Ji-Ping Huang

Theoretical Thermotics

Transformation Thermotics and Extended Theories for Thermal Metamaterials

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Preface

Metamaterial Physics Deserves a Nobel Prize

During the release of 2019 Nobel Prize in Physics, I was finalizing the book. This reminds me to think about an interesting (or tongue-in-cheek) problem in order to attract the reader: Does metamaterial physics deserve to be issued a Nobel prize? Absolutely, my answer is "YES". See Fig. 1. Since the seminal article by V. G. Veselago (June 13, 1929—September 15, 2018) in 1968 and especially the two other seminal articles by J. B. Pendry and coauthors in 1996 and 1999, the field of metamaterial physics has grown vigorously until today. With the aid of the

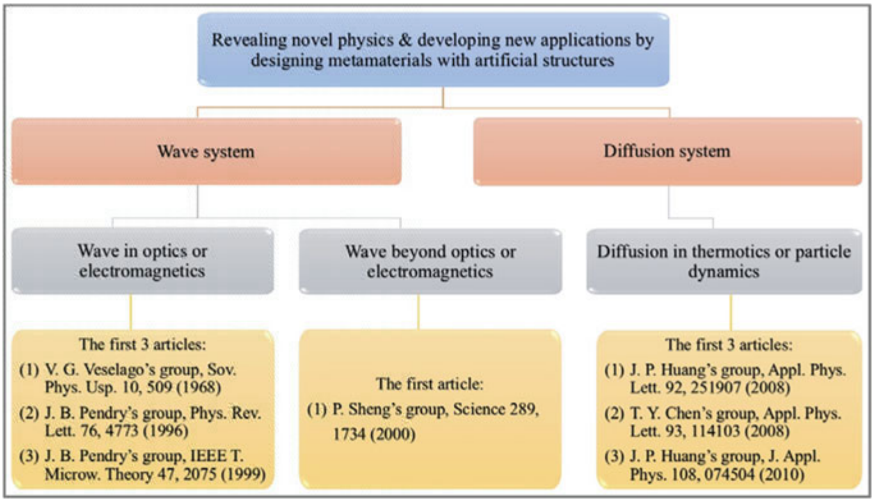

Fig. 1 A large number of novel physics and applications have arisen from metamaterials with artificial structures for wave systems and diffusion systems since 1968 and 2008, respectively. Both waves and diffusion are two important methods for transferring energy. See also Appendix: Brief History of the First Ten Years of Thermal Metamaterials

concept of metamaterial, many fundamental physics have been discovered in various branches of physics, ranging from optics/electromagnetics to elasticity/acoustics/mechanics/··· for wave systems, and from thermotics to particle dynamics for diffusion systems. As a result, various kinds of metamaterials were theoretically designed and experimentally fabricated in such branches. This book focuses on the branch of thermotics, namely, thermal metamaterials. The phrase "thermal metamaterial" was first adopted in Ref. [1] to name thermal cloaks (shields) and relevant devices designed by using transformation thermotics for heat conduction (diffusion) studied in the five references [2–6]. Owing to the existence of three ways of heat transfer (i.e., conduction, convection, and radiation), nowadays the connotation of "thermal metamaterial" has naturally been extended to include metamaterials for controlling heat convection and radiation. Incidentally, in this book, thermal metamaterials also contain some thermal metadevices (whose novel functions are realized mainly because of specific geometric structures), to comply with the common usage in the literature.

Thermal Metamaterial: Past, Present, and Future

In 2008, my group and Chen's group predicted the concept of novel thermal phenomena including thermal cloaking one after another [2, 3]. At the early stage (loosely speaking, before 2014) of thermal metamaterials, many experiments have been conducted to demonstrate the phenomenon of thermal cloaking under various conditions, see Refs. [5–9]. Accordingly, this field received plenty of popular attention [10–12] (see also https://www.sciencemag.org/news/2012/05/heat-trickery-paves-way-thermal-computers). These impacts attracted me to come back to the field of thermal metamaterials. Since the end of 2014, my group has completely returned to this field. So far, we have published dozens of articles.

