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Preface

SocProS stands for *Soft Computing for Problem Solving*. It is an Eight years old series of International Conferences held annually under the joint collaboration among a group of faculty members from the institutes of repute like NIT Silchar, IIT Roorkee, South Asian University Delhi, Liverpool Hope University, UK and VIT Vellore.

For the first time, SocProS was held at IE(I), RLC, Roorkee, India during Dec 20-22, 2011, with General Chairs as Prof Kusum Deep, Indian Institute of Technology Roorkee and Prof Atulya K. Nagar, Liverpool Hope University, UK. The second SocProS was held at JKLU, Jaipur, India during Dec 28–20, 2012. Similarly, the third SocProS was held at the Greater Noida Extension Centre of IIT Roorkee during December 26–28, 2013, fourth SocProS was held at NIT Silchar, Assam during December 27–29, 2014, Fifth SocProS was held at Saharanpur Campus of IIT Roorkee, during December 18–20, 2015, Sixth SocProS was held at Thapar University, Patiala, Punjab, during December 23–24, 2016, Seventh SocProS was held at IIT Bhubaneswar, Odisha, During December 23–24, 2017, Now the name 'SocProS' became a brand name which has already established its benchmark in last eight years through its successful milestones every time in attracting many participants from all over the world like UK, US, Korea, France, Dubai, South Africa etc.

This time, the Eighth SocProS has been held at VIT Vellore, India during Dec 17–19, 2018. Like earlier SocProS conferences, the focus of SocProS 2018 lies in Soft Computing and its applications to solve real life problems occurring in different domains in the field of medical and health care, supply chain management, signal processing and multimedia, industrial optimization, image processing, cryptanalysis etc. SocProS 2018 attracted a wide spectrum of thought-provoking research papers on various aspects of Soft Computing with umpteen applications, theories and techniques. A total 176 quality research papers are selected for publication in the form of proceedings in its Volume 1 and Volume 2.

We are sure that the research findings in the novel papers contained in this proceeding will be much fruitful and may inspire more and more researchers to work in the field of *soft computing*. The topics that are presented in this proceedings are Fuzzy logic & Fuzzy controller, Artificial Neural Network, Face Recognition & Classification, Feature Extraction, Machine learning, Reinforcement learning, Deep learning, Supervised learning, Different optimization techniques like Spider-Monkey Optimization, Particle Swarm Optimization, Meta heuristic Optimization, Artificial Bee Colony Optimization, Walk Grey Wolf Optimization, Algorithms like Flower Pollination Algorithm, Parallel Random Forest Algorithm, C-mode Clustering Algorithm, Crow Search Algorithm, Genetic Algorithm, Artificial Bee Colony Algorithm, Adaptive Multi-Swarm Bat Algorithm etc. Therefore this proceeding must provide an excellent platform to explore the assorted soft computing techniques to the readers.

The editors would like to express their sincere gratitude to its Patron, Plenary Speakers, Invited Speakers, Reviewers, Programme Committee Members, International Advisory Committee, and Local Organizing Committee; without whose support the quality and standards of the Conference could not be maintained. Special thanks to Springer and its team for this valuable publication.

Over and above, we would like to express our deepest sense of gratitude to 'VIT Vellore' for hosting this conference. Also, sincere thanks to all sponsors of SocProS' 2018.

Silchar, India	Kedar Nath Das
New Delhi, India	Jagdish Chand Bansal
Roorkee, India	Kusum Deep
Liverpool, UK	Atulya K. Nagar
Vellore, India	Ponnambalam Pathipooranam
Vellore, India	Rani Chinnappa Naidu

About This Book

The proceedings of SocProS 2018 will serve as an academic bonanza for scientists and researchers working in the field of Soft Computing. This book contains theoretical as well as practical aspects using fuzzy logic, neural networks, evolutionary algorithms, swarm intelligence algorithms, etc. with many applications under the umbrella of 'Soft Computing'. This book is beneficial for the young as well as experienced researchers dealing across complex and intricate real world problems for which finding a solution by traditional methods is a difficult task.

The different application areas covered in the proceedings are: Image Processing, Cryptanalysis, Industrial Optimization, Supply Chain Management, Newly Proposed Nature Inspired Algorithms, Signal Processing, Problems related to Medical and Health Care, Networking Optimization Problems etc. This will surely helpfully for the researchers/scientists working in similar fields of optimization.

