatorial/Probabilistic Approach" Aparicio and Monge adopt a probabilistic approach to variable selection, by calculating robust efficiency scores based on these two recommended strategies. Using data from the OECD PISA international educational achievement database, the authors calculate computational times and distances among radial scores, cross-efficiency scores, unconditional scores, and entropy scores.

3.2 Empirical Applications

The empirical applications of efficiency and productivity analysis appearing in this section examine a number of industries, sectors, and countries, illustrating the widespread empirical relevance of the analytical techniques appearing in Sect. 1.3.1. The three studies in Sect. 1.3.2.1 explore three very different economic issues using three disparate collections of individual production units from the USA, Spain, and the EU. The four studies in Sect. 1.3.2.2 explore a similarly wide range of economic issues using aggregate data from Peru, Spain, the EU and 39 countries.

3.2.1 Individual Production Units

Corporate social responsibility (CSR) has attracted intense interest in academe, particularly as it relates to corporate financial performance. A Google search on the linkage between the two returned about 160,000 results. In chapter "Corporate Social Responsibility and Firms' Dynamic Productivity Change" Kapelko changes the orientation from financial to economic by examining the impact of CSR on corporate dynamic productivity change. CSR data are obtained from a popular database and contain the usual environmental, social, and governance (ESG) components. Productivity change is estimated from a novel specification of a dynamic Luenberger productivity indicator that incorporates costly partial adjustment of quasi-fixed inputs. A bootstrap regression model is used to relate dynamic productivity change to CSR and its components. Using a large unbalanced panel of US firms during 2004–2015, Kapelko finds a modest positive impact of CSR and two of its components, S and G, and a modest negative impact of E, on dynamic productivity change, after controlling for a number of firm and other effects.

An important research topic concerns how markets value the ability of firms to increase intangible assets that create a gap between their market value and their book value and enhance the information value of the former. Intangible assets may include a risk indicator, as an outcome of a firm's business model, and a bank economic performance indicator, as an outcome of a firm's production plan. In chapter "Probability of Default and Banking Efficiency: How Does the Market Respond?" Curi and Lozano-Vivas examine a large sample of European banks before and after the financial crisis. They measure market value with Tobin's Q (the ratio of the sum of the market value of equity and book value of liabilities to the asset replacement cost), they specify a business model of minimizing probability of default, and they specify a production plan of maximizing cost-efficiency. Probability of default is obtained from a popular database, and cost-efficiency is estimated using DEA. Empirical findings are based on a regression of Tobin's Q on probability of default and cost-efficiency. Controlling for bank and other effects, they find bank valuation to be significantly influenced by both probability of default and cost-efficiency, with probability of default becoming more important and cost-efficiency less important, after the crisis.

Land consolidation (LC) involves land reallocation to reduce fragmentation, and rural planning to enhance infrastructure provision, and has been proposed as a way of adding new farmland, improving land productivity and agricultural competitiveness, and promoting sustainable land use. In chapter "The Impact of Land Consolidation on Livestock Production in Asturias' Parishes: A Spatial Production Analysis" Álvarez, Orea and Pérez-Méndez study the impact of LC processes on milk and beef production, and the number of farms, in a detailed geospatial sample of Asturian parishes since 2000. They model production technology with a set of translog distance functions, which they estimate using spatial panel data econometric techniques with controls for three LC indicators. They find a positive impact of LC processes on production, due to both direct effects (within parish) and indirect (spatial effects generated in neighboring parishes) spillover effects.

3.2.2 Aggregates of Production Units

Gross domestic product (GDP) is a narrow measure of economic well-being, and GDP growth is an equally narrow measure of social economic progress. The relative merits of the two measures have been debated for nearly a century. One popular measure of social economic progress is the social progress index (SPI), defined as the arithmetic mean of three noneconomic component factors of human needs, well-being, and opportunity. Each factor has four sub-factors, each having multiple indicators. The SPI typically is used to compare the performance of nations. However in chapter "A Novel Approach to

Computing a Regional Social Progress Index" Charles, Gherman and Tsolas apply it to 26 regions in Peru in 2015. They develop a novel two-phase method for computing a regional SPI. The first phase aggregates indicators into sub-factors and sub-factors into three factors for each region, using a two-stage objective general index. In the second phase, radial and non-radial DEA programs are used to construct regional SPIs. The two rankings are significantly positively correlated, especially at the upper (coastal regions) and lower (jungle regions) tails of the two distributions.

Estimating the impact of public infrastructure on private productivity has been a popular research topic for decades. Infrastructure can influence productivity directly, as an input at least partly under the control of individual producers, and indirectly by promoting resource reallocation among producers, preferably from less productive to more productive producers. In chapter "A Two-Level Top-Down Decomposition of Aggregate Productivity Growth: The Role of Infrastructure" Orea, Álvarez-Ayuso and Servén decompose aggregate labor productivity growth into direct and indirect effects. They then apply production and duality theory to a stochastic translog production frontier to implement the decomposition of labor productivity growth. They decompose labor productivity growth into size and substitution effects, a technical change effect, and direct and indirect infrastructure effects, the latter captured by the effect of infrastructure on the production efficiency of individual producers. They use a balanced panel of 5 industries in 39 countries over 15 years to estimate the effects of within-industry variables (size, substitution, and technical change) and 4 reallocation variables associated with infrastructure provision. Among their many findings is that telecommunications networks facilitate both within-industry productivity and resource reallocation among industries.

The Kyoto Protocol requires nations to establish binding targets for greenhouse gas emissions, in an effort to keep global warming within 2 °C of preindustrial levels. The EU has adopted such a target, which requires its reliance on fossil fuels to generate electricity and derived heat. In chapter "European Energy Efficiency Evaluation Based on the Use of Superefficiency Under Undesirable Outputs in SBM Models" R. Gómez-Calvet, Conesa, A. R. Gómez-Calvet and Tortosa-Ausina study the productivity performance of the electricity and derived heat sector in 28 EU nations during 2008–2012. Unlike many productivity studies, they include both desirable output, electricity and derived heat obtained from nonrenewable sources, and undesirable output, CO_2-equivalent greenhouse gas emissions, together with three inputs, primary energy consumed, installed capacity, and the number of employees. Analytically the authors follow Tone by specifying production technology with a superefficiency variant of a slacks-based DEA model. They then specify a Malmquist productivity index derived from the efficiencies obtained from the DEA model, which they decompose into efficiency change and technical change. They find wide variation in estimated productivity change, and its components, across countries.

The efficiency with which municipalities deliver public goods is an important research topic. Two requirements for credible inference are a representative and comprehensive set of performance indicators and a reliable estimation strategy. In chapter "Measuring Global Municipal Performance in Heterogeneous Contexts: A Semi-nonparametric Frontier Approach" Cordero, Diaz-Caro and Polo study the service delivery performance of a sample of medium-size Catalonian municipalities during 2005–2012 containing the financial crisis. Their data set contains five services (and a composite indicator of all five), three inputs, and four contextual variables (including the size of municipal debt) that may influence service delivery efficiency. Their estimation technique is the increasingly popular StoNED, which combines some virtues of DEA and SFA, as modified to incorporate contextual variables. Among their empirical findings are (i) efficiency varies directly with municipality size, (ii) efficiency has declined through time for all municipality sizes, and (iii) coastal municipalities are significantly more efficient than other municipalities.