Thermal metamaterials mean those materials or devices with artificial structures that can be used to control heat conduction, convection, and radiation in novel manners. In this case, geometric structure (rather than physical property) plays a dominating role. This fact makes thermal metamaterials different from other materials including thermoelectric materials, pyroelectric materials, magnetocaloric materials, and photothermal conversion materials; for the latter, physical property (rather than geometric structure) plays a dominating role instead. For a brief history of the first 10 years (2008–2018) for thermal metamaterials, I would refer the reader to the Appendix at the end of this book, which is a celebration article I was invited to write.

So far, thermal metamaterials have aroused enormous research interests, as also evidenced by Google search that shows the search of "thermal metamaterials" occupies 29.6% of all kinds of "metamaterials" as of August 13, 2019.

To celebrate the fruitful progress of thermal metamaterials and to prepare for the future challenges, I launched and chaired a National Conference on Thermodynamics and Thermal Metamaterials on July 18–19, 2019, in Fudan

Fig. 2 Group photo: 2019 National Conference on Thermodynamics and Thermal Metamaterials, held on July 18–19, 2019, in Fudan University, Shanghai, China

University, Shanghai, China. About 40 participants attended the first national conference, see Fig. 2. Due to the success of this national conference, I plan to not only continue the holding of the national conference, but also launch an international conference on the same topic starting from 2020.

Useful Theoretical Physics and Useful Theoretical Thermotics

To design thermal metamaterials in the literature, analytic theories have been extensively developed with a special focus on transformation thermotics. In this book, I would prefer to call the transformation thermotics and its extended theories together as "theoretical thermotics", with an attempt to contribute them to the discipline of "theoretical physics (statistical physics)" that is developing very well in China due to the efforts of many good researchers. This name could also remind the colleagues and latecomers to figure out the microscopic mechanisms for "theoretical thermotics" (that, after all, mainly describes macroscopic thermal theories for the time being), rather than to satisfy with the existing macroscopic theories; in this direction, Chap. 8 already gives a good example. Consequently, the name "theoretical thermotics" looks more suitable than other candidates like "structural thermotics" or "artificial thermotics" (the latter have been kindly suggested to me by some friends of mine).

In a word, theoretical thermotics describes the theory of transformation thermotics and its extended theories for the *active control* of macroscopic thermal properties of artificial systems, namely, metamaterials with artificial structures. Thus, theoretical thermotics is in sharp contrast to classical thermodynamics, which mainly comprises the four thermodynamic laws with a particular emphasis on the *passive description* of macroscopic thermal properties of natural systems. Incidentally, because the transformation method in transformation thermotics and theoretical thermotics is not intended to transform (or actually cannot transform) the four thermodynamic laws in thermodynamics, for the sake of clarity I choose the wording "thermotics" instead of "thermodynamics" for naming transformation thermotics or theoretical thermotics.

Clearly, theoretical thermotics can help to design thermal metamaterials, which are further useful for engineering techniques and applications [13], say, for designing standard printed circuit board [14, 15], daytime radiative cooling [16], and so on. This book focuses on fundamental theories, rather than engineering techniques and applications, and it introduces 18 theories including 7 general theories and 11 special theories.

Acknowledgement and Some Additional Notes

The main content of this book mainly comes from the articles published by my group. The current members of my group are Mr. C. R. Jiang, Mr. G. L. Dai, Mr. J. Wang, Ms. S. Yang, Mr. B. Y. Tian, Mr. L. J. Xu, Mr. F. B. Yang, Dr. B. Wang, Mr. P. Jin, and Mr. C. Q. Wang, and they also helped to improve the English of this book. Some former group members working in the field include Dr. C. Z. Fan, Dr. Y. Gao, Mr. J. Y. Li, Dr. X. Y. Shen, and Dr. Y. Li. I must thank all the current and past members for their fruitful contributions, which make this book possible.

