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Advanced Computational Methods for Knowledge Engineering

Proceedings of the 6th International
Conference on Computer Science,
Applied Mathematics and
Applications, ICCSAMA 2019

Advances in Intelligent Systems and Computing

Volume 1121

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Editors

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 Springer

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ISSN 2194-5357

ISSN 2194-5365 (electronic)

Advances in Intelligent Systems and Computing

ISBN 978-3-030-38363-3

ISBN 978-3-030-38364-0 (eBook)

<https://doi.org/10.1007/978-3-030-38364-0>

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Preface

ICCSAMA 2019, held on December 19–20, 2019, in Hanoi, Vietnam, was the sixth event of the series of international scientific conferences on computer science, applied mathematics and applications. The conference is co-organized by the Computer Science and Applications Department, LGIPM, University of Lorraine, France; the Institute for research and applications of optimization (VinOptima), VinTech, Vingroup; the International School, Vietnam National University, Hanoi, Vietnam; the Laboratory of Mathematics, National Institute for Applied Sciences-Rouen, France, and the Department of Information Systems Wrocław University of Science and Technology, Poland.

The aim of ICCSAMA 2019 is to bring together leading academic scientists, researchers and scholars to discuss and share their newest results in the fields of computer science, applied mathematics and their applications. These two fields are very close and related to each other. It is also clear that the potentials of computational methods for knowledge engineering and optimization algorithms are to be exploited, and this is an opportunity and a challenge for researchers.

For the ICCSAMA 2019 edition, the Program Committee received more than 75 submissions. Each paper was peer-reviewed by at least two members of the International Program Committee and the International Reviewer board. After the review process, 37 high-quality papers were selected for oral presentation and publication in this book. The selected papers cover several topics in applied mathematics and computer science, and they are divided into four parts: nonconvex optimization, DC programming and DCA and applications; data mining and data processing; machine learning methods and applications, and knowledge information and engineering systems. Extended versions of selected papers will be considered for publication in post-conference special issues including *Journal of Global Optimization*.

ICCSAMA 2019 was attended by about 100 scientists and practitioners. The conference program is composed of four plenary lectures and one semi-plenary lecture of world-class speakers and the oral presentation of 37 selected papers as well as several selected abstracts.

ICCSAMA 2019 has created numerous interesting interactions between two communities computer science and applied mathematics, and we hope that researchers and practitioners can find here many inspiring ideas and useful tools and techniques for their works. Many such challenges are suggested by particular approaches and models presented in individual chapters of this book.

We would like to thank the chairs and the members of International Program Committee as well as the reviewers for their hard work in the review process, which helped us to guarantee the highest quality of the selected papers for the conference. We also would like to express our thanks to the keynote speakers for their interesting and informative talks. Our sincere thanks go to all the authors for their valuable contributions and to the other participants who enriched the conference success.

We wish to thank all members of the Organizing Committee for their excellent work to make the conference a success.

We cordially thank Prof. Janusz Kacprzyk and Dr. Thomas Ditzinger from Springer for their help in publishing this book.

Finally, we would like to express our special thanks to the main sponsor VinTech City, VinTech, VinGroup for their considerable support.

December 2019

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**Nonconvex Optimization, DC
Programming and DCA, and
Applications**



A New Efficient Algorithm for Maximizing the Profit and the Compactness in Land Use Planing Problem

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Abstract. This paper deals with a land-use planing problem in which the objective is to maximize the profit (or to minimize the cost) while ensuring the compactness. The original mathematical model is a multi-objective optimization problem with binary integer variables. It is then transformed to a single objective optimization problem. One may use a commercial software to solve such problem but the computation time is expensive especially in large scale problem. Hence, finding new efficient algorithms for the problem is necessary. Recently, two alternatives method based on genetic algorithm (GA) and non dominated sorting genetic algorithm (NSGA-II) are proposed. In this work, we propose a new local method based on difference of convex functions algorithm (DCA). The numerical results are compared with the one provided by GA. It shows that the proposed algorithm is much better and the obtained solutions are close to the global solutions.

Keywords: DCA · Mixed integer linear optimization · Land use planing problem · Profit · Compactness

1 Introduction

Land use planing problem is an important problem because the land area is limited while the population is continuously increasing. The area of agricultural land is about 46% of the earth's land. It may decrease and the food demand is increasing [10] because of the population's augmentation. It is estimated that the food demand in 2050 will increase by 70% compared to the present. Therefore, finding a solution to optimize the use of agricultural land attired the interest of scientists in mathematics, computer science and agronomy. In literature, the researchers often formulate the problem in the form of optimization problem and then develop solution methods for it. In recent 20 years, many mathematical models have been proposed. Each model considers a specific case, objective and constraint. We can classify the proposed models by 3 groups [10]: maximizing the profit [3] optimizing the management of water resources [1], optimizing

the protect of the environment and ecosystem [2]. Some research simultaneously consider 2 or 3 objectives, we then have multiple objective optimization problems.