Acknowledgments We gratefully acknowledge the support of *Banco Santander* (*Santander Universidades*) for the organization of the 2018 Workshop on Efficiency and Productivity in Alicante, as well as the assistance of the members of the Advisory Board of the Chair who were engaged in its design and development. We would also like to express our gratitude to Springer Nature and to Lidia Ortiz for the help provided during the editing of the book. Finally, we thank our authors for their participation at the Workshop and their contributions to this book.

Part II
Methodological Advances

New Definitions of Economic Cross-efficiency

Juan Aparicio and José L. Zofío

Abstract Overall efficiency measures were introduced in the literature for evaluating the economic performance of firms when reference prices are available. These references are usually observed market prices. Recently, Aparicio and Zofío (*Economic cross-efficiency: Theory and DEA methods*. ERIM Report Series Research in Management, No. ERS-2019-001-LIS. Erasmus Research Institute of Management (ERIM). Erasmus University Rotterdam, The Netherlands. http://hdl.handle.net/1765/115479, 2019) have shown that the result of applying cross-efficiency methods (Sexton, T. R., Silkman, R. H., & Hogan, A. J. (1986). Data envelopment analysis: Critique and extensions. In R. H. Silkman (Ed.), *Measuring efficiency: An assessment of data envelopment analysis, new directions for program evaluation* (Vol. 32, pp. 73–105). San Francisco/London: Jossey-Bass), yielding an aggregate multilateral index that compares the technical performance of firms using the shadow prices of competitors, can be precisely reinterpreted as a measure of economic efficiency. They termed the new approach "economic cross-efficiency." However, these authors restrict their analysis to the basic definitions corresponding to the Farrell (*Journal of the Royal Statistical Society, Series A, General 120*, 253–281, 1957) and Nerlove (*Estimation and identification of Cobb-Douglas production functions*. Chicago: Rand McNally, 1965) approaches, i.e., based on the duality between the cost function and the input distance function and between the profit function and the directional distance function, respectively. Here we complete their proposal by introducing new economic cross-efficiency measures related to other popular approaches for measuring economic performance, specifically those based on the duality between the profitability (maximum revenue to cost) and the generalized (hyperbolic) distance function and between the profit function and either the weighted additive or the Hölder distance function. Additionally, we introduce panel data extensions related to the so-called cost-Malmquist index and the profit-Luenberger indicator. Finally, we illustrate the models resorting to data envelopment analysis techniques—from which shadow prices are obtained and considering a banking industry dataset previously used in the cross-efficiency literature.

Keywords Data envelopment analysis · Overall efficiency · Cross-efficiency

1 Introduction

In a recent contribution, Aparicio and Zofío (2019) link the notions of overall economic efficiency and cross-efficiency by introducing the concept of economic cross-efficiency. Overall economic efficiency compares optimal and actual economic performance. From a cost perspective and following Farrell (1957), cost-efficiency is the ratio of minimum to actual (observed) cost, conditional on a certain quantity of output and input prices. From a profit perspective, Chambers et al. (1998) define the so-called Nerlovian inefficiency as the normalized difference between maximum profit and actual (observed) profit, conditional on both output and input prices.

Cost and profit efficiencies can in turn be decomposed into technical and allocative efficiencies by resorting to duality theory. In the former case, it can be shown that Shephard input distance function is dual to the cost function and, for *any*

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J. Aparicio et al. (eds.), *Advances in Efficiency and Productivity II*, International Series in Operations Research & Management Science 287, https://doi.org/10.1007/978-3-030-41618-8_2

reference prices, cost-efficiency is always smaller or equal to the value of the input distance function (Färe and Primont 1995). Consequently, as the distance function can be regarded as a measure of technical efficiency, whatever (residual) difference may exist between the two can be attributed to allocative efficiency. Likewise, in the case of profit inefficiency, Chambers et al. (1998) show that the directional distance function introduced by Luenberger (1992) is dual to the profit function and, for *any* reference prices, (normalized) maximum profit minus observed profit is always greater than or equal to the directional distance function. Again, since the directional distance function can be regarded as a measure of technical inefficiency, any difference corresponds to allocative inefficiency.

In this evaluation framework of economic performance, the reference output and input prices play a key role. In applied studies, the use of market prices allows studying the economic performance of firms empirically. However, in the duality approach just summarized above, reference prices correspond to those shadow prices that equate the supporting economic functions (cost and profit functions) to their duals (input or directional distance functions). Yet there are many other alternative reference prices, such as those that are assigned to each particular observation when calculating the input and directional distance functions in empirical studies. An example are the optimal weights that are obtained when solving the "multiplier" formulations of data envelopment analysis (DEA) programs that, approximating the production technology, yield the values of the technical efficiencies.

This set of weights can be used to cross-evaluate the technical performance of a particular observation with respect to its counterparts. That is, rather than using its own weights, the technical efficiency of an observation can be reevaluated using the weights corresponding to other units.[1] This constitutes the basis of the cross-efficiency methods initiated by Sexton et al. (1986). Taking the mean of all bilateral cross-evaluations using the vector of all (individual) optimal weights results in the cross-efficiency measure. Aparicio and Zofío (2019) realized that if these weights were brought into the duality analysis underlying economic efficiency, by considering them as specific shadow prices, the cross-efficiency measure can be consistently reinterpreted as a measure of economic efficiency and, consequently, could be further decomposed into technical and allocative efficiencies.

In particular, and under the customary assumption of input homotheticity (see Aparicio and Zofío 2019), cross-efficiency analysis based on the shadow prices obtained when calculating the input distance function results in the definition of the Farrell cost cross-efficiency. Likewise, it is possible to define the Nerlovian profit cross-inefficiency considering the vector of optimal shadow prices obtained when calculating the directional distance function. One fundamental advantage of the new approach based on shadow prices is that these measures are well-defined under the assumption of variable returns to scale; i.e., they always range between zero and one, in contrast to conventional cross-efficiency methods that may result in negative values. This drawback of the cross-efficiency methodology is addressed by Lim and Zhu (2015), who devise an ad hoc method to solve it, based on the translation of the data. The proposal by Aparicio and Zofío (2019) also takes care of the anomaly effortlessly while opening a new research path that connects the economic efficiency and cross-efficiency literatures.

This chapter follows up this new avenue of research by extending the economic cross-efficiency model to a number of multiplicative and additive definitions of economic behavior and their associated technological duals. From an economic perspective, this is quite relevant since rather than minimizing cost or maximizing profit, and due to market, managerial, or technological constraints, firms may be interested, for example, in maximizing revenue or maximizing profitability. As the economic goal is different, the underlying duality that allows a consistent measurement of economic cross-efficiency is different. For example, for the revenue function, the dual representation of the technology is the output distance function (Shephard 1953), while for the profitability function it is the generalized distance function (Zofío and Prieto 2006). Moreover, since the generalized distance function nests the input and output distance functions as particular cases (as well as the hyperbolic distance function), we can relate the cost, revenue, and profitability cross-efficiency models. Also, since a duality relationship may exist between a given supporting economic function and several distance functions, alternative economic cross-efficiency models may coexist. We explore this situation for the profit function. Besides the already mentioned model for profit efficiency measurement and its decomposition based on the directional distance function, an alternative evaluation can be performance relying on the weighted additive distance function (Cooper et al. 2011) or the Hölder distance function (Briec and Lesourd 1999). We present these last two models and compare them to the one based on the directional distance function. We remark the results of these models differ because of the alternative normalizing constraints that the duality relationship imposes. Hence researchers and practitioners need to decide first on the economic approach that is relevant for their study—cost, revenue, profit, and profitability—and then, among the set of suitable distance functions complying with the required duality conditions, choose the one that better characterizes the production process. Related to the DEA

[1]This cross-efficiency evaluation with respect to alternative peers results in smaller technical efficiency scores, because DEA searches for the most favorable weights when performing own evaluations.

methods that we consider in this chapter to implement the economic cross-efficiency models, it is well-known that the use of radial (multiplicative) distance functions projects observations to subsets of the production possibility set that are not Pareto-efficient because nonradial input reductions and output increases may be feasible (i.e., slacks). As for additive distance functions, the use of the weighted additive distance function in a DEA context ensures that efficiency is measured against the strongly efficient subset of the production possibility set, while its directional and Hölder counterparts do not. Thus, the choice of distance function is also critical when interpreting results. For example, in the event that slacks are prevalent, this source of technical inefficiency will be confounded with allocative inefficiency when decomposing profit inefficiency. Of course, other alternative models of economic cross-efficiency could be developed in terms of alternative distance functions. And some of them could even generalize the proposals presented here, such as the profit model based on the loss distance function introduced by Aparicio et al. (2016), which nests all the above additive functions.