In particular, together with me, Mr. G. L. Dai wrote Chaps. 2, 3, 6, and 8, Mr. L. J. Xu and Mr. G. L. Dai wrote Chaps. 4 and 5, and Ms. S. Yang translated the Appendix from Chinese to English. Also, Mr. G. L. Dai, Mr. L. J. Xu, and Ms. S. Yang helped to prepare the section "Exercises and Solutions" after each chapter. The reason why we add "Exercises and Solutions" is that we hope this book could be not only a monograph for experts to read, but also a textbook for newcomers to practice (so that he/she could engage in this new field as soon as possible). Incidentally, in order to facilitate reading, each chapter in the book has its own symbols. In this sense, to read the book, the reader can start with any chapter of the book (especially, Chaps. 6–19).

In the last years, I have had many invaluable opportunities to present our research progress on theoretical thermotics and thermal metamaterials to many top professors (in no particular order): Prof. Chang-Pu Sun, Prof. Rong-Gen Cai, Prof. Min-Xing Luo, Prof. Zhong-Xian Zhao, Prof. Qi Ou-Yang, Prof. Yu-Gang Ma, Prof. Xiao-Ping Ou-Yang, Prof. Ji Zhou, Prof. Zhong-Can Ou-Yang, Prof. Ding-Yu Xing, Prof. Shi-Ning Zhu, Prof. Yi-Peng Jing, Prof. Hong-Xing Xu, Prof. Bao-Wen Li, Prof. Qi-Kun Xue, Prof. Xin-Cheng Xie, Prof. Rui-Bao Tao, Prof. Xin-Gao Gong, Prof. Lu Yu, Prof. Tao Xiang, Prof. Xian-Hui Chen, Prof. Xian-Gang Luo, Prof. Tie-Jun Cui, Prof. Wei Wang, Prof. Zheng-You Liu, Prof. Hai-Qing Lin, Prof. Ke-Qing Xia, Prof. Mu Wang, Prof. Xiao-Peng Zhao, Prof. Shu-Xin Bai, Prof. Hong Zhao, Prof. Bo Zheng, Prof. Yuan-Ning Gao, Prof. Dong-Lai Feng, and Prof. Hai-Ping Fang. Here, I want to thank all of them from the bottom of my heart for their critical comments, inspiring encouragement, and unceasing support.

I am also indebted to my family members, especially two daughters (Ji-Yan Huang with a nickname of Qian-Qian and Ji-Yang Huang with a nickname of Yue-Yue), for bringing me happiness.

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Shanghai, China Ji-Ping Huang
October 2019

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Contents

Chapter 1
Introduction

Abstract Classical thermodynamics pays a special attention to the passive description of macroscopic heat phenomena of natural systems with the theoretical framework of the four thermodynamic laws. In contrast, theoretical thermotics, introduced in this book, allows one to achieve the active control of macroscopic heat phenomena of artificial systems with the theoretical framework of transformation thermotics and extended theories. As a result, thermal metamaterials can be theoretically designed at will, which have abundant application values. Thus, a hot field comes to appear.

Keywords Thermodynamics · Theoretical thermotics · Passive description · Active control · Transformation thermotics · Thermal metamaterials

1.1 Thermodynamics Versus Theoretical Thermotics

1.1.1 Thermodynamics Concentrating on a Passive Description of Macroscopic Heat Phenomena of Natural Systems

The framework of thermodynamics is composed of the four laws of thermodynamics. Let us take the second law of thermodynamics as an example, which states "the total entropy of an isolated system can never decrease over time". The statement indicates an intrinsic property of isolated systems, and this property can not be changed by humans at all. Thus, we would say that classical thermodynamics pays a special attention on the passive description of macroscopic heat phenomena of natural systems.

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1.1.2 Theoretical Thermotics Concentrating on an Active Control of Macroscopic Heat Phenomena of Artificial Systems

The above-mentioned passive description of macroscopic heat phenomena means that humans can not break the four laws, but only obey them. In this regard, if one can control heat flow at will, this control would be definitely useful for human life. This is just the goal of theoretical thermotics. Certainly, the four laws of thermodynamics also work for theoretical thermotics, but we try to establish and develop different kinds of theories to manipulate and control the flow of heat purposefully. Consequently, we achieve the active control of macroscopic heat phenomena of artificial systems.

