Fei Yan
Salvador E. Venegas-Andraca

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Fei Yan
Changchun University of Science and Technology
Changchun, China

Salvador E. Venegas-Andraca
Tecnologico de Monterrey
Escuela de Ingenieria y Ciencias
Monterrey, Mexico

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Preface

Quantum information science is a deeply interdisciplinary field that involves physics, mathematics, and computer science. It is devoted to finding methods to exploit quantum mechanical effects in nature, notably superposition and entanglement, to perform information processing beyond the limits of conventional computation. Over the past several decades, various research groups around the world have strived to achieve the ambitious goal of building a quantum computer that could dramatically improve computational power for particular tasks.

Responding to the growing need to extract information from images and video, image processing is a fundamental task in many branches of science and engineering. Due to the restricted architecture of classical computers and the computational complexity of state-of-the-art classical algorithms in image processing and its applications, developing efficient algorithms to store and manipulate visual information has become an important and challenging research area.

Quantum image processing focuses on quantum algorithms for storing, processing, and retrieving visual information. Due to some of the astounding properties inherent to quantum information, for instance, computational parallelism, it is anticipated that quantum image processing technologies will offer capabilities and performance that are currently unrivaled by their traditional equivalents in areas such as computing speed, tamperproof security, and minimal storage requirements.

This book is divided into seven chapters. In Chap. 1, the key fundamentals of quantum computation and information are reviewed, and the history of quantum image processing is introduced. The widely used quantum image representations and their well-designed operations are presented in Chaps. 2 and 3. The outline of quantum image security technologies and a few quantum image understanding algorithms are suggested in Chaps. 4 and 5. The two emerging subtopics of quantum movies and quantum audio are elaborated in Chap. 6. Chapter 7 discusses open questions identified in the literature, along with future development trends in quantum image processing.

It is hoped that this book offers a rigorous introduction to quantum image processing and some thought-provoking snapshots of prospective developments. The completion of this book relied greatly on the research achievements published in the field and the two bibles: *Quantum Computation and Quantum Information* (Michael A. Nielsen et al.) and *Digital Image Processing* (Rafael C. Gonzalez et al.).

Immense gratitude is due to the emeritus professor of the Tokyo Institute of Technology, Kaoru Hirota, and Professor Zhengang Jiang at Changchun University of Science and Technology for their enlightenment and ongoing help that turned this book from an idea into reality. In addition, special commendation goes to Kehan Chen, Nianqiao Li, and Shan Zhao for their contributions to the timely and thorough organization of the figures and references in the book.

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Being subject to the limits of the authors' ability and because quantum image processing is still in its primary stage, it is hard to avoid errors and omissions. The authors apologize for this and welcome criticism and suggestions.

Changchun, China — Fei Yan
Monterrey, Mexico — Salvador E. Venegas-Andraca

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Chapter 1
Introduction and Overview

Quantum computation and quantum information comprise the study of information processing tasks that can be accomplished using quantum mechanical systems [3]. Goals include to study how information is represented and communicated using quantum states, and how to describe and handle the corruption of quantum and classical information [38]. Quantum computers, quantum cryptography, and quantum teleportation are among the most celebrated topics that have emerged from this field. These techniques rely distinctively on the quantum properties such as uncertainty, interference, and entanglement [4, 59].

The disciplines of computer science and computer engineering have transformed every aspect of human endeavors [50]. In these fields, exciting and cutting-edge research into new computational models, materials, and techniques for building computing hardware has been broached and/or realized [27]. Novel methods have been proposed to speed up certain tasks, and to build bridges between computer science and other scientific fields, allowing scientists to think of natural phenomena as computational procedures and simulate them [52, 56].

In its canonical form, theoretical computer science takes no account of the physical properties of the devices used to perform computational or information processing tasks [35]. This could be perceived as a drawback because the behavior of any physical device used for computation or information processing must ultimately be predicted within the ambit of the laws of physics [28].

In 1982, Feynman proposed a novel computational model [19], quantum computation, which was based on the principles of quantum physics. Quantum computation constitutes a truly innovative paradigm of computation, which offers new perspectives in many regards, among them future encouraging scenarios for high performance computing as well as novel algorithms that solve seemingly intractable problems in today's advanced classical computer models and technologies. The mathematical formulation and physical realization of quantum technology ensure improved miniaturization, massively accelerated performance of certain tasks, and new levels of secure communication, information processing, and ultra-precise

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measurement [10]. These are some of the theoretical discoveries and promising conjectures that have positioned quantum computation as a key element in modern science.