Finally, in this chapter we also extend the economic cross-efficiency model to a panel data setting where firms are observed over time. For this we rely on existing models that decompose cost or profit change into productivity indices or indicators based on quantities, i.e., the Malmquist productivity index or Luenberger productivity indicator, and their counterpart price formulations. As the Malmquist index or Luenberger indicator can be further decomposed into efficiency change and technological change components, we can further learn about the sources of cost or profit change. As for the price indices and indicators, they can also be decomposed so as to learn about the role played by allocative efficiency. We relate this panel data framework to the cross-efficiency model and, by doing so, introduce the concept of economic cross-efficiency change. In this model, the cost-Malmquist and profit-Luenberger definitions proposed by Maniadakis and Thanassoulis (2004) and Juo et al. (2015), using market prices to determine cost change and profit change, are modified following the economic cross-efficiency rationale that replaces the former by the set of shadow prices corresponding to all observations, which results in a complete evaluation of the economic performance observations over time—to the extent that a complete set of alternative prices is considered.

This chapter is structured as follows. In the next section, we introduce the notation and recall the economic cross-efficiency model proposed by Aparicio and Zofío (2019). In the third section, we present the duality results that allow us to extend the analytical framework to the notion of profitability cross-efficiency based on the generalized distance function and how it relates to the partially oriented Farrell cost and revenue cross-efficiencies. We also introduce two alternative models of profit cross-efficiency based on the weighted additive and Hölder distance functions. A first proposal of economic cross-inefficiency for panel data models based on the cost-Malmquist index and profit-Luenberger indicator is proposed in Sect. 4. In Sect. 5, we illustrate the empirical implementation of the existing and new definitions of economic cross-efficiency through data envelopment analysis and using a dataset of bank branches previously used in the literature. Finally, relevant conclusions are drawn in Sect. 6, along with future venues of research in this field.

2 Background

In this section, we briefly introduce the notion of (standard) cross-efficiency in data envelopment analysis and review the concept of economic cross-efficiency. Let us consider a set of n observations (e.g., firms or decision-making units, DMUs) that use m inputs, whose (nonnegative) quantities are represented by the vector $X \equiv (x_1, \ldots, x_m)$, to produce s outputs, whose (nonnegative) quantities are represented by the vector $Y \equiv (y_1, \ldots, y_s)$. The set of data is denoted as $\{(X_j, Y_j), j = 1, \ldots, n\}$. The technology or production possibility set is defined, in general, as $T = \left\{(X, Y) \in R_+^{m+s} : X \text{ can produce Y}\right\}$.

Relaying on data envelopment analysis (DEA) techniques, T is approximated as $T_c = \left\{(X, Y) \in R_+^{m+s} : \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_i, \forall i, \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_r, \forall r, \lambda_j \geq 0, \forall j\right\}$. This corresponds to a production possibility set characterized by constant returns to scale (CRS). Allowing for variable returns to scale (VRS) results in the following definition: $T_v = \left\{(X, Y) \in R_+^{m+s} : \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_i, \forall i, \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_r, \forall r, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, \forall j\right\}$—see Banker et al. (1984).[2]

Let us now introduce the notion of Farrell cross-efficiency.

[2]Based on these technological characterizations, in what follows we define several measures that allow the decomposition of economic cross-efficiency into technical and allocative components. As it is now well-established in the literature, we rely on the following terminology: We refer to the different *factors* in which economic cross-efficiency can be decomposed *multiplicatively* as efficiency measures (e.g., Farrell cost-efficiency). Numerically, the greater their value, the more efficient observations are. For these measures, one is the upper bound signaling an efficient behavior. Alternatively, we refer to the different *terms* in which economic cross-*in*efficiency can be decomposed *additively* as *in*efficiency measures (e.g.,

2.1 Farrell (Cost) Cross-efficiency

In DEA, for firm k, the radial input technical efficiency assuming CRS is calculated through the following program:

$$
\begin{aligned}
ITE_c(X_k, Y_k) = \max_{U,V} & \ \frac{\sum_{r=1}^{s} u_r y_{rk}}{\sum_{i=1}^{m} v_i x_{ik}} \\
s.t. & \\
& \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \ j = 1, \ldots, n, \ (1.1) \\
& u_r \geq 0, \qquad r = 1, \ldots, s, \ (1.2) \\
& v_i \geq 0, \qquad i = 1, \ldots, m. \ (1.3)
\end{aligned}
\tag{1}
$$

Although (1) is a fractional problem, it can be linearized as shown by Charnes et al. (1978). $ITE_c(X_k, Y_k)$ ranges between zero and one. Hereinafter, we denote the optimal solution obtained when solving (1) as $\left(V_k^*, U_k^*\right)$.

Model (1) allows firms to choose their own weights on inputs and outputs in order to maximize the ratio of a weighted (virtual) sum of outputs to a weighted (virtual) sum of inputs. In this manner, the assessed observation is evaluated in the most favorable way, and DEA provides a self-evaluation of the observation by using input and output weights that are unit-specific. Unfortunately, this fact hinders obtaining a suitable ranking of firms based on their efficiency score, particularly for efficient observations whose $ITE_c(X_k, Y_k)$. In contrast to standard DEA, a cross-evaluation strategy is suggested in the literature (Sexton et al. 1986, and Doyle and Green 1994). In particular, the (bilateral) cross input technical efficiency of unit l with respect to unit k is defined by:

$$
CITE_c(X_l, Y_l \,|k) = \frac{U_k^* \cdot Y_l}{V_k^* \cdot X_l} = \frac{\sum_{r=1}^{s} u_{rk}^* y_{rl}}{\sum_{i=1}^{m} v_{ik}^* x_{il}}. \tag{2}
$$

$CITE_c(X_l, Y_l|k)$ also takes values between zero and one and satisfies $CITE_c(X_l, Y_l|l) = ITE_c(X_l, Y_l)$.[3]

Given the observed n units in the data sample, the traditional literature on cross-efficiency postulates the aggregation of the bilateral cross input technical efficiencies of unit l with respect to all units k, $k = 1, \ldots, n$, through the arithmetic mean. This results in the definition of the multilateral notion of cross input technical efficiency of unit l:

$$
CITE_c(X_l, Y_l) = \frac{1}{n}\sum_{k=1}^{n} CITE_c(X_l, Y_l \,|k) = \frac{1}{n}\sum_{k=1}^{n} \frac{U_k^* \cdot Y_l}{V_k^* \cdot X_l} = \frac{1}{n}\sum_{k=1}^{n} \frac{\sum_{r=1}^{s} u_{rk}^* y_{rl}}{\sum_{i=1}^{m} v_{ik}^* x_{il}}. \tag{3}
$$