1.2 Two Features of Theoretical Thermotics

1.2.1 Theoretical Framework: Transformation Thermotics and Extended Theories

In theoretical framework, we establish and develop the theory of transformation thermotics and its extended theories (all are analytical theories). Such theories allow us to design artificial systems or structures (thermal metamaterials), in order to control heat transfer arbitrarily.

1.2.2 Application Value: Design Thermal Metamaterials for Macroscopic Heat-Flow Control

Thermal metamaterials pave a new way to control the transfer of heat (conduction, convection, and radiation). For the sake of comprehensiveness, below we present more relevant backgrounds and details according to Ref. [1].

With the advent of energy crisis, energy sources like coal, oil and natural gas are becoming less and less. However, more and more low-grade heat energy is produced and wasted due to various reasons including inefficient utilization. Therefore, how to efficiently control the flow of heat energy becomes particularly important.

Heat transfer at microscopic scale has been deeply explored by many scholars, such as Refs. [2–8], which have helped to develop the field significantly. For the existing research at microscopic scale, a delicate review has been made by [5]. In contrast, the topic of this chapter and this book is mainly on theories and experiments for controlling heat transfer at macroscopic scale. Certainly, traditional Fourier's law (bridging heat flux and temperature gradient in a material), established by Joseph

Fourier in his treatise "Théorie analytique de la chaleur" (1822), can be seen as the first quantitative theory for studying heat conduction at macroscopic scale. After 1822, about two hundred years have witnessed much more developments, such as, applying effective medium theories from optics/electromagnetics [9, 10] to thermotics due to the mathematical similarity between dielectric permittivities and thermal conductivities. Such theories have been reviewed by many researchers including [11, 12]. Meanwhile, many other macroscopic methods have also been proposed to study heat transfer, such as phonon hydrodynamics models [13, 14], the dual-phase-lag model [15, 16], the ballistic-diffusive model [17, 18], and so on. Such methods can be referred to a comprehensive review by Guo and Wang [19].

Starting from ten years ago, researchers started to develop new theories for controlling macroscopic heat transfer again. Reference [20] first introduced the theory of coordinate transformation from optics/electromagnetics [21, 22] to thermotics (steady-state heat conduction), and predicted the concept of "thermal cloak", which helps to guide the flow of heat around an object as if the object does not exist. Such a thermal cloak has potential applications in thermal protection, misleading infrared detection, and heat preservation/dissipation. As a result, a new direction forms, which is called "transformation thermotics" (or equivalently "transformation thermodynamics" as occasionally used by some other researchers) in the literature.

With the establishment of transformation thermotics and extended theories, there comes a research upsurge of achieving novel thermal transport phenomena via designing artificial structures or devices. The theoretical proposals of thermal cloaks [20, 23–27] have further motivated experimental demonstrations [28–32] and popular attention [33–35] (see also http://www.sciencemag.org/news/2012/05/heat-trickery-paves-way-thermal-computers). In this book, we call transformation thermotics and extended theories as theoretical thermotics, which has been explained in Part III of Preface.

The so-called "thermal metamaterial" was first adopted by [36] to name thermal cloaks (shields) and relevant devices designed by using transformation thermotics in the five references [20, 23, 26, 28, 29], thus causing the formation of the direction of thermal metamaterials. Incidentally, the phrase "thermal metamaterial" was originally used for thermal conduction only [36], but its connotation has been significantly extended afterwards. So far, thermal metamaterials also cover those artificial structural materials for controlling thermal convection [37–39] and radiation [40–43] with novel properties. Nowadays, as defined by [44], "thermal metamaterials are materials composed of engineered, microscopic structures that exhibit unique thermal performance characteristics based primarily on their physical structures and patterning, rather than just their chemical composition or bulk material properties".