In addition, a growing number of quantum computing applications in several branches of science and technology have been suggested. One such emerging area is the field of quantum image processing (QIMP) [51]. In its early stage, the field is bedeviled with many questions [33]. To begin, what is the best way to represent images on quantum computers, and how one should prepare, process, and retrieve them? Then, to really say the field has matured, one should be capable of performing some basic image processing tasks and realizing some high-level applications using quantum computing hardware, before gradually accomplishing more advanced and robust image processing tasks. The advances highlighted in the following chapters indicate a promising role for QIMP in facilitating the acceleration, security, and integrity of traditional (digital) image processing tasks [60].

The discussion in this chapter is twofold. First, some fundamental concepts and theories of quantum computation and information are introduced. Further on, the birth and development of QIMP as a background are discussed.

1.1 Quantum Computation and Information

A recent study [37] echoed a longstanding claim that quantum computing technologies would usher in unprecedented accuracy and sophistication to solve numerous problems considered intractable using the best of today's classical (i.e., digital or nonquantum) computing resources. While acceptable large-scale quantum devices are still unavailable, the immense potential of quantum computing has attracted interest and investments aimed at the commercialization of its hardware and software. These make quantum computation and information become a cynosure among emerging computing paradigms [65].

1.1.1 Quantum Computers

An important law in the computer industry, Moore's law states that the number of transistors in a dense integrated circuit doubles roughly every 2 years [8]. This observation of Gordon Moore, co-founder of the Intel Corporation, proved to be accurate for several decades, and it has been used to guide long-term planning and to set targets for research and development in the semiconductor industry [15]. Advances in digital electronics are strongly tied to Moore's law, including quality-adjusted microprocessor prices, memory capacity, sensors, and even the number and size of pixels in digital cameras. Digital electronics have been consistent contributors to world economic growth in the late twentieth and early twenty-first centuries. Thus, Moore's law embodies a driver of technological and social change, productivity, and economic growth [30].

Moore's law comprises an observation and projection of an historical trend and is not a physical or natural law. Although the semiconductor industry's growth rate was steady from 1975 until approximately 2012, it was faster during the first decade of the new millennium. In general, it does not sound logic to extrapolate from an historical growth rate to an undefined future. For example, in the 2010 update to the International Technology Roadmap for Semiconductors it was predicted that growth would slow around 2013. Moreover, Gordon Moore himself, in 2015, foresaw that the rate of progress would reach saturation: "I see Moore's law dying here in the next decade or so" [11]. This is mainly because transistors are made of silicon. According to theoretical physicist Michio Kaku, when transistors are too closely packed (layers are 20 atoms across now, and this will likely decrease to five atoms), there will be two main problems [29]:

- The heat generated will be sufficient to melt the silicon.
- Quantum theory will dominate in the resultant small distance between atoms. From the Heisenberg uncertainty principle, it will be impossible to accurately locate electrons, which will result in leakage.

Therefore, while Moore's law gave good predictions, for further advancement, one should develop new technology and quantum computers are future candidates. A quantum computer is a device that performs quantum computing [31]. Such a computer is different from binary digital electronic computers based on transistors. Although common digital computing requires that the data be encoded into binary digits (bits), each of which is always in one of two definite states (0 or 1), quantum computation uses quantum bits or qubits, which can be in *superpositions* of states, i.e., linear combinations of their basis states.

As of 2019, quantum computer development is still in its infancy, but experiments are being performed in which quantum computational operations are executed on a very small number of quantum bits. Practical and theoretical research continues, and many governments and military agencies are funding quantum computing research in the hope of developing quantum computers for civilian, business, trade, environmental, and national security purposes.

Several significant advances have occurred in recent years. In January 2017, since its second-generation system (the 512-qubit D-Wave Two in May 2013) was bought by Google and NASA for research and practical use, D-Wave is reportedly selling a 2,000-qubit quantum computer (the D-Wave 2000Q [13], see Fig. 1.1), whose special-purpose processor was designed to implement quantum annealing, rather than operating as a universal-gate quantum computer [12]. In December 2017, Microsoft released a preview version of a Quantum Development Kit, including a programming language, *Q*#, which can be used to write programs for an emulated quantum computer [36]. In late 2017 and early 2018, IBM, Intel, and Google reported testing quantum processors containing 50, 49, and 72 qubits, respectively, all realized using superconducting circuits [41]. These circuits are approaching the number of qubits for which simulation of their quantum dynamics is expected to

Fig. 1.1 D-Wave 2000Q quantum computer

become prohibitive on classical computers, although it has been argued that further improvements in error rates are required to put classical simulation out of reach. In February 2018, scientists reported the discovery of a new form of light, which possibly involves polaritons, which could be useful in the development of quantum computers [34]. In July 2018, a team led by the University of Sydney achieved the world's first multi-qubit demonstration of a quantum chemistry calculation performed on a system of trapped ions, one of the leading hardware platforms in the race to develop a universal quantum computer [26].