Before presenting the notion of economic cross-efficiency, we need to briefly recall the main concepts related to the measurement of economic efficiency through frontier analysis, both in multiplicative form (Farrell 1957) and in additive manner (Chambers et al. 1998). We start considering the Farrell radial paradigm for measuring and decomposing cost-efficiency. For the sake of brevity, we state our discussion in the input space, defining the input requirement set $L(Y)$ as the set of nonnegative inputs $X \in R_+^m$ that can produce nonnegative output $Y \in R_+^s$, formally $L(Y) = \left\{X \in R_+^m : (\mathrm{X}, \mathrm{Y}) \in T\right\}$, and the isoquant of $L(Y) : IsoqL(Y) = \{X \in L(Y) : \varepsilon < 1 \Rightarrow \varepsilon x \notin L(Y)\}$. Let us also denote by $C_L(Y, W)$ the minimum cost of producing the output level Y given the input market price vector $W \in R_{++}^m : C_L(Y, W) = \min\left\{\sum_{i=1}^{m} w_i x_i : X \in L(Y)\right\}$.

Nerlovian profit inefficiency). Now the greater their numerical value, the greater the *in*efficiency, with zero being the lower bound associated to an efficient behavior.

[3]For a list of relevant properties, see Aparicio and Zofío (2019).

The standard (multiplicative) Farrell approach views cost-efficiency as originating from technical efficiency and allocative efficiency. Specifically, we have:

$$\underbrace{\frac{C_L(Y,W)}{\sum_{i=1}^{m} w_i x_i}}_{\text{Cost Efficiency}} = \underbrace{\frac{1}{D_L^I(X,Y)}}_{\text{Technical Efficiency}} \cdot \underbrace{AE_L^F(X,Y;W)}_{\text{Allocative Efficiency}}, \tag{4}$$

where $D_L^I(X,Y) = \sup\{\theta > 0 : X/\theta \in L(Y)\}$ is the Shephard input distance function (Shephard 1953) and allocative efficiency is defined residually. We use the subscript L to denote that we do not assume a specific type of returns to scale. Nevertheless, we will refer to $C_c(Y,W)$ and $D_c^I(X,Y)$ for CRS and $C_v(Y,W)$ and $D_v^I(X,Y)$ for variable returns to scale (VRS) when needed. Additionally, it is well-known in DEA that the inverse of $D_L^I(X,Y)$ coincides with $ITE_L(X_k,Y_k)$. For the particular case of CRS program (1), $ITE_c(X_k,Y_k) = D_c^I(X,Y)^{-1}$.

Considering actual common market prices for all firms within an industry, then the natural way of comparing the performance of each one would be using the left-hand side in (4). We then could assess the obtained values for each firm since we were using the same reference weights (prices) for all the observations, creating a market-based ranking. This idea inspired Aparicio and Zofío (2019), who suggest that cross-efficiency in DEA could be also defined based on the notion of Farrell's cost-efficiency. In particular, for a given set of *any* reference prices (e.g., shadow prices, market prices, or other imputed prices), they define the Farrell (cost) cross-efficiency of unit l with respect to unit k as:

$$FCE_L(X_l,Y_l|k) = \frac{C_L\left(Y_l,V_k^*\right)}{\sum_{i=1}^{m} v_{ik}^* x_{il}}, \tag{5}$$

where $L\in\{c,v\}$ denotes either constant or variable returns to scale.

As in (4), $FCE_L(X_l,Y_l|k) = \frac{1}{D_L(X_l,Y_l)} \cdot AE_L^F\left(X_l,Y_l;V_k^*\right)$. Therefore, Farrell cross-efficiency of unit l with respect to unit k corrects the usual technical efficiency, the inverse of the Shephard distance function, through a term with meaning of (shadow) allocative efficiency.

Given the observed n units, the traditional literature on cross-efficiency suggests to aggregate bilateral cross-efficiencies through the arithmetic mean to obtain the multilateral notion of cross-efficiency. In the case of the Farrell cross-efficiency, this yields:

$$FCE_L(X_l,Y_l) = \frac{1}{n}\sum_{k=1}^{n} FCE_L(X_l,Y_l|k) = \frac{1}{n}\sum_{k=1}^{n} \frac{C_L\left(Y_l,V_k^*\right)}{\sum_{i=1}^{m} v_{ik}^* x_{il}}. \tag{6}$$

Additionally, $FCE_L(X_l,Y_l)$ can be always decomposed (under any returns to scale) into (radial) technical efficiency and a correction factor defined as the arithmetic mean of n shadow allocative efficiency terms. That is,

$$FCE_L(X_l,Y_l) = \frac{1}{n}\sum_{k=1}^{n} FCE_L(X_l,Y_l|k) = \frac{1}{n}\sum_{k=1}^{n} \frac{C_L\left(Y_l,V_k^*\right)}{\sum_{i=1}^{m} v_{ik}^* x_{il}} = ITE_L(X_l,Y_l) \cdot \frac{1}{n}\sum_{k=1}^{n} AE_L^F\left(X_l,Y_l;V_k^*\right), \tag{7}$$

with $ITE_L(X_l,Y_l)$ and $AE_L^F\left(X_l,Y_l;V_k^*\right)$, $L\in\{c,v\}$, denoting constant and variable returns to scale technical and (shadow) allocative efficiencies, respectively.

We note that $FCE_L(X_l,Y_l)$ satisfies two very interesting properties:

First, assuming the existence of perfectly competitive input markets resulting in a single equilibrium price for each input (i.e., firms are price takers), if we substitute (shadow) prices by these market prices in (7), then $FCE_L(X_l,Y_l)$ precisely coincides with $C_L(Y_l,W) / \sum_{i=1}^{m} w_i x_{il}$, which is Farrell's measure of cost inefficiency (4). Hence, economic cross-efficiency offers a "natural" counterpart to consistently rank units when reference prices are unique for all units. This property is not satisfied in general by the standard measure of cross-efficiency, if both input and output market prices are used as weights;

i.e., $CITE_c(X_l, Y_l) = \frac{\sum_{r=1}^{s} p_r y_{rl}}{\sum_{i=1}^{m} w_i x_{il}} \neq \frac{C_c(Y_l, W)}{\sum_{i=1}^{m} w_i x_{il}}$. Indeed Aparicio and Zofío (2019) show that besides market prices, input homotheticity is required for the equality to hold; otherwise $CITE_c(X_l, Y_l) \geq FCE_c(X_l, Y_l)$. Nevertheless, we also remark that the concept of economic cross-efficiency can accommodate firm-specific market prices if some degree of market power exists and firms are price makers in the inputs markets. In that case, individual firms' shadow prices would be substituted by their market counterparts in (7). This connects our proposal to the extensive theoretical and empirical economic efficiency literature considering individual market prices, e.g., Ali and Seiford (1993).

Second, as previously remarked, $FCE_L(X_l, Y_l)$ is well-defined, ranging between zero and one, even under variable returns to scale. This property is not verified in general by the standard cross-efficiency measures (see Wu et al. 2009; Lim and Zhu 2015). This is quite relevant because traditional measures may yield negative values under variable returns to scale, which is inconsistent and hinders the extension of cross-efficiency methods to technologies characterized by VRS.