In our eyes, the existing materials for macroscopic heat control can be generally classified into two types. One is based on physical properties, such as thermoelectric materials, pyroelectric materials, magnetocaloric materials, photo thermal conversion materials, etc. The other is based upon geometric structures rather than physical properties (namely, geometric structures play a more important role than materials' physical properties). Among geometric structures, (normal) structural materials can be used to realize normal control of heat flow, but thermal metamaterials can be utilized to achieve novel controls. So far, the field of thermal metamaterials has aroused enormous research interests, as also evidenced by Google search that shows the search of "thermal metamaterials" occupies 29.6% of all kinds of "metamaterials" as of August 13, 2019.

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Part I
General Theories

Chapter 2
Transformation Thermotics for Thermal Conduction

Abstract This chapter describes the theory of transformation thermotics for thermal conduction. We begin with the relationship between coordinate transformation and geometric transformation and then give some basic tools of tensor analysis. Based on Fourier's law for heat conduction, we show how the form-invariance of an equation under arbitrary coordinate transformation can result in a new technique to manipulate temperature field and heat flux. As a model application, we design a thermal cloak to show how transformation thermotics works.

Keywords Transformation thermotics · Coordinate transformation · Geometric transformation · Form invariance · Heat conduction

2.1 Opening Remarks

"Transformation thermotics is based on the form-invariance of the governing equations of heat transfer under coordinate transformations. It engineers thermal properties of materials like thermal conductivity, to modulate the heat flux in novel manners like cloaking, concentrating and rotating."

We can find similar descriptions about transformation thermotics [1, 2] in the literature today. If one is not familiar with transformation theory on thermotics, optics or acoustics, he/she might be puzzled by some concepts like "form-invariance under coordinate transformations" and why this invariance can be used for heat management. Here, we shall firstly talk about the motivation of transforming theory and introduce some basic concepts.

Suppose light is traveling on a uniform plane and the trace of movement is a straight line. Now one wants to let the light move on a curve, a simple idea is just to bend the plane and then he/she may expect the light is bent accordingly. However, is this enough and how to bend the space like bending a paper? Luckily, we have been told in general relativity that the change of energy-momentum tensor can bend the space so we can have a more general guess here that if one wants to manipulate some physical fields as if the space is changed, he/she can change some important properties of the space or the material on it, for example, the thermal conductivity tensor.

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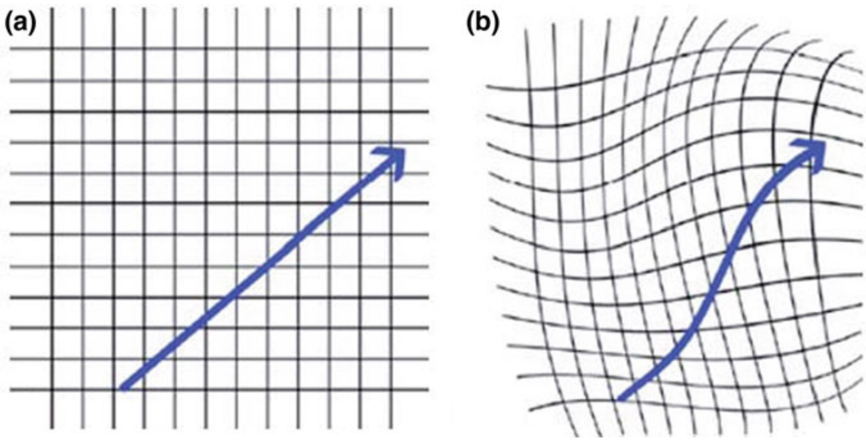

Fig. 2.1 Schematic diagram showing how transformation works, **a** is the original coordinate system shown by black uniform grids, and the blue arrow represents a straight physical field, **b** is the new coordinate system shown by uneven grids, which can also be seen as a twisted space so the blue arrow is curved

Now one may ask: "Where is the coordinate transformation? You seem be talking about geometric transformation when mentioning bending the light. What's more, why can this idea work for heat transfer?" To answer these questions, we should introduce transformation theory which tells how to change space or material properties based on coordinate transformation to achieve the desired effect as the fields change under geometric transformation; see Fig. 2.1. Also, we shall discuss the condition when transformation theory is valid.

