

Paolo Serafini

Mathematics to the Rescue of Democracy

What does Voting
Mean and How
can it be Improved?



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*Dedicated to the dear memory of my friend
and colleague Bruno Simeone (1946–2010),
who, some years ago, introduced me to the
mathematics of electoral systems.*

Preface

The idea of writing this book came to my mind as a result of the Italian political elections in 2018. The election outcome has produced a host of debates in the media that conveyed the impression of a general confusion of ideas. The complexity, both theoretical and technical, of the voting activity seemed to be totally absent within these debates. This confusion continues, notably not only in Italy, and the voting proposals that periodically enter into the political discussion are based on emotion and whatever issues currently perceived as being important or convenient, never on sound scientific reasoning.

The aim of this book is to provide at least some knowledge to a wider audience on the fundamentals of voting theory in the illusive hope of making more people aware of the very meaning of voting and of the various possibilities for improving the present voting systems. Maybe this could be of some help in dealing with delicate questions that are better answered outside of the realm of partisan points of view.

Pieris, Italy
September 2019

Paolo Serafini

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A particular thanks is due to my wife Viviana, who encouraged me to write this book on a subject that is new to me, one of ‘almost’ scientific-popular writing, and who later contributed to improving the drafting of some chapters.

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About the Author

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Chapter 1

Introduction



In 1925, the famous and influential journalist Walter Lippmann wrote in his book *The Phantom Public* ([59] p. 46): “What in fact is an election? We call it an expression of the popular will. But is it? We go into a polling booth and mark a cross on a piece of paper for one of two, or perhaps three or four names. Have we expressed our thoughts on the public policy of the United States? Presumably we have a number of thoughts on this and that with many buts and ifs and ors. Surely the cross on a piece of paper does not express them. It would take us hours to express our thoughts, and calling a vote the expression of our mind is an empty fiction.”

Lippmann was right. The present way of voting is rough and may produce electoral outcomes that are puzzling either because they don’t seem to reflect the wishes of the majority of voters or because they lead to deadlock situations.

Voting is a fundamental activity in the life of a society, and it would be appropriate to know exactly what kind of outcome we want from an election, what the theoretical limitations of voting are, what kind of information would be helpful for the electors to pass along and, finally, how the efficacy of voting could be improved.

In general, people have some minimum knowledge about medicine, economics or other disciplines, and this enables them, at least partially, to direct their actions. Instead, in the practice of voting, the impression is that there is very little to understand, because everything seems obvious and elementary. On the contrary, it is much more complex than it appears.

The aim of this short book is to explain, in a hopefully simple way, the foundations upon which electoral techniques are based. Although one may think that voting pertains social or juridical sciences, the fact is that the basis of every electoral system is mathematics. What’s more, it is not a particularly abstruse form of mathematics. It can be made comprehensible to non-mathematicians. But neither is it elementary. In any case, the effort to understand it can garner the significant reward of a greater awareness of what we do when we vote. The title of the book highlights this hope.

The aim is not to provide a detailed description of certain electoral systems, either in Italy or elsewhere. There will, however, be more than a few references to existing or past situations, either as examples or to underline problems and defects.

It is important to know that, no matter how an electoral system has been devised, there are goals that are impossible to pursue, independent of the presumptions of the system's authors, but there are also opportunities toward improving the situation in an informed way.

It may be surprising to see that many electoral systems that are currently employed in various countries have evident defects and, yet are, nonetheless, still in use, justified for reasons of habit and familiarity with a particular method. For instance, one of the most fallacious ideas is that the alternative that receives most votes should be chosen even in cases in which it has received less than half of the total votes. Yet, many electoral systems are based on this idea.

A quite common bit of confusion is related to the aim of voting. Voting for parliamentary representation is very different from voting for a president or a govern. If these two aims overlap, the outcomes can be problematic.

The vote that we are accustomed to is the simple indication of a preference for one alternative among many. What we think of the alternatives that we have not chosen is not transmitted as information, and this is unfortunate, because the vote outcome would be in far greater compliance with the collective will than the current method of voting. Is it conceivable to modify a secular habit? However difficult this notion may seem, the time has probably come to make the vote more informative. In this book, we will try to illustrate the advantages of voting in a more completely engaged way. To this purpose, we will make explicit reference in Chap. 8 to the method devised by Balinski and Laraki [12, 15], the so-called *Majority judgement*.

