Electronic Circuits with MATLAB®, PSpice®, and Smith Chart

Won Y. Yang | Jaekwon Kim | Kyung W. Park
Donghyun Baek | Sungjoon Lim | Jingon Joung
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To our parents and families
who love and support us
and
to our teachers and students
who enriched our knowledge

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Preface

The aim of this book is not to let the readers drowned into a sea of computations. More hopefully, it aims to inspire the readers with mind and strength to make full use of the MATLAB and PSpice softwares so that they can feel comfortable with mathematical equations without caring about how to solve them and further can enjoy developing their ability to analyze/design electronic circuits. It aims also to present the readers with a steppingstone to radio frequency (RF) circuit design from junior–senior level to senior-graduate level by demonstrating how MATLAB can be used for the design and implementation of microstrip filters. The features of this book can be summarized as follows:

1) For representative examples of designing/analyzing electronic circuits, the analytical solutions are presented together with the results of MATLAB design and analysis (based on the theory) and PSpice simulation (similar to the experiment) in the form of trinity. This approach gives the readers not only information about the state of the art, but also confidence in the legitimacy of the solution as long as the solutions obtained by using the two software tools agree with each other.
2) For representative examples of impedance matching and filter design, the solution using MATLAB and that using Smith chart have been presented for comparison/crosscheck. This approach is expected to give the readers not only confidence in the legitimacy of the solution, but also deeper understanding of the solution.
3) The purposes of the two softwares, MATLAB and PSpice, seem to be overlapped and it is partly true. However, they can be differentiated since MATLAB is mainly used to design circuits and perform a preliminary analysis of (designed) circuits while PSpice is mainly used for detailed and almost real-world simulation of (designed) circuits.
4) Especially, it presents how to use MATLAB and PSpice not only for designing/analyzing electronic and RF circuits but also for understanding the underlying processes and related equations without having to struggle with time-consuming/error-prone computations.

The contents of this book are derived from the works of many (known or unknown) great scientists, scholars, and researchers, all of whom are deeply appreciated. We would like to thank the reviewers for their valuable comments and suggestions, which contribute to enriching this book.

We also thank the people of the School of Electronic and Electrical Engineering, Chung-Ang University for giving us an academic environment. Without affections and supports of our families and friends, this book could not be written. We gratefully acknowledge the editorial, Brett Kurzman and production staff of John Wiley & Sons, Inc. including Project Editor Antony Sami and Production Editor Viniprammia Premkumar for their kind, efficient, and encouraging guide.

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There you will find valuable material designed to enhance your learning, including the following:

- Learning Outcomes for all chapters
- Exercises for all chapters
- References for all chapters
- Further reading for all chapters
- Figures for Chapters 16, 22, and 30

1

Load Line Analysis and Fourier Series

CHAPTER OUTLINE

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1.1 Load Line Analysis

The v-i characteristic of a nonlinear resistor such as a diode or a transistor is often described by a curve on the v-i plane rather than by a mathematical relation. The v-i characteristic curve can be obtained by using a curve tracer for nonlinear resistors. To analyze circuits containing a nonlinear resistor, we should use the *load line analysis*. To grasp the concept of the load line, consider the graphical analysis of the circuit in Figure 1.1(a), which consists of a linear resistor R_1, a nonlinear resistor R_2, a DC voltage source V_s, and an AC voltage source of small amplitude $v_\delta \ll V_s$. Kirchhoff's voltage law (KVL) can be applied around the mesh to yield the mesh equation as