2.2 Coordinate Transformation and Geometric Transformation

Let us start from the relationship between coordinate transformation and geometric transformation. For clarity, we have to talk about some basic knowledge on tensor analysis. Using Cartesian coordinate system in three-dimensional Euclidean space $\mathbb{E}^3$, a vector $\boldsymbol{r}$ with coordinates (x, y, z) can be written as

$$\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k} \tag{2.1}$$

where $\{\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}\}$ is the standard orthogonal basis of Cartesian coordinate system. Consider a mapping $f : \mathbb{E}^3 \to \mathbb{E}^3$, satisfying

$$f(\boldsymbol{r}) = (2x)\boldsymbol{i} + (2y)\boldsymbol{j} + (2z)\boldsymbol{k}. \tag{2.2}$$

It can be easily checked that f is a bijection or one-to-one correspondence on $\mathbb{E}^3$. The meaning of f is that the length of each vector doubles in $\mathbb{E}^3$ while the direction keeps unchanged. For a unit-ball in $\mathbb{E}^3$, its volume becomes 8 times under f. This is a simple example of geometric transformation, which changes the vector $\boldsymbol{r}$.

Naturally, we have another bijection $\hat{f} : \mathbb{R}^3 \rightarrow \mathbb{R}^3$, satisfying

$$\hat{f}((x, y, z)) = (2x, 2y, 2z). \tag{2.3}$$

If we use a new set of basis $\{\boldsymbol{g}_u, \boldsymbol{g}_v, \boldsymbol{g}_w\} = \{\boldsymbol{i}/2, \boldsymbol{j}/2, \boldsymbol{k}/2\}$, we can see $\hat{f}$ just gives the new coordinates under this basis,

$$(x, y, z) \cdot (\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k})' = \hat{f}((x, y, z)) \cdot (\boldsymbol{g}_u, \boldsymbol{g}_v, \boldsymbol{g}_w)'. \tag{2.4}$$

Here we should point out that a set of vectors $\{\boldsymbol{g}_u, \boldsymbol{g}_v, \boldsymbol{g}_w\}$ can be a basis in $\mathbb{E}^3$ if and only if they are linearly unrelated (the 3 vectors are not in the same plane). In other words, orthogonality and normality are unnecessary. $\{\boldsymbol{g}_u, \boldsymbol{g}_v, \boldsymbol{g}_w\}$ is also called covariant basis. In tensor analysis, contravariant basis $\{\boldsymbol{g}^u, \boldsymbol{g}^v, \boldsymbol{g}^w\}$ is another set of vectors satisfying

$$\boldsymbol{g}_u \cdot \boldsymbol{g}^v = \delta_{uv}, \tag{2.5}$$

where δ_{uv} is the Kronecker delta

$$\delta_{uv} = \begin{cases} 0 & \text{if } u \neq v, \\ 1 & \text{if } u = v. \end{cases} \tag{2.6}$$

It is obvious to see the existence of this contravariant basis and we may decompose the vector r as

$$\boldsymbol{r} = x^u \boldsymbol{g}_u + x^v \boldsymbol{g}_v + x^w \boldsymbol{g}_w = x_u \boldsymbol{g}^u + x_v \boldsymbol{g}^v + x_w \boldsymbol{g}^w, \tag{2.7}$$

or by using Einstein summation convention, we can simplify it as

$$\boldsymbol{r} = x^u \boldsymbol{g}_u = x_u \boldsymbol{g}^u. \tag{2.8}$$

Here $\{x^u, x^v, x^w\}$ is also known as contravariant components and $\{x_u, x_v, x_w\}$ is called covariant components, which can be obtained by

$$x^u = \boldsymbol{r} \cdot \boldsymbol{g}^u, \quad x_u = \boldsymbol{r} \cdot \boldsymbol{g}_u. \tag{2.9}$$

In Cartesian coordinate systems, both covariant basis and contravariant basis are $\{\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}\}$, so covariant and contravariant components are also the same.

To sum up, coordinate transformation means choosing a different basis while the vector $\boldsymbol{r}$ itself is not changed. In fact, invariance under coordinate transformation is a necessary condition for vectors.