Large-scale quantum computers would theoretically be able to solve certain problems much more quickly than any classical computers that use even the best current algorithms, such as integer factorization using Shor's algorithm [48] and the simulation of quantum many-body systems [57]. There exist quantum algorithms, such as Simon's algorithm [49], that run faster than any possible probabilistic classical algorithm. A classical computer could in principle (with exponential resources) simulate a quantum algorithm, as quantum computation does not violate the Church–Turing thesis [66]. However, quantum computers may be able to efficiently solve problems that are not feasible on classical computers.

1.1.2 Quantum Bits and Quantum Registers

1.1.2.1 Quantum Bits

Analogous to the fundamental concept of classical computation and information, the bit, a quantum bit (or qubit) is the smallest unit of information in a quantum system [38]. The difference between bits and qubits is that a qubit can be in a superposition state, which can be described as a unit vector in two-dimensional Hilbert space (see appendix for more mathematical descriptions). As shown in Fig. 1.2, the vector can

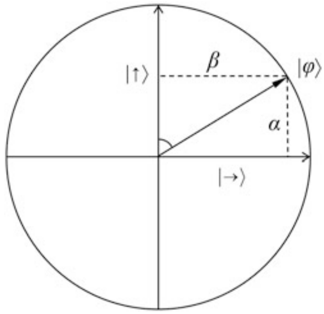


Fig. 1.2 Diagram of a qubit's superposition state

always be written as $|\varphi\rangle = \alpha|\uparrow\rangle + \beta|\rightarrow\rangle$, where $|\uparrow\rangle$ and $|\rightarrow\rangle$ are orthogonal basis states and α and β are complex numbers for probability amplitudes. The probabilities for $|\varphi\rangle$ to be in the $|\uparrow\rangle$ and $|\rightarrow\rangle$ states are, respectively, $|\alpha|^2$ and $|\beta|^2$, where $|\alpha|^2 + |\beta|^2 = 1$. Geometrically, this can be interpreted as the condition that the qubit's state is *normalized* to length 1 [38].

When $|\varphi\rangle$ is projected onto $|\uparrow\rangle$, $|\varphi\rangle$ becomes $|\varphi\rangle_{\uparrow}$ $(= \alpha|\uparrow\rangle)$, which is equivalent to measuring $|\varphi\rangle$ in the $|\uparrow\rangle$ direction. Similarly, when $|\varphi\rangle$ is projected onto $|\rightarrow\rangle$, $|\varphi\rangle$ becomes $|\varphi\rangle_{\rightarrow}$ $(= \beta|\rightarrow\rangle)$, which is equivalent to measuring $|\varphi\rangle$ in the $|\rightarrow\rangle$ direction. Therefore, when observing or measuring a qubit in a superposition state, the state will be disturbed and changed; this phenomenon is called *collapse*. If one lets $|\uparrow\rangle = |0\rangle$ and $|\rightarrow\rangle = |1\rangle$, then $|\varphi\rangle = \alpha|0\rangle + \beta|1\rangle$, so the states $|0\rangle$ and $|1\rangle$ are known as computational basis states, and they form an orthonormal basis for this vector space.

If one lets $u \in \{0, 1\}$, then $|u\rangle$ is a column vector (known as *ket*) with two components in two-dimensional Hilbert space; that is, $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. In addition, $\langle u|$ is the conjugate transpose of $|u\rangle$ and is a row vector (known as *bra*) with two components: $\langle 0| = \begin{pmatrix} 1 & 0 \end{pmatrix}$ and $\langle 1| = \begin{pmatrix} 0 & 1 \end{pmatrix}$. If one lets $v \in \{0, 1\}$, then $\langle u|v\rangle$ $(= \langle u||v\rangle)$ is the inner product of $|u\rangle$ and $|v\rangle$. The inner product is a scalar, for example, $\langle 0|0\rangle = \langle 1|1\rangle = 1$ and $\langle 0|1\rangle = \langle 1|0\rangle = 0$. It is interesting that *bra* and *ket* constitute bra(c)ket, so that when "⟨" and "⟩" match to form a complete bracket, the bracket as a whole always represents a number [42].