Contents

About the Editors

Dr. Kedar Nath Das is an Assistant Professor at the Department of Mathematics, National Institute of Technology, Silchar, Assam, India. Over the past 10 years, he has made substantial contributions to research on soft computing, and has published several research papers in prominent national and international journals. His chief area of interest is in evolutionary and bio-inspired algorithms for optimization.

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Prof. Rani Chinnappa Naidu received the B.Eng. and M.Tech. degrees from VIT University, Vellore, India, and Ph.D. degree from Northumbria University, Newcastle upon Tyne, UK., all in Electrical Engineering. After that, she joined as a Postdoctoral Researcher in Northumbria Photovoltaic Applications Centre, Northumbria University, UK. She is currently an Associate Professor at VIT University. She is an Senior member in IEEE. She leads an appreciable number of research groups and projects in the areas such as solar photovoltaic, wind energy, power generation dispatch, power system optimization, and artificial intelligence techniques.

Analysis of Fractional-Order Deterministic HIV/AIDS Model During Drug Therapy Treatment

Ajoy Dutta, Asish Adak and Praveen Kumar Gupta

Abstract In this study, we discussed the Caputo sense fractional-order HIV/AIDS model including the drug therapy, and mathematically examined the dynamic behaviour of the model. We have discussed qualitative analysis of the proposed mathematical model and defined the existence and uniqueness conditions. Local stability is also checked for HIV-free equilibrium point. We have given some facts about the growth rate of HIV/AIDS, the source of HIV virus, as well as death rate of CD4$^+$ T cells, which play a vital role in HIV dynamics. The numerical simulations are demonstrated to reveal the analytical results.

Keywords Mathematical modelling · Caputo derivative · HIV/AIDS model · Stability analysis

1 Introduction

At present, more than 50 million citizens worldwide are living with Human Immunodeficiency Virus, and most of the citizens have become resistant to the existing antiretroviral therapies. In the current scenario, there has been a lot of progress in the HIV treatment due to tremendous use of cART (combined antiretroviral therapy) and HAART (highly active antiretroviral therapy). These treatments have yielded considerable improvements in diagnosis and have moderated the rate of infections in citizens who follow their drug treatment. Therefore, scientists need to build up new antiretroviral drugs to fight HIV while it is located in the vital fact that replication of this virus is a very unproductive process [1, 2]. The impact of the traditional drug has to conquer the drug deficiency in lymphoid cells and tissues (CD4 T lymphocytes, host HIV infection). Ho et al. [3] constructed a 'Systems Approach' for HIV treatment through multi-drug-involved nanoparticles. Otunuga [4] has defined and

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examined a 2n + 1-dimensional differential equation model with the introducing noise in the transmission rate and treatment of HIV disease.

In the current scenario, fractional calculus is in this phase where numerous models are going to be proposed, described, and applied to real-world problems in the area of physical, biological, engineering sciences and many more branches [5, 6]. However, the researchers have previously accounted for many outstanding results. But, in real situation, many non-local phenomena are unexplored and it will be discovered in the near future. Recently, Pinto and Carvalho [7] analysed the effect of screening and pre-exposure prophylaxis on HIV dynamics in infected citizens, and the said model reported that the fractional derivative order has an influential role during the HIV epidemics. Recently, Pinto and Carvalho [8] studied a mathematical model for HIV infection with fractional-order derivatives where latent T helper cells are incorporated. With these motivations of application of fractional derivatives, we analysed a fractional-order HIV/AIDS dynamical model with the impact of drug therapy.

2 Fractional HIV/AIDS Model

In this part of the study, we constructed a mathematical model which has four compartments: uninfected, HIV-infected CD4$^+$ T cells, virus cells, and drugs concentration as $T,\ I,\ V,\ C$, respectively. More precisely, we constructed an HIV/AIDS dynamic model that describes the relationship between all the said compartments, and it is formulated by the following fractional-order non-linear system of differential equations in the Caputo sense

$$_C D_t^{\alpha} T = s - \beta V T + \gamma I - d_1 T - f_1 C T, \tag{1}$$

$$_C D_t^{\alpha} I = \beta V T - \gamma I - d_2 I - f_2 (1 - \eta) C I, \tag{2}$$

$$_C D_t^{\alpha} V = b d_2 I - d_3 V - f_3 (1 - \eta) C V, \tag{3}$$

$$_C D_t^{\alpha} C = \nu - \delta C, \tag{4}$$

with

$$T(0) = T_0,\ I(0) = I_0,\ V(0) = V_0,\ C(0) = C_0. \tag{5}$$

Here, the inflow rate and the die rate are s , $d_1 T$ of uninfected CD4$^+$ T cells. The uninfected CD4$^+$ T cells converted into infected CD4$^+$ T cells by a virus as $\beta V T$, recovered or cured infected CD4$^+$ T cells as a rate γI, obliterated at a rate $f_1 C T$ due to injecting of drugs. Infected cells might be killed because of virion in their nucleus.