The argument is a very large one and, in order to be thorough, four times as many pages would be necessary. Hence, we have opted to focus on a few select principles in order to make the fundamental aspects understandable and possibly direct the reader to more specific essays for an in-depth study.

Chapter 2

Voting



In the idea of the State that we inherited from the Age of Enlightenment, there are three public powers that belong to the citizens who constitute the State: the legislative power, the executive power and the judicial power. Clearly, there are other powers in the society that can condition, for better or for worse, the life of the State, but it is the three aforementioned ones that belong to the community of citizens. The practice of these powers cannot be carried out by the community as a whole, and thus it is necessary to delegate these tasks.¹

If we want the three powers to be independent of each other, it is important to understand how the machinery of mandate allows for such independence. There can be differences among nations. For instance, the Italian Constitution guarantees the independence of the judicial power from the other two in a precise way. The mandate goes through a selection that is an expression of the citizens inasmuch it is guaranteed by public laws. It is not the expression of a direct vote. If it were so, it would follow the same logic of the mandate for the legislative and executive powers, thereby jeopardizing the very concept of independence. This separation of the judicial power from the other two is not as sharp in every nation and the very fact that there are often attempts to weaken the separation shows why it is so necessary.

The question is different for the legislative and executive powers. The mandate is direct and is carried out through a vote. The choice of how to implement the mandate changes according to the nation. Basically, there are three possibilities: the vote is for the legislative power and the mandate for the executive power is left to the legislative power, or the vote is for the executive power and the mandate for the legislative power is left to the executive power, or there are two distinct votes, one for the legislative power and another for the executive power.

¹Concerning the idea of 'direct democracy', without mandate and intermediate bodies, see the considerations in the last Chap. 15.

The first method is the one used in so-called parliamentary democracies: a parliament is elected and the executive power is the expression of the will of the parliament. The second method is not formally present in any democratic country. However, it can, in practical terms, happen. The recent history of the Italian parliament shows that an inversion of the two roles has been attempted. The third method is in effect in a number of important countries, like France and the United States of America.

Having an ‘effective’ mandate for both the legislative power and the executive power at the same time is problematic, because the legislative power is well delegated when the society’s many articulated requests are being addressed, whereas the executive power is well delegated if the government is granted a certain degree of stability, at least for a set period of time. It is not possible to govern well if the government’s actions are frequently interrupted or weakened.

Representation and governmental stability are therefore the two opposing requirements of a parliamentary democracy. If the method of voting favors representation, the governing becomes less stable and, vice versa, if governmental stability is favored, the very essence of the legislative power is undermined. In designing a voting method, it must be clear which goal we want to pursue, within the bounds of the constitutional law.

The Italian Constitution has been designed to give ample representation to the citizens, after two decades of dictatorship, with the idea in mind that the parliament would be able to form stable governments. Indeed, this is what happened from 1948 (when the Constitution was approved) until the beginning of the nineties. A fact that has often been misunderstood concerns the large numbers of different governments that have been formed throughout these years. In most cases, each change of government consisted in a turnover of power positions in the administration. The political course of action, contrastingly, was always stable. Things began to change starting with the 1994 elections, when the electoral mechanism was forced to address the choice of the prime minister as well, in spite of the fact that the Constitution had not been changed. This created enormous confusion that has still not dispelled. The idea has been spread that political elections are meant for choosing governments directly. It has been presumed that the vote percentage ranking could constitute the basis for obtaining a government mandate,² and there has been a lot of misguided talk about ‘winners’ and ‘losers’, terms that do not exist in the Constitution.

The mechanisms for forming a government in a parliamentary democracy are not based on rankings. The scenario involves parties that have been apportioned a certain number of seats and the goal is to form coalitions that can gather at least half of the total number of seats (unless a single party gets the absolute majority of seats, a very unlikely possibility for many countries, let alone Italy). How this can happen is more pertinent to Game Theory rather than it is to Social Choice Theory. For this purpose, particular indices, like the Shapley–Shubik value [91, 92], have been proposed. These indices evaluate the strength of the parties toward the formation

²On the danger of considering relative majorities as being eligible to form governments, see Chap. 6.