In this research, we tackle a model used in [13] that is based on the one introduced by Jeroen et al. [4]. The aim to maximize the total profit while ensuring that the cells with the same land use are close as possible (compactness). The original mathematical model is a bi-objective optimization problem. One can transform the original model to a single objective optimization problem by using scalar technique. The objective of the resulting problem is the combination of the profit and the compactness. The difficulty of the problem comes from the mixed binary variables. The solution method often need a large executing time. Thus, developing efficient local methods for it is necessary. In [13], the author proposed two local methods called GA and NSGA-II to solve the problem. The experimentation showed that NSGA-II is better than GA by 9% but the computation time of NSGA-II is much longer. In this work, we develop a deterministic method based on DC programming and DCA to solve the mixed integer linear optimization model in [13]. The idea is to reformulate the problem as a DC program by using penalty technique and then develop DC algorithm (DCA) for solving it.

To evaluate the efficiency of the proposed algorithm, we consider 15 instances and compare the results provided by DCA and local method GA. The gap between the objective value obtained by DCA and the optimal value is also estimated. The results on simulation data show that the gap of DCA is smaller than 5%. It is quite good result with a local method.

The paper is organized as follows. In Sect. 2, we state the problem and present the mathematical model. Section 3 presents the solution method via DC programming and DCA. The computational results are reported in Sects. 4 and 5 concludes the paper.

2 Problem Statement

We consider the mathematical model of land use planing problem that has been addressed in [13]. It is a variant of the one in [4]. The difference is the replacement of minimizing the cost by maximizing the profit and we do not use the buffer for the cells in borders. The problem is stated as follows: consider a rectangular area which has to be allocated with different land uses. First, we divide the area into $N.M$ cells by N rows and M columns, the cell in row i and column j will be called (i, j) . Suppose there are K different land uses, symbol k indicates a specific land use, $k \in 1, \dots, K$. The following parameters are known:

- B_{ijk} : the profit generated by cell (i, j) if it is allocated to land use k .
- T_k : the total number of cell will be allocated to land use k .

The problem is to find the allocation such that the total profit generated by the considered area is the largest and the cells with the same allocated land use are placed close together to form a block (*compactness*).

In [4], the author proposed a mathematical model in the form of bi-objective linear optimization problem with binary 0–1 variables. Let x_{ijk} be the decision variables which equal to 1 if cell (i, j) is used for land use k , 0 otherwise. It is easy to see that the total profit is expressed as:

$$Profit = \sum_{i=1}^N \sum_{j=1}^M \sum_{k=1}^K B_{ijk} x_{ijk}$$

There are some following constraints

$$\sum_{k=1}^K x_{ijk} = 1 \quad \forall (i, j) \quad (1)$$

Constraint (1) ensures that each cell is allocated to only one land use.

$$\sum_{i=1}^N \sum_{j=1}^M x_{ijk} = T_k \quad \forall k \quad (2)$$

Constraint (2) ensures that the number of cells allocated to land use k is T_k .

To measure the compactness, variables y_{ijk} are introduced. The value of y_{ijk} equals to 0 if cell (i, j) is not allocated to land use k ($x_{ijk} = 0$). In the case where cell (i, j) is allocated to land use k ($x_{ijk} = 1$) then y_{ijk} is the number of cells close to cell (i, j) by row or collum, which are allocated to land use k . Variable y_{ijk} can be expressed as:

In the case where cell (i, j) is not on the borders.

$$y_{ijk} \leq 4.x_{ijk} \quad \forall i, j, k \quad (3)$$

$$y_{ijk} \leq x_{i-1jk} + x_{i+1jk} + x_{ij-1k} + x_{ij+1k} \quad \forall k, 2 \leq i \leq N-1, 2 \leq j \leq M-1 \quad (4)$$

$$y_{ijk} \geq x_{i-1jk} + x_{i+1jk} + x_{ij-1k} + x_{ij+1k} - 4.(1 - x_{ijk}) \quad \forall k, 2 \leq i \leq N-1, 2 \leq j \leq M-1 \quad (5)$$

In the case where cell (i, j) is on the borders but it is not a corner.