An interesting by-product of the economic cross-efficiency approach is that by incorporating the economic behavior of firms in the formulations (e.g., cost minimizers in $FCE_v(X_l, Y_l)$), the set of weights represented by the shadow prices are reinterpreted as market prices, rather than their usual reading in terms of the alternative supporting technological hyperplanes that they define and against which technical inefficiency is measured. This solves some recent criticism raised against the cross-efficiency methods, since shadow prices could be then considered as specific realizations of market prices, e.g., see Førsund (2018a, b) and Olesen (2018).

Next, we briefly introduce the Nerlovian cross-inefficiency.

2.2 *Nerlovian (Profit) Cross-inefficiency*

Now, we recall the concepts of profit inefficiency and its dual graph measure corresponding to the directional distance function (Chambers et al. 1998).

Given the vector of input and output market prices $(W, P) \in R_+^{m+s}$, and the production possibility set T, the profit function is defined as: $\Pi_T(W, P) = \max_{X,Y} \left\{ \sum_{r=1}^{s} p_r y_r - \sum_{i=1}^{m} w_i x_i : (X, Y) \in T \right\}$. In what follows, let $\Pi_c(W, P)$ and $\Pi_v(W, P)$ be the maximum profit given the CRS technology T_c and the VRS technology T_v, respectively.

Profit inefficiency *à la Nerlove* for firm k is defined as maximum profit (i.e., the value of the profit function given market prices) minus observed profit, normalized by the value of a prefixed reference vector $(G^x, G^y) \in R_+^{m+s}$. By duality, the following inequality is obtained (Chambers et al. 1998):

$$\frac{\Pi_T(W, P) - \left(\sum_{r=1}^{s} p_r y_{rk} - \sum_{i=1}^{m} w_i x_{ik} \right)}{\sum_{r=1}^{s} p_r g_r^y + \sum_{i=1}^{m} w_i g_i^x} \geq \vec{D}_T\left(X_k, Y_k; G^x, G^y\right). \tag{8}$$

where $\vec{D}_T(X_k, Y_k; G^x, G^y) = \max_{\beta} \{\beta : (X_k - \beta G^x, Y_k + \beta G^y) \in T\}$ is the directional distance function. As for the Farrell approach, profit inefficiency can be also decomposed into technical inefficiency and allocative inefficiency, where the former corresponds to the directional distance function:

$$\frac{\Pi_T(W, P) - \left(\sum_{r=1}^{s} p_r y_{rk} - \sum_{i=1}^{m} w_i x_{ik} \right)}{\sum_{r=1}^{s} p_r g_r^y + \sum_{i=1}^{m} w_i g_i^x} = \vec{D}_T\left(X_k, Y_k; G^x, G^y\right) + AI_T^N\left(X_k, Y_k; W, P; G^x, G^y\right). \tag{9}$$

The subscript T in $\Pi_T(W, P)$, $\vec{D}_T(X_k, Y_k; G^x, G^y)$ and $AI_T^N(X_k, Y_k; W, P; G^x, G^y)$ implies that we do not assume a specific type of returns to scale. Nevertheless, as before we will use $\vec{D}_c(X_k, Y_k; G^x, G^y)$ and $AI_c^N(X_k, Y_k; W, P; G^x, G^y)$ for CRS and $\vec{D}_v(X_k, Y_k; G^x, G^y)$ and $AI_v^N(X_k, Y_k; W, P; G^x, G^y)$ for VRS.

In the case of DEA, when VRS is assumed, the directional distance function is determined through (10):

$$\begin{aligned}
\overrightarrow{D}_v\left(X_k, Y_k; G^x, G^y\right) = \max_{\beta,\lambda} \ & \beta \\
s.t \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{ik} - \beta g_i^x, \qquad i = 1, \ldots, m, \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{rk} + \beta g_r^y, \qquad r = 1, \ldots, s, \\
& \sum_{j=1}^{n} \lambda_j = 1, \\
& \lambda_j \geq 0, \qquad\qquad j = 1, \ldots, n.
\end{aligned} \tag{10}$$

whose dual is:

$$\begin{aligned}
\min_{U,V,\alpha} \ & -\sum_{r=1}^{s} u_r y_{rk} + \sum_{i=1}^{m} v_i x_{ik} + \alpha \\
s.t. \quad & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - \alpha \leq 0, \ j = 1, \ldots, n, \\
& \sum_{r=1}^{s} u_r g_r^y + \sum_{i=1}^{m} v_i g_i^x = 1, \\
& U \geq 0_s, V \geq 0_m.
\end{aligned} \tag{11}$$

Let us also denote the optimal solutions of problem (11) as $\left(\overrightarrow{V}_k^*, \overrightarrow{U}_k^*, \overrightarrow{\alpha}_k^*\right)$.

Aparicio and Zofío (2019) defined the Nerlovian cross-inefficiency of unit l with respect to unit k as:

$$NCI_v\left(X_l, Y_l; G^x, G^y \,|k\right) = \frac{\Pi\left(\overrightarrow{V}_k^*, \overrightarrow{U}_k^*\right) - \left(\sum_{r=1}^{s} \overrightarrow{u}_{rk}^* y_{rl} - \sum_{i=1}^{m} \overrightarrow{v}_{ik}^* x_{il}\right)}{\sum_{r=1}^{s} \overrightarrow{u}_{rk}^* g_r^y + \sum_{i=1}^{m} \overrightarrow{v}_{ik}^* g_i^x} = \frac{\overrightarrow{\alpha}_k^* - \left(\sum_{r=1}^{s} \overrightarrow{u}_{rk}^* y_{rl} - \sum_{i=1}^{m} \overrightarrow{v}_{ik}^* x_{il}\right)}{\sum_{r=1}^{s} \overrightarrow{u}_{rk}^* g_r^y + \sum_{i=1}^{m} \overrightarrow{v}_{ik}^* g_i^x}. \tag{12}$$

As usual, the arithmetic mean of (12) for all observed units yields the final Nerlovian cross-inefficiency of unit l:

$$NCI_v\left(X_l, Y_l; G^x, G^y\right) = \frac{1}{n}\sum_{k=1}^{n} NCI_v\left(X_l, Y_l; G^x, G^y \,|k\right). \tag{13}$$

Invoking (9), we observed once again that the Nerlovian cross-inefficiency of firm l is a “correction” of the original directional distance function value for the unit under evaluation, where the modifying factor can be interpreted as (shadow) allocative inefficiency:

$$\begin{aligned}
NCI_v\left(X_l, Y_l; G^x, G^y\right) = & \ \overrightarrow{D}_v\left(X_0, Y_0; G^x, G^y\right) \\
& + \frac{1}{n}\sum_{k=1}^{n} AI_v^N\left(X_l, Y_l; \overrightarrow{V}_k^*, \overrightarrow{U}_k^*; G^x, G^y\right).
\end{aligned} \tag{14}$$

Finally, these authors showed that the approach by Ruiz (2013), based on the directional distance function under CRS, is a particular case of (14).

3 New Economic Cross-(in)efficiency Measures

3.1 Profitability Cross-efficiency

We now extend the previous framework of economic cross-(in)efficiency to a set of new measures which can be decomposed either multiplicatively or additively. We start with the notion of profitability—corresponding to Georgescu-Roegen's (1951) "return to the dollar," defined as the ratio of observed revenue to observed cost. We then show that it can be decomposed into a measure of economic efficiency represented by the generalized distance function introduced by Chavas and Cox (1999) and a factor defined as the geometric mean of the allocative efficiencies corresponding to the n shadow prices. Let us define maximum profitability as $\mathrm{P}_T(W, P) = \max_{X,Y} \left\{ \sum_{r=1}^{s} p_r y_r / \sum_{i=1}^{m} w_i x_i : (X, Y) \in T \right\}$. Zofío and Prieto (2006) proved that:

$$\frac{P \cdot Y_k / W \cdot X_k}{\mathrm{P}_T(W, P)} \leq D_c^G(X_k, Y_k; \gamma), \tag{15}$$

where $D_c^G(X_k, Y_k; \gamma) = \inf\left\{\delta : \left(\delta^{\gamma} X_k, Y_k/\delta^{1-\gamma}\right) \in T\right\}$, $0 \leq \gamma \leq 1$, is the generalized distance function and $P \cdot Y_k = \sum_{r=1}^{s} p_r y_{rk}$ and $W \cdot X_k = \sum_{i=1}^{m} w_i x_{ik}$.