So far, we can see geometric transformation and coordinate transformation are two different concepts. However, it can be observed that the mapping f in geometric transformation and mapping $\hat{f}$ in coordinate transformation have some relationships. Mapping f can naturally induce mapping $\hat{f}$ and vice versa. They both change the coordinates (and thus length) of a vector: f changes the vector itself while $\hat{f}$ changes the measure of space instead. So, we can take f and $\hat{f}$ as the same if we only care about the mathematical forms of new coordinates after the mappings, although they have different physical explanations indeed.

For most curvilinear coordinate systems, $\{\boldsymbol{g}_u, \boldsymbol{g}_v, \boldsymbol{g}_w\}$ is not a set of constant vectors and can vary with the elements in $\mathbb{E}^3$. Unless otherwise stated in this chapter, indices u, v, w are used for general (curvilinear) coordinate systems while i, j, k for Cartesian coordinate systems. For example, in spherical coordinate systems, we have $\boldsymbol{r} = r\boldsymbol{g}_u + \theta\boldsymbol{g}_v + \varphi\boldsymbol{g}_w$ where

$$\begin{aligned} \boldsymbol{g}_u &= \sin\theta\cos\varphi\boldsymbol{i} + \sin\theta\sin\varphi\boldsymbol{j} + \cos\theta\boldsymbol{k}, \\ \boldsymbol{g}_v &= r(\cos\theta\cos\varphi\boldsymbol{i} + \cos\theta\sin\varphi\boldsymbol{j} - \sin\theta\boldsymbol{k}), \\ \boldsymbol{g}_w &= r\sin\theta(-\sin\varphi\boldsymbol{i} + \cos\varphi\boldsymbol{j}). \end{aligned} \tag{2.10}$$

In addition, we can see only $\boldsymbol{g}_u$ is a unit vector. Here $\{\boldsymbol{g}_u, \boldsymbol{g}_v, \boldsymbol{g}_w\}$ is also called local covariant basis and we shall give the derivation for general cases below. Let (x^u, x^v, x^w) denote the coordinates for a vector in a curvilinear coordinate system which has the relationship with Cartesian coordinates as

$$\begin{aligned} x^u &= x^u(x, y, z), \\ x^v &= x^v(x, y, z), \\ x^w &= x^w(x, y, z). \end{aligned} \tag{2.11}$$

To ensure (x^u, x^v, x^w) can be a curvilinear coordinate, the map $\hat{f} : (x, y, z) \to (x^u, x^v, x^w)$ should be a smooth bijection, which is equivalent to the condition

$$\det\mathbf{J} = \begin{vmatrix} \dfrac{\partial x^u}{\partial x} & \dfrac{\partial x^u}{\partial y} & \dfrac{\partial x^u}{\partial z} \\ \dfrac{\partial x^v}{\partial x} & \dfrac{\partial x^v}{\partial y} & \dfrac{\partial x^v}{\partial z} \\ \dfrac{\partial x^w}{\partial x} & \dfrac{\partial x^w}{\partial y} & \dfrac{\partial x^w}{\partial z} \end{vmatrix} \neq 0, \quad \det\mathbf{J}^{-1} = \begin{vmatrix} \dfrac{\partial x}{\partial x^u} & \dfrac{\partial x}{\partial x^v} & \dfrac{\partial x}{\partial x^w} \\ \dfrac{\partial y}{\partial x^u} & \dfrac{\partial y}{\partial x^v} & \dfrac{\partial y}{\partial x^w} \\ \dfrac{\partial z}{\partial x^u} & \dfrac{\partial z}{\partial x^v} & \dfrac{\partial z}{\partial x^w} \end{vmatrix} \neq 0, \tag{2.12}$$

where $\mathbf{J}$ is the Jacobian matrix (we use a different font to distinguish between tensors) from coordinate (x, y, z) to (x^u, x^v, x^w). Here, the domain (for (x, y, z)) and the range (for (x^u, x^v, x^w)) of $\hat{f}$ are both $\mathbb{R}^3$.