$$y_{ijk} \leq x_{i+1jk} + x_{ij-1k} + x_{ij+1k} \quad \forall k, i = 1, 2 \leq j \leq M-1 \quad (6)$$

$$y_{ijk} \geq x_{i+1jk} + x_{ij-1k} + x_{ij+1k} - 3.(1 - x_{ijk}) \quad \forall k, i = 1, 2 \leq j \leq M-1 \quad (7)$$

$$y_{ijk} \leq x_{i-1jk} + x_{ij-1k} + x_{ij+1k} \quad \forall k, i = N, 2 \leq j \leq M-1 \quad (8)$$

$$y_{ijk} \geq x_{i-1jk} + x_{ij-1k} + x_{ij+1k} - 3.(1 - x_{ijk}) \quad \forall k, i = N, 2 \leq j \leq M-1 \quad (9)$$

$$y_{ijk} \leq x_{i-1jk} + x_{i+1jk} + x_{ij+1k} \quad \forall k, 2 \leq i \leq N-1, j = 1 \quad (10)$$

$$y_{ijk} \geq x_{i-1jk} + x_{i+1jk} + x_{ij+1k} - 3.(1 - x_{ijk}) \quad \forall k, 2 \leq i \leq N-1, j = 1 \quad (11)$$

$$y_{ijk} \leq x_{i-1jk} + x_{i+1jk} + x_{ij-1k} \quad \forall k, 2 \leq i \leq N-1, j = M \quad (12)$$

$$y_{ijk} \geq x_{i-1jk} + x_{i+1jk} + x_{ij-1k} - 3.(1 - x_{ijk}) \quad \forall k, 2 \leq i \leq N-1, j = M \quad (13)$$

In the case where cell (i, j) is a corner.

$$y_{ijk} \leq x_{i+1jk} + x_{ij+1k} \quad \forall k, i = 1, j = 1 \quad (14)$$

$$y_{ijk} \geq x_{i+1jk} + x_{ij+1k} - 2.(1 - x_{ijk}) \quad \forall k, i = 1, j = 1 \quad (15)$$

$$y_{ij1k} \leq x_{i+1jk} + x_{ij-1k} \quad \forall k, i = 1, j = M \quad (16)$$

$$y_{ijk} \geq x_{i+1jk} + x_{ij-1k} - 2.(1 - x_{ijk}) \quad \forall k, i = 1, j = M \quad (17)$$

$$y_{ij1k} \leq x_{i-1jk} + x_{ij+1k} \quad \forall k, i = N, j = 1 \quad (18)$$

$$y_{ijk} \geq x_{i-1jk} + x_{ij+1k} - 2.(1 - x_{ijk}) \quad \forall k, i = N, j = 1 \quad (19)$$

$$y_{ijk} \leq x_{i-1jk} + x_{ij-1k} \quad \forall k, i = N, j = M \quad (20)$$

$$y_{ijk} \geq x_{i-1jk} + x_{ij-1k} - 2.(1 - x_{ijk}) \quad \forall k, i = N, j = M. \quad (21)$$

The function that measures the compactness is given by

$$f_2(x, y) = \sum_{i=1}^N \sum_{j=1}^M \sum_{k=1}^K y_{ijk}.$$

We can see that the measurement of compactness $f_2(x, y)$ is calculated based on the number of pair of two consecutive cells (by row or column) which are allocated the same land use. The aim is to maximize the compactness.

We also need the non-negativity and binary constraints

$$x_{ijk}, y_{ijk} \geq 0 \quad \forall i, j, k. \quad (22)$$

$$x_{ijk} \in \{0, 1\} \quad \forall i, j, k. \quad (23)$$

Hence, we obtain a multi-objective optimization problem

$$\begin{aligned} \max f_1(x, y) &= \sum_{i=1}^N \sum_{j=1}^M \sum_{k=1}^K B_{ijk} x_{ijk} \\ \max f_2(x, y) &= \sum_{i=1}^N \sum_{j=1}^M \sum_{k=1}^K y_{ijk} \quad (P) \\ \text{s.t.} & \quad (3) - (23) \end{aligned}$$

A technique to solve multi-objective optimization problem is to transform it to a single optimization one. By using a coefficient $w > 0$, the single objective optimization problem is written as follows:

$$\begin{aligned} \max f(x, y) &= f_1(x, y) + w.f_2(x, y) \quad (P') \\ \text{s.t.} & \quad (3) - (23) \end{aligned}$$

Problem (P') is a mixed integer linear program. It can be solved by using a commercial software but the computation time is very long in the case of large number of integer variables. In [13], the author proposed two methods based on genetic scheme to solve the two objectives optimization problem and the single one. In this work, we propose a local approach based on DC programming and DCA. The work is motivated by the rapidity and the efficiency of DCA.