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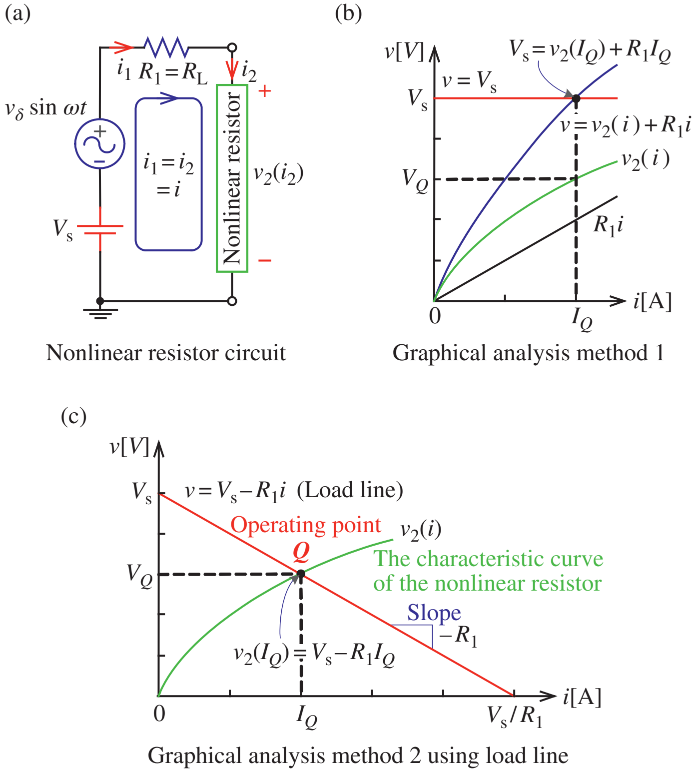

Figure 1.1 Graphical analysis of a linear/nonlinear resistor circuit.

$$R_1 i + v_2(i) = V_s \tag{1.1.1}$$

where the v-i relationship of R_2 is denoted by $v_2(i)$ and represented by the characteristic curve in Figure 1.1(b). We will consider a graphical method, which yields the *quiescent, operating*, or *bias point* Q = (I_Q, V_Q)*, that is, a pair of the current through and the voltage across* R_2 *for* $v_\delta = 0$.

Since no specific mathematical expression of $v_2(i)$ is given, we cannot use any analytical method to solve this equation and that is why we are going to resort to a graphical method. First, we may think of plotting the graph for the LHS (left-hand side) of Eq. (1.1.1) and finding its intersection with a horizontal line for the RHS (right-hand side), that is, $v = V_s$ as depicted in Figure 1.1(b). Another way is to leave only the nonlinear term on the LHS and move the other term(s) into the RHS to rewrite Eq. (1.1.1) as

$$v_2(i) = V_s - R_1 i \tag{1.1.2}$$

and find the intersection, called the *operating point* and denoted by Q (quiescent point), of the graphs for both sides as depicted in Figure 1.1(c). The straight line with the slope of $-R_1$ is called the *load line*. This graphical method is better than the first one in the aspect that it does not require us to plot a new curve for $v_2(i) + R_1 i$. That is why it is widely used to analyze nonlinear resistor circuits in the name of 'load line analysis'. Note the following:

- Most resistors appearing in this book are linear in the sense that their voltages are linearly proportional to their currents so that their voltage-current relationships (VCRs) are described by Ohm's law

$$v = R\,i \tag{1.1.3}$$

  and consequently, their v-i characteristics are described by straight lines passing through the origin with the slopes corresponding to their resistances on the i-v plane. However, they may have been modeled or approximated to be linear just for simplicity and convenience, because all physical resistors more or less exhibit some nonlinear characteristic. The problem is whether or not the modeling is valid in the range of practical operation so that it may yield the solution with sufficient accuracy to serve the objective of analysis and design.
- A curve tracer is an instrument that displays the v-i characteristic curve of an electric element on a cathode-ray tube (CRT) when the element is inserted into an appropriate receptacle.

1.1.1 Load Line Analysis of a Nonlinear Resistor Circuit

Consider the circuit in Figure 1.1(a), where a linear load resistor $R_1 = R_L$ and a nonlinear resistor R connected in series are driven by a DC voltage source V_s in series with a small-amplitude AC voltage source producing the virtual voltage as

$$v_s(t) = V_s + v_\delta \sin\omega t \tag{1.1.4}$$

The VCR $v(i)$ of the nonlinear resistor R is described by the characteristic curve in Figure 1.2.