Since we want to obtain the local basis for vector $\boldsymbol{r}$ with coordinate (x^u, x^v, x^w), we write the line element for an infinitesimal displacement from $\boldsymbol{r}$ to $\boldsymbol{r} + \mathrm{d}\boldsymbol{r}$,

$$\mathrm{d}\boldsymbol{r} = \frac{\partial\boldsymbol{r}}{\partial x^u}\mathrm{d}x^u + \frac{\partial\boldsymbol{r}}{\partial x^v}\mathrm{d}x^v + \frac{\partial\boldsymbol{r}}{\partial x^w}\mathrm{d}x^w. \tag{2.13}$$

On the other hand, for vector d$\boldsymbol{r}$, its coordinate is set as $(\mathrm{d}x^u, \mathrm{d}x^v, \mathrm{d}x^w)$, meaning

$$\mathrm{d}\boldsymbol{r} = \boldsymbol{g}_u \mathrm{d}x^u + \boldsymbol{g}_v \mathrm{d}x^v + \boldsymbol{g}_w \mathrm{d}x^w. \tag{2.14}$$

So the local covariant basis is just

$$\boldsymbol{g}_u = \frac{\partial \boldsymbol{r}}{\partial x^u}, \quad \boldsymbol{g}_v = \frac{\partial \boldsymbol{r}}{\partial x^v}, \quad \boldsymbol{g}_w = \frac{\partial \boldsymbol{r}}{\partial x^w}. \tag{2.15}$$

It is clear that $\{\boldsymbol{g}_u, \boldsymbol{g}_v, \boldsymbol{g}_w\}$ points out the directions in which (u, v, w) increases. Finally we have

$$\begin{aligned}
\boldsymbol{g}_u &= \frac{\partial x}{\partial x^u}\boldsymbol{i} + \frac{\partial y}{\partial x^u}\boldsymbol{j} + \frac{\partial z}{\partial x^u}\boldsymbol{k}, \\
\boldsymbol{g}_v &= \frac{\partial x}{\partial x^v}\boldsymbol{i} + \frac{\partial y}{\partial x^v}\boldsymbol{j} + \frac{\partial z}{\partial x^v}\boldsymbol{k}, \\
\boldsymbol{g}_w &= \frac{\partial x}{\partial x^w}\boldsymbol{i} + \frac{\partial y}{\partial x^w}\boldsymbol{j} + \frac{\partial z}{\partial x^w}\boldsymbol{k}.
\end{aligned} \tag{2.16}$$

This is a very convenient choice of the basis and we can use other basis. With local basis, we can introduce metric tensor $\boldsymbol{G}$, whose covariant components are defined as

$$g_{uv} = \boldsymbol{g}_u \cdot \boldsymbol{g}_v. \tag{2.17}$$

Then we can use the form of tensor product $\otimes$ (the Cartesian product) as

$$\boldsymbol{G} = g_{uv} \boldsymbol{g}^u \otimes \boldsymbol{g}^v. \tag{2.18}$$

The determinant of $\left[g_{uv}\right]$ is

$$g = \left|\left[g_{uv}\right]\right| \tag{2.19}$$

and it is a function with $\boldsymbol{r}$ or (x^u, x^v, x^w). Since we can also write

$$\boldsymbol{G} = g^{uv} \boldsymbol{g}_u \otimes \boldsymbol{g}_v = g^u_v \boldsymbol{g}_u \otimes \boldsymbol{g}^v = g^v_u \boldsymbol{g}^u \otimes \boldsymbol{g}_v, \tag{2.20}$$

we obtain

$$\left|\left[g^{uv}\right]\right| = \frac{1}{g}, \quad \left|\left[g^u_v\right]\right| = \left|\left[g^v_u\right]\right| = 1. \tag{2.21}$$

Here what we want to emphasize is that the determinant of a rank-2 tensor is different from the determinant of a matrix. In tensor analysis, the determinant of a rank-2 tensor $\boldsymbol{A}$ is

$$\det \boldsymbol{A} = \left|\left[A^u_v\right]\right| = \left|\left[A^v_u\right]\right|. \tag{2.22}$$

For metric tensor $\boldsymbol{G}$, we have

$$\det \boldsymbol{G} = 1, \tag{2.23}$$