3 DC Programming and Solution Method

3.1 A Brief Presentation of DC Programming and DCA

DC programming and DCA is backbone of non convex programming. DCA was first introduced by Pham Dinh Tao in 1985 and has been extensively developed since 1994 by Le Thi Hoai An and Pham Dinh Tao in their common works. It has been successfully applied to many large-scale (smooth or nonsmooth) nonconvex programs in various domains of applied science, and has now become classic and popular. In this section, we briefly present DC programming and DCA (see [5–7] and references therein for more detail).

Let $\Gamma_0(\mathbb{R}^n)$ denotes the convex cone of all lower semi-continuous proper convex functions on \mathbb{R}^n . Consider the following primal DC program:

$$(P_{dc}) \quad \alpha = \inf\{f(z) := g(z) - h(z) : z \in \mathbb{R}^n\}, \quad (24)$$

where $g, h \in \Gamma_0(\mathbb{R}^n)$ and function $f(z)$ is called a DC function (difference of convex functions).

Let C be a nonempty closed convex set. The indicator function on C , denoted χ_C , is defined by $\chi_C(z) = 0$ if $z \in C$, ∞ otherwise. Then, the problem

$$\inf\{f(z) := g(z) - h(z) : x \in C\}, \quad (25)$$

can be transformed into an unconstrained DC program by using the indicator function of C , i.e.,

$$\inf\{f(z) := \phi(z) - h(z) : z \in \mathbb{R}^n\}, \quad (26)$$

where $\phi := g + \chi_C$ is in $\Gamma_0(\mathbb{R}^n)$.

Recall that, for $h \in \Gamma_0(\mathbb{R}^n)$ and $z_0 \in \text{dom } h := \{z \in \mathbb{R}^n | h(z_0) < +\infty\}$, the subdifferential of h at z_0 , denoted $\partial h(z_0)$, is defined as

$$\partial h(z_0) := \{\xi \in \mathbb{R}^n : h(z) \geq h(z_0) + \langle z - z_0, \xi \rangle, \forall z \in \mathbb{R}^n\}, \quad (27)$$

which is a closed convex set in \mathbb{R}^n . It generalizes the derivative in the sense that h is differentiable at z_0 if and only if $\partial h(z_0)$ is reduced to a singleton which is exactly $\{\nabla h(z_0)\}$.

The idea of DCA is simple: each iteration of DCA approximates the concave part $-h$ by its affine majorization (that corresponds to taking $\xi^k \in \partial h(z^k)$) and minimizes the resulting convex problem (P_k) .

Generic DCA scheme

Initialization: Let $z^0 \in \mathbb{R}^n$ be a best guess, $0 \leftarrow k$.

Repeat

 Calculate $\xi^k \in \partial h(z^k)$

 Calculate $z^{k+1} \in \arg \min\{g(z) - h(z^k) - \langle z - z^k, \xi^k \rangle : x \in \mathbb{R}^n\} \quad (P_k)$

$k + 1 \leftarrow k$

Until convergence of z^k .

Convergence properties of the DCA and its theoretical bases are described in [5, 9, 11, 12].

3.2 Reformulation and DC Algorithm

To use DCA for solving (P'), we transform it into a DC program by using a penalty technique given in [8]. The work is based on the following theorem.

Theorem 1. [8] *Let Ω be a nonempty bounded polyhedral convex set, f be a finite DC function on Ω and p be a finite nonnegative concave function on Ω . Then there exists $\eta_0 \geq 0$ such that for $\eta > \eta_0$ the following problems have the same optimal value and the same solution set*

$$(P_\eta) \quad \alpha(\eta) = \min\{f(z) + \eta.p(z) : z \in \Omega\},$$

$$(P) \quad \alpha = \min\{f(z) : z \in \Omega, p(z) \leq 0\}.$$

Proof. see [8].

Denote by L the number of variables of problem (P'), $L = 2.N.M.K$ and $S = \{z = (x, y) \in \mathbb{R}^L \text{ s.t. (3) - (23)}\}$. Set D is the relaxed domain of S , say $D = \{z = (x, y) \in \mathbb{R}^L \text{ s.t. (3) - (22); } 0 \leq x \leq 1\}$.