As depicted in Figure 1.2, the upper/lower limits as well as the equilibrium value of the current i through the circuit can be obtained from the three operating points, that is, the intersections (Q_1, Q, and Q_2) of the characteristic curve with the following three load lines.

$$v = V_s + v_\delta - R_L i \tag{1.1.5a}$$

$$v = V_s - R_L i \tag{1.1.5b}$$

$$v = V_s - v_\delta - R_L i \tag{1.1.5c}$$

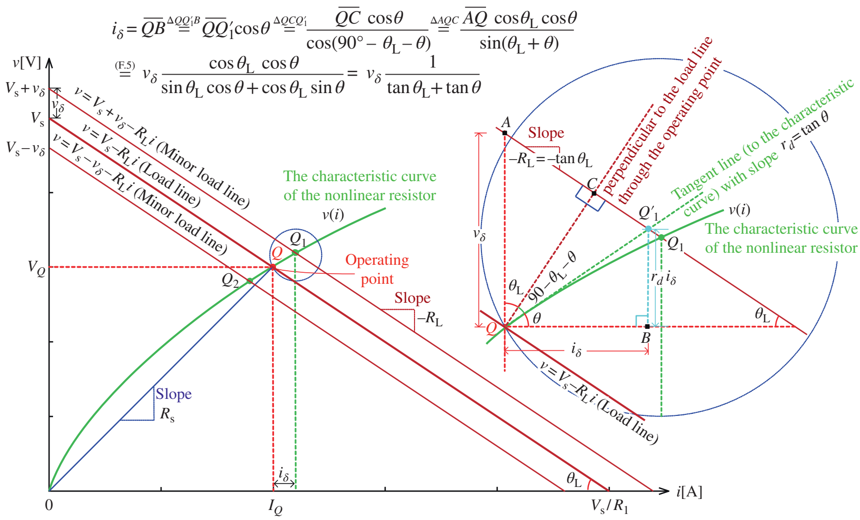

Figure 1.2 Variation of the voltage and current of a nonlinear resistor around the operating point ***Q***.

Although this approach gives the exact solution, we gain no insight into the solution from it. Instead, we take a rather approximate approach, which consists of the following two steps.

- Find the equilibrium (I_Q, V_Q) at the major operating point Q, which is the intersection of the characteristic curve with the DC load line (1.1.5b).
- Find the two approximate minor operating points Q_1' and Q_2' from the intersections of the tangent to the characteristic curve at Q with the two minor load lines (1.1.5a) and (1.1.5c).

Then we will have the current as

$$i(t) = I_Q + i_\delta \sin\omega t \tag{1.1.6}$$

With the *dynamic, small-signal,* or *AC resistance* r_d defined to be the slope of the tangent to the characteristic curve at Q as

$$r_d = \left.\frac{dv}{di}\right|_Q \tag{1.1.7}$$

let us find the analytical expressions of I_Q and i_δ in terms of V_s and v_δ, respectively. Referring to the encircled area around the operating point in Figure 1.2, we can express i_δ in terms of v_δ as

$$i_\delta = \overline{QB} \overset{\Delta QQ_1'B}{=} \overline{QQ_1'}\cos\theta \overset{\Delta QCQ_1'}{=} \frac{\overline{QC}\cos\theta}{\cos(90^\circ - \theta_L - \theta)} \overset{\Delta AQC}{=} \frac{\overline{AQ}\cos\theta_L\cos\theta}{\sin(\theta_L + \theta)}$$

$$\overset{(F.5)}{=} v_\delta \frac{\cos\theta_L\cos\theta}{\sin\theta_L\cos\theta + \cos\theta_L\sin\theta} = v_\delta \frac{1}{\tan\theta_L + \tan\theta} \tag{1.1.8}$$

This corresponds to approximating the characteristic curve in the operation range by its tangent at the operating point. Noting that