The loss rate of an HIV infected cell is considered as $(\gamma + d_2)I$, where $d_2 I$ is the elimination rate of HIV-infected cells, γI is the cure rate of infected cells into the uninfected compartment, and infected cells are destroyed at a rate $f_2(1 - \eta)CV$ due to injecting of drugs. Virions are generated by infected cells at a rate $b\, d_2 I$, decayed at a rate $d_3 V$, and destroyed at a rate $f_3(1 - \eta)CV$ due to injecting of drugs.

3 Analysis of the Model

Before starting the study of stability analysis of the fractional-order system (1)–(5), we begin with some basic results: definition of Caputo fractional-order differentiation, theorems, and lemmas. Let us consider $x(t) = [T(t), I(t), V(t), C(t)]^T$ and $\Re^4_+ = \{x \in \Re^4 : x \geq 0\}$.

Definition 1 (see [9]). Consider that $\alpha > 0,\ t > a$ where $\alpha, a, t \in \mathrm{R}$. Therefore, Caputo fractional operator formula is

$$ {}_C D_t^\alpha f(t) = \begin{cases} \frac{1}{\Gamma(n-\alpha)} \int\limits_a^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha+1-n}} d\tau, & (n-1) < \alpha \leq n \\ \frac{d^n f(t)}{dt^n}, & \alpha = n \end{cases} $$

of order α, where $\Gamma(\cdot)$ is the Euler Gamma function.

3.1 Positivity and Boundedness

Assume that $f : \mathrm{R}^n \to \mathrm{R}^n$ for n = 4. Consider the fractional-order system

$$ {}_C D_t^\alpha x(t) = F(x),\ 0 < \alpha \leq 1,\ \text{and}\, x(0) = x_0, \tag{6} $$

where $F(x) = [f_1,\ f_2,\ f_3,\ f_4]^T$ and $x_0 \in R^n$. For the global existence of the solution for the system (1)–(4) with initial conditions (5), we have to define the subsequent lemmas.

Lemma 2 (see [10]). *If* $F(x)$ *and* $\frac{\partial F}{\partial x}(x)$ *are continuous and* $\|F(x)\| \leq \lambda + \mu\|x\|$ *for all* $x \in R^n$, *where* λ *and* μ *are two positive constants. Then, the system (1)–(4) has the one and only solution on* $[0,\ +\infty)$.

Theorem 3 (see [10]). *Consider that* $f(t) \in \mathrm{C}[a,\ b]$ *and Caputo derivative* ${}_C D_t^\alpha f(t) \in \mathrm{C}[a, b]$ *for* $0 < \alpha \leq 1$, *so we define*

$$ f(t) = f(a) + \frac{1}{\Gamma(\alpha)} \big({}_C D_t^\alpha f\big)(\xi)(t-a)^\alpha, \tag{7} $$

for $a < \xi < t, \forall t \in (a,\ b]$.

Lemma 4 (see [10]). *Consider that* $f(t) \in \mathrm{C}[a, b]$ *and Caputo derivative* ${}_C D_t^{\alpha} f(t) \in \mathrm{C}[a, b]$ *for* $0 < \alpha \le 1$. If ${}_C D_t^{\alpha} f(t) \ge 0, \forall t \in [a, b]$, *then* $f(t)$ *is increasing, moreover* ${}_C D_t^{\alpha} f(t) \le 0, \forall t \in [a, b]$, *then* $f(t)$ *is decreasing,* $\forall t \in [a, b]$.