We consider function $p(z) = \sum_{i=1}^N \sum_{j=1}^M \sum_{k=1}^K (1 - x_{ijk})x_{ijk}$. It is clear that $p(z) \geq 0 \quad \forall z \in D$. Problem (P') can be written as:

$$(P') \quad \begin{aligned} \min -f(z) &= - \sum_{i=1}^N \sum_{j=1}^M \sum_{k=1}^K B_{ijk}x_{ijk} - w. \sum_{i=1}^N \sum_{j=1}^M \sum_{k=1}^K y_{ijk} \\ \text{s.t. } z &\in D \\ p(z) &\leq 0. \end{aligned}$$

By using Theorem 1, Problem (P') is transformed to the equivalent one

$$(P_{eq}) \quad \begin{aligned} \min F(z) &= -f(z) + \eta p(z) \\ \text{s.t. } z &\in D \end{aligned}$$

where η is a sufficiently large number. It can be seen that (P_{eq}) is a DC program. The DC decomposition $F(z) = G(z) - H(z)$ is described as

$$\begin{aligned} G(z) &= - \sum_{i=1}^N \sum_{j=1}^M \sum_{k=1}^K B_{ijk}x_{ijk} - w. \sum_{i=1}^N \sum_{j=1}^M \sum_{k=1}^K y_{ijk} \\ H(z) &= \eta \sum_{i=1}^N \sum_{j=1}^M \sum_{k=1}^K (x_{ijk}^2 - x_{ijk}). \end{aligned}$$

From the definition of H , it is easy to see that H is differentiable and

$$\begin{cases} \frac{\partial H}{\partial x_{ijk}} = 2.\eta.x_{ijk} - \eta & \forall i, j, k, \\ \frac{\partial H}{\partial y_{ijk}} = 0 & \forall i, j, k. \end{cases} \quad (28)$$

DCA applied to land use problem (P_{eq}) can be described as follows:

DCA-LU

Initialization

Let ϵ be a sufficiently small positive number. Set $\ell = 0$ and the initial point $z^0 \in \mathbb{R}^L$.

Repeat

Calculate $\beta_{ijk}^\ell = \frac{\partial H}{\partial x_{ijk}} = 2\eta \cdot x_{ijk} - \eta \quad \forall i, j, k$.

Solve the linear program

$$\begin{aligned} \min & - \sum_{i=1}^N \sum_{j=1}^M \sum_{k=1}^K (-B_{ijk} - \beta_{ijk}^\ell) x_{ijk} - w \cdot \sum_{i=1}^N \sum_{j=1}^M \sum_{k=1}^K y_{ijk} \\ \text{s.t. } & z \in D \end{aligned}$$

to obtain $z^{\ell+1}$.

$\ell \leftarrow \ell + 1$

Until $\|z^{\ell+1} - z^\ell\| \leq \epsilon$ or $\|F(z^{\ell+1}) - F(z^\ell)\| \leq \epsilon$.

In the case where the solution provided by DCA does not satisfy the integer constraints, we change the value of penalty coefficient η and the initial point and then rerun **DCA-LU**. We obtain a multi-restart DC algorithm as follows:

ResDCA-LU

Initialization

Let η^0 be the initial value of the penalty coefficient. Set $\ell = 0$ and the initial point $z^0 = (x^0, y^0) \in \mathbb{R}^L$.

Repeat

Launch DCA-LU with the initial point z^ℓ to obtain $z^{\ell+1} = (x^{\ell+1}, y^{\ell+1})$. Set $IntVar = x^{\ell+1}$.

If $x_{ijk}^{\ell+1}$ is not integer then reset $x_{ijk}^{\ell+1}$ by the rule

$$x_{ijk}^{\ell+1} = \begin{cases} 0 & \text{if } x_{ijk}^{\ell+1} < 0.5 \\ 1 & \text{otherwise} \end{cases} \quad \forall i, j, k.$$

$\eta^{\ell+1} = 10 * \eta^\ell$

$\ell \leftarrow \ell + 1$

Until $IntVar$ is integer.

4 Numerical Results

To evaluate the efficiency of the proposed algorithm, we compare the result provided by ResDCA-LU and GA. Because of the lack of the real data, we use 15 simulation instances by changing the size of the area and profits generated by each land use. There are 3 sizes ($N = 10, M = 10$), ($N = 20, M = 20$) and ($N = 50, M = 50$). For all instances, we suppose that there are 4 land uses ($K = 4$). If cell (i, j) is suitable for land use k then the corresponding profit $B_{ijk} = cof > 1$ and $B_{ijk} = 1$ otherwise. Five cases corresponding to $(cof = 1.5; 2; 3; 4; 5)$ are investigated. Assume that the top left corner, the top right corner, the bottom

left corner, the bottom right corner are suitable for the first land use, the second land use, the third land use, the fourth land use respectively. Both algorithms ResDCA-LU and GA are implemented in Matlab 2017, run on CPU Intel core i5 2.8 GHz, RAM 8 GB. The free software CVX is used to solve the linear programs. The setting for GA is similar to the one in [13].