- the load line and the tangent to the characteristic curve at Q are at angles of $(180^\circ - \theta_L)$ and θ to the positive i-axis,
- the slope of the load line is $\tan(180^\circ - \theta_L) = -\tan\theta_L$ and it must be $-R_L$, which is the proportionality coefficient in i of the load line Eq. (1.1.2); $\tan\theta_L = R_L$, and
- the slope of the tangent to the characteristic curve at Q is the dynamic resistance r_d defined by Eq. (1.1.7); $\tan\theta = r_d$,

we can write Eq. (1.1.8) as

$$i_\delta = \frac{v_\delta}{R_L + r_d} \tag{1.1.9}$$

Now we define the *static* or *DC resistance* of the nonlinear resistor R to be the ratio of the voltage V_Q to the current I_Q at the operating point Q as

$$R_s = \frac{V_Q}{I_Q} = \frac{V_s - R_L I_Q}{I_Q} \tag{1.1.10}$$

so that the DC component of the current, I_Q, can be written as

$$I_Q = \frac{V_s}{R_L + R_s} \tag{1.1.11}$$

Finally, we combine the above results to write the current through and the voltage across the nonlinear resistor R as follows.

$$i(t) = I_Q + i_\delta \sin\omega t = \frac{V_s}{R_L + R_s} + \frac{v_\delta}{R_L + r_d}\sin\omega t \tag{1.1.12}$$

$$v(t) = R_s I_Q + r_d i_\delta \sin\omega t = \frac{R_s}{R_L + R_s}V_s + \frac{r_d}{R_L + r_d}v_\delta \sin\omega t \tag{1.1.13}$$

This result implies that the nonlinear resistor exhibits twofold resistance, that is, the *static resistance* R_s to a DC input and the *dynamic resistance* r_d to an AC input of small amplitude. That is why r_d is also called the (*small-signal*) *AC resistance*, while R_s is called the *DC resistance*.

Remark 1.1 Operating Point and Static/Dynamic Resistances of a Nonlinear Resistor

1) For a nonlinear resistor R_2 connected with linear resistors in a circuit excited by a DC source and a small-amplitude AC source, its *operating point* $Q = (V_Q, I_Q)$ is the intersection of its characteristic curve $v(i)$ and the load line.
2) The v-intercept of the load line ($v = V_s - R_L i$) is determined by the DC component (V_s) of the voltage source. The slope of the load line is determined by the equivalent resistance (R_L) of the linear part seen from the pair of terminals of the nonlinear resistor (see Problem 1.2).
3) The *static* or *DC resistance* (R_s) is the ratio of the voltage V_Q to the current I_Q at the operating point Q.
4) The *dynamic*, *small-signal*, *AC*, or *incremental resistance* (r_d) is the slope of the tangent to the characteristic curve at Q.
5) Once we have R_L, R_s, and r_d, we can use Formulas (1.1.12) and (1.1.13) to find approximate expressions for the voltage and current of the nonlinear resistor.
6) As for linear resistors, we do not say the static or dynamic resistance, since they are identical.
7) The relationship between the AC (small-signal) components of voltage across and current through the nonlinear resistor can be attributed to the Taylor series expansion of its VCR $v(i)$ up to the first-order term around the operating point $Q =(V_Q, I_Q)$.

$$v(i) \approx V_Q + \left.\frac{dv}{di}\right|_Q (i - I_Q) = V_Q + r_d i_\delta \quad \text{with } r_d = \left.\frac{dv}{di}\right|_Q \tag{1.1.14}$$

Remark 1.2 DC Analysis and Small-Signal (AC) Analysis

1) The procedure to analyze a circuit (which contains nonlinear resistors like a diode or a transistor and is driven by a high DC voltage source V_s [for biasing] and a low AC voltage source $v_\delta \sin \omega t$ [for amplification]) consists of two steps. The first step, called DC analysis, is to remove the AC voltage source $v_\delta \sin \omega t$ and find the operating point $Q = (V_Q, I_Q)$ of the nonlinear resistor, which corresponds to the load line analysis. The second step, called small-signal (AC) analysis, is to find the dynamic resistance r_d of the nonlinear resistor (from the slope of its i-v characteristic curve or the derivative of its VCR equation at Q-point), remove the DC voltage source V_s, regard the nonlinear resistor as a linear resistor r_d (corresponding to a linear

approximation of the characteristic curve), and find the AC components (i_δ sin ωt, $r_d i_\delta$ sin ωt) of the current through and voltage across the nonlinear resistor. The DC solution and AC solution can be added up to yield the complete solution.