We remark that the generalized distance function in expression (15), rather than being defined to allow for either constant or variable returns to scale as in the previous models, is characterized by the former. The reason is that the production technology exhibits local constant returns to scale at the optimum; hence maximum profitability is achieved at loci representing most productive scale sizes in Banker et al. (1984) terminology. This provides the rationale to develop the duality underlying expression (15) departing from such technological specification. We further justify this choice in what follows when recalling the variable returns to scale technology so as to account for scale efficiency.

The generalized distance function $D_c^G(X_k, X_k; \gamma)$ can be calculated relying on DEA by solving the following nonlinear problem:

$$\begin{aligned} D_c^G(X_k, Y_k; \gamma) = \min_{\delta,\lambda} \ & \delta \\ s.t. \ & \\ & \sum_{j=1}^{n} \lambda_j x_{ij} \leq \delta^{1-\gamma} x_{ik}, \quad && i = 1, \ldots, m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geq \tfrac{y_{rk}}{\delta^{\gamma}}, && r = 1, \ldots, s, \\ & \lambda_j \geq 0, && j = 1, \ldots, n, \end{aligned} \tag{16}$$

Following the Farrell and Nerlovian decompositions (7) and (14), it is possible to define allocative efficiency as a residual from expression (15):

$$\frac{P \cdot Y_k / W \cdot X_k}{\mathrm{P}_T(W, P)} = D_c^G(X_k, Y_k; \gamma) \cdot AE_c^G(X_k, Y_k; W, P; \gamma), \tag{17}$$

where $AE_c^G(X_K, Y_K; W, P; \gamma) = \frac{P \cdot \hat{Y}_k / W \cdot \hat{X}_k}{\mathrm{P}_T(W,P)}$ with $\hat{X}_k = D_c^G(X_k, Y_k; \gamma) X_k$ and $\hat{Y}_k = Y_k / D_c^G(X_k, Y_k; \gamma)$.[4] So, allocative efficiency, which is a measure that in the Farrell approach essentially captures the comparison of the rate of substitution between production inputs with the ratio of market prices at the production isoquant given the output level Y_k, is, in this case, the profitability calculated at the (efficient) projection linked to the generalized model.

As previously mentioned, since the technology may be characterized by variable returns to scale, it is possible to bring its associated directional distance function $D_v^G(X_k, X_k; \gamma)$ into (17)—calculated as in (16) but adding the VRS constraint $\sum_{j=1}^{n} \lambda_j = 1$. This allows decomposing productive efficiency into two factors, one representing "pure" VRS technical

[4] Färe et al. (2002) defined this relationship in terms of the hyperbolic distance function; i.e., $D_c^H(X_k, Y_k) = \min_{\delta,z} \left\{ \delta : \sum_{j=1}^{n} \lambda_j X_j \leq \delta X_k, \frac{Y_k}{\delta} \leq \sum_{j=1}^{n} \lambda_j Y_j, \lambda_j \geq 0, j = 1, \ldots, n \right\}$.

efficiency and a second one capturing scale efficiency: i.e., $D_c^G(X_k, X_k; \gamma) = D_v^G(X_k, X_k; \gamma) \cdot SE^G(X_k, X_k; \gamma)$, where $SE^G(X_k, X_k; \gamma) = D_c^G(X_k, X_k; \gamma)/D_v^G(X_k, X_k; \gamma)$. Defining expression (15) under constant returns to scale enables us to individualize the contribution that scale efficiency makes to profitability efficiency. Otherwise, had we directly relied on the directional distance function defined under variable returns to scale in (15), scale inefficiency would had been confounded with allocative efficiency in (17).

Reinterpreting the left-hand side of (15) in the framework of cross-efficiency, we next define a new economic cross-efficiency approach that allows us to compare the (bilateral) performance of firms l with respect to firm k using the notion of profitability:

$$PCE_c(X_l, Y_l; \ \gamma |k) = \frac{U_k^* \cdot Y_l / V_k^* \cdot X_l}{\mathrm{P}_T(V_k^*, U_k^*)}, \tag{18}$$

where, once again, (V_k^*, U_k^*) are the shadow prices associated with the frontier projections generated by $D_c^G(X_k, X_k; \ \gamma)$.

To aggregate all cross-efficiencies in a multiplicative framework, we depart on this occasion from standard practice and use the geometric mean, whose properties make the aggregation meaningful when consistent (transitive) bilateral comparisons of performance in terms of productivity are pursued; see Aczél and Roberts (1989) and Balk et al. (2017). Hence:

$$PCE_c(X_l, Y_l; \ \gamma) = \left(\prod_{k=1}^{n} \frac{U_k^* \cdot Y_l / V_k^* \cdot X_l}{\mathrm{P}_T(V_k^*, U_k^*)}\right)^{1/n}, \tag{19}$$

As in the Farrell and Nerlovian models (7) and (14), we can decompose $PCE_c(X_l, Y_l; \ \gamma)$ according to technical and allocative criteria, thereby obtaining:

$$PCE_c(X_l, Y_l; \ \gamma) = D_c^G(X_k, Y_k; \ \gamma) \cdot \left(\prod_{k=1}^{n} AE_c^G(X_k, Y_k; V_k^*, U_k^*; \gamma)\right)^{1/n} \tag{20}$$

Based on this decomposition, the role played by VRS technical efficiency and scale efficiency can be further individualized since $D_C^G(X_k, X_k; \gamma) = D_v^G(X_k, X_k; \gamma) \cdot SE^G(X_k, X_k; \gamma)$.

We now obtain some relevant relationships between the profit and profitability cross-(in)efficiencies. Relaying on Färe et al. (2002) and Zofío and Prieto (2006), it is possible to show that under constant returns to scale, maximum feasible profit is zero, $\Pi_c(W, P) = 0$ (if $\Pi_c(W, P) < +\infty$), and, therefore, maximum profitability is one, $\mathrm{P}_c(W, P) = 1$.[5] Also, it is a well-known result that, under CRS, $ITE_c(X_k, Y_k) = D_c^G(X_k, Y_k; \ 0)$.[6] Combining both conditions, it is possible to express (17) as follows:

$$\frac{P \cdot Y_k}{W \cdot X_k} = ITE_c(X_k, Y_k) \cdot AE_c^G(X_k, Y_k; W, P; \ 0). \tag{21}$$