We run ResDCA-LU with the initial penalty coefficient of 200. The initial point for the first run of DCA is $z^0 = 0$, parameter w is fixed 0.5 for all runs. For each instance, we run 10 times of GA and pick up the highest quality solution to compared with ResDCA. Table 1 presents the results given by ResDCA-LU and GA. In the table, some notations are used:

- ◇ $Size$: the size of the area. It is given by the number of rows and columns.
- ◇ T_k : the number of cells being allocated to land use k .
- ◇ cof : the coefficient reflects the suitability of cells for land uses. It is described in the first paragraph.
- ◇ val_{DCA} : the objective value given by ResDCA-LU.
- ◇ RN : the number of rerunning DCA in ResDCA-LU.
- ◇ LB : the objective value of the relaxed problem that is obtained from problem (P') by removing integer constraints. It is a lower bound of the optimal objective value.
- ◇ T_{DCA} : the executing time in seconds of ResDCA-LU.
- ◇ G_{DCA} : the gap of DCA. It is calculated by $G_{DCA} = 100 \left| \frac{val_{DCA} - LB}{LB} \right|$.
- ◇ val_{GA} : the best objective value given by GA.
- ◇ T_{GA} : the executing time in seconds of GA.
- ◇ G_{GA} : the gap of GA. It is calculated by $G_{GA} = 100 \left| \frac{val_{GA} - LB}{LB} \right|$.

Table 1. Results provided by ResDCA and GA

$Size$	$T_1; T_2; T_3; T_4$	cof	val_{DCA}	LB	RN	T_{DCA}	G_{DCA}	val_{GA}	T_{GA}	G_{GA}
10×10	20; 30; 30; 20	1.5	-298.0	-316.6	1	43.7	5.9	-212.0	161.8	33.0
10×10	20; 30; 30; 20	2	-345.0	-360.3	0	25.2	4.2	-245.0	162.8	32.0
10×10	20; 30; 30; 20	3	-435.0	-450.0	3	82.9	3.3	-302.0	163.3	32.9
10×10	20; 30; 30; 20	4	-525.0	-540.3	3	85.3	2.8	-376.0	161.9	30.4
10×10	20; 30; 30; 20	5	-615.0	-630.3	3	81.2	2.4	-438.0	163.3	30.5
20×20	80; 120; 120; 80	1.5	-1291.0	-1322.8	0	158.0	2.4	-748.0	660.3	43.5
20×20	80; 120; 120; 80	2	-1471.0	-1502.5	0	119.5	2.1	-827.0	663.4	45.0
20×20	80; 120; 120; 80	3	-1829.0	-1862.5	3	473.9	1.8	-993.0	657.6	46.7
20×20	80; 120; 120; 80	4	-2193.0	-2222.5	0	154.6	1.3	-1166.0	667.4	47.5
20×20	80; 120; 120; 80	5	-2553.0	-2580.5	0	158.0	1.1	-1344.0	667.0	47.9
50×50	500; 750; 750; 500	1.5	-8388.0	-8489.8	1	2390.0	1.2	-4314.0	9304.7	49.2
50×50	500; 750; 750; 500	2	-9517.0	-9614.5	1	2387.9	1.0	-4687.0	9277.7	51.3
50×50	500; 750; 750; 500	3	-11767.0	-11864.5	1	2245.0	0.8	-5479.0	9323.1	53.8
50×50	500; 750; 750; 500	4	-14010.0	-14114.5	3	3229.2	0.7	-6244.0	9154.1	55.8
50×50	500; 750; 750; 500	5	-16260.0	-16364.6	3	2901.0	0.6	-7033.0	9259.1	57.0

From the results, we observe that:

- ResDCA-LU provides an integer solution for all instances although DCA-LU is a local algorithm and works on continuous domain.
- The number of rerunning DCA-LU is less than or equal to 3. In some cases, It does not need to recall DCA-LU.
- The quality of solution given by ResDCA-LU is much higher than the one furnished by GA. The DCA's solutions are very close to the global optimal solutions. The gap is smaller than 3% for almost instances (12/15 instances). We can consider the obtained solutions as a global solution.
- ResDCA-LU is much faster than GA. The executing time of GA is about 4 times of the executing time of DCA.