2) As the magnitude v_δ of the AC voltage becomes large, the large-signal model (see Section 2.1.1 for a diode) or the characteristic curve itself might have to be used for analysis since the nonlinear behavior of the nonlinear resistor may become conspicuous, leading to unignorable distortion of the voltage/current waveforms obtained using the small-signal analysis.

1.1.2 Load Line Analysis of a Nonlinear *RL* circuit

As an example of applying the load line analysis for a nonlinear first-order circuit, consider the circuit of Figure 1.3.1(a), which consists of a nonlinear resistor, a linear resistor $R = 2\,\Omega$, and an inductor $L = 14\,$H, and is driven by a DC voltage source of $V_s = 12\,$V and an AC voltage source v_δ sin $\omega t = 2.8$ sin t[V]. The v-i relationship of the nonlinear resistor is $v(i) = i^3$ and described by the characteristic curve in Figure 1.3.1(b). Applying KVL yields the following mesh equation:

$$L\frac{di(t)}{dt} + Ri(t) + i^3(t) = V_s + v_\delta \sin t;$$
$$14\frac{di(t)}{dt} + 2i(t) + i^3(t) = 12 + 2.8\sin t \tag{1.1.15}$$

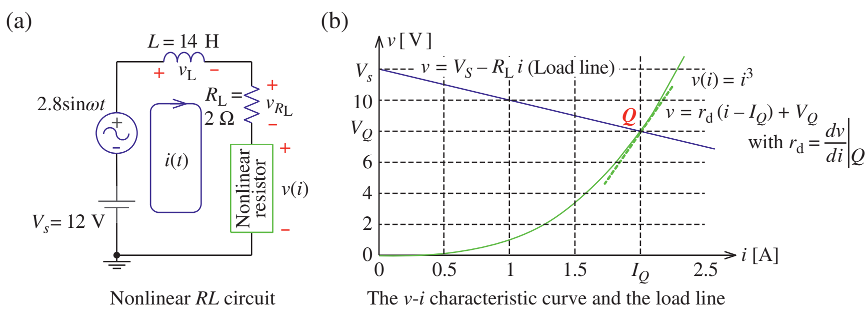

Figure 1.3.1 A nonlinear *RL* circuit and its load line analysis.

First, we can draw the load line on the graph of Figure 1.3.1(b) to find the operating point Q from the intersection of the load line and the v-i characteristic curve of the nonlinear resistor. If the function $v(i) = i^3$ of the characteristic curve is available, we can also find the operating point Q as the DC solution to Eq. (1.1.15) by removing the AC source and the time derivative term to write

$$2I_Q + I_Q^3 = 12 \tag{1.1.16}$$

and solving it as

$$I_Q = 2\,\text{A}, V_Q = v_2(I_Q) = I_Q^3 = 8\,\text{V} \rightarrow Q = (I_Q, V_Q) = (2\,\text{A}, 8\,\text{V}) \tag{1.1.17}$$

Eq. (1.1.16) can be solved by running the following MATLAB statements:

```
>>eq_dc=@(i)2*i+i.^3-12; I0=0; IQ=fsolve(eq_dc,I0)
```