Now, in the usual cross-efficiency context considering k's shadow prices $(V_k^*, U_k^*)$when evaluating the performance of firm l, we first have that the standard input-oriented bilateral cross-efficiency can be interpreted as a profitability measure: $CITE_c(X_l, Y_l |k) = \frac{U_k^* \cdot Y_l}{V_k^* \cdot Y_l}$. Second, $CITE_c(X_l, Y_l) = \frac{1}{n}\sum_{k=1}^{n} \frac{U_k^* \cdot Y_l}{V_k^* \cdot X_l}$ is the arithmetic mean of the n individual profitabilities [see (3)]. Additionally, by (21), we obtain the following decomposition of $CITE_c(X_l, Y_l)$:

$$CITE_c(X_l, Y_l) = \frac{1}{n}\sum_{k=1}^{n} \frac{U_k^* \cdot Y_l}{V_k^* \cdot X_l} = ITE_c(X_l, Y_l) \cdot \left[\frac{1}{n}\sum_{k=1}^{n} AE_c^G(X_l, Y_l; V_k^*, U_k^*; \ \gamma = 0)\right]. \tag{22}$$

Hence, under the assumption of CRS, $CITE_c(X_l, Y_l)$ can be decomposed as $FCE_L(X_l, Y_l)$ into two technical and allocative factors, expression (7). Indeed, CRS implies that the production technology is input homothetic, and Aparicio and Zofío (2019) show in their Theorem 1 that in this (less restrictive) case, $CITE_c(X_l, Y_l) = FCE_c(X_l, Y_l)$, and therefore (22) coincides

[5] Aparicio and Zofío (2019) show in their Lemma 2 that given an optimal solution to problem (1), (V_k^*, U_k^*), then $\Pi_c(V_k^*, U_k^*) = 0$, i.e., maximum profit equal to infinitum can be discarded.

[6] In terms of the hyperbolic distance function, $ITE_c(X_k, Y_k) = D_c^H(X_k, Y_k)^2$.

with (7). Consequently, as in the latter expression, the classical input cross-inefficiency measure is equal to the self-appraisal score of firm l, $ITE_c(X_l, Y_l)$, modified by the mean of its (shadow) generalized allocative efficiencies. Note also that, as per (20), technical efficiency can be decomposed into VRS and scale efficiencies: $ITE_c(X_l, Y_l) = ITE_v(X_l, Y_l) \cdot SE^F(X_l, Y_l)$.

Finally, it is also worth mentioning that the profit and profitability dualities and their associated economic cross-inefficiencies, including their decompositions, can be directly related in the case of CRS. Following Färe et al. (2002, 673), the precursor of expression (15) in terms of the profit function is:

$$\Pi_T(W, P) \geq \frac{P \cdot Y_l}{D_c^G(X_l, Y_l;\ \gamma)^{\gamma}} - D_c^G(X_l, Y_l;\ \gamma)^{1-\gamma} W \cdot X_l. \tag{23}$$

Since $\Pi_T(W, P) = 0$ in the case of CRS, expression (15) is easily derived from (23) and vice versa. However, under VRS, $\Pi_T(W, P)$ is not nil and we cannot obtain the duality-based inequality (15), with the left-hand side not depending on any efficiency measure (distance function) and the right-hand side not depending on prices. This shows, once again, the importance of defining multiplicative economic cross-efficiency measures under the assumption of VRS.

3.2 *Farrell (Revenue) Cross-efficiency*

Following the same procedure set out to define the Farrell (cost) cross-efficiency, $FCE_L(X_l, Y_l)$in (6), we can develop an output-oriented approach in terms of the radial output technical efficiency, $OTE_c(X_k, Y_k)$under CRS calculated through a DEA program corresponding to the inverse of (1)—see Ali and Seiford (1993)—and the revenue function. As usual, $OTE_v(X_k, Y_k)$ may be computed under VRS adding the constraint $\sum_{j=1}^{n} \lambda_j = 1$.

The standard output technical cross-efficiency of l based on the optimal weights—shadow prices—of k, $\left(V_k^*, U_k^*\right)$, defines as:

$$COTE_c(X_l, Y_l \,|k) = \frac{V_k^* \cdot X_l}{U_k^* \cdot Y_l} = \frac{\sum_{i=1}^{m} v_{ik}^* x_{il}}{\sum_{r=1}^{s} u_{rk}^* y_{rl}}, \tag{24}$$

The introduction of the Farrell (revenue) cross-efficiency requires defining the output requirement set $P(X)$ as the set of nonnegative outputs $Y \in R_+^s$ that can be produced with nonnegative inputs $X \in R_+^m$, formally $P(X) = \left\{Y \in R_+^s : (\text{X, Y}) \in T\right\}$, and the isoquant of $P(X)$: $Isoq\ P(X) :\ = \{Y \in P(X) : \varepsilon > 1 \Rightarrow \varepsilon Y \notin P(X)\}$. Let us also denote by $R_L(X, P)$ the maximum revenue obtained from using input level X given the output market price vector $P \in R_{++}^s$: $R_L(X, P) = \max_Y \left\{\sum_{i=1}^{s} p_s y_s : Y \in P(X)\right\}$. The standard revenue definition and decomposition is given by:

$$\underbrace{\frac{R_L(X, P)}{\sum_{i=1}^{s} p_s y_s}}_{\text{Revenue Efficiency}} = \underbrace{\frac{1}{D_L^O(X, Y)}}_{\text{Technical Efficiency}} \cdot \underbrace{AE_L^F(X, Y; P)}_{\text{Allocative Efficiency}}, \tag{25}$$

where $D_L^O(X, Y) = \inf\{\phi > 0 : Y/\phi \in P(X)\}$ is the Shephard output distance function (Shephard 1953) and allocative efficiency is defined residually. Again, we use the subscript L to stress that revenue efficiency can be defined with respect to different returns to scale.

Consequently, considering shadow prices, the Farrell (revenue) cross-efficiency of firm l with respect to firm k is:

$$FRE_L(X_l, Y_l \,|k) = \frac{R_L\left(X_l, U_k^*\right)}{U_k^* \cdot Y_l} = \frac{R_L\left(X_l, U_k^*\right)}{\sum_{r=1}^{s} u_{rk}^* y_{rl}}, \tag{26}$$

with $L \in \{c, v\}$ denoting constant and variable returns to scale, respectively.

As in (25), $FRE_L(X_l, Y_l|k) = \frac{1}{D_L^O(X_l, Y_l)} \cdot AE_L^F(X_l, Y_l; U_k^*)$. Therefore, Farrell revenue cross-efficiency corrects the usual technical efficiency, the inverse of Shephard output distance function, through a term capturing (shadow) allocative efficiency.

As in the case of the Farrell cost cross-efficiency (6), we could aggregate all individual revenue cross-efficiencies following the standard approach that relies on the arithmetic mean. However, in the current multiplicative framework, we rely on our preferred choice for the geometric mean, already used in the profitability approach. This yields:

$$FRE_L(X_l, Y_l) = \left(\prod_{k=1}^{n} FRE_L(X_l, Y_l|k)\right)^{1/n} = \left(\prod_{k=1}^{n} \frac{R_L(X_l, U_k^*)}{U_k^* \cdot Y_l}\right)^{1/n}, \tag{27}$$

which can be further decomposed into technical and allocative components:

$$FRE_L(X_l, Y_l) = \left(\prod_{k=1}^{n} \frac{R_L(X_l, U_k^*)}{U_k^* \cdot Y_l}\right)^{1/n} = OTE_L(X_l, Y_l) \cdot \left(\prod_{k=1}^{n} AE_L^F(X_l, Y_l; U_k^*)\right)^{1/n}. \tag{28}$$

We now combine the cost and revenue approaches of economic cross-efficiency and relate it to the profitability cross-efficiency definition. Assume first that the $FCE_L(X_l, Y_l)$ in (6) is defined using the geometric mean as aggregator—so it is consistent with $FRE_L(X_l, Y_l)$ in (27). Then, given that $FCE_L(X_l, Y_l)$ depends on (shadow) input prices but not on (shadow) output prices and vice versa for $FRE_L(X_l, Y_l)$, we suggest to mix both approaches to introduce yet another new cross-efficiency measure under the Farrell paradigm.