Theorem 5 (see [11]). *Let* $x^* = [T^*, I^*, V^*, C^*]^T$ *is one of the equilibrium points of the fractional-order system* ${}_C D_t^{\alpha} x(t) = F(x), 0 < \alpha \le 1$ *and* $x(0) = x_0$. *Then* x^* *is asymptotically stable locally if the spectrum of the Jacobian matrix* $J(x^*)$ *for the system (1)–(5) satisfies the following*:

$$\left|\arg\left(Eigenvalues\ J(x^*)\right)\right| > \frac{\alpha\pi}{2},$$

where $J(x^*) = [b_{i\,j}]_{x=x^*}, i, j = 1, 2, 3, 4,$ *and* $b_{i\,j} = \frac{\partial f_i}{\partial x_j}$.

3.2 Equilibrium Points and Reproduction Number

Equilibrium Points. In this subsection, we define all the equilibria for the system (1)–(4) after solving the following non-linear algebraic equations:

$$ {}_C D_t^{\alpha} T = {}_C D_t^{\alpha} I = {}_C D_t^{\alpha} V = {}_C D_t^{\alpha} C = 0. \tag{8}$$

Then, we get two equilibrium points:

(i) the first one, infection-free equilibrium point, i.e.

$$E_0 = \left[\frac{s\delta}{d_1\delta + f_1\nu}, 0, 0, \frac{\nu}{\delta}\right]^T. \tag{9}$$

(ii) the second one, endemic equilibrium point, i.e.

$$E_1 = [T_1, I_1, V_1, C_1]^T, \tag{10}$$

where

$$T_1 = \frac{\big[(d_2 + \gamma)\delta + f_2(1 - \eta)\nu\big][d_3\delta + f_3(1 - \eta)\nu]}{d_2 b\beta\,\delta^2},$$

$$I_1 = \frac{s d_2 b\beta\delta^3 - (d_1\delta + f_1\nu)\big[(d_2 + \gamma)\delta + f_2(1 - \eta)\nu\big][d_3\delta + f_3(1 - \eta)\nu]}{d_2 b\beta\delta^2[d_2\delta + f_2(1 - \eta)\nu]},$$

$$V_1 = \frac{s d_2 b\beta\delta^3 - (d_1\delta + f_1\nu)\big[(d_2 + \gamma)\delta + f_2(1 - \eta)\nu\big][d_3\delta + f_3(1 - \eta)\nu]}{\beta\delta[d_2\delta + f_2(1 - \eta)\nu][d_3\delta + f_3(1 - \eta)\nu]},$$

and $C_1 = \frac{\nu}{\delta}$.

Reproduction Number. Afterwards, we calculate the basic reproduction number ($\Re_0$) of system (1)–(4). Biologically, let $\Re_0$ be a symbol of the average number of new infections developed by single infected cell during infection.

$$\Re_0 = \frac{s d_2 b \beta \delta^3}{(d_1\delta + f_1\nu)\big[(d_2+\gamma)\delta + f_2(1-\eta)\nu\big][d_3\delta + f_3(1-\eta)\nu]}. \tag{11}$$

3.3 Local Stability

In this subsection, we define the local stability of system (1)–(4) with the help of the Jacobian matrix,

$$J(x)|_{E_0} = \begin{pmatrix} -\beta V - d_1 - f_1 C & \gamma & -\beta T & -f_1 T \\ \beta V & -d_2 - \gamma - f_2(1-\eta)C & \beta T & -f_2(1-\eta)I \\ 0 & bd_2 & -d_3 - f_3(1-\eta)C & -f_3(1-\eta)V \\ 0 & 0 & 0 & -\delta \end{pmatrix}$$

Hence, the associated transcendental equation for the above matrix is

$$\big| J(x)|_{E_0} - \lambda I \big| = 0, \tag{12}$$

where I is the 4 × 4 identity matrix.

Now, we defined the stability behaviour of the infection-free equilibrium point, i.e. $E_0 = \left[\frac{s\delta}{d_1\delta + f_1\nu}, 0, 0, \frac{\nu}{\delta}\right]^T$ in the following theorem:

Theorem 6 *The infection-free equilibrium point E_0 of fractional-order system (1)–(4) is locally asymptotically stable for $\Re_0 < 1$ if all eigenvalues λ_i of the Jacobian matrix $J(E_0)$ satisfy the condition $|\arg(\lambda_i)| > \alpha\frac{\pi}{2}$.*