In addition to the inner product, *bra* and *ket* may also be multiplied in reverse order, and one can call $|u\rangle\langle v|$ the outer product of $|u\rangle$ and $|v\rangle$; it is an operator in the matrix form. If $|0\rangle\langle 0|$ is operated on $|\varphi\rangle$, the result of $\alpha|0\rangle$ is obtained, which indicates that $|0\rangle\langle 0|$ has extracted the $|0\rangle$ component from $|\varphi\rangle$, or that $|0\rangle\langle 0|$ projects $|\varphi\rangle$ onto $|0\rangle$, and $|\varphi\rangle$ is measured in the $|0\rangle$ direction. Similarly, $|1\rangle\langle 1|$ has extracted the $|1\rangle$ component from $|\varphi\rangle$, or that $|1\rangle\langle 1|$ projects $|\varphi\rangle$ onto $|1\rangle$, and $|\varphi\rangle$ is measured in the $|1\rangle$ direction.

Even though a qubit can represent many states, when it is observed, the measurement results can only be either 0 or 1, and each result exists with a certain probability [38]. The measurement operation is represented by a "meter" symbol, as shown in Fig. 1.3. As previously described, this operation converts a single-qubit

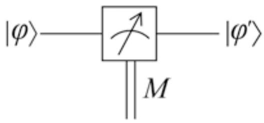


Fig. 1.3 Quantum circuit symbol for measurement

state $|\varphi\rangle$ to a probabilistic classical bit M (distinguished from a qubit by drawing it as a double-line wire).

1.1.2.2 Quantum Registers

A quantum register is a system comprising multiple qubits [38]. It is the quantum analog of the classical processor register. Quantum computers perform calculations by manipulating qubits within a quantum register. While an n-size classical register can store a single value of the 2^n possibilities spanned by n bits, a quantum register can store all 2^n possibilities spanned by n qubits.

The state of a quantum register is the tensor product of n qubits' states. The tensor product is a way of combining vector spaces to form larger vector spaces [38]. This formation is crucial in understanding the quantum mechanics of multiparticle systems. The notation for the tensor product, $\otimes$, is used to express the composition of quantum systems. The short notation for the tensor product $|u\rangle \otimes |v\rangle$ of two vectors or two kets, $|u\rangle$ and $|v\rangle$, is $|uv\rangle$ or $|u\rangle|v\rangle$, and $A^{\otimes n} = A \otimes A \otimes \cdots \otimes A$ denotes the tensor product of a matrix A for n times.

Suppose there are two qubits in a quantum register. A two-qubit system has four computational basis states denoted as $|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$. A pair of qubits can also exist in superpositions of these four states, so the quantum state of two qubits involves associating a complex coefficient with each computational basis state, such that

$$|\varphi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle. \tag{1.1}$$

Similar to the case for a single qubit, the measurement result 00, 01, 10, and 11 occurs with probability $|\alpha_{00}|^2$, $|\alpha_{01}|^2$, $|\alpha_{10}|^2$, $|\alpha_{11}|^2$, and these probabilities sum to one. For a two-qubit system, one could measure just a subset of the qubits, say the first qubit. Measuring the first qubit alone gives 0 with probability $|a_{00}|^2 + |a_{01}|^2$, leaving the post-measurement state:

$$|\varphi'\rangle = \frac{\alpha_{00}|00\rangle + \alpha_{01}|01\rangle}{\sqrt{|a_{00}|^2 + |a_{01}|^2}}. \tag{1.2}$$

If the state of multiple qubits cannot be presented as a tensor product, then these qubits are in the *entangled* state [38]. For instance, $|\varphi\rangle_A = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ and $|\varphi\rangle_B = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$ are in entangled states. In such a case, measurement of one qubit will affect the measurement of the other qubits. For example, when one measures the state of $|\varphi\rangle_A$, then the measurement of one qubit will make the state of the other qubit the same, while when one measures the state of $|\varphi\rangle_B$, the measurement of one qubit will make the state of the other qubit the opposite.