Then, as a preparation for analytical approach, we linearize the nonlinear differential equation (1.1.15) around the operating point Q by substituting $i = I_Q + \delta i = 2 + \delta i$ into it and neglecting the second or higher degree terms in δi as

$$\begin{aligned} &14\frac{d(2+\delta i)}{dt} + 2(2+\delta i) + (2+\delta i)^3 = 12 + 2.8\sin t \\ &; \frac{d}{dt}\delta i(t) = -\delta i(t) + 0.2\sin t \end{aligned} \tag{1.1.18}$$

Note that we can set the slope of the characteristic curve as the dynamic resistance r_d of the nonlinear resistor:

$$r_d = \left.\frac{dv}{di}\right|_Q = 3I_Q^2 = 12\ \Omega \tag{1.1.19}$$

and apply KVL to the circuit with the DC source V_s removed and the nonlinear resistor replaced by r_d to write the same linearized equation as Eq. (1.1.18):

$$\begin{aligned} &14\frac{di(t)}{dt} + 2i(t) + i^3(t) = V_s + 2.8\sin t \\ &\underset{V_s \to 0}{\overset{i^3(t)\to r_d i(t)}{\longrightarrow}} 14\frac{di(t)}{dt} + (2+12)i(t) = 2.8\sin \omega t \end{aligned} \tag{1.1.20}$$

We solve the first-order linear differential equation with zero initial condition $\delta i(0) = 0$ to get $\delta i(t)$ by running the following MATLAB statements:

```
>>syms s
  dIs=2.8/(s^2+1)/14/(s+1); dit_linearized=ilaplace(dIs)
  dit1_linearized=dsolve('Dx=-x+0.2*sin(t)','x(0)=0') % Alternatively
```

This yields

```
dit_linearized = exp(-t)/10 - cos(t)/10 + sin(t)/10
```

which means

$$\delta i(t) = \frac{1}{10}e^{-t} - \frac{1}{10}\cos t + \frac{1}{10}\sin t \tag{1.1.21}$$

We add this AC solution to the DC solution I_Q to write the approximate analytical solution for $i(t)$ as

$$i(t) = I_Q + \delta i(t) \overset{(1.1.17)}{\underset{(1.1.21)}{=}} 2 + 0.1(e^{-t} - \cos t + \sin t) \tag{1.1.22}$$

Now, referring to Appendix D, we use the MATLAB numerical differential equation (DE) solver ‘`ode45()`’ to solve the first-order nonlinear differential equation (1.1.15) by defining it as an anonymous function handle:

```
>>di=@(t,i)(12+2.8*sin(t)-2*i-i.^3)/14; % Eq.(1.1.15)
```

and then running the following MATLAB statements:

```
>>i0=IQ; tspan=[0 10]; % Initial value and Time span
  [t,i_numerical]=ode45(di,tspan,i0); % Numerical solution
```

We can also plot the numerical solution together with the analytical solution as black and red lines, respectively, by running the following MATLAB statements:

```
>>i_linearized=eval(IQ+dit_linearized); %Analytical solution (1.1.22)
  plot(t,i_numerical,'k', t,i_linearized,'r')
```