Figure 1 presents the gap provided by ResDCA-LU and GA. The gap of ResDCA-LU decreases when the size of the problem is augmented and the gap of GA increases. It reflects that ResDCA-LU is more efficient for larger scale problems.

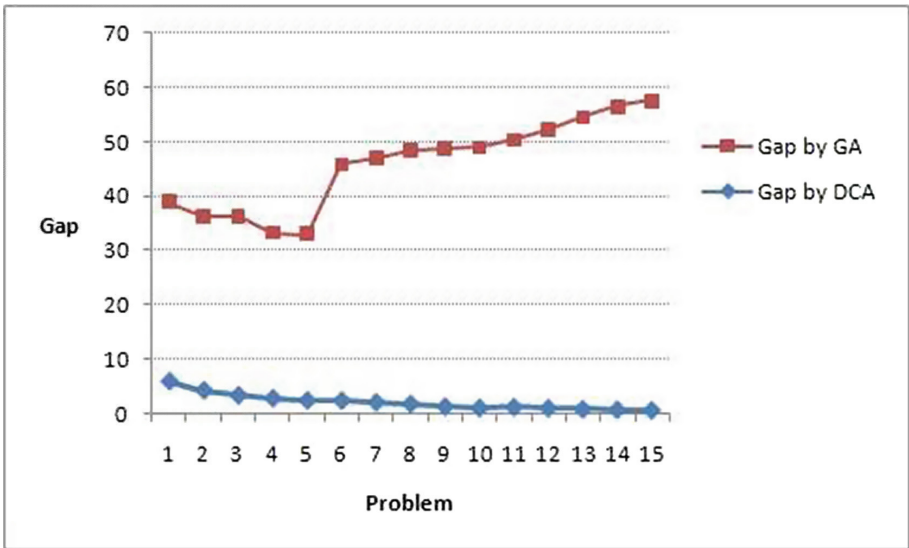


Fig. 1. The gaps by DCA and GA.

5 Conclusion

In this paper, we investigate a mixed integer linear model for land use planning problem in which the objective is to maximize the combination of the profit and

the compactness. A local algorithm based on DC programming is proposed by using the reformulation and exact penalty techniques. The new algorithm is compared with a genetic algorithm (a recent stochastic local algorithm). The experimentation shows that the results are promising. For 15 simulation instances, DCA dominates GA for both objective value and executing time. The solutions provided by DCA are very close to the global optimal solutions. The limitation of this research is only the lack of results on real data. In future work, we plan to investigate more deeply DCA by considering some others data scenarios, combine DCA with a global scheme to globally solve the problem, or develop a variant of the existing model by integrating some others criterion.

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A New Solution Method for a Mean-Risk Mixed Integer Nonlinear Program in Transportation Network Protection

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Abstract. The paper deals with a transportation network protection problem. The aim is to limit losses due to disasters by choosing an optimal retrofitting plan. The mathematical model given by Lu, Gupte, Huang [11] is a mixed integer non linear optimization problem. Existing solution methods are complicated and their computing time is long. Hence, it is necessary to develop efficient solution methods for the considered model. Our approach is based on DC (difference of two convex functions) programming and DC algorithm (DCA). The original model is first reformulated as a DC program by using exact penalty techniques. We then apply DCA to solve the resulting problem. Numerical results on a small network are reported to see the behavior of DCA. It shows that DCA is fast and the proposed approach is promising.

Keywords: DC programming · DC algorithm · Penalty function · Transportation · Retrofitting · CVaR

1 Introduction

In a transportation network, on roads the bridges are built to cross rivers or places with uneven terrain. Due to long-term use or outdated construction structures, these bridges are at risk of serious damage or collapse when natural disasters occur. Once the bridges are damaged as a result of extreme phenomena, they will lead to economic and social losses due to the cost of repairing and restoring. Moreover, the transportation network is affected by repair activities. These losses can be avoided or reduced if the risk bridges are identified and evaluated, and thus a proactive implementation strategy can be proposed. However, due to limited resources, it is not possible to retrofit all completed bridges in practice. So there should be a plan to improve bridges in the direction of priority to have economic efficiency. Choosing which risk bridge to retrofit should consider the impact on other risk bridges in the transportation network because of a

change in redistribution of traffic flows in the network. Therefore, it is necessary to consider strategies for retrofitting bridges at the network level.