Proof The Jacobian matrix $J(E_0)$ for the systems (1)–(4) calculated at the infection-free equilibrium point E_0 is

$$J(E_0) = \begin{pmatrix} -d_1 - f_1\left(\frac{\nu}{\delta}\right) & \gamma & -\beta\left(\frac{s\delta}{d_1\delta+f_1\nu}\right) & -f_1\left(\frac{s\delta}{d_1\delta+f_1\nu}\right) \\ 0 & -d_2 - \gamma - f_2(1-\eta)\left(\frac{\nu}{\delta}\right) & \beta\left(\frac{s\delta}{d_1\delta+f_1\nu}\right) & 0 \\ 0 & bd_2 & -d_3 - f_3(1-\eta)\left(\frac{\nu}{\delta}\right) & 0 \\ 0 & 0 & 0 & -\delta \end{pmatrix}.$$

The characteristic equation of the Jacobian matrix $J(E_0)$ is

$$(\lambda + \delta)\Big(\lambda + d_1 + f_1\frac{\nu}{\delta}\Big)(\lambda^2 + a_1\lambda + a_2) = 0 \tag{13}$$

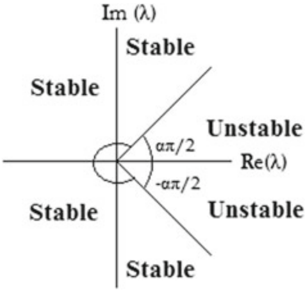

Fig. 1 Stability region of the fractional-order system (1)–(4) is enlarged when 0 < α≤1, where λ is the root of the characteristic equation

where

$$a_1 = \left(d_2 + d_3 + \gamma + (f_2 + f_3)(1 - \eta)\frac{\nu}{\delta}\right) > 0$$

and

$$a_2 = \frac{1}{\delta^2}\big[(d_2 + \gamma)\delta + f_2(1 - \eta)\nu\big][d_3\delta + f_3(1 - \eta)\nu] - \left(\frac{sd_2b\beta\delta}{d_1\delta + f_1\nu}\right),$$

or

$$a_2 = \frac{1}{\delta^2}\big[(d_2 + \gamma)\delta + f_2(1 - \eta)\nu\big][d_3\delta + f_3(1 - \eta)\nu]\,(1 - \Re_0) \tag{14}$$

The literature suggests the Routh–Hurwitz stability criterion for fractional-order systems [7, 10], and describes the necessary and sufficient condition $|\arg(\lambda_i)| > \alpha\frac{\pi}{2}$ for various models. According to this criterion, it is clear that all roots of the characteristic Eq. (13) have negative real parts if and only if $a_1 > 0$ and $a_2 > 0$. Equation (14) implies that if $\Re_0 < 1$, then all roots will be negative and for $\Re_0 < 1$ the necessary and sufficient condition will satisfy (Fig. 1).

Hence, the system (1)–(4) is asymptotically stable at E_0 if $\Re_0 < 1$; otherwise, if $\Re_0 > 1$, the characteristic Eq. (13) has given at least one positive eigenvalues, therefore, E_0 is unstable.

4 Numerical Solution and Discussion

In this paper, we presented a numerical solution of the fractional-order HIV/AIDS model (1)–(5). Here, we solve this system using Mathematica 8.0. Consider that $s = 3\,\text{mm}^{-3}\text{day}^{-1}$, $\beta = 0.000024\,\text{mm}^3\text{day}^{-1}$, $\gamma = 0.2\,\text{day}^{-1}$, $b = 1940$, $d_1 = 0.01\,\text{day}^{-1}$, $d_2 = 0.5\,\text{day}^{-1}$, $d_3 = 3.4\,\text{day}^{-1}$ (since removal rate of infected cells will be higher than uninfected cells), $\alpha = 1$, $f_1 = 0.009\,\text{day}^{-1}$, $f_2 = 2 \times 10^{-10}\,\text{day}^{-1}$ and $f_3 = 10^{-4}\,\text{day}^{-1}$ [10, 11].

In view of the reality that the recovery rate (γ) will also depend on drugs which are given to the patient, we are defining the variation of γ with initial conditions $T(0) = 1000$, $I(0) = 0$, $V(0) = 0.001$ and $C(0) = 0.5$ in Fig. 2.