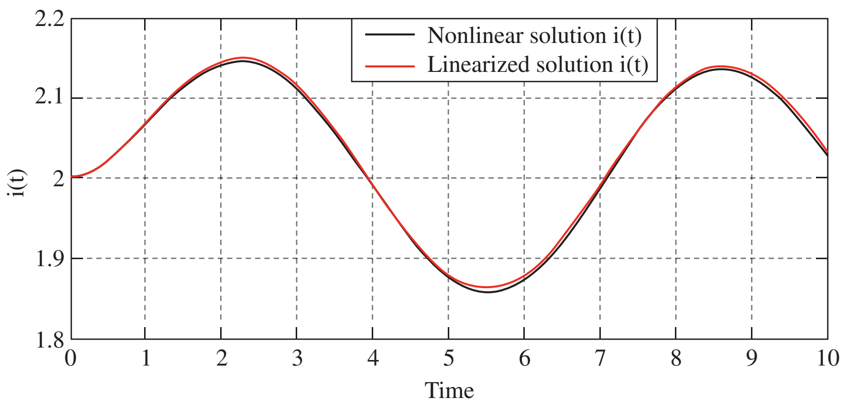

Figure 1.3.2 The linearized solution and nonlinear solution for the nonlinear *RL* circuit of Figure 1.3.1.

```
%elec01f03.m - for the analysis of a nonlinear RL Circuit
clear, clf
global RL L Vs vd
N=1000; i_step=0.003; i=[0:N]*i_step; % Range on the current axis
RL=2; L=14; Vs=12; vd=2.8;
eq_dc=@(i,RL,Vs)Vs-RL*i-i.^3; % Eq.(1.1.16): DC part of Eq.(1.1.20)
IQ=fsolve(eq_dc,0,optimset('fsolve'),RL,Vs) % Current at operating point Q
VQ=IQ^3; % Voltage at the Q-point
v= Vs - RL*i; % Load line
v2= i.^3; % Characteristic curve
i1=1.7:i_step:2.3; % Range on which to plot the tangent line
v4 = 3*IQ^2*(i1-IQ)+VQ; % Tangent line to the characteristic curve at Q
subplot(211), plot(i,v,'k', i,v2,'b', i1,v4,'r', IQ,VQ,'mo')
% Use the Laplace transform to solve the linearized differential eq.
syms s
dIs=0.2/(s^2+1)/(s+1); dit_linearized=ilaplace(dIs)
% This yields 0.1*(exp(-t) -cos(t) +sin(t)) +2; Eq.(1.1.22).
% Alternatively, use the symbolic differential solver dsolve() as
dit1_linearized = dsolve('Dx=-x+0.2*sin(t)','x(0)=0')
% Use nonlinear ODE solver ode45() to solve Eq.(1.1.15).
di=@(t,i)(12+2.8*sin(t)-2*i-i.^3)/14; % Eq.(1.1.15)
[t,i]=ode45(di,[0 10],IQ); % Numerical sol to Eq.(1.1.15)
i_linearized=eval(IQ+dit_linearized); % Eq.(1.1.22) for time range t
% Plot the Analytical (linearized) and Numerical (nonlinear) solutions.
subplot(212), plot(t,i,'k', t,i_linearized,'r'), ylabel('i(t)')
legend('Nonlinear solution i(t)','Linearized solution i(t)')
title('Analytical (linearized) and Numerical (nonlinear) solutions')
```

This will yield the plots of the numerical solution $i(t)$ and the approximate analytical solution (1.1.22) for the time interval [0,10 s] as depicted in Figure 1.3.2.

Overall, we can run the above MATLAB script "elec01f03.m" to get Figure 1.3.1(b) (the load line together with the characteristic curve) and Figure 1.3.2 (the numerical nonlinear solution together with the analytical linearized solution) together.

1.2 Voltage-Current Source Transformation

Two electric circuits are said to be *externally equivalent* with respect to a pair of terminals if their terminal voltage-current relationships are identical so that they are indistinguishable from outside. The *source transformation* refers to

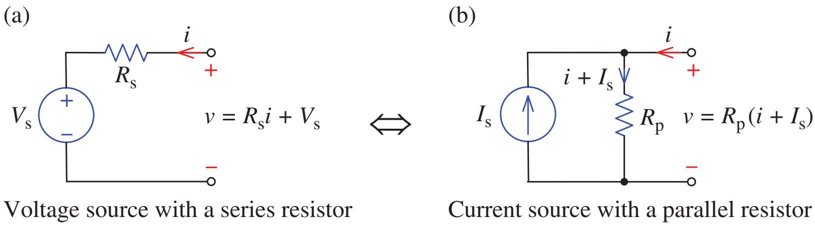

Figure 1.4 Equivalence of voltage and current sources.