Network-based bridge retrofitting problem is a general transportation network protection problem, and it can be divided into two broad categories, depending on whether bridges are considered as links or as paths. Therefore, in essence, the problem of transportation network protection is a network design problem. Typically, a network design is a bi-level mathematical optimal model. The upper level problem involves the retrofit decisions that are optimal for the best social wellfares while the lower-level one is concerned about the behavior of network users, which often present demand performance equilibrium.

Scenarios of natural phenomena are considered to be included in the transportation protection problems. Because we do not know for sure which scenario will occur, a method that can consider a lot of possible scenarios should be developed such as stochastic programming (SP) [10] or robust optimization (RO) method [1] to take into handle all scenarios. Stochastic programming methods take into account the expectation of a series of all scenarios. So it is suitable for problems with the goal of achieving long-term economic efficiency. However, it does not work well for extreme events. Therefore, when extreme events occur, the network will be affected. Meanwhile, RO methods consider the worst cases with low probability of occurrence and often offers costly solutions. Thus, it can be seen that SP and RO methods are not the best methods to consider the change of risk problem.

In [11], Lu, Gupte and Huang developed a mean-risk two-stage stochastic programming model that is more flexible in handling risks in a favorable way when resources are limited. The first stage minimizes the retrofitting cost by making strategic retrofit decisions whereas the second stage minimizes the travel cost. The conditional value-at-risk (CVaR) is included as the risk measure for the total system cost. The considered model is equivalent to a nonconvex mixed integer nonlinear program (MINLP), where the travel cost for bridge links is a nonlinear and non-convex function of retrofit decisions. According to [2], nonconvex MINLPs can be very difficult to solve. In [11], the model was solved by the Generalized Benders Decomposition method [3]. The authors derived a convex reformulation of the second-stage problem to overcome algorithmic challenges embedded in the non-convexity, nonlinearity, and non-separability of first- and second-stage variables. Thus, the model of the transportation protection problem is formulated as a convex mixed integer nonlinear program (CMINLP).

In [11], the authors proposed a method called generalized Benders decomposition to solve (CMINLP). We also use a commercial software for solving it but the executing time is quite long even for a small network. Therefore, developing efficient solution methods for CMINLP is still a challenge.

In this work, we introduce a new alternative solution method based on the mathematical technique in non-convex optimization, namely, DC programming and DC algorithm in conjunction with the use of the penalty function technique for solving Problem (CMINLP). This technique has been successfully applied to many non-convex optimization problems and showed the efficiency in particular

for large-scale problems [5, 8, 9, 12]. We tested on a nine-node network and found the algorithm running very fast. Moreover, we analyze the factors affecting the convergence time and optimal value of the DC algorithm such as choosing penalty functions, penalty parameters, starting point.

The structure of the paper is organized as follows. After the introduction section, we present the problem description in Sect. 2. Section 3 introduces the solution method. Experimental results are presented in Sect. 4. The conclusion is showed in the last section.

2 Problem Description

In this section we redescribe the model presented in [11]. This model focuses on transport network protection to prevent against extreme disasters such as earthquakes.

2.1 Parameters and Variables

To describe the problem, we use the following notations:

- A transportation network with the set of nodes N and the set of directed arcs (or links) A , denoted by $G = (N, A)$;
- R : the set of origins in the network;
- S : the set of destinations in the network;
- \mathcal{OD} : the set of network origin-destination (O-D) pairs;
- $d^{rs} \in \mathbb{R}_+$: the given travel demand between O-D pair (r, s) , $(r, s) \in \mathcal{OD}$;
- \bar{A} ($\bar{A} \subset A, \bar{A} \neq \emptyset$): the set of arcs that are directedly affected by hazards, primarily including risk bridges;
- c_a : the practical capacity of arc a ;
- H : the finite set representing a list of retrofit strategies that can be applied to at-risk bridges to mitigate the adverse effects caused by future disaster events;
- b_a^h : the retrofit cost for $a \in \bar{A}$ with strategy h ;
- b_0 : the total budget is used for retrofitting bridges;
- K : the set of hazard scenarios which can happen to the network;
- $p_k \in (0, 1)$: the given probability of scenario k , $k \in K$;
- $\theta_a^{h,k}$: the ratio of post-disaster arc capacity to the full arc capacity, with each $k \in K$ and for every $a \in \bar{A}$ $h \in H$, $\theta_a^{h,k} \in (0, 1]$. When a disaster occurs, the post-disaster capacity of arc $a \in \bar{A}$ that has been retrofitted with strategy $h \in H$ equals $c_a \theta_a^{h,k}$;
- δ : the experimental data;
- γ : the parameter converts the travel time into monetary value;
- t_{0a} : the parameter indicates the travel time in case of the free-flow-rate of arc a .