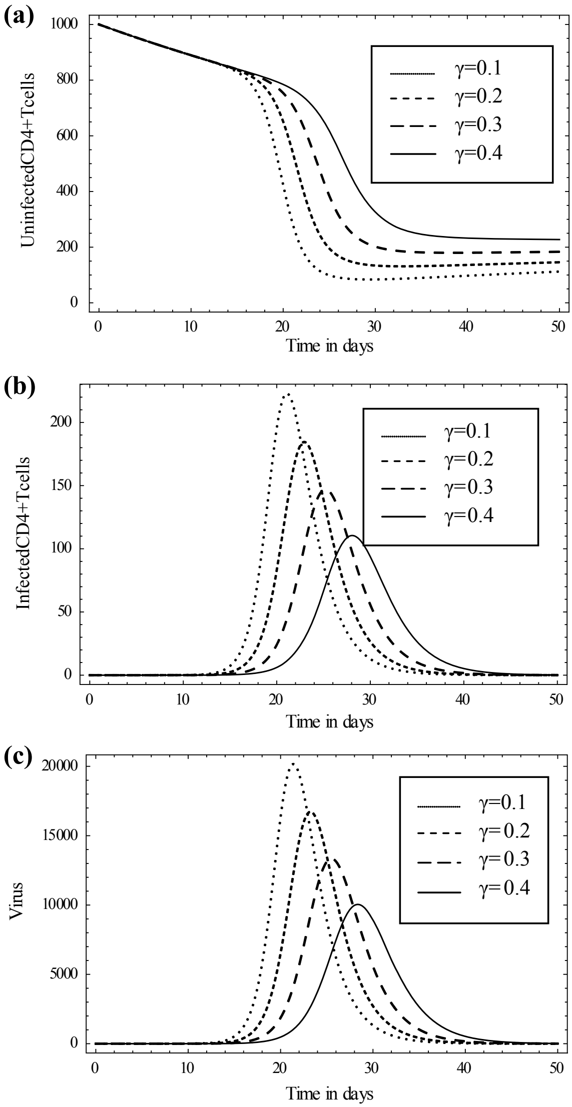

Fig. 2 Plot of **a** uninfected CD4 + T cells **b** infected CD4 + T cells **c** virus versus time for various values of γ

5 Conclusion

In this paper, we presented a Caputo sense fractional-order HIV/AIDS dynamics model with drug therapy. The authors have defined the equilibrium points and reproduction number by Jacobian-based spectral radius method for the proposed model. The recent appearance of fractional differential equations as models makes it necessary to investigate analysis of solution for such equations. So, the authors describe the stability analysis on a proposed fractional order model, and obtained a sufficient condition on the parameters for asymptotically stable infection-free steady state. The numerical solutions have demonstrated the impact of drugs in the patients and it is depicted through Fig. 2 for various values of γ.

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Load Bearing Capacity for a Ferrofluid Squeeze Film in Double Layered Porous Rough Conical Plates

Yogini D. Vashi, Rakesh M. Patel and Gunamani B. Deheri

Abstract This article goals to determine the enactment of double layered porous rough conical plates with ferrofluid based squeeze film lubrication. The Neuringer–Roseinweig model has been employed for magnetic fluid flow. For the characterization of roughness two different forms of polynomial distribution function have been used and comparison is made between both roughness structure. The stochastic model of Christensen and Tonder regarding transverse roughness has been invoked to develop the associated Reynolds' equation from which the pressure circulation is found. This provides growth to the calculation of load-bearing capacity. From the graphical appearance it is established that from the design point of view roughness pattern G1 is more suitable compared to G2. The results presented here confirm that the introduction of double layered plates results in improved load carrying capacity. This is further enhanced by the ferrofluid lubrication. Further, the roughness affects the bearing system significantly, however, the situation enhanced in the case of negatively skewed roughness. A noticeable fact is that the porosity of the outer layer influences more as compared to the inner layer even in the presence of mild magnetic strength.

Keywords Squeeze film · Conical plates · Roughness · Ferrofluid · Load carrying capacity