1.1.3 Quantum Circuits and Quantum Gates

1.1.3.1 Quantum Circuits

Changes to a quantum state can be described in the language of quantum computation. A quantum circuit is a quantum computation model in which a computation is a sequence of quantum logic gates (or simply quantum gates) [38]. Figure 1.4 shows a simple quantum circuit containing three quantum gates. The circuit is read from left to right, and each line represents a wire in the quantum circuit. While this may not correspond to a physical wire, it may instead correspond to the passage of time, or perhaps to a physical particle, e.g., a photon moving through space from one location to another [38]. It is conventionally assumed that the state input to the circuit is a computational basis state that is usually the state consisting of all $|0\rangle$s.

The state of qubits in quantum circuits evolves naturally over time, different combinations of quantum gates can implement specific quantum algorithms, and, finally, the results are presented with quantum measurements. In quantum computing, and specifically the quantum circuit model of computation, a quantum gate is a rudimentary quantum circuit operating on a small number of qubits [38]. Quantum gates are represented by *unitary* matrices. The number of qubits in the input and output of the gate must be equal; a gate which acts on n qubits is represented by a $2^n \times 2^n$ unitary matrix. It is noteworthy that it is impossible to make a copy of an unknown quantum state by using a circuit. This property, namely that qubits cannot be copied, is known as the *no-cloning* theorem [38], and is one of the primary differences between quantum and classical information.

1.1.3.2 Quantum Gates

Suppose an operator U_f on a quantum state is a unitary matrix, i.e.,

$$U_f U_f^\dagger = I, \tag{1.3}$$

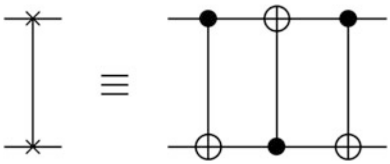
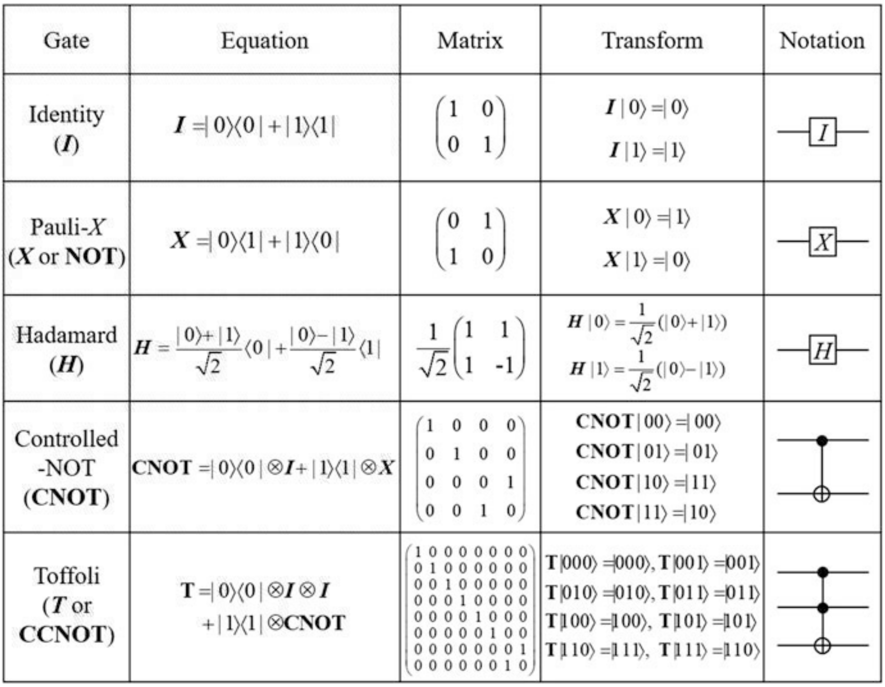