$$FE_L(X_l, Y_l) = \frac{FCE_L(X_l, Y_l)}{FRE_L(X_l, Y_l)} = \frac{\left(\prod_{k=1}^{n} \frac{C_L(Y_l, V_k^*)}{V_k^* \cdot X_l}\right)^{1/n}}{\left(\prod_{k=1}^{n} \frac{R_L(X_l, U_k^*)}{U_k^* \cdot Y_l}\right)^{1/n}} = \frac{ITE_L(X_l, Y_l) \cdot \left(\prod_{k=1}^{n} AE_L^F(X_l, Y_l; V_k^*)\right)^{1/n}}{OTE_L(X_l, Y_l) \cdot \left(\prod_{k=1}^{n} AE_L^F(X_l, Y_l; U_k^*)\right)^{1/n}}. \tag{29}$$

$FE_L(X_l, Y_l)$ is related to $CITE_c(X_l, Y_l|k)$ under CRS:

$$FE_c(X_l, Y_l) = \frac{\left(\prod_{k=1}^{n} \frac{U_k^* \cdot Y_l}{V_k^* \cdot X_l}\right)^{1/n}}{\left(\prod_{k=1}^{n} \frac{R_c(X_l, U_k^*)}{C_c(Y_l, V_k^*)}\right)^{1/n}} = \frac{\left(\prod_{k=1}^{n} CITE_c(X_l, Y|k)\right)^{1/n}}{\left(\prod_{k=1}^{n} \frac{R_c(X_l, U_k^*)}{C_c(Y_l, V_k^*)}\right)^{1/n}}. \tag{30}$$

The value of (30) must be closed to:

$$\left(\prod_{k=1}^{n} CITE_c(X_l, Y|k)\right)^{1/n} / \left(\prod_{k=1}^{n} \mathrm{P}_T(V_k^*, U_k^*)\right)^{1/n}. \tag{31}$$

Additionally, $FE_L(X_l, Y_l)$ always takes values between zero and one, while $FE_L(X_l, Y_l) \leq \frac{ITE_L(X_l, Y_l)}{OTE_L(X_l, Y_l)}$, under any returns to scale assumed.

At this point, it is worth mentioning that analogous results to the Farrell cost cross-efficiency can be derived for the cross output technical efficiency and revenue efficiency when output homotheticity is assumed; i.e., $COTE_c(X_l, Y_l|k) = FRE_c(X_l, Y_l|k)$. However, $COTE_c(X_l, Y_l) \neq FRE_c(X_l, Y_l)$ in general if $COTE_c(X_l, Y_l)$ is defined as usual by additive aggregation, and $FRE_c(X_l, Y_l)$ is defined through multiplicative aggregation.

3.3 *Profit Cross-inefficiency Based on the (Weighted) Additive Distance Function*

This section introduces a measure of economic cross-efficiency based on the weighted additive distance function, which constitutes an alternative to the Nerlovian definition based on the directional distance function.

Cooper et al. (2011) proved that:

$$\frac{\Pi_T(W,P)-\left(\sum_{r=1}^{s}p_r y_{rk}-\sum_{i=1}^{m}w_i x_{ik}\right)}{\min\left\{\frac{w_1}{a_{1k}},\ldots,\frac{w_m}{a_{mk}},\frac{p_1}{b_{1k}},\ldots,\frac{p_s}{b_{sk}}\right\}}\geq WA_T(X_k,Y_k;A_k,B_k), \tag{32}$$

where:

$$WA_v(X_k,Y_k;A_k,B_k)=\max_{S,H,\lambda_j}\left\{A_k\cdot S+B_k\cdot H:\sum_{j=1}^{n}\lambda_j x_{ij}\leq x_{ik}-s_i,\forall i,\ y_{rk}+h_r\leq\sum_{j=1}^{n}\lambda_j y_{rj},\right.$$
$$\left.\sum_{j=1}^{n}\lambda_j=1,\lambda_j\geq 0,\ j=1,\ldots,n,\quad S\geq 0_m,\ H\geq 0_s\right\} \tag{33}$$

is the weighted additive model in DEA. In particular, A_k and B_k are prefixed input and output weights, respectively. As in the Nerlovian approach (8), the left-hand side of (32) measures profit inefficiency, defined as maximum profit (i.e., the value of the profit function at the market prices) minus observed profit, normalized by the minimum of the ratios of market prices to their corresponding prefixed weights. Based on (32), and assuming variable returns to scale, profit inefficiency for firm k can be decomposed as follows:

$$\frac{\Pi_V(W,P)-\left(\sum_{r=1}^{s}p_r y_{rk}-\sum_{i=1}^{m}w_i x_{ik}\right)}{\min\left\{\frac{w_1}{a_{1k}},\ldots,\frac{w_m}{a_{mk}},\frac{p_1}{b_{1k}},\ldots,\frac{p_s}{b_{sk}}\right\}}=WA_v(X_k,Y_k;A_k,B_k)+AI_V^W(X_k,Y_k;W,P;A_k,B_k). \tag{34}$$

Substituting market prices by shadow prices[7] in evaluating firm l with respect to firm k, we obtain:

$$WACI_V(X_l,Y_l;A_l,B_l\,|k)=\frac{\Pi_v\left(V_k^*,U_k^*\right)-\left(\sum_{r=1}^{s}u_{rk}^* y_{rl}-\sum_{i=1}^{m}v_{ik}^* x_{il}\right)}{\min\left\{\frac{v_{1k}^*}{a_{1l}},\ldots,\frac{v_{mk}^*}{a_{ml}},\frac{u_{1k}^*}{b_{1l}},\ldots,\frac{u_{sk}^*}{b_{sl}}\right\}}. \tag{35}$$

Aggregating all profit cross-inefficiencies through the arithmetic mean, given the additive framework, allows us to define the new profit cross-inefficiency measure based on the weighted additive approach:

$$WACI_v(X_l,Y_l;A_l,B_l)=\frac{1}{n}\sum_{k=1}^{n}\frac{\Pi_v\left(V_k^*,U_k^*\right)-\left(\sum_{r=1}^{s}u_{rk}^* y_{rl}-\sum_{i=1}^{m}v_{ik}^* x_{il}\right)}{\min\left\{\frac{v_{1k}^*}{a_{1l}},\ldots,\frac{v_{mk}^*}{a_{ml}},\frac{u_{1k}^*}{b_{1l}},\ldots,\frac{u_{sk}^*}{b_{sl}}\right\}} \tag{36}$$

which can be decomposed as (34), yielding:

$$WACI_v(X_l,Y_l;A_l,B_l)=WA_v(X_l,Y_l;A_l,B_l)+\frac{1}{n}\sum_{k=1}^{n}AI_V^W\left(X_l,Y_l;V_k^*,U_k^*;A_l,B_l\right) \tag{37}$$

Therefore $WACI_T(X_l, Y_l; A_l, B_l)$ coincides with the sum of the original technical inefficiency measure of firm l, determined by the weighted additive model, and a correction factor capturing (shadow) allocative inefficiencies.

It is worth mentioning that, among all the approaches mentioned in this chapter, the weighted additive model is unique such that it measures technical efficiency with respect to the strongly efficient frontier, resorting to the notion of Pareto-Koopmans efficiency.

[7] Shadow prices are obtained for $\mathrm{DMU_k}$ through the linear dual of program (33).