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Nomenclature

a	Dimension of bearing (mm)
$\boldsymbol{h}$	Uniform fluid film thickness(mm)
h	Mean film thickness
h_s	Deviation from the mean film thickness
$\dot{h}_0$	Normal velocity of bearing surface
H_1	The thickness of the inner layer of the porous plate (mm)
H_2	The thickness of the outer layer of the porous plate (mm)
$\overline{H}$	Magnetic field vector
p	Pressure distribution(N/m^2)
$\overline{p}$	Non dimensional Pressure distribution
W	Load carrying capacity (N)
$\overline{W}$	Non dimensional Load bearing capacity
η	Dynamic viscosity of fluid (NS/m^2)
μ_0	Permeability of free space (N/A^2)
$\overline{\mu}$	Magnetic susceptibility of magnetic field
σ	Standard deviation (mm)
α	Variance (mm)
ε	Skewness (mm)
ϕ_1	The permeability of the inner layer (m^2)
ϕ_2	The permeability of the outer layer (m^2)
ψ_1	Porosity of inner layer
ψ_2	Porosity of outer layer
α^*	Non dimensional variance
ε^*	Non dimensional skewness
σ^*	Non dimensional standard deviation
ρ	Density of fluid
$\overline{q}$	Velocity of fluid
η	Fluid viscosity
$\overline{M}$	Magnetization vector

1 Introduction

Porous bearing is used very widely in many devices such as Vacuum cleaners, extractor fans, motorcar starters, hair dryer, etc. They are also used in business machines, farm and construction equipment, and aircraft automotive accessories. In addition, porous bearing can work hydrodynamically longer without maintenance and more stable than the equivalent conventional bearing. Also, in these bearings friction is less as compared to the non-porous bearings. so many researchers have studied the effect of double layered porous bearings of various shapes. Uma Srinivasan [1] worked to study double layered slider bearing with a porous surface. The double layered surface enhanced the bearing's load carrying capacity as well as the friction drag. However, it reduced the friction coefficient. Verma [2] investigated the influence of double-layered porous slider bearing. The study of Rao et al. [3] focused on the relation between Brinkman model and a double layered porous journal bearing's performance. The results suggested that in a double layered bearing, the low permeability layer stuck to the high permeability one, leading to increased bearing capacity and as a result, a decreased friction coefficient. Uma Srinivasan [4] intended to study the impact of time-height of squeeze films on a bearing's load capacity. Various geometrical aspects like circular, elliptical, rectangular, etc. were used for the purpose. It was a comparative study focusing on two-layered porous bearing and conventional bearings. The results suggested that double layered plates enhance a bearing's load carrying capacity. Cusano [5] analyzed an infinitely long two-layered porous bearing.

Conical bearings have been developed for use in agricultural and construction machinery, for the suspension of jolts and insulation of engine vibrations from cabins. Lin et al. [6] studied the behavior of non-Newtonian micropolar fluid squeeze film between conical plates. The non-Newtonian effects of micropolar fluid were found to be better in comparison with the Newtonian case also its effect lengthened the approaching time of squeeze film conical plates. Dinesh Kumar et al. [7] studied the effect of ferrofluid squeeze film for spherical and conical bearings using perturbation analysis. Prakash and Vij [8] analyzed the effect of the shape of the plate and porosity on the performance of squeeze films between porous plates of various shapes.

Practically, a perfectly smooth surface does not exist as all surfaces are rough to some extent. In applied settings, a smooth surface bearing does not provide an optimum idea of performance and bearing life span. Thus, in the recent year studies have focused on correlating surface roughness with the bearing capacity. Christensen and Tonder [9–11] worked on the stochastic surface roughness theory with hydrodynamic lubrication. Many authors have used this technique to understand the impact of surface roughness on performance. Patel and Deheri [12] made a comparative study of different porous configurations and their impact on double layered slider bearing with roughness and magnetic fluid base. The results suggested that Kozeny carman model is more effective than Irmay's model. Deheri et al. [13] made a theoretical study of the influence of squeeze film with a magnetic base on porous rough conical plates. The results showed that an appropriate semi-vertical angle can revert the negative impacts of porosity and standard deviations for negatively skewed roughness. Patel and Deheri [14] deliberated the impact of slip velocity on a squeeze film with ferrofluid in conical plates with longitudinal roughness. It was found that standard deviation and magnetization can substantially neutralize the negative impact created by slip velocity and surface roughness on bearing performance, provided that the negatively skewed roughness was appropriate. Vashi et al. [21] studied the impact of ferrofluid based rough porous parallel plates with couple stress effect.

Various good research articles are available in the literature for the study of squeeze film lubrication of conical bearings and truncated conical bearings. For examples Shimpi and Deheri [15] in porous truncated conical plates, Patel and Deheri [16] in porous conical plates, Vadher et al. [17] in porous rough conical plates, Patel et al. [20] in rough conical bearing with deformation effect.

At present no work has been made to study the influence of surface roughness with two different patterns on ferrofluid based squeeze film in double layered porous conical plates. So in this current study the investigation of Patel and Deheri [13] is extended to the double layered porous conical plates with two different forms of the transverse surface roughness patterns.

2 Analysis

All the traditions of hydrodynamic lubrication are reserved here. The lubricant is an incompressible ferrofluid lubrication, considered for the analysis. Both the porous facings are supposed to be homogeneous and isotropic and porosity is directed by a generalized form of Darcy's law.

Figure 1 displays the geometrical structure of squeeze film lubrication of porous rough conical plates bearing.

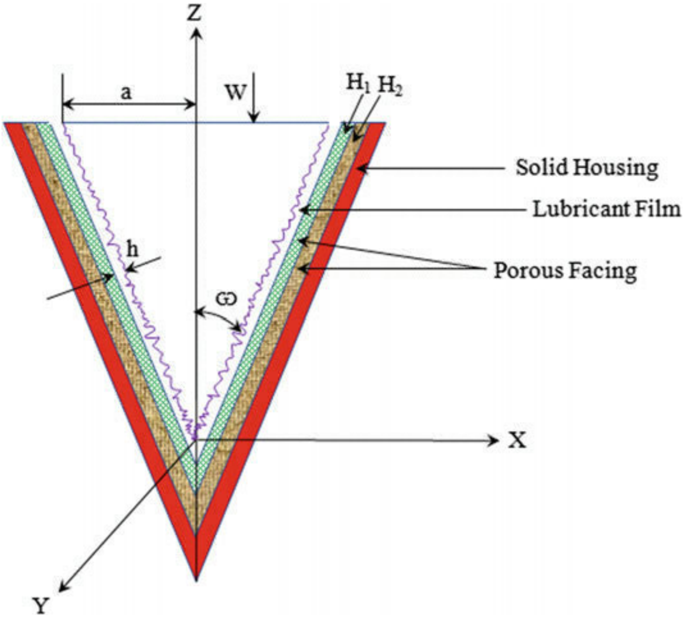

Fig. 1 Physical structure of the bearing system

In the sought of the discussion of Uma Srinivasan [1] the modified Reynolds equation comes out to be

$$\frac{1}{x}\frac{d}{dx}\left(\frac{dp}{dx}\right) = \frac{12\eta \overset{\bullet}{h}_0 \sin\omega}{\boldsymbol{h}^3 \sin^3\omega + 12\phi_1 H_1 + 12\phi_2 H_2} \tag{1}$$

The bearing faces are deliberated to be transversely rough in the context of Christensen and Tonder's [9–11] discussion the lubricant film thickness is taken as

$$\boldsymbol{h} = h + h_s$$

where h is mean film thickness and h_s is the part due to the surface roughness as measured from nominal film thickness. According to Christensen and Tonder [9–11] Stochastic part h_s is defined by the polynomial probability distribution function $f(h_s)_1$ for the domain $-c \le h_s \le c$,where c represents the maximum deviation from the mean film thickness.

$$f(h_s)_1 = \begin{cases} \frac{35}{32c^7}\left(c^2 - h_s^2\right)^3, & -c \le h_s \le c \\ 0, & \text{elsewhere} \end{cases} \tag{2}$$

Further, a different form of this type of polynomial distribution from Prajapati [18] is

$$f(h_s)_2 = \begin{cases} \frac{15}{16c^5}\left(c^2 - h_s^2\right)^2, & -c \le h_s \le c \\ 0, & \text{elsewhere} \end{cases} \tag{3}$$

The measure of the symmetry of the random variable h_s are mean α the standard deviation σ and the parameter ε defined by the relations

$$\alpha = E(h_s) \quad \sigma^2 = E\left[(h_s - \alpha)^2\right] \quad \varepsilon = E\left[(h_s - \alpha)^3\right]$$

where $E(\bullet)$ is the expectancy operator given by the formula

$$E(\bullet) = \int_{-\infty}^{\infty} (\bullet) f(h_s) dh_s \tag{4}$$

The detailed study regarding the roughness model can be observed in Christensen and Tonder [9–11].

Neuringer and Rosensweig [19] established a model to designate the stable flow of magnetic fluid. This model involves the following equations.

Equation of motion:

$$\rho(\overline{q}.\nabla)\overline{q} = -\nabla p + \eta\nabla^2\overline{q} + \mu_0\left(\overline{M}.\nabla\right)\overline{H} \tag{5}$$