Fig. 1.4 Quantum swapping gate



Gate	Equation	Matrix	Transform	Notation
Identity ($\boldsymbol{I}$)	$\boldsymbol{I} = \|0\rangle\langle 0\| + \|1\rangle\langle 1\|$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\boldsymbol{I}\|0\rangle = \|0\rangle$ $\boldsymbol{I}\|1\rangle = \|1\rangle$	I
Pauli-X ($\boldsymbol{X}$ or **NOT**)	$\boldsymbol{X} = \|0\rangle\langle 1\| + \|1\rangle\langle 0\|$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\boldsymbol{X}\|0\rangle = \|1\rangle$ $\boldsymbol{X}\|1\rangle = \|0\rangle$	X
Hadamard ($\boldsymbol{H}$)	$\boldsymbol{H} = \frac{\|0\rangle + \|1\rangle}{\sqrt{2}}\langle 0\| + \frac{\|0\rangle - \|1\rangle}{\sqrt{2}}\langle 1\|$	$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	$\boldsymbol{H}\|0\rangle = \frac{1}{\sqrt{2}}(\|0\rangle + \|1\rangle)$ $\boldsymbol{H}\|1\rangle = \frac{1}{\sqrt{2}}(\|0\rangle - \|1\rangle)$	H
Controlled -NOT (**CNOT**)	$\mathbf{CNOT} = \|0\rangle\langle 0\| \otimes \boldsymbol{I} + \|1\rangle\langle 1\| \otimes \boldsymbol{X}$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	$\mathbf{CNOT}\|00\rangle = \|00\rangle$ $\mathbf{CNOT}\|01\rangle = \|01\rangle$ $\mathbf{CNOT}\|10\rangle = \|11\rangle$ $\mathbf{CNOT}\|11\rangle = \|10\rangle$	
Toffoli ($\boldsymbol{T}$ or **CCNOT**)	$\mathbf{T} = \|0\rangle\langle 0\| \otimes \boldsymbol{I} \otimes \boldsymbol{I} + \|1\rangle\langle 1\| \otimes \mathbf{CNOT}$	$\begin{pmatrix} 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&1&0 \end{pmatrix}$	$\mathbf{T}\|000\rangle = \|000\rangle, \mathbf{T}\|001\rangle = \|001\rangle$ $\mathbf{T}\|010\rangle = \|010\rangle, \mathbf{T}\|011\rangle = \|011\rangle$ $\mathbf{T}\|100\rangle = \|100\rangle, \mathbf{T}\|101\rangle = \|101\rangle$ $\mathbf{T}\|110\rangle = \|111\rangle, \mathbf{T}\|111\rangle = \|110\rangle$	

Fig. 1.5 Commonly used quantum gates

where $U_f^\dagger$ is the conjugate-transpose matrix of U_f and I is an identity matrix as shown in Fig. 1.5 when it is in the two-dimensional format.

The unitary transform uses the unitary matrix as the operator. The qubit is still in its normalized state after the unitary transform. Since the unitary transform is reversible, so is the quantum gate, i.e., the input state is turned into the output state by using the quantum gate (composed of the U_f transform), and the quantum gate (composed of the $U_f^\dagger$ transform), can turn the output state into the input state, i.e.,

$$U_f|x\rangle = |f(x)\rangle, \tag{1.4}$$

and

$$U_f^\dagger|f(x)\rangle = |x\rangle. \tag{1.5}$$

It is noteworthy that quantum *parallelism* allows quantum computers to simultaneously evaluate a function $f(x)$ for many values of x [38].

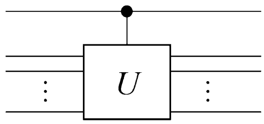


Fig. 1.6 Quantum controlled-U gate

Let us recall Fig. 1.4 to find that this circuit completes the swap operation, noting that these gates have the following sequence of effects on a computational basis state $|a, b\rangle$:

$$
\begin{aligned}
|a, b\rangle &\xrightarrow{CNOT} |a, a \oplus b\rangle \\
&\xrightarrow{CNOT} |a \oplus (a \oplus b), (a \oplus b)\rangle = |b, a \oplus b\rangle \\
&\xrightarrow{CNOT} |b, (a \oplus b) \oplus b\rangle = |b, a\rangle,
\end{aligned} \tag{1.6}
$$

where all additions are of modulo 2. The circuit's effect, therefore, is interchanging the states of the two qubits.

Figure 1.5 presents some commonly used quantum gates, their matrix representations, and their circuits. In addition, supposing U is any unitary matrix acting on some number n of qubits (in Fig. 1.6), U can be considered a quantum gate on these qubits. One can then define a controlled-U gate, which is a natural extension of the CNOT gate. This type of gate has a single control qubit, indicated by the line with the solid black circle, and n target qubits, indicated by the boxed U. Setting the control qubit to 0 has no effect on the target qubits. Setting it to 1, however, the gate U is applied to the target qubits.

1.2 Background of Quantum Image Processing

Quantum computation and information are transitioning from emerging branches of physics to mature research fields in science and engineering. Besides advancing their mathematical and physical foundations, a growing number of scientists and engineers are identifying and developing cross-fertilizing initiatives in quantum information processing in fields, such as artificial intelligence, pattern recognition, machine learning, neural network, cognition, and image processing [53].

1.2.1 Quantum Interdisciplinary Research

Several areas of quantum interdisciplinary research are now introduced and some studies on these efforts are noted.