the conversion of a voltage source in series with an element like a resistor (Figure 1.4(a)) to a current source in parallel with the element (Figure 1.4(b)), or vice versa in such a way that the two circuits are (externally) equivalent w.r.t. their terminal characteristics. What is the relationship among the values of the voltage source V_s, the current source I_s, the series resistor R_s, and the parallel resistor R_p required for the external equivalence of the two source models? To find it out, we write the VCR of each circuit as

$$v = R_s i + V_s \tag{1.2.1a}$$

$$v = R_p(i + I_s) = R_p i + R_p I_s \tag{1.2.1b}$$

where the current through R_p is found to be $(i + I_s)$ by applying KCL at the top node of the resistor R_p in Figure 1.4(b). In order for these two polynomial equations (in i) to be identical for any value of v and i, their coefficients (including the constant term) should be the same:

$$R_p = R_s = R \tag{1.2.2a}$$

$$V_s = R_p I_s \text{ or, equivalently, } I_s = \frac{V_s}{R_s} \tag{1.2.2b}$$

This *source equivalence condition* is used for voltage-to-current or current-to-voltage source transformation.

1.3 Thevenin/Norton Equivalent Circuits

Thevenin's theorem says that any network consisting of linear elements and independent/dependent sources as shown in Figure 1.5(a) may be replaced at a pair of its terminals (nodes) by the Thevenin equivalent circuit, which consists of a single element of impedance Z_{Th} in series with a single independent voltage source V_{Th} (see Figure 1.5(b)), where the values of Z_{Th} and V_{Th} are determined as follows:

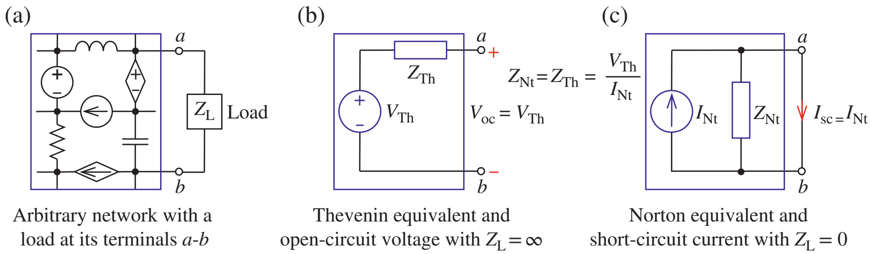

Figure 1.5 Thevenin/Norton equivalents of an arbitrary circuit seen from terminals *a* to *b*.

T1. Thevenin eqavuivalent voltage source V_{Th}:
The *open-circuit voltage* across the terminals, that is, the voltage across the open-circuited terminals *a*-*b* (with $Z_L = \infty$).

T2. Thevenin equivalent impedance Z_{Th}:
The equivalent impedance of the circuit (with all the independent sources removed) seen from the terminals *a*-*b*, where an impedance is a 'generalized' resistance.

Norton's theorem says that any linear network may be replaced at a pair of its terminals by the Norton equivalent circuit, which consists of a single element of impedance Z_{Nt} in parallel with a single independent current source I_{Nt} (see Figure 1.5(c)), where the values of Z_{Nt} and I_{Nt} are determined as follows:

N1. Norton equivalent current source I_{Nt}:
The *short-circuit current* through the terminals, that is, the current through the short-circuited terminals *a*-*b* (with $Z_L = 0$).

N2. Norton equivalent impedance Z_{Nt}:
The equivalent impedance of the circuit (with all the independent sources removed) seen from the terminals *a*-*b*

Since Thevenin and Norton equivalents are equivalent in representing a linear circuit seen from a pair of two terminals, one can be obtained from the other by using the source transformation introduced in Section 1.2. This suggests another formula for finding the equivalent impedance as

$$Z_{Th} = Z_{Nt} = \frac{V_{Th}}{I_{Nt}} = \frac{V_{oc}(\text{the open-circuit voltage})}{I_{sc}(\text{the short-circuit current})} \tag{1.3.1}$$

Note that we should find the equivalent impedance after removing every independent source, that is, with every voltage/current source short-/open-circuited. For networks having no dependent source, the series/parallel combination and Δ-Y/Y-Δ conversion formulas often suffice